Modeling Extreme Industrial Events under Indeterminacy Using Neutrosophic Fréchet Distribution

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Abstract

This work presents a neutrosophic extension of the Fréchet distribution to enhance the modeling of extreme values under conditions of indeterminacy and uncertainty. While the classical Fréchet distribution is widely used in fields such as finance, hydrology, and environmental sciences to model extreme maximum values, it does not fully accommodate imprecise, vague, or conflicting data commonly encountered in real-world scenarios. By incorporating the principles of neutrosophic logic the proposed neutrosophic Fréchet distribution provides a more flexible and realistic approach to representing extreme phenomena. The paper introduces its theoretical formulation, outlines key statistical properties, and proposes an estimation method based on maximum likelihood. Through simulations and numerical illustrations, the robustness and applicability of the model are described, especially in contexts where data is incomplete, uncertain, or contradictory. A real industrial dataset is employed to illustrate the applicability of the proposed model.

Keywords: Probabilistic model; Neutrosophic probability; Neutrosophic measures; Estimation; Simulation

1. Introduction

In the context of extreme value theory (EVT), probability distributions are central part as they are used as mathematical models to depict the behavior of extreme deviations from the median or average in a dataset [1]. EVT is concerned with the statistical behavior of the maximum or minimum values of a sample, that are of interest in the evaluation of risks associated with rare but extremal events like floods, earthquakes, financial crashes or structural collapses [2]. The heavy beautiful, tailed behavior of these extreme values is known to be modeled by specific probability distributions like Gumbel, Fréchet and Weibull distributions based on the tail behavior of the underlying data [3]. For instance, the Fréchet distribution is commonly employed in hydrology to describe maximum rainfall or flood levels, and the Gumbel distribution is applied in environmental science to model temperature extremes [4]. In finance, it is used to predict the risk of market crashes by examining extreme losses, while in engineering it enables one to design structures, which can support extreme stress or load situations [5]. Statisticians can make more reliable and realistic predictions for rare but important events by using EVT and its accompanying distributions [6]. These distributions are useful for estimating return levels and exceedances under these distributions for making improved predictions and decisions in areas such as engineering, finance, environmental science, and insurance. Fréchet is one of the cornerstone distributions in extreme value theory, ideal for modeling the behavior of extreme maximums of a given data set [7]. It is important for expressing processes with heavy tails, where the probability of very large values matters. As reasons in hydrology (e.g., for modelling peak river discharges or flood heights), finance (e.g., in modelling extreme market losses) and environmental sciences (e.g., for investigating maximum wind speeds or precipitation) [8]. The Fréchet distribution provides the risk and potential severity of extreme events, which is useful for robust systems design and decision making in high-risk environments [9]. Its versatility to model tail behavior has established it as a fundamental tool in

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reliability theory and the investigation of material strength, for in such circumstances knowledge of the probability of failure in the tail is crucial [10]. Fuzzy random variables are useful in the development of EVT under real-life situations in which, the uncertainty does not only result from randomness, but also from lack of precision and vagueness of values in data [11]. In practice, when it comes to input from the physical and social sciences about phenomena and events such as environmental risk, structural behavior, and financial prediction it is quite common for the input variables of a model to be imprecisely specified because of the limitations of the measurement procedure, expert judgment or information availability [12]. Fuzzy random variables make it possible to jointly integrate probabilistic and fuzzy uncertainties, therefore EVT models become more robust and realistic [13]. For instance, in hydrology, the maximum level of rainfall observed in history could be imprecise because of the limitations of sensors or the expert's subjective opinion and so modeled as a fuzzy random variable. Likewise, in structural design, manufacturability variation can lead to similarities in describing the maximum stress a material can sustain. EVT with fuzzy random variables improves the extreme event modeling under hybrid uncertainty, helping decision-makers to make more intelligent and reliable decisions in critical systems. Similarly, when modeling extreme temperature or precipitation events, there may be limited historical records or contradictory sources (e.g., missing sensor data or subjective historical estimates). There might be an idea about the highest temperature of the year (e.g., ranging between 48°C to 50°C), for which we can model using either interval (or neutrosophic) random variables in EVT. Seismic events also commonly exhibit magnitudes that are variably reported by independent sources, including regional and global seismological agencies. Such fluctuations bring indeterminacy, neutrosophic or fuzzy modeling is more realistic than crisp EVT itself [14]. In finance, extreme losses (such as Value at Risk or maximal daily drawdown) are influenced by news, sentiment, and speculative factors that are not strictly random [15]. The neutrosophic variables can describe this truth (probability), indeterminacy (unknowns), and falsity (noise or error) at estimating the extreme risks. Neutrosophic statistics and neutrosophic probability distributions are essential in the process of modern data analysis for providing a powerful tool for dealing with imprecise, inconsistent, and indeterminate information that traditional statistics cannot represent completely [16-20]. In reality scenarios as engineering, medicine, social sciences or environmental studies, data can be contaminated by a considerable amount of uncertainty, stemming not only from statistical fluctuations but also from vagueness (fuzziness) and conflicting sources [21]. Definition Neutrosophic distributions consist of three parts - truth, indeterminacy and falseness, which permits the possibility to study systems where data is partially true, partially unknown or partially false. These distributions are especially helpful in complicated decision-making scenarios, risk analysis, and the exploration of uncertain phenomena such as extreme weather, fault diagnosis, or the analysis of opinions [22-24]. By generalizing the workings of classical and fuzzy statistics, neutrosophic statistics prove to be a more viable and versatile alternative approach when it comes to uncertainty modeling, providing us with more sustainable and consistent results [25].

In this paper, the neutrosophic version of the Fréchet distribution is proposed to widen its applicability in modelling extreme value data in the space of uncertainty and indeterminacy. Classical Fréchet distribution is commonly used in extreme value theory to describe extreme values across various domains such as hydrology, finance, and environmental science. However, the real-world data from these domains is frequently uncertain or incomplete and often classical models are not able to capture this. By using neutrosophic logic that includes truth, indeterminacy and falsity components, the proposed neutrosophic Fréchet distribution is a more flexible and realistic framework to model uncertain and imprecise data. Such a method ensures that extreme value modeling is reinforced and maintains robustness and risk assessments and decisions become better in the case of uncertainties in the data.

The remaining of this paper is organized as follows: A basic review of the classical Fréchet distribution properties is presented in Section 2. Based on this classical setting, our new contribution is presented in Section 3. The Neutrosophic Fréchet distribution, which integrates indeterminacy, factors into the previous model. Section 4 presents specialized estimation procedures of the parameters. Finally, Section 5 presents a synthesis of our main contributions and discusses their broader impact on statistical modelling.

2. Fréchet Classical Structure

In this section, the basic structure of the Fréchet distribution under the classical with some key statistical properties are discussed. Consider a set of independent and identically distributed (iid) random variables $\{Y_i: 1 \leq i \leq n\}$, which describe a critical quantity in an engineering or physical system. The statistical properties of the maximum $\mathcal{M}_n = \max\{Y_i: 1 \leq i \leq n\}$ and the minimum $m_n = \{Y_i: 1 \leq i \leq n\}$ are often crucial, especially as n grows large.

Classical EVT analyzes the limiting distributions of $\mathcal{M}_n$ and m_n as $n \to \infty$. The theory establishes that, under proper normalization (with constants $a_n > 0$ and b_n), if the scaled maximum $a_n(\mathcal{M}_n - b_n)$ converges to a nondegenerate distribution $\mathcal{G}(y)$, then $\mathcal{G}(y)$ must belong to one of three possible types (Gumbel, Fréchet, or Weibull). A parallel result holds for the minimum m_n.

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Among these, only the Fréchet distribution $cdfs$ for $\mathcal{M}_n$ (maxima) and the Weibull distribution $cdfs$ for m_n (minima) are naturally restricted to nonnegative values.

The two-parameter Fréchet distribution is particularly important in modeling extreme events with heavy-tailed behavior, such as catastrophic failures or extreme environmental conditions. The Fréchet cdf is

$$\mathcal{F}(y) = \exp\left[-\left(\frac{y}{\mathcal{B}}\right)^{-\mathcal{A}}\right], \quad y \geq 0,\ \mathcal{A} > 0,\ \mathcal{B} > 0, \tag{1}$$

where $\mathcal{A}$ is the shape parameter, controlling the tail behavior of the distribution, and $\mathcal{B}$ is the scale parameter, determining the spread of the distribution.

The Fréchet and Weibull cumulative distribution functions (cdf) share a direct analytical relationship through inversion. Specifically, if Y is a random variable following the Weibull CDF then the transformed variable X follows a Fréchet distribution.

$$X = \frac{1}{Y}. \tag{2}$$

By applying basic transformations of random variables, we can readily demonstrate that X follows a Fréchet distribution with shape parameter $\mathcal{A} = \alpha$ and scale parameters $\mathcal{B} = 1/\beta$. As a result, the analytical properties of the Fréchet cdf can be directly obtained from the properties of the Weibull cdf via inverse transformation.

Despite this, the key properties of the Fréchet cdf can be derived straightforwardly. The probability density function (PDF) is

$$f(y) = \left({}^{\mathcal{A}}/_{\mathcal{B}}\right)\left({}^{y}/_{\mathcal{B}}\right)^{-(\mathcal{A}+1)} e^{[-({}^{y}/_{\mathcal{B}})^{-\mathcal{A}}]}. \tag{3}$$

The $100q - th$ percentile y_q , can be directly obtained as

$$y_q = \mathcal{B}[-lnq]^{-1/\mathcal{A}}. \tag{4}$$

The model value can be obtained as:

$$y_m = \mathcal{B}\left[\frac{\mathcal{A}}{(\mathcal{A}+1)}\right]^{-1/\mathcal{A}}. \tag{5}$$

Observe that the mode y_m approaches 0 as $\mathcal{A} \to 0$, and tends to infinity as $\mathcal{A} \to \infty$.

The moments represent the most fundamental quantities characterizing the distribution. For a random variable Y follows the Fréchet distribution, the r^{th} moment is given by:

$$E[Y^r] = \int_0^\infty y^r f(y)dy = \mathcal{B}^r \Gamma(1 - r/\mathcal{A}) \tag{6}$$

where $\Gamma(\cdot)$ denotes the gamma function. Thus utilizing Eq (6), mean and variance of classical model can be written as:

$$\mu = \mathcal{B}\Gamma\left(1 - {}^{1}/_{\mathcal{A}}\right)$$

$$\sigma^2 = \mathcal{B}^2\left[\Gamma\left(1 - {}^{2}/_{\mathcal{A}}\right) - \Gamma^2\left(1 - {}^{1}/_{\mathcal{A}}\right)\right]$$

Based on mean and variance, the coefficient of variation (cv) can be yielded as:

$$cv = \sqrt{\left\{\Gamma\left(1 - {}^{2}/_{\mathcal{A}}\right) - \Gamma^2\left(1 - {}^{1}/_{\mathcal{A}}\right)\right\} \Big/ \Gamma^2\left(1 - {}^{1}/_{\mathcal{A}}\right)}. \tag{7}$$

From Eq. (7), we observe that the cv depends solely on the shape parameter $\mathcal{A}$, making it a direct measure of dispersion. The variability decreases monotonically as $\mathcal{A}$ increases and conversely grows when $\mathcal{A}$ decreases. For large values of $\mathcal{A}$, the scale parameter $\mathcal{B}$ closely approximates the mean μ.

The classical Fréchet distribution also has very important statistical properties and applications in reliability analysis

The hazard function associated with the Fréchet distribution is given by:

$$\mathcal{k}(y) = \frac{\left({}^{\mathcal{A}}/_{\mathcal{B}}\right)\left({}^{y}/_{\mathcal{B}}\right)^{-(\mathcal{A}+1)} e^{\left[-({}^{y}/_{\mathcal{B}})^{-\mathcal{A}}\right]}}{1 - e^{\left[-({}^{y}/_{\mathcal{B}})^{-\mathcal{A}}\right]}}, \quad y \geq 0, \tag{8}$$

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The hazard function exhibits non-monotonic behavior. It first increases to a single maximum before decreasing asymptotically. While the uniqueness of this maximum can be established analytically, its exact location requires numerical computation. This characteristic makes the Fréchet distribution unsuitable for conventional reliability applications where monotonically increasing failure rates are typically assumed.

3. Neutrosophic Fréchet Distribution

The fundamental model in EVT is the which plays a significant role in characterizing the distribution of maxima in data, and many application fields, e.g., hydrology, finance, reliability, and environmental science. Its classical form is mentioned for its heavy tail property and is applicable to the rare extreme events modeling. Yet, in most applications not all available information about the observations is correct, ambiguous, or even contradictory information must be considered. To overcome these problems, neutrosophic logic generalizing the fuzzy and intuitionistic the logics provides a robust platform by considering data indeterminacy. In this section, we introduce the neutrosophic Fréchet distribution and state some parallel properties of this distribution that may be applied to extreme situation under uncertainty.

A random variable y following the Fréchet distribution characterized by $\mathcal{A}_n$ and $\mathcal{B}_n$, is defined by its CDF.

$$\mathcal{F}_N(y) = \exp\left[-\left(\frac{y}{\mathcal{B}_n}\right)^{-\mathcal{A}_n}\right], \quad y \geq 0,\ \mathcal{A}_n > 0,\ \mathcal{B}_n > 0, \qquad (9)$$

Here $\mathcal{A}_n$ and $\mathcal{B}_n$ are not the single value parameters but follow the interval values. For visualizing this idea, some plots of CDF curves are given in Figure 1 for different values of scale and shape neutrosophic parameters.

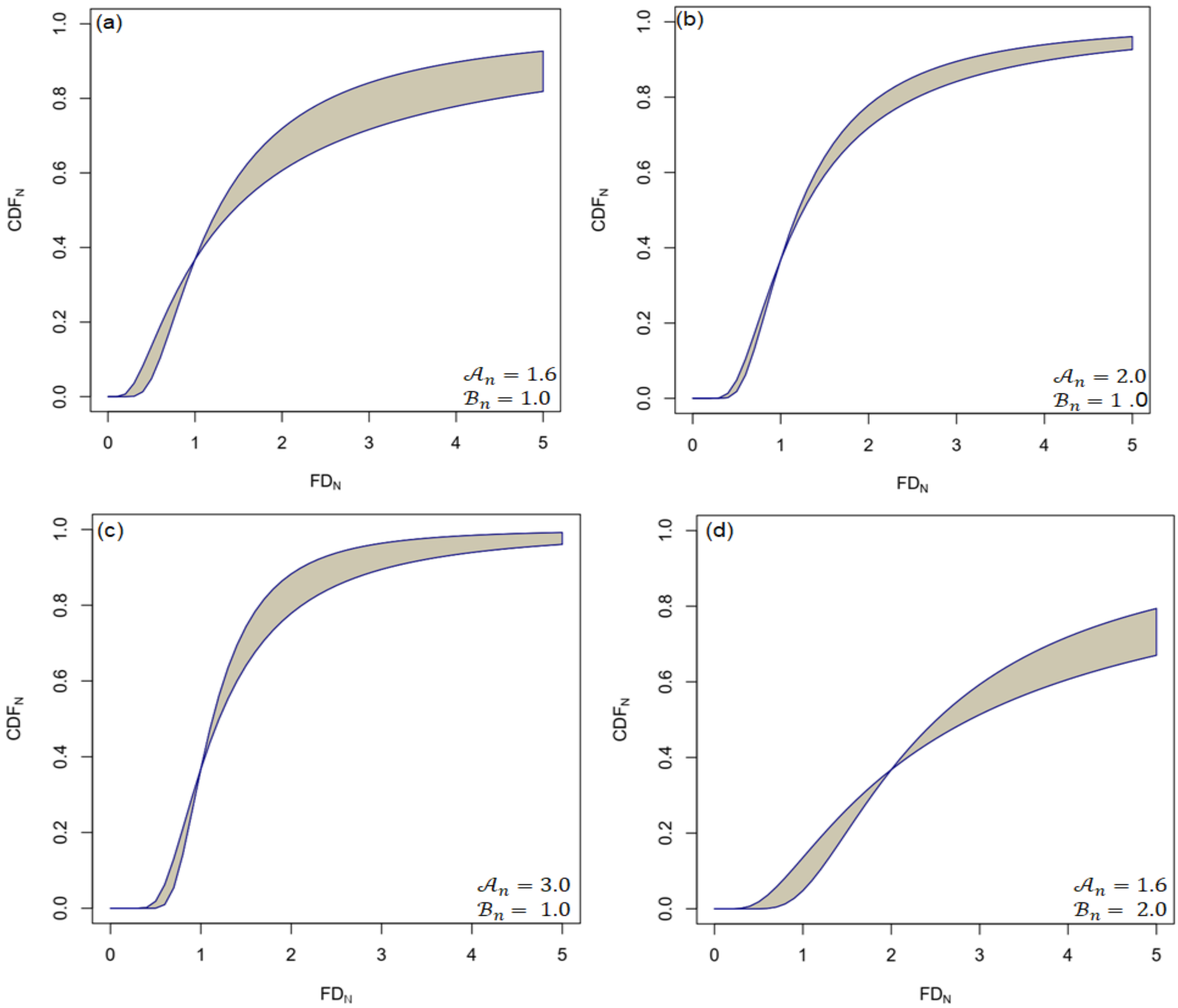

Figure 1. PDF curves of the neutrosophic Fréchet distribution for different parameters values

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The parameter $\mathcal{A}_n$ is the neutrosophic shape parameter, controlling the tail behavior of the distribution, and $\mathcal{B}_n$ is the neutrosophic scale parameter, determining the spread of the distribution. The indeterminacy region is represented by shaded region in Figure1. With zero indeterminacy in parameters, results covert to classical model. The PDF of the neutrosophic Fréchet distribution can be expressed as:

$$f_N(y) = \left(\mathcal{A}_n/\mathcal{B}_n\right)\left(y/\mathcal{B}_n\right)^{-(\mathcal{A}_n+1)} e^{\left[-\left(y/\mathcal{B}_n\right)^{-\mathcal{A}_n}\right]}. \qquad (10)$$

The PDF of the neutrosophic Fréchet distribution tells us how likely it is that a variable that follows this distribution will take on a particular value. It is determined by two primary factors a shape parameter and a scale parameter. Specifically, if we have values for these 2 parameters, then we can control how fast or slow the probabilities should decline for larger values or how wide or narrow the distribution should be. Generally, the suggested distribution is right-skewed heavy tailed, i.e., it is assigning higher priority to the very large values than many other distributions. This is particularly attractive for modeling extremes, for example maximum rainfall in a year, financial losses, or times to structural failure. When we consider the neutrosophic logic, the shape and the scale do not have any more crisp values. They are considered neutrosophic numbers, which are constructed by indeterminacy data values. The PDF curves for selected values are shown in Figure 2.

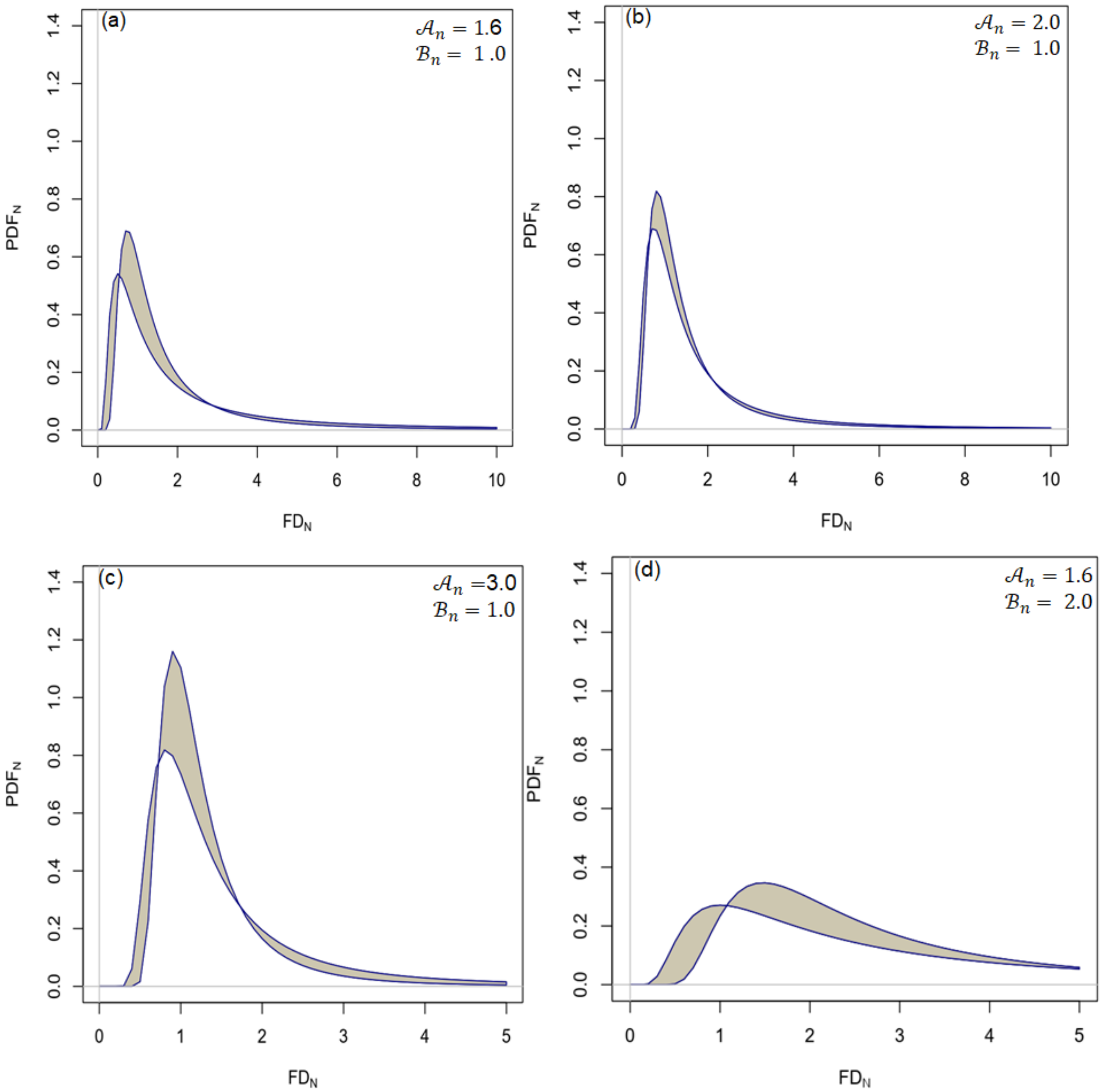

Figure 2. PDF curves of curves distribution for selected values of neutrosophic parameters.

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The panel of Figure 2 shows the PDF curves of the neutrosophic Fréchet distributions in some shape and scale parameter intervals. There is a subplot within the panel for each range of interval for the shape and scale parameters, which manifests the uncertainty of real data from the perspective of being described with neutrosophic logic.

As seen, the steepness and the tail of the distribution are affected by the shape parameter. The diminution of the shape parameter interval leads to the heavier-tailed distribution which follows the larger probabilities in the extreme phenomena. On the other hand, sharper peaks and tails are obtained when the shape values are larger.

Parallel to classical results, the quantile function of the proposed can be derived as:

$$y_q = \mathcal{B}_n[-lnq]^{-1/\mathcal{A}_n}. \quad (11)$$

where the value y_q is 100th quantile point of the distribution.

Using Eq (11) we can draw random samples from the proposed distribution. To generate the random samples from the neutrosophic Fréchet distribution by its quantile function. We choose two uncertain quantities where $\mathcal{A}_n$and $\mathcal{B}_n$ both are intervals characterizing its vagueness. Next, we generate two sets of random numbers in the range [0,1] as probabilities. For all these values we compute two outputs: one by setting the parameters to their lower bound and one by setting the parameters to their upper bound using the quantile function as defined in Eq (16). This produces an interval for each sample, capturing the possible range of outcomes due to both randomness and uncertainty in the parameter values. Repeating this process for a desired sample size results in a dataset composed of interval-valued observations that reflect the neutrosophic nature of the distribution. At specific seed equal to 120, 40 random samples from the proposed model with $\mathcal{A}_n = [2.5, 3.5]$and $\mathcal{B}_n = [1.5,2.5]$ are shown in Table 1.

Table 1: Neutrosophic samples from the proposed model

Samples from neutrosophic Fréchet distribution				
[1.540, 2.547]	[1.121, 2.030]	[1.463, 2.455]	[2.706, 3.810]	[1.234, 2.174]
[1.707, 2.741]	[7.296, 7.738]	[1.064, 1.956]	[3.128, 4.226]	[3.928, 4.972]
[1.936, 2.999]	[5.377, 6.222]	[1.213, 2.148]	[1.232, 2.172]	[5.198, 6.074]
[1.359, 2.330]	[1.819, 2.869]	[1.856, 2.910]	[2.657, 3.761]	[1.237, 2.178]
[2.095, 3.174]	[1.592, 2.609]	[2.188, 3.273]	[3.105, 4.204]	[2.234, 3.323]
[1.439, 2.427]	[1.052, 1.941]	[1.429, 2.414]	[1.388, 2.366]	[1.479, 2.475]
[1.152, 2.071]	[1.800, 2.847]	[5.350, 6.201]	[4.274, 5.281]	[1.481, 2.477]
[1.558, 2.568]	[2.615, 3.718]	[1.609, 2.628]	[1.236, 2.178]	[2.260, 3.350]

Table 1 shows the neutrosophic samples from the proposed model using program written in R. Interval data indicate the imprecision due to assume vagueness in the parameters.

Similarly, the model point in neutrosophic framework can be written as:

$$y_m = \mathcal{B}_n\left[\frac{\mathcal{A}_n}{(\mathcal{A}_n+1)}\right]^{-1/\mathcal{A}_n}. \quad (12)$$

Note that the mode y_m asymptotically approaches to 0 as $\mathcal{A}_n \to 0$. The neutrosophic quantile function provides the value below which a given proportion of the data is expected to fall, considering uncertainty and indeterminacy in the distribution's parameters.

The moments represent the most fundamental quantities characterizing the distribution. For a random variable Y follows the Fréchet distribution, the r^{th} moment is given by:

$$E_N[Y^r] = \int_0^\infty y^r f(y)dy = \mathcal{B}_n{}^r\Gamma(1 - r/\mathcal{A}_n) \quad (13)$$

Thus, neutrosophic mean and variance can be written as:

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$$\mu_N = \mathcal{B}_n \Gamma\left(1 - {}^{1}/_{\mathcal{A}_n}\right)$$

$${\sigma_N}^2 = {\mathcal{B}_n}^2 \left[\Gamma\left(1 - {}^{2}/_{\mathcal{A}_n}\right) - \Gamma^2\left(1 - {}^{1}/_{\mathcal{A}_n}\right)\right]$$

$$cv_N = \sqrt{\left\{\Gamma\left(1 - {}^{2}/_{\mathcal{A}_n}\right) - \Gamma^2\left(1 - {}^{1}/_{\mathcal{A}_n}\right)\right\} \Big/ \Gamma^2\left(1 - {}^{1}/_{\mathcal{A}_n}\right)}. \quad (14)$$

The hazard function (or failure rate) associated with the Neutrosophic Fréchet distribution (FD_N) is given by:

$$\mathcal{k}_N(y) = \frac{\left({}^{\mathcal{A}_n}/_{\mathcal{B}_n}\right)\left({}^{y}/_{\mathcal{B}_n}\right)^{-(\mathcal{A}_n+1)} e^{\left[-\left({}^{y}/_{\mathcal{B}_n}\right)^{-\mathcal{A}_n}\right]}}{1 - e^{\left[-\left({}^{y}/_{\mathcal{B}_n}\right)^{-\mathcal{A}_n}\right]}}, \; y \geq 0, \quad (15)$$

The hazard function, which is the same as the failure-rate function, indicates the instantaneous rate of an event at a given time during the study period, providing the event has not yet occurred. This function is useful to measure the level of risk or rate of failure when parameters are uncertain or indeterminate for neutrosophic Fréchet distribution. In the neutrosophic sense, the shape and scale parameters are not fixed constants, but they are all in interval valued neutrosophic numbers of senses, which embody truth, indeterminacy and falsehood degrees; these coincide with the case of traditional sense with zero indeterminacy. Therefore, the hazard function itself is an interval-valued measure, which gives an interval as a possible range of failure rates at each time instead of a unique deterministic value. Now if we assume the different values of neutrosophic parameters, the basic characteristics of the proposed distribution are given in Table 2.

Table 2: Numerical characteristics of the proposed model

$\mathcal{A}_n$	$\mathcal{B}_n$	μ_N	${\sigma_N}^2$	$\boldsymbol{cv}$	$\boldsymbol{y_m}$
[2.5, 3.0]	[1.5, 2.0]	[2.233, 2.708]	[3.381, 5.339]	[0.678, 1.034]	[1.716, 2.201]
[3.0, 4.0]	[2.0, 3.0]	[2.708, 3.676]	[2.437, 3.381]	[0.424, 0.678]	[2.201, 3.172]
[2.8, 3.5]	[1.0, 2.5]	[1.398, 3.189]	[1.193, 2.745]	[0.519, 0.781]	[1.115, 2.686]
[3.2, 4.5]	[2.5, 4.0]	[3.295, 4.760]	[2.948, 3.953]	[0.360, 0.603]	[2.721, 4.182]
[2.6, 3.2]	[1.8, 2.8]	[2.618, 3.691]	[4.959, 5.926]	[0.603, 0.929]	[2.040, 3.048]

Table 2 summarizes some of the properties of the neutrosophic Fréchet distribution under different scenarios. Each line represents a pair of two parameters, and the columns tell what type of characteristic is to be expected. More specifically, the table shows how the mean, the variability or spread, the proportion of variability relative to the mean, and the most probable event (or peak point) would change as a function of these parameters. It reveals that, depending on how the input values are tuned, the distribution may be spread out, and the most typical value may move within quite a range. This serves to comprehend the flexibility and sensitivity of distribution in uncertain or imprecise environments.

4. Parameter Estimation

In this section, we discuss the maximum likelihood estimation (MLE) approach for estimating unknown parameters. The MLE is an elementary statistical technique that allows us to estimate the unknown parameters in our model by finding the values, which make our observed data most likely. It does so by comparing various potential values for parameters and choosing the ones that do the best of explaining our data. This methodology is popular because it is generally effective, applicable to many model types, and versatile, especially as the amount of data grows with no storage restrictions. MLE is a primary tool in statistical modeling, machine learning and data analysis, in enabling researchers and analysts to make reasonable conclusions based on observed patterns.

The model parameters are estimated via maximum likelihood, obtained by optimizing the likelihood function.

$$L(\mathcal{A}_n, \mathcal{B}_n) = \prod_{i=1}^{n} f_N(y_i) \quad (16)$$

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which further can be written as:

$$L(\mathcal{A}_n,\mathcal{B}_n)=\prod_{i=1}^{n}\left(\frac{\mathcal{A}_n}{\mathcal{B}_n}\right)\left(\frac{y_i}{\mathcal{B}_n}\right)^{-(\mathcal{A}_n+1)}\exp\left[-\left(\frac{y_i}{\mathcal{B}_n}\right)^{-\mathcal{A}_n}\right] \quad (17)$$

To simplify the calculation, the log likelihood function can be written as:

$$\ln L\,(\mathcal{A}_n,\mathcal{B}_n)=n\ln\mathcal{A}_n-n\ln\mathcal{B}_n-(\mathcal{A}_n+1)\sum_{i=1}^{n}\ln\left(\frac{y_i}{\mathcal{B}_n}\right)-\sum_{i=1}^{n}\left(\frac{y_i}{\mathcal{B}_n}\right)^{-\mathcal{A}_n} \quad (18)$$

Partial derivate with respect to unknown values yielded:

$$\frac{\partial\ln L}{\partial\mathcal{A}_n}=\frac{n}{\mathcal{A}_n}-\sum_{i=1}^{n}\ln\left(\frac{y_i}{\mathcal{B}_n}\right)+\sum_{i=1}^{n}\left(\frac{y_i}{\mathcal{B}_n}\right)^{-\mathcal{A}_n}\ln\left(\frac{y_i}{\mathcal{B}_n}\right)=0 \quad (19)$$

$$\frac{\partial\ln L}{\partial\mathcal{B}_n}=-\frac{n\mathcal{A}_n}{\mathcal{B}_n}+\frac{(\mathcal{A}_n+1)}{\mathcal{B}_n}\sum_{i=1}^{n}\left(\frac{y_i}{\mathcal{B}_n}\right)^{-1}-\mathcal{A}_n\sum_{i=1}^{n}\left(\frac{y_i}{\mathcal{B}_n}\right)^{-\mathcal{A}_n-1}=0 \quad (20)$$

A closed-form solution of these Eq (19) and Eq (20) does not exist so they can be solved numerically using iterative methods such as Newton-Raphson or optimization routines in R software. Using random data from the proposed model with parameter settings $\mathcal{A}_n=[2.5,3.5]$and $\mathcal{B}_n=[1.5,2.5]$ are drawn and estimated values of unknown parameters across different sample sizes are shown in Table 3.

Table 3: Estimated values of unknown parameters of the proposed model for simulated data

Sample size	$\mathcal{A}_n$	$\mathcal{B}_n$
20	[2.2476 , 3.1463]	[1.6088 , 2.6281]
40	[2.2535 , 3.1554]	[1.6473 , 2.6732]
60	[2.5433 , 3.5607]	[1.5233 , 2.5276]
80	[2.4914 , 3.4882]	[1.4989 , 2.4987]
100	[2.5496 , 3.5697]	[1.5005 , 2.5005]
120	[2.5122 , 3.5175]	[1.5288 , 2.5342]

Table 3 shows the estimated values of the two unknown parameters of the proposed model from the simulated datasets. For each given sample size, Table 3 includes a specific interval of values for each parameter. It is not difficult to see that these intervals are consequences of imprecise or neutrosophic data being dealt with in estimating the parameters. With a growing number of samples, the values for the estimate stabilize and the results become more consistent, leading to more reliable parameter values. Table 3 serves to illustrate how the model behaves when presented with varying amounts of data, and how the estimates of the parameters can fluctuate within a particular interval due to inherent data vagueness.

5. Application to Real Data

In the present work model is implemented to investigate electricity demand data of Saudi Arabia in period 2000-2020. The data, collected from [26], corresponds to annual electricity demand on a gigawatt-hour (GWh) scale and portrays the patterns exhibited by country's energy segment. Proper analysis of this kind of data is important for policymakers and strategic decision makers in order to fulfil the needs of such a rapid growing population and economy. Electricity usage grew in tune with the transforming nation's rapid urbanization and population explosion during that time. This corresponds to having a dynamic Saudi Arabia that is currently being developed part of this Vision 2030, with the focus on residential, commercial and industrial expansion plus economic diversification and modernization. Reliable statistics for demand on electricity and generation are a prerequisite to

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the national energy planning, where important infrastructure investment decisions (renewable vs conventional power plant capacities) and choices on sustainable energy sources as well as efficient grid operations will be influenced by informed data. Such knowledge contributes to solving issues of peak demand, energy security and environmental sustainability by supporting the prosperity of Saudi Arabia's oil/gas industry in long-term.

Electricity consumption volumes are by nature uncertain given data errors, prediction differences, seasons and the business cycle. In order to cope with this vagueness, original data was converted to neutrosophic data based on methodology defined in the work [27]. Such a transformation accounts for the truth, falsity and various levels of indeterminacy. Consequently, it provides consistent instruments to deal with uncertainty in energy planning. The associated neutrosophic electricity consumption data are given in Table 3 of the work [27].

Now utilizing the method of neutrosophic MLE given in section 4, we can estimate the parameters of the proposed distribution, which are given below:

$\hat{\mathcal{A}}_n = [2.7791, 2.779]$ and $\hat{\mathcal{B}}_n = [182696.8, 182697.8]$

Interval estimates for the parameters of the model were derived by separately fitting the proposed model to lower and upper observed bounds. The resulting ranges for the parameters reflect possible variation of the distribution in shape and scale owing to data fuzziness. The lower limits of these problem intervals are estimates from the lowest values, while the upper limits are estimates from the highest values. This gives a much stronger reflection of the uncertainty in the data and permits better quality modelling for energy sector planning.

6. Conclusion

In this study, we have introduced a new version of the neutrosophic Fréchet distribution as an attempt to overcome the difficult aspects related to modeling extreme value data under uncertainty and indeterminacy. By generalizing the classical model via neutrosophic logic, the new model possesses a strong modeling power and can express the randomness and imprecision of actual circumstances well. With the neutrosophic parameters that commonly observed in practical data, the distribution properties including its probability density function, quantile function, and hazard rate were investigated. The method of maximum likelihood estimation for interval-valued data also demonstrated the practicability of the suggested approach. Simulation results revealed that the estimated intervals became smaller and more stable with the increment of the sample size, which confirmed the model reliability. In general, the proposed neutrosophic Fréchet distribution strengthens the capabilities of the extreme value theory for handling real-life complexities and provides promising routes to more robust statistical modeling in engineering, industrial, environmental, and risk analysis.

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