



Neutrosophic of Capacitated and Uncapacitated Stochastic Facility Location Problems

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Abstract

Facility location problems assigned for determining the location of different types of facilities as factories, warehouses, hospitals, ..., etc. It also helps to find the quantity of products and goods delivered to customers from the assigned facilities. As in other fields, uncertainty occurs in facility location problems, when the cost, time and other information seem in deterministic and unknown. The uncertainty in facility location problems promoted scientists to apply robust optimization such as stochastic techniques for solving complex locations problems. However, in stochastic problems some uncertain parameters need highly approaches such as neutrosophic sets, which is an extension of fuzzy sets to tackle the stochastic parameters. In this paper, a neutrosophic approaches based on neutrosophic sets applied for solving capacitated and uncapacitated stochastic facility location problems. The normal and neutrosophic models designed and some applications illustrated for testing the neutrosophic stochastic facility location problems in two cases capacitated and uncapacitated facilities. The result various for the two different situations and shows that decision maker therefore offers flexibility of various solutions when applying the neutrosophic case under different situations.

Keywords: Facility Location; Stochastic; Capacitated; Uncapacitated; Neutrosophic

1. Introduction

Facility location problems considered as one of important optimization problems for determining the location of different types of facilities such as hospitals, healthcare centers, factories, ..., etc in order to minimize the total cost [2,3,6]. In addition, the facility location problems also helps in determining the quantity of demand delivered from facilities locations to customers to minimize cost and delivery time, which also leads to maximize profits [12,20,23]. This type of problems classified for two types: capacitated and uncapacitated facility locations, based on existing of facility capacity [9,16,18]. The capacitated facility location problems refers to the limited capacity of facilities for serving customers, in other words it means that each single facility can satisfy a limited number of customers [4,11,13]. However, in the uncapacitated facility location problems, unlimited customers can be served from each single facility [10].

Usually information or parameters or data such as demand, cost and facility capacity of the facility location problems known and deterministic, however, sometimes uncertainty appears in the facility location problems when some parameters as demand, delivery time and cost seem unknown [5]. To tackle challenges of uncertainty, many researchers created and improved stochastic approaches to solve and model the uncertainty of demand and cost [15,17]. In this case, uncertainty in the facility location problems can be described as stochastic facility location problem and many approaches can be applied aiming to find the uncertain parameters [21]. Neutrosophic, represented by neutrosophic sets, which is an extension of fuzzy sets, is recently applied for solving uncertainty of stochastic facility location problems [1]. In the neutrosophic sets, three types of memberships functions: truth memberships, indeterminacy memberships and falsity memberships, have been introduced to illustrate number of

uncertain and unknown data [1,8,14]. Incorporation and merging the neutrosophic stochastic approach in facility location problems field is vital for solving uncertainty and unknown information. Recently, the neutrosophic set applied widely for talking uncertainty in many situation.

In this paper, we solve the uncertainty of both cases (capacitated and uncapacitated stochastic facility location problem) by applying the neutrosophic stochastic approach based on the use of neutrosophic sets. We illustrate the neutrosophic parameters and mathematical models of the problem and we test the difference between normal and neutrosophic models, we also illustrate examples of different stochastic facility location problems and neutrosophic as well.

This paper is organizes as follow: in section 2 the deterministic (certain) and neutrosophic uncapacitated stochastic facility location problem is considered. While in section 3, the n deterministic (certain) and neutrosophic capacitated stochastic facility location problem is considered. Finally, the conclusion is illustrated in section 4.

2. Main Discussion

2.1. Uncapacitated Stochastic Facility Location Problem

In the case of uncapacitated facility location problems, unlimited number of customers can be served from each single facility location [19,23]. Uncertainty exists when parameters as cost, demand and time unknown [18]. In this case the stochastic facility location problems (SFLP) parameters can be described as follows:

F: refers to the set of the facility locations, where $\mathbf{F}=\{1, 2, 3, \dots, n\}$

C: refers to the set of the customer locations, where $\mathbf{C}=\{1, 2, 3, \dots, m\}$

D_{fc} : represents the demand delivered from facility f to customer c

S_{fc} : represents the cost for serving customer c from facility f

P_f : represents the fixed cost for opening facility f

The decision variables of the problems are as follows:

$$x_f = \begin{cases} 1, & \text{if facility } f \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

y_{fc} : the quantity of demand delivered to customer c from facility f

The mathematical model of the USFLP is as follows:

$$\text{Min. } Z = \sum_{f \in \mathbf{F}} P_f x_f + \sum_{f \in \mathbf{F}} \sum_{c \in \mathbf{C}} S_{fc} y_{fc} \quad (1)$$

Subject to:

$$\sum_{f \in \mathbf{F}} y_{fc} = 1 \quad \forall \quad c \in \mathbf{C} \quad (2)$$

$$x_f \in \{0, 1\}, \quad y_{fc} \geq 0 \quad \forall \quad f \in \mathbf{F}, c \in \mathbf{C} \quad (3)$$

Constraint (1) is the objective function refers to the minimized total cost of the opened facilities and the required demand of customers. The delivered demand to customers is restricted by constraint (2) and constraint (3) refers to non-negativity and integrality of variables.

2.2. Neutrosophic Uncapacitated Stochastic Facility Location Problem

When the cost or time or any other information seem in deterministic and unknown, this refers to the uncertainty in the uncapacitated stochastic facility location problems. In this case, neutrosophic, which is an extension of fuzzy sets, can be applied to solve the uncertainty case. The Neutrosophic set N refers to the space of points \mathbf{X} , where $\mathbf{x} \in \mathbf{X}$ and it can be defined as three membership functions: Truth $\mathbf{T}_N(\mathbf{x})$, Indeterminacy $\mathbf{I}_N(\mathbf{x})$ and Falsity $\mathbf{F}_N(\mathbf{x})$ [8]. Based on this case, the the neutrosophic for uncapacitated stochastic facility location problems (NUSFLP) involves indeterminacy, truth and falsity of the neutrosophic parameters which become as follow:

$D_{fc} = (T_{D_{fc}}, I_{D_{fc}}, F_{D_{fc}})$: represents the neutrosophic demand delivered from facility f to customer c

$S_{fc} = (T_{S_{fc}}, I_{S_{fc}}, F_{S_{fc}})$: represents the neutrosophic cost for serving customer c from facility f

$P_f = (T_{P_f}, I_{P_f}, F_{P_f})$: represents the neutrosophic fixed cost for opening facility f

The first decision variable of the neutrosophic x_f is the same as in the stochastic case refers if facility f is opened or not. While the second decision variable for the neutrosophic case: $y_{fc} = (T_{y_{fc}}, I_{y_{fc}}, F_{y_{fc}})$ represents the

quantity of demand delivered to customer c from facility f . The three values in the neutrosophic facility location problem (T, I, F) denote to the truth, indeterminacy and falsity of the objective function for the NUSFLP, which expressed as follow:

$$T_M = \sum_{f \in F} T_{P_f} x_f + \sum_{f \in F} \sum_{c \in C} T_{S_{fc}} y_{fc} \tag{4}$$

$$I_M = \sum_{f \in F} I_{P_f} x_f + \sum_{f \in F} \sum_{c \in C} I_{S_{fc}} y_{fc} \tag{5}$$

$$F_M = \sum_{f \in F} F_{P_f} x_f + \sum_{f \in F} \sum_{c \in C} F_{S_{fc}} y_{fc} \tag{6}$$

The mathematical model of the neutrosophic uncapacitated stochastic facility location problem is as follow:

$$\text{Min. } \tilde{Z} = T_M + I_M + F_M \tag{7}$$

Subject to:

$$T_{y_{fc}} \leq y_{fc} \leq F_{y_{fc}} \quad \forall f \in F, c \in C \tag{8}$$

$$T_{y_{fc}} \leq y_{fc} \leq F_{y_{fc}} \quad \forall f \in F, c \in C \tag{9}$$

Constraint (7) is the objective function refers to the neutrosophic minimized total cost of the opened facilities and the required demand of customers. The neutrosophic delivered demand to customers is restricted by constraint (8) and constraint (9) refers to non-negativity and integrality of variables in the neutrosophic situation.

Now, we test an application of 4 customer locations and 3 facility locations with a fixed cost (50, 60, 70) and serving cost (10,15,20,25;12,18,24,30;15,20,25,35) and the demand is (1,2,1,3;2,1,3,2;1,3,2,1).

Solving the above example using the USFLP model yields the optimal solution of: $x_1 = 1, x_2 = 1, x_3 = 0$, referring that facilities 1 and 2 need to be opened and the optimal required demand as in Table 1:

Table 1: The optimal required demand as in Table 1

Customer \ Facility	C ₁	C ₂	C ₃	C ₄
F ₁	0.2	1.8	0	3
F ₂	0.8	0	1	0
F ₃	0	0.2	0	0

To solve the same example for the neutrosophic case, parameters defined as follow:

the fixed cost for the NUSFLP defined as: $\tilde{P}_1 = (40, 50, 60), \tilde{P}_2 = (50, 60, 70), \tilde{P}_3 = (60, 70, 80)$, and the serving cost as in Table 2:

Table 2: Optimal Solution of NUSFLP

Customer \ \tilde{S}_{fc}	C ₁	C ₂	C ₃	C ₄
\tilde{S}_{1c}	8,10,12	13,15,17	18,20,22	23,25,27
\tilde{S}_{2c}	10,12,14	16,18,20	22,24,26	28,30,32
\tilde{S}_{3c}	13,15,17	18,20,22	23,25,27	33,35,37

Table 3: Optimal Demand of NUSFLP

\widetilde{D}_{fc} \ Customer	C ₁	C ₂	C ₃	C ₄
\widetilde{D}_{1c}	0,1,2	1,2,3	0,1,2	2,3,4
\widetilde{D}_{2c}	1,2,3	0,1,2	2,3,4	1,2,3
\widetilde{D}_{3c}	0,1,2	2,3,4	1,2,3	0,1,2

The required demand for the NSFLP as in Table 3. Then we solve the same example using the NUSFLP model, the result for the first decision variable $x_f = x_1 = 1, x_2 = 1, x_3 = 0$ which is the same as the SFLP case. While the optimal required demand of the NSFLP as in Table 4:

Table 4: Optimal Demand of NUSFLP

C \ \widetilde{D}_{fc}	C ₁	C ₂	C ₃	C ₄
\widetilde{D}_{1c}	$y_{11} = (0.5, 1, 1.5)$	$y_{12} = (1.8, 2, 2.2)$	$y_{13} = (0.2, 1, 1.8)$	$y_{14} = (2.5, 3, 3.5)$
\widetilde{D}_{2c}	$y_{21} = (1.5, 2, 2.5)$	$y_{22} = (0.8, 1, 1.2)$	$y_{23} = (2, 3, 4)$	$y_{24} = (0.8, 1, 1.2)$
\widetilde{D}_{3c}	0	0	0	0

Comparing the result of the two cases, we see that the optimal demand for the NSFLP offers three values (truth, indeterminacy, falsity), while the decision for choosing facilities to opened is the same for the two cases. As we know, in the stochastic situation all information is known, so that it is normal to find the exact solution of the problems. However, under the uncertainty information is unknown, which is tackled in this research using the neutrosophic technique. The decision maker therefore offers flexibility of various solutions when applying the neutrosophic case under different situations. Next, we solve large instances consist of 10 facilities and 50 customers as in Figures 1,2 and the optimal solution of the two cases are the same for assigning facilities to customers. However, the costs is different for the two cases USFLP and NUSFLP.

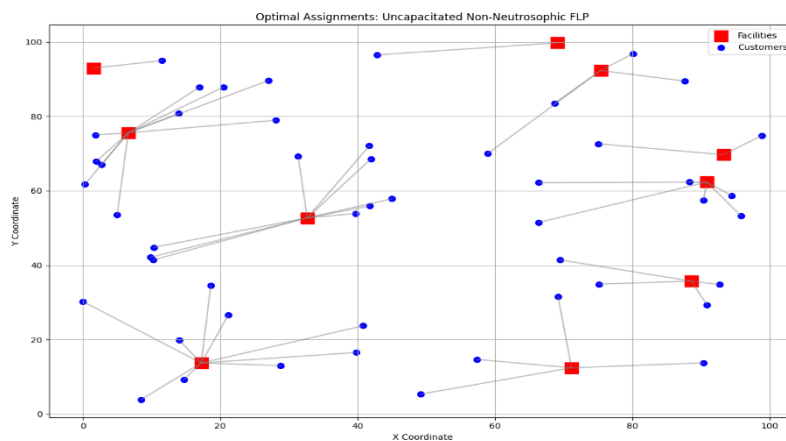


Figure 1. Large instance of USFLP

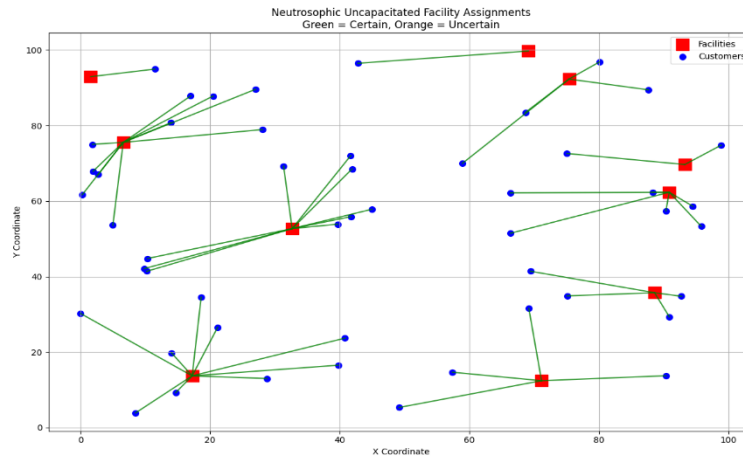


Figure 2. Large instance of NUSFLP

3.1. Capacitated Stochastic Facility Location Problem

In this type of facility location problems, facilities are restricted to specific capacity (K) for each facility f [15,22,24]. In this case, demand of customers effected by the capacity of facility served that customer and many scenarios (S) occurs with probability (β) referring that some demand customers are satisfied but other demand customers are not satisfied. The capacity constraint add to the general mathematical model of the capacitated stochastic facility location problem and the model becomes as follows:

$$\text{Min. } Z = \sum_{f \in F} P_f x_f + \sum_{s \in S} \beta_s [\sum_{f \in F} \sum_{c \in C} S_{fc} y_{fc}] \tag{10}$$

Subject to:

$$\sum_{f \in F} y_{fc} = 1 \quad \forall c \in C \tag{11}$$

$$y_{fc} \leq D_{fc} x_f \quad \forall f \in F, c \in C \tag{12}$$

$$\sum_{f \in F} y_{fc} \leq K_f x_f \quad \forall f \in F \tag{13}$$

$$x_f \in \{0, 1\}, y_{fc} \geq 0 \quad \forall f \in F, c \in C \tag{14}$$

Constraint (11) is the objective function refers to the minimized total cost of the opened facilities and the required demand of customers of the CSFLP. The delivered demand to customers is restricted by constraint (12) and constraint (13) refers the capacity of facilities. Constraint (14) is the non-negativity and integrality of variables.

3.2. Neutrosophic Capacitated Stochastic Facility Location Problem

As in the UCSFLP, when the cost or time or any other information seem in deterministic and unknown, this refers to the uncertainty in the capacitated stochastic facility location problems. In this case, neutrosophic, which is an extension of fuzzy sets, can be applied to solve the uncertainty case. The Neutrosophic set N refers to the space of points X , where $x \in X$ and it can be defined as three membership functions: Truth $T_N(x)$, Indeterminacy $I_N(x)$ and Falsity $F_N(x)$ [8]. Based on this case, the the neutrosophic for capacitated stochastic facility location problems (NCSFLP) involves indeterminacy, truth and falsity of the neutrosophic parameters. The Neutrosophic case of the CSFLP considering the uncertainty involves the neutrosophic parameters for the CSFLP, however, the difference here is the demand of customer c for scenarios (S) represented as $D_{fc} = (T_{D_{fc}}, I_{D_{fc}}, F_{D_{fc}})$. The three values in the neutrosophic facility location problem (T, I, F) denoted to the truth, indeterminacy and falsity of the objective function for the NUSFLP as in equations also applied in this case.

The mathematical model of the neutrosophic capacitated stochastic facility location problem is as follow:

$$\text{Min. } \check{Z} = T_M + I_M + F_M \tag{15}$$

Subject to:

$$T_{y_{fc}} \leq y_{fc} \leq F_{y_{fc}} \quad \forall f \in F, c \in C \tag{16}$$

$$T_{y_{fc}} \leq y_{fc} \leq F_{y_{fc}} \quad \forall f \in F, c \in C \tag{17}$$

Constraint (15) is the objective function refers to the neutrosophic minimized total cost of the opened facilities and the required demand of customers. The neutrosophic delivered demand to customers is restricted by constraint (16) is the neutrosophic quantity of demand delivered to customers. Constraint (17) represent the non-negativity and integrality of variables in the neutrosophic situation.

We solve example of 10 customer locations and 5 facility locations with X and Y coordinates. Capacities of facilities are: 200, 180, 220 and the stochastic demand of customers is approximated to (50, 40, 30, 70, 25, 40, 40, 55, 70, 40). We solve this example and the result of the CSFLP show that all facilities opened and facility 1 is assigned to customers (1,2,3,5), facility 2 is assigned to customers (4,7,8), facility 3 is assigned to customers (6,9,10) as shown in **Figure 3**. The cost of opening facilities is 3300 and the demand serving cost is 191, thus the total minimized cost of the CSFLP is 3491. Then we solve the same example using the NCSFLP and the result shows that all facilities also opened such as in the CSFLP and the same assignment of facilities for customers as in the CSFLP. However, the neutrosophic cost of opening facilities is 3300 and the neutrosophic demand serving cost is 235, thus the total minimized cost of the NCSFLP is 3535 as shown in **Figure 4**. We can see that the neutrosophic solution is larger than the normal case, sine data and information are imprecise which is however differs from the certain case where the data and the information determined and constant for the CSFLP.

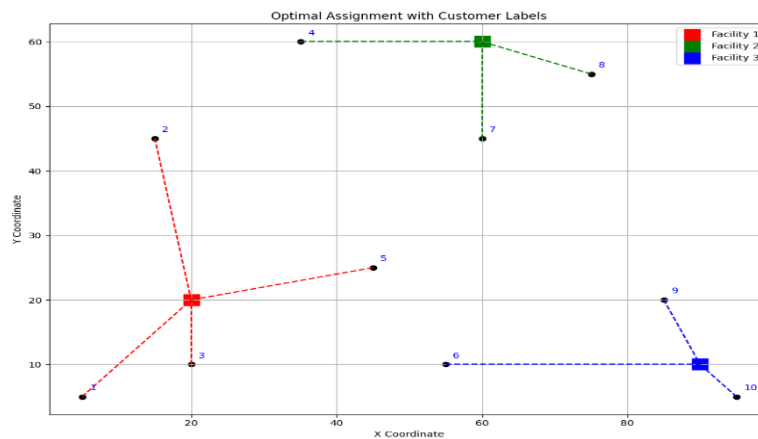


Figure 3. CSFLP Solution of the Example

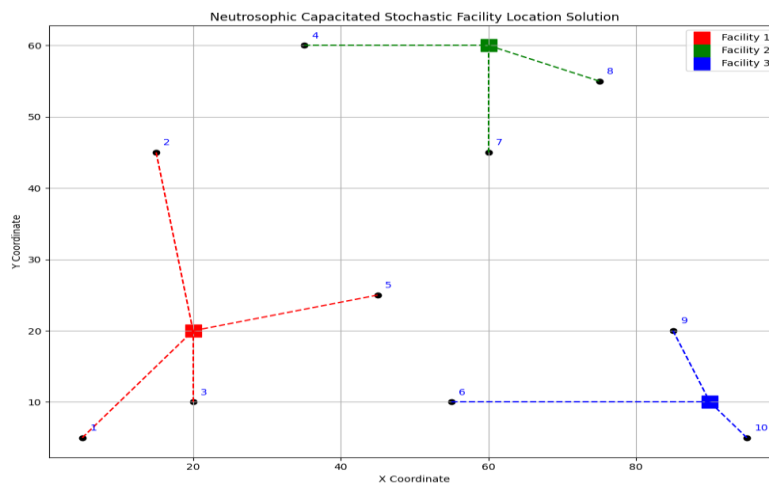


Figure 4. NCSFLP Solution of the Example

Next, we solve large instances consist of 10 facilities and 50 customers as in Figures 5,6 and as shown in the figures , the optimal solution of the two cases are the same for assigning facilities to customers. However, the costs is different for the two cases CSFLP and N CSFLP.

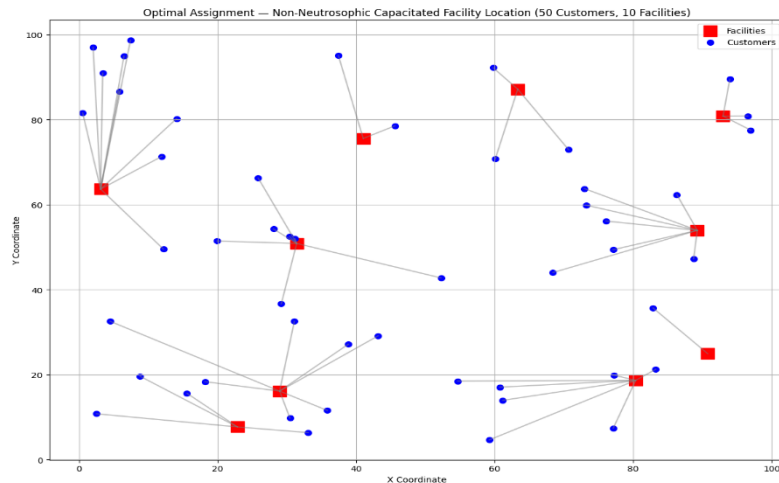


Figure 5. Large instance of CSFLP

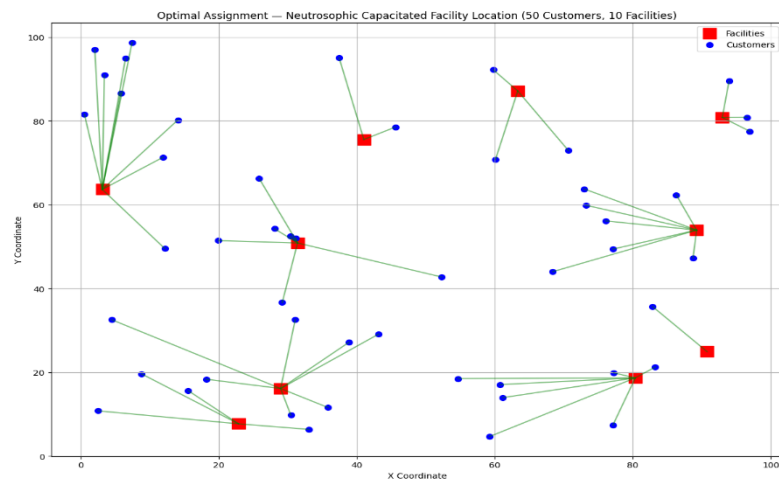


Figure 6. Large instance of NCSFLP

4. Conclusion

In this paper, we consider the stochastic facility location problems where information and data are uncertain and unknown. In stochastic facility location problems uncertainty exists when parameters as cost, demand and time are unknown. Neutrosophic, which is an extension of fuzzy sets, is applied in this case for two types of the stochastic facility location problems: capacitated and uncapacitated facilities in order to tackle the uncertainty by involving indeterminacy, truth and falsity memberships of the neutrosophic parameters. The mathematical models of the neutrosophic capacitated and incapacitated illustrated in this paper and tested for applications and the results various for the certain and uncertain situations. This work considers the general case of the capacitated and capacitated stochastic facility location problems, however, more research can be executed with constraints that are more exclusive and parameters based on each situation of the problem.

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