Separation Axioms Defined by Four Different Points in Neutrosophic Crisp Spaces

Nour M. Easi[1,*], L. A. A. Jabar[1], Ali H. M. Al-Obaidi[1]

[1]Department of Mathematics, College of Education for Pure Science, University of Babylon, Iraq

Emails: edu238.nour.maki@student.uobabylon.edu.iq; l.h.jabar64@gmail.com; aalobaidi@uobabylon.edu.iq

Abstract

In this paper, separation axioms are discussed in neutrosophic crisp topological spaces from a new perspective. This is generally useless because any neutrosophic set does not necessarily have a union of its neutrosophic points under any union and for any kind of points. Hence, the separation properties are studied concerning stable neutrosophic crisp topological spaces, which are determined by two special types of complement. Moreover, various examples are illustrated in these cases.

Keywords: Neutrosophic crisp sets; Neutrosophic crisp points; Stable neutrosophic crisp topological space; Stable closure neutrosophic; T_i-Space, $i = 0,1,2$

1. Introduction

The important aspect of all scientific and life problems from a mathematical perspective is that it is classified based on points and sets. Given the overlap of these problems and the complexity of life resulting from evolution, it is necessary to define new types of sets and, subsequently, define different types of points. For example, the Fuzzy set has different types of Fuzzy points: $\left(P_A^\lambda, \ni P(a) = \lambda, \forall a \in A \subset X\right); (P_x^\lambda, \ni P(x) = \lambda,\ for\ some\ x \in X)$ such that $0 < \lambda < 1$ [1]. In addition, there are three types of soft points in soft sets [1]: $(\ F_x^A\ ,\ F(x) = A\ , F(y) = \emptyset\ , x \neq y)$; $(F_A, (x, A)), (F_x^A\ \ni\ F(x) = A\ \ \forall\ x \in X)$. Moreover, there are two types of points in the neutrosophic set [1]: $< x, \lambda, \lambda, \lambda >; < x, \lambda, 0{,}0 >\ \ s.t. 0 < \lambda < 1$. Furthermore, there are three types of points in the neutrosophic axial set [1]:

$$S_A =< A, A_1, A_2 >, s.t.\ A \cap A_i = \emptyset, i = 1{,}2, ;\ \ NP_A =< \emptyset, A, \emptyset >;\ NP_A^\emptyset =< A, \emptyset, \emptyset >.$$

When studying the continuity of a function on a neutrosophic space, it varies from function to function depending on the type of points on which it is built. This affects the concept of topology, such as [1], compact, and other concepts. This opens new possibilities for studies that consider the space of functions after fixing the type of points.

Salama and Florentin [2] defined three distinct kinds of neutrosophic crisp sets. The first type requires the condition $(H_1 \cap H_2 = \emptyset, H_1 \cap H_3 = \emptyset, H_2 \cap H_3 = \emptyset)$, to which we give the symbol $(NC_1 - set)$, while the second type, symbolized $(NC_2 - set)$, requires the condition $(H_1 \cap H_2 = \emptyset, H_1 \cap H_3 = \emptyset, H_2 \cap H_3 = \emptyset\ and\ H_1 \cup H_2 \cup H_3 = X)$, and the third type $(NC_3 - set)$ requires the condition $(H_1 \cap H_2 \cap H_3 = \emptyset\ and\ H_1 \cup H_2 \cup H_3 = X)$).

Moreover, through their research they defined types of empty sets, which are:

$\emptyset_{N_1} =\ll \emptyset, \emptyset, X \gg, \emptyset_{N_2} =\ll \emptyset, X, \emptyset \gg, \emptyset_{N_3} =\ll \emptyset, X, X \gg, \emptyset_{N_4} =\ll \emptyset, \emptyset, \emptyset \gg$

and synonymously they defined types of universal sets and their forms, these are:

$X_{N_1} =\ll X, \emptyset, \emptyset \gg . X_{N_2} =\ll X, X, \emptyset \gg, X_{N_3} =\ll X, \emptyset, X \gg, X_{N_4} =\ll X, X, X \gg.$

Likewise, the complement of any neutrosophic crisp set, which has three types:

$(H_N)^{C_1} =\ll {H_1}^C, {H_2}^C, {H_3}^C \gg, (H_N)^{C_2} =\ll H_3, H_2, H_1 \gg . (H_N)^{C_3} =\ll H_3, {H_2}^C, H_1 \gg.$

Salama and Florentin [2] defined the first and second subsets as follows:

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$(H_N \subseteq_1 F_N) =\ll H_1 \subseteq F_1, H_2 \subseteq F_2, F_3 \subseteq H_3 \gg$,

$(H_N \subseteq_2 F_N) =\ll H_1 \subseteq F_1, F_2 \subseteq H_2, F_3 \subseteq H_3 \gg)$),

while the union and intersection were defined as follows:

$$(H_N \cup_1 F_N) =\ll H_1 \cup F_1, H_2 \cup F_2, H_3 \cap F_3 \gg,$$

$$(H_N \cup_2 F_N) =\ll H_1 \cup F_1, H_2 \cap F_2, H_3 \cap F_3 \gg,$$

$$(H_N \cap_1 F_N) =\ll H_1 \cap F_1, H_2 \cap F_2, H_3 \cup F_3 \gg,$$

$$(H_N \cap_2 F_N) =\ll H_1 \cap F_1, H_2 \cup F_2, H_3 \cup F_3 \gg.$$

Similarly, they defined two types of points:

$$P_{N_1} =\ll \{P\}, \emptyset, \{P\}^C \gg,$$

$$P_{N_2} =\ll \emptyset, \{P\}, \{P\}^C \gg.$$

However, they do not cover a complete area in relation to space. Therefore, we decided to add two additional definitions, namely

$$P_{N_3} =\ll \{P\}, \emptyset, \emptyset \gg, P_{N_4} =\ll \emptyset, \{P\}, \emptyset \gg$$

where $\{P\}$ is singleton [3]. As for the belonging of points to sets, they were used as follows:

$$P_{N_1} \in_1 H_N \leftrightarrow P \in H_1,$$

$$P_{N_2} \in_2 H_N \leftrightarrow P \notin H_3 and\ P \in H_2,$$

$$P_{N_3} \in_3 H_N \leftrightarrow P \in H_1,$$

$$P_{N_4} \in_4 H_N \leftrightarrow P \in H_2.$$

A new type of topological space is defined based on neutrosophic crisp sets with an intersection of the second type, a union of the second type, and complement $C_1$and C_3. Thus, the belonging of sets to families will be normal, and equality is considered normal equality

$$A_N \neq B_N \leftrightarrow P_N \in A_N, P_N \notin B_N \text{ or } P_N \notin A_N, P_N \in B_N$$

then $P \neq Q \leftrightarrow P_{N_i} \neq Q_{N_i}, i = 1,2,3,4.$

In the space (X, ϑ) is said to be satisfying the complementary properties

$$\forall\ NCP\ \ P_N \notin\ A_N \leftrightarrow P_N\ \in (A_N)^C$$

defined the space (X, ϑ) is NCTS then the somth if $\forall\ G_N, L_N \in\ \vartheta$ then $G_N \cap L_N \in \vartheta$ and the space (X, ϑ) is NCTS then the totalitarian property if $\forall \curlywedge \in \curlywedge, G_{\curlywedge} \in \vartheta\ then\ \cup_{\curlywedge \in \curlywedge} G_{\curlywedge} \in \vartheta$.

The stable interior of A_N is denoted by $\delta i_{ij}(A_N)$ and given as follows

$$\delta i_{ij}(A_N) =\cup_i \{H_N \in \vartheta, H_N \subseteq_j A_N\},\ \ i, j = 1,2$$

then the stable exterior of A_N is denoted by $\delta e_{ij}(A_N)$ and given as follows

$$\delta e_{ij}(A_N) = \delta i_{ij}(A_N)^{C_m}\ \ i, j = 1,2,\ \ \ m = 1,3$$

Finally, some required definitions are given for any function $f: X \to Y$ such that X and Y are NCS, then the following facts are true (all found in [3]):

1- If $B_N =\ll B_1, B_2, B_3 \gg$ is a NCS in Y, then the preimage of B_N under f denoted by $f^{-1}(B_N)$ is a NCS in Xs.t. $f^{-1}(B_N) =\ll f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \gg$.
2- If $A_N =\ll A_1, A_2, A_3 \gg$ is a NCS in X, then the image of A under f denoted by $f(A_N)$ is a NCS in Y s.t. $f(A_N) =\ll f(A_1), f(A_2), f(A_3) \gg$.
3- If (X, ϑ_1) and (Y, ϑ_2) be two NCTS, then f is said to be continuous if the preimage of each NCS in ϑ_2 is a NCS in ϑ_1.
4- If (X, ϑ_1) and (Y, ϑ_2) be two NCTS, then f is said to be open if the image of each NCS in ϑ_2 is a NCS in ϑ_1.

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2. Stable Neutrosophic Crisp Topological Space (SNCT-space)

Definition 2.1:

Let X be a fixed set that is not empty, a (SNCT-space) is a family ϑ satisfies the following condition:

1. $X_{N1}, \emptyset_{N3} \in \vartheta$

2. $\forall H_N, F_N \in \vartheta, \exists S_N \in \vartheta, \ni S_N \subseteq_1 H_N \cap_2 F_N$

3. $\forall H_{Ni} \in \vartheta, \exists L_N \in \vartheta, \ni L_N \subseteq_1 \cup_2 H_{Ni}, i = 1,2,\dots,n$

Then (X, ϑ) is a (SNCT-space). For any $A_N \in \vartheta$ is a stable neutrosophic crisp open set and its denoted by ($SNCO - set$), the complement of type 1 or 3 for ($SNCO - set$) is stable neutrosophic crisp closed set and denoted by ($SNCC - set$).

Example 2.2:

Let $X = \{q, w, e, r, t, y,\}$ be a nonempty fixed set and $A_N, B_N, C_N, D_N, E_N, F_N, G_N, H_N$ are a NC-sets, such that:

$A_N = \ll \{q\}, \{w, e, r\}, \{t\} \gg$ $\quad$ $B_N = \ll \{q\}, \{t, y\}, \{e\} \gg$

$C_N = \ll \{q\}, \{w, e, r, t, y\}, \{t, e\} \gg$ $\quad$ $D_N = \ll \{q\}, \emptyset, \{t\} \gg$

$E_N = \ll \{q\}, \{t, y\}, \{e, t\} \gg$ $\quad$ $F_N = \ll \{q\}, \emptyset, \{e\} \gg$

$G_N = \ll \{q\}, \{w, e, r\}, \{t, e\} \gg$ $\quad$ $H_N = \ll \{q\}, \emptyset, \{e, t\} \gg$

$\vartheta = \{A_N, B_N, C_N, D_N, E_N, F_N, G_N, H_N, \emptyset_{N_3}, X_{N_1}\}$

Then (X, ϑ) is SNCT-space.

Definition 2.3:

Let (X, ϑ) be a SNCT-space and A_N be a NCS, then the closure of A_N is denoted by $CL_{ij}(A_N)$ and define as:

$$CL_{ij}(A_N) = (\delta e_{ij}(A_N))^{C_m}, i, j = 1,2, \qquad m = 1,3.$$

We will rely on the first complement, and in the same way, the third complement can be used.

Example 2.4:

From example 2.2, let

$$K_N = \ll \{e, t\}, \{y, t, q\}, \{y, q, r, w\} \gg$$

and

$$(K_N)^{C_1} = \ll \{q, r, y, w\}, \{w, e, r\}, \{e, t\} \gg$$

then

$$CL_{11}(K_N) = (\delta e_{11}(K_N))^{C_1} = \ll \{w, e, r, t, y\}, \{q, t, y\}, \{q, w, r, y\} \gg$$

$$CL_{22}(K_N) = (\delta e_{22}(K_N))^{C_1} = \ll \{w, e, r, t, y\}, \{q, t, y\}, \{q, w, r, y\} \gg$$

Preposition 2.5:

Let (X, ϑ) be a SNCT-space H_N, F_N $are\ a\ NC - set$ of any type. Then the properties hold for $i, j = 1,2$

(i) $CL_{ij}(\emptyset_{N3}) = \emptyset_{N3}$, $CL_{ij}(X_{N1}) = X_{N1}$

(ii) $H_N \subseteq_1 CL_{ij}(H_N)$

(iii) $F_N \subseteq_j H_N, then\ CL_{ij}(F_N) \subseteq_j CL_{ij}(H_N)$

(iv) $CL_{ij}(F_N \cup_i H_N) = CL_{ij}(F_N) \cup_i CL_{ij}(H_N)$

(v) $CL_{ij}(F_N \cap_i H_N) \subseteq_j CL_{ij}(F_N) \cap_i CL_{ij}(H_N)$

(x) $CL_{ij}(CL_{ij}(H_N)) = CL_{ij}(H_N)$

Proof:

(i) we have

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$$CL_{ij}(\emptyset_{N3}) = (\delta e_{ij}(\emptyset_{N3}))^{C_1} = (X_{N1})^{C_1} = \emptyset_{N3}$$

$$CL_{ij}(X_{N1}) = (\delta e_{ij}(X_{N1}))^{C_1} = (\emptyset_{N3})^{C_1} = X_{N1}$$

(ii) Since $\delta e_{ij}(H_N) \subseteq_j (H_N)^{C_1}$, then

$$((H_N)^{C_1})^{C_1} \subseteq_j (\delta e_{ij}(H_N))^{C_1} \rightarrow H_N \subseteq_1 CL_{ij}(H_N)$$

(iii) since $F_N \subseteq_j H_N$,then $\delta e_{ij}(H_N) \subseteq_j \delta e_{ij}(F_N)$

$$(\delta e_{ij}(F_N))^{C_1} \subseteq_j \delta e_{ij}(H_N)^{C_1} \rightarrow CL_{ij}(F_N) \subseteq_j CL_{ij}(H_N)$$

(iv) $CL_{ij}(F_N \cup_i H_N) = (\delta e_{ij}(F_N \cup_i H_N))^{C_1} = (\delta e_{ij}(F_N) \cap_i \delta e_{ij}(H_N))^{C_1}$

$$= (\delta e_{ij}(F_N))^{C_1} \cup_i (\delta e_{ij}(H_N))^{C_1} = CL_{ij}(F_N) \cup_i CL_{ij}(H_N)$$

(v) $F_N \cap_i H_N \subset_j F_N$ $\quad$ *by* (iii)

$$\rightarrow CL_{ij}(F_N \cup_i H_N) \subseteq_j CL_{ij}(F_N) \ldots (1)$$

and $F_N \cap_i H_N \subset_j H_N$ $\quad$ *by* (iii)

$$\rightarrow CL_{ij}(F_N \cup_i H_N) \subseteq_j CL_{ij}(H_N) \ldots (2)$$

$$Hance \quad CL_{ij}(F_N \cap_i H_N) \subseteq_j CL_{ij}(F_N) \cap_i CL_{ij}(H_N)$$

(x) $CL_{ij}\left(CL_{ij}(H_N)\right) = \left(\delta e_{ij}(\delta e_{ij}(H_N))^{C_1}\right)^{C_1}$

$$= \left(\delta i_{ij}\left(\delta i_{ij}((H_N)^{C_1})\right)^{C_1}\right)^{C_1} = (\delta i_{ij}(\delta i_{ij}(H_N)^{C_1}))^{C_1}$$

Since $(\delta i_{ij}(\delta i_{ij}(H_N)) = \delta i_{ij}((H_N))$, then

$$CL_{ij}\left(CL_{ij}(H_N)\right) = \left(\delta i_{ij}(H_N))^{C_1}\right)^{C_1} = (\delta e_{ij}(H_N))^{C_1} = CL_{ij}(H_N)$$

Preposition 2.6:

Let (X, ϑ) be a SNCT-space, and H_N be a NCS. Then

(i) H_N is a SNCO – set iff $(H_N)^{C_1} \cup_i CL_{ij}(H_N) = X_{N1}$.

(ii) H_N is a SNCC – set iff $CL_{ij}(H_N)^{C_1} \cup_i (H_N) = X_{N1}$.

Proof:

(i) H_N is a SNCO iff $\delta i_{ij}(H_N) = H_N$

$$H_N \cap_i (H_N)^{C_1} = \emptyset_{N_3} \Longrightarrow \delta i_{ij}(H_N) \cap_i \delta i_{ij}(H_N)^{C_1} = \emptyset_{N_3} \Longrightarrow H_N \cap_i \delta e_{ij}(H_N) = \emptyset_{N_3}$$

$$\Longrightarrow (H_N)^{C_1} \cup_i CL_{ij}(H_N) = X_{N1}$$

(ii) H_N is a SNCC iff $(H_N)^{C_1}$is SNCO iff $\delta i_{ij}(H_N)^{C_1} = (H_N)^{C_1}$ $\quad H_N \cap_i (H_N)^{C_1} = \emptyset_{N_3} \Longrightarrow \delta i_{ij}(H_N) \cap_i \delta i_{ij}(H_N)^{C_1} = \emptyset_{N_3}$ $\quad \delta e_{ij}(H_N)^{C_1} \cap_i (H_N)^{C_1} = \emptyset_{N_3} \Longrightarrow$ $CL_{ij}(H_N)^{C_1} \cup_i (H_N) = X_{N1}$

Preposition 2.7:

For any NCS H_N is *SNCO-set* (X, ϑ) , the following statement equivalent

(i) $CL_{ij}(\delta i_{ij}(H_N)) = CL_{ij}((H_N))$

(ii) $\delta e_{ij}(CL_{ij}(H_N)) \subseteq_j \delta e_{ij}((H_N))$

Proof:

(i) Since H_N is a *SNCO-set* we have $\delta i_{ij}(H_N) = H_N$

$$\delta e_{ij}\left(\delta i_{ij}(H_N)\right) = \delta e_{ij}(H_N) \Longrightarrow \left(\delta e_{ij}\left(\delta i_{ij}(H_N)\right)\right)^{C_1} = \left(\delta e_{ij}(H_N)\right)^{C_1}$$

$$\Longrightarrow CL_{ij}(\delta i_{ij}(H_N)) = CL_{ij}((H_N))$$

(ii) For $j = 1,2$, since $\delta e_{ij}((H_N)) \subseteq_j (H_N)^{C_1} \Longrightarrow (H_N) \subseteq_j \left(\delta e_{ij}(H_N)\right)^{C_1}$

$$\Longrightarrow (H_N) \subseteq_j CL_{ij}((H_N)) \Longrightarrow \delta e_{ij}(CL_{ij}(H_N)) \subseteq_j \delta e_{ij}((H_N)).$$

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Preposition 2.8:

Let (X, ϑ) be a SNCT-space and H_N is a NCS of any type. Then if $\delta e_{ij}(H_N)^{C_1} = \emptyset_{N_3}$ *Then* $CL_{ij}(H_N)^{C_1} = X_{N1}$

Proof:

$$\delta e_{ij}(H_N)^{C_1} = \left((\delta e_{ij}(H_N)^{C_1})^{C_1}\right)^{C_1} \Longrightarrow \left((\delta e_{ij}(H_N)^{C_1})^{C_1}\right)^{C_1} = \emptyset_{N_3}$$

$$\left(\delta e_{ij}(H_N)^{C_1}\right)^{C_1} = X_{N1} \Longrightarrow CL_{ij}(H_N)^{C_1} = X_{N1}.$$

3. Neutrosophic Crisp Topological Space (NCT-space), $T_i, i = 0, 1, 2$

In this part of the research, we will address the definition and study of the properties of separation in the general and special cases with the four points.

Definition 3.1: $T_0 -$ Neutrosophic Crisp Topological Space (NCT-space)

$$\forall P_{N_i} \neq q_{N_i}, P_{N_i}, q_{N_i} are\ PNCS, \exists\ G_N \varepsilon\, \vartheta \ni P_{N_i} \in_i G_N\ but\ q_{N_i} \notin_i G_N,\ \ i = 1,2,3,4$$

Preposition 3.2:

Every subspace (NCSS) T_0 –NCTs is a T_0 –NCTs and hence the property is hereditary.

Example 3.3:

Every discrete NCTs is a T_0 –NCTs.

Example 3.4:

Let ϑ_1 and ϑ_2 be two topologies on a non-empty set X and let $\vartheta_1 \subset \vartheta_2$ if ϑ_1 is a T_0 –NCTs, then ϑ_2 is a T_0 –NCTs

Note 3.5:

When defining T_0 at a particular point, the space does not need to be T_0 at the rest of the other types of points, as shown in the example.

Example 3.6:

Let $X = \{k, g, s\}$, $\vartheta = \{A_N, B_N, C_N, D_N, E_N, \emptyset_{N_3}, X_{N_1}\}$and let

$$A_N = \ll \{k, s\}, \emptyset, \{g\} \gg \quad B_N = \ll \{s\}, \emptyset, \{k, g\} \gg$$

$$C_N = \ll \emptyset, \emptyset, \{k, g\} \gg \quad D_N = \ll \{g\}, \emptyset, \{k\} \gg$$

$$E_N = \ll \{k\}, \emptyset, \{g\} \gg$$

Note 3.7:

There is also an important note If $P_N \in A_N$ is not necessarily $P_N \in (A_N)^C$ unless certain conditions are met, as follows:

Preposition 3.8:

A Stable neutrosophic crisp topological Space$(X, \vartheta)$with complementary properties and totalitarian property. Then(X, ϑ) is a T_0 –NCTs iff any $P_{1_{N_i}} \neq P_{2_{N_i}}$ of X.Then $CL_{ij}(P_{1_{N_i}}) \neq CL_{ij}(P_{2_{N_i}})$

Preposition 3.9:

The property of a space being a T_0 –NCTs is preserved under one-one open mapping and hance is a neutrosophic crisp topological property

Definition 3.10: T_1-NCTS

$$\forall P_{N_i} \neq q_{N_i}, P_{N_i}, q_{N_i} are\ PNCS, \exists\ G_N, H_N\, \varepsilon\, \vartheta \ni P_{N_i} \in_i G_N\ but\ q_{N_i} \notin_i G_N\ and\ P_{N_i} \notin_i H_N\ but\ q_{N_i} \in_i H_N,\ \ i = 1,2,3,4$$

Preposition 3.11:

Every subspace (NCSS) of T_1 –NCTs is T_1 –NCTs and hence the property is hereditary.

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Preposition 3.12:

The property of a space being a T_1 –NCTs is preserved under one-one open mapping and hance is a neutrosophic crisp topological property

1. Proof:

Let (X, ϑ_1) be a T_1 –NCTs and let f be a bijective mapping of (X, ϑ_1) to(Y, ϑ_2) . Let $q_{1_{N_i}} \neq q_{2_{N_i}} \in_i Y$. Since f is a bijective mapping there exists $P_{1_{N_i}} \neq P_{2_{N_i}} \in_i X$ such that

$$f\left(P_{1_{N_i}}\right) = q_{1_{N_i}} \, and \, f\left(P_{2_{N_i}}\right) = q_{2_{N_i}}.$$

Since (X, ϑ_1) is a T_1 –NCTs, then

$$\exists \; G_N, H_N \; \varepsilon \; \vartheta \ni \; P_{N_i} \in_i G_N \; but \; q_{N_i} \notin_i G_N \; and \; P_{N_i} \notin_i H_N \; but \; q_{N_i} \in_i H_N \, , i = 1,2,3,4.$$

Since f is an open mapping $f(G_N)$ and $f(H_N)$ ε ϑ such that

$q_{1_{N_i}} = f\left(P_{1_{N_i}}\right) \in_i f(G_N) \; but \; q_{2_{N_i}} = f\left(P_{2_{N_i}}\right) \notin_i f(G_N)$

and

$$q_{1_{N_i}} = f\left(P_{1_{N_i}}\right) \notin_i f(H_N) \; but \; q_{2_{N_i}} = f\left(P_{2_{N_i}}\right) \in_i f(H_N)$$

Hance (Y, ϑ_2) is a T_1 –NCTs.

Note 3.13:

When defining T_1 at a particular point, the space does not need to be T_1 at the rest of the other types of points, as shown in the example.

Example 3.14:

From Example 3.6

Definition 3.15:

T_2-NCTS or Hausdroff space iff $\forall \, P_{1_{N_i}} \neq P_{2_{N_i}} \in_i X \, , i = 1,2,3,4$

$\exists \; G_N, \; H_N \; \varepsilon \; \vartheta \; , \ni P_{1_{N_i}} \in_i G_N \; and \; P_{2_{N_i}} \in_i H_N \; and \; G_N \cap_2 H_N = \emptyset_{N_3}$

Preposition 3.16:

Every subspace (NCSS) of T_2 –NCTs is T_2 –NCTs and hence the property is hereditary.

Preposition 3.17:

Every T_2 –NCTs is T_1 –NCTs

Proof:

Let (X, ϑ_1)be T_2 –NCTs and Let $P_{1_{N_i}} \neq P_{2_{N_i}} \in_i X \, , i = 1,2,3,4$. Since the NCTS is T_2 $\exists \; G_N, \; H_N \; \varepsilon \; \vartheta \; , \ni$ $P_{1_{N_i}} \in_i G_N \; and \; P_{2_{N_i}} \in_i H_N \; and \; G_N \cap_2 H_N = \emptyset_{N_3}$

This implies that $P_{1_{N_i}} \in_i G_N \; but \; P_{2_{N_i}} \notin_i G_N$ and $P_{1_{N_i}} \notin_i H_N \; but \; P_{2_{N_i}} \in_i H_N$. Hence the NCTS is T_1

Preposition 3.18:

The property of a space being a T_2 –NCTs is preserved under one-one open mapping and hence is a neutrosophic crisp topological property

Preposition 3.19:

Let(X, ϑ_1) be a neutrosophic crisp topological space, and let (X, ϑ_2) be Hausdroff NCTS. let $f: X \to Y$ be a one–to–one continuous map. Then (X, ϑ_2) is also Hausdroff NCTS.

Example 3.20:

Let $X = \{n, m, k\}$, $\vartheta = \{A_N, B_N, C_N, X_{N_1}, \emptyset_{N_3}\}$, then

$A_N = \ll \{n\}, \{n, k\}, \{m, k\} \gg, B_N = \ll \{m\}, \{n, m\}, \{n, k\} \gg, C_N = \ll \{k\}, \{m, k\}, \{n, m\} \gg$

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4. Conclusion

This paper presents the newfound concepts of separation axioms such as T_0 –NCT, T_1 –NCT, T_2 –NCT and their properties under diverse four points in stable neutrosophic crisp topological space. Other related essential concepts are given such as stable neutrosophic crisp topological space depending on four different points and their related concepts such as the closure and its properties. The work can be extended in view of other works for example ideal grill compactness space [3], fuzzy soft [4], and weakly and genderized neutrosophic crisp [5].

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