Bipolar Interval Valued Fuzzy Subgroups

Ammar Al-Khateeb[1], Methaq A. Abdlwahid[2], Fawzi Noori Nassar[3], Faisal Al-Sharqi[3, *]

[1]Department of Studies and Planning, University of Anbar, Ramadi, Anbar, Iraq

[2]College of Applied Sciences-Hit, University of Anbar, Iraq

[3]Department of Mathematics, Faculty of Education for Pure Sciences, University of Anbar, Ramadi, Anbar, Iraq

Emails: ammar.r.a@uoanbar.edu.iq; methaq90alheety@uoanbar.edu.iq; faszinalheeti@uoanbar.edu.iq; faisal.ghazi@uoanbar.edu.iq

Abstract

Group theory is one of the significant parts of mathematical algebra. This theory is characterized by its ability to address various applications, including the classification of the symmetry of crystals, atoms, molecules, and polyhedral structures. In this work, we study a newly introduced concept, namely BIVFSs, which is an extension of previous concepts discussed in the previous studies section of this work. In this work, we establish and apply basic algebraic concepts applicable to this concept. We combine this concept with group theory, which has important properties and applications, generating important results, which are explained in the third section of this work. An important result of this work is BIVF-level set, support, BIVF-kernel and bipolar BIVF- characteristic function, and BCF point. Then, we interpret the BIVF-subgroup. Furthermore, we present the associated examples and theorems and prove these associated theorems.

Keywords: Fuzzy set; Bipolar fuzzy set; Interval valued fuzzy set; Fuzzy group theory; Interval valued group theory

1. Introduction

Group theory is one of the significant parts of mathematical algebra. This theory is characterized by its ability to address various applications, including the classification of the symmetry of crystals, atoms, molecules, and polyhedral structures. To resolve the ambiguity and uncertainty that plagues our world today, Zadeh [1] set out to present fuzzy sets as an extension of the fragile set. The fuzzy set was then extended into an interval-valued fuzzy set [2] to give additional flexibility in dealing with uncertainty issues. Again, to resolve the ambiguity and uncertainty that plagues our world today, Rosenfeld [3] set out to present fuzzy group theory. Bhattacharya and Mukherjee [4] explored many concepts over fuzzy group ideas like fuzzy-relations, fuzzy level-subgrops, fuzzy-functions, and fuzzy-equivalnce relations. Jin-Xuan [5] interpreted fuzzy homomorphism and fuzzy isomorphism.

However, human thinking has two aspects: positive pole and negative pole of human beings. To follow this aspect Zhang [6] modified the FS to the conception of bipolar fuzzy sets (BFSs) by converting [0, 1] to ([0, 1], [1, 0]). The bipolar fuzzy set (BF) structure contains degrees of support for both positive and negative aspects of human opinion or expression. The BF valued set presented by Lee [7]. BF-valued subgroups interpreted by Mahmood and Munir [8]. The structure of BF subrings by Alolaivan et al. [9]. Recently, Hammad et al. [10] introduced the bipolar fuzzy set with interval values. Because of the high flexibility of this concept, Hammad et al. [10] presented a wide range of applications for it. However, he did not address the algebraic issues and the mechanism for linking this concept with group theory, which we mentioned and its importance at the outset. This gave us an incentive to study this concept in this work, clarify its link with group theory, and present some important results.

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2. Basic Concepts

Definition 2.1. [1]A fuzzy set (FS) A on Ω given as a following form:

$$A_{FS} = \{(\dot{\rho}, \Phi(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$$

Where $\Phi(\dot{\rho})$ stretch as a membrship function from Ω to [0,1].

Definition 2.2 [1]**:** Let $\mathrm{A}_{1,FS} = \{(\dot{\rho}, \Phi_1(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$ and $\mathrm{A}_{2,FS} = \{(\dot{\tau}, \Phi_2(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$ be two FSs on Ω, Then the following properties are done,

i. Complement:

The $(\mathrm{A}_{1,FS})^C = \{(\dot{\rho}, \Phi_1^C(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$=$\{(\dot{\rho}, 1 - \Phi_1(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$

ii. Union:

The $\mathrm{A}_{1,FS} \cup \mathrm{A}_{2,FS} = \{max[\Phi_1(\dot{\rho}), \Phi_2(\dot{\rho})],, \forall \dot{\rho} \in \Omega\}$.

iii. Intersection:

The $\mathrm{A}_{1,FS} \cap \mathrm{A}_{2,FS} = \{min[\Phi_1(\dot{\rho}), \Phi_2(\dot{\rho})],, \forall \dot{\rho} \in \Omega\}$.

iv. Subset:

The $\mathrm{A}_{1,FS} \subseteq \mathrm{A}_{2,FS}$ if $\Phi_1(\dot{\rho}) \leq \Phi_2(\dot{\rho})$.

v. Equals:

The $\mathrm{A}_{1,FS} = \mathrm{A}_{2,FS}$ if $\Phi_1(\dot{\rho}) = \Phi_2(\dot{\rho})$.

Example 2.3 [11]: Let $\Omega = \{\dot{\rho}_1, \dot{\rho}_2, \dot{\rho}_3\}$ given as a universe set. Then the two FSs $\mathrm{A}_{1,FS}$ and $\mathrm{A}_{2,FS}$ given as following:

$$\mathrm{A}_{1,FS} = \left\{\frac{\langle 0.2\rangle}{\dot{\rho}_1}, \frac{\langle 0.3\rangle}{\dot{\rho}_2}, \frac{\langle 0.7\rangle}{\dot{\rho}_3}\right\}$$

$$\Gamma_{2,FS} = \left\{\frac{\langle 0.3\rangle}{\dot{\rho}_1}, \frac{\langle 0.4\rangle}{\dot{\rho}_2}, \frac{\langle 0.8\rangle}{\dot{\rho}_3}\right\}$$

Then, the operations specified as:

i. Complement:

$$(\mathrm{A}_{1,FS})^C = \left\{\frac{\langle 1-0.2\rangle}{\dot{\rho}_1}, \frac{\langle 1-0.3\rangle}{\dot{\rho}_2}, \frac{\langle 1-0.7\rangle}{\dot{\rho}_3}\right\}$$

$$= \left\{\frac{\langle 0{,}8\rangle}{\dot{\rho}_1}, \frac{\langle 0.7\rangle}{\dot{\rho}_2}, \frac{\langle 0.3\rangle}{\dot{\rho}_3}\right\}$$

ii. Union:

The $\mathrm{A}_{1,FS} \cup \mathrm{A}_{2,FS} =$

$$\left\{\frac{\langle 0.3\rangle}{\dot{\rho}_1}, \frac{\langle 0.4\rangle}{\dot{\rho}_2}, \frac{\langle 0.8\rangle}{\dot{\rho}_3}\right\}$$

vi. Intersection:

The $\mathrm{A}_{1,FS} \cap \mathrm{A}_{2,FS} =$

$$\left\{\frac{\langle 0.2\rangle}{\dot{\rho}_1}, \frac{\langle 0.3\rangle}{\dot{\rho}_2}, \frac{\langle 0.7\rangle}{\dot{\rho}_3}\right\}$$

vii. Subset:

The $\mathrm{A}_{1,FS} \subseteq \mathrm{A}_{2,FS}$ since $\Phi_1(\dot{\rho}) \leq \Phi_2(\dot{\rho})$ in both sets.

Definition 2.4. [2] A interval valued fuzzy set (FS) B on Ω given as a following form:

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$$B_{IVFS} = \{(\dot{\rho}, \Phi(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$$

Where $\Phi(\dot{\rho}) = [\Phi^L(\dot{\rho}), \Phi^U(\dot{\rho})]$ stretch as a lower-and upper membership function from Ω to $[0,1]$.

Definition 2.5 [2] Let $B_{1,IVFS} = \{(\dot{\rho}, \Phi_1(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$ where $\Phi_1(\dot{\rho}) = [\Phi_1^L(\dot{\rho}), \Phi_1^U(\dot{\rho})]$and $B_{2,IVFS} = \{(\dot{\rho}, \Phi_2(\dot{\rho})), \forall \dot{\rho} \in \Omega\}$ where $\Phi_2(\dot{\rho}) = [\Phi_2^L(\dot{\rho}), \Phi_2^U(\dot{\rho})]$,

Then the following properties are done,

viii. Complement:

The $(A_{1,FS})^C = \{(\dot{\rho}, \Phi_1^C(\dot{\rho})), \forall \dot{\rho} \in X\}$

$$= \{(\dot{\rho}, [1 - \Phi_1^U(\dot{\rho}), 1 - \Phi(\dot{\rho})]), \forall \dot{\rho} \in \Omega\}$$

ix. Union:

The $A_{1,FS} \cup A_{2,FS} = \{max[\Phi_1(\dot{\rho}), \Phi_2(\dot{\rho})],, \forall \dot{\rho} \in \Omega\}$.

$= \{max[\Phi_1^L(\dot{\rho}), \Phi_2^L(\dot{\rho})], max[\Phi_1^U(\dot{\rho}), \Phi_2^U(\dot{\rho})], \forall \dot{\rho} \in \Omega\}$.

x. Intersection:

The $A_{1,FS} \cap A_{2,FS} = \{min[\Phi_1(\dot{\rho}), \Phi_2(\dot{\rho})],, \forall \dot{\rho} \in \Omega\}$.

$= \{min[\Phi_1^L(\dot{\rho}), \Phi_2^L(\dot{\rho})], min[\Phi_1^U(\dot{\rho}), \Phi_2^U(\dot{\rho})], \forall \dot{\rho} \in \Omega\}$.

xi. Subset:

The $A_{1,FS} \subseteq A_{2,FS}$ if $\Phi_1(\dot{\tau}) \leq \Phi_2(\dot{\tau})$ such that

$\Phi_1^L(\dot{\tau}) \leq \Phi_2^L(\dot{\tau})$ and $\Phi_1^U(\dot{\tau}) \leq \Phi_2^U(\dot{\tau})$

xii. Equals:

The $A_{1,FS} = A_{2,FS}$ if $\Phi_1(\dot{\tau}) = \Phi_2(\dot{\tau})$.

such that

$\Phi_1^L(\dot{\tau}) = \Phi_2^L(\dot{\tau})$ and $\Phi_1^U(\dot{\tau}) = \Phi_2^U(\dot{\tau})$.

Example 2.7 [2]: Let $\Omega = \{\dot{\rho}_1, \dot{\rho}_2, \dot{\rho}_3\}$ given as a universe set. Then the two IVFSs $B_{1,FS}$ and $B_{2,FS}$ given as following:

$$B_{1,FS} = \left\{\frac{\langle[0.2,0,4]\rangle}{\dot{\rho}_1}, \frac{\langle[0.4,0,4]\rangle}{\dot{\rho}_2}, \frac{\langle[0.3,0,7]\rangle}{\dot{\rho}_3}\right\}$$

$$B_{2,FS} = \left\{\frac{\langle[0.2,0,4]\rangle}{\dot{\rho}_1}, \frac{\langle[0.5,0,6]\rangle}{\dot{\rho}_2}, \frac{\langle[0.7,0,8]\rangle}{\dot{\rho}_3}\right\}$$

Then the above operations given as following:

i. Complement:

$$(B_{1,FS})^C = \left\{\frac{\langle[0.2,0,4]\rangle}{\dot{\rho}_1}, \frac{\langle[0.4,0,4]\rangle}{\dot{\rho}_2}, \frac{\langle[0.3,0,7]\rangle}{\dot{\rho}_3}\right\}$$

$$= \left\{\frac{\langle[0.6,0,8]\rangle}{\dot{\rho}_1}, \frac{\langle[0.6,0,6]\rangle}{\dot{\rho}_2}, \frac{\langle[0.3,0,7]\rangle}{\dot{\rho}_3}\right\}$$

ii. Union:

The $B_{1,FS} \cup B_{2,FS} =$

$$\frac{\langle[0.2,0,4]\rangle}{\dot{\rho}_1}, \frac{\langle[0.5,0,6]\rangle}{\dot{\rho}_2}, \frac{\langle[0.7,0,8]\rangle}{\dot{\rho}_3}\}$$

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iii. Intersection:

The $B_{1,FS} \cap B_{2,FS} =$

$$\left\{\frac{\langle[0.2,0,4]\rangle}{\dot{\rho}_1}, \frac{\langle[0.4,0,4]\rangle}{\dot{\rho}_2}, \frac{\langle[0.3,0,7]\rangle}{\dot{\rho}_3}\right\}$$

iv. Subset:

The $B_{1,FS} \subseteq B_{2,FS}$ since $\Phi_1(\dot{\tau}) \leq \Phi_2(\dot{\tau})$ in both sets.

Definition 2.8. A bipolar fuzzy set (BFS) A on Ω given as a following form:

$$A_{BFS} = \left\{\left(\dot{\rho}, \Phi^+(\dot{\rho}), \Phi^-(\dot{\rho})\right), \forall \dot{\rho} \in \Omega\right\}$$

Where $\Phi^+(\dot{\rho}), \Phi^-(\dot{\rho})$ stretch as a positive membership and negative membership functions from Ω to $[0,1]$ and $[-1,0]$ respectivily.

Definition 2.9. [10] An interval valued-bipolar fuzzy set (IV-BFS) A on Ω given as a following form:

$$A_{BFS} = \left\{\left(\dot{\rho}, \Phi^+(\dot{\rho}), \Phi^-(\dot{\rho})\right), \forall \dot{\rho} \in \Omega\right\}$$

Where $\Phi^+(\dot{\rho}) = [\Phi^{+,L}(\dot{\rho}), \Phi^{+,U}(\dot{\rho})], \Phi^-(\dot{\rho}) = [\Phi^{-,L}(\dot{\rho}), \Phi^{-,U}(\dot{\rho})]$ stretch as a lower and upper positive membership and lower and upper positive negative membership functions from Ω to $[0,1]$ and $[-1,0]$ respectively.

Definition 2.10 [12] Let $A_{FS} = \left\{\left(\dot{\rho}, \Phi(\dot{\rho})\right), \forall \dot{\rho} \in \Omega\right\}$ be a fuzzy sub set of non – empty universe set Ω. For $\alpha \in [0,1]$, the α −Lower level subsets of A_{FS} is the set $(A_{FS})_\alpha$ and given as following:

$(A_{FS})_\alpha = \{(\dot{\rho}, \mathfrak{B}(\dot{\rho}) \geq \alpha)\}$.

Definition 2.11 [13]Let $A_{FS} = \left\{\left(\dot{\rho}, \Phi(\dot{\rho})\right), \forall \dot{\rho} \in \Omega\right\}$ be a fuzzy sub set of non – empty universe set Ω. For $\alpha \in [0,1]$, the level subsets of A_{FS} is the set $(A_{FS})_\alpha$ and given as following:

$(A_{FS})_\alpha = \{(\dot{\rho}, \Phi(\dot{\rho}) \leq \alpha)\}$.

Definition 2.12. [3] Assume that $(G,*)$ is a group. A F-subset A_{FS} is denotes as a fuzzy subgroup (FSG) of G. If the following point satisfied. For all $\dot{\rho}_1$ and $\dot{\rho}_2 \in G$:

i. $\Phi\left(\dot{\rho}_1\dot{\rho}_2\right) \geq min\left\{\Phi\left(\dot{\rho}_1\right), \Phi\left(\dot{\rho}_2\right)\right\}$.

ii. $\Phi(\dot{\rho}^{-1}) = \Phi(\dot{\rho})$.

Example 2.13 [14]Let G be the Klein 4 −Group such that $G = \{e, a, b, ab\}$ and $a^2 = b^2 = e$ with $ab = ba$.

Here the fuzzy sub set of G Γ_{FS} given as following:

$\Phi_{FS}(e) = 0.8, \Phi_{FS}(a) = 0.4, \Phi_{FS}(b) = 0.4$ and $\Phi_{FS}(ab) = 0.6$.

Then, here based on **Definition 2.12** its clearly Γ_{FS} is fuzzy sub group of G.

Definition 2.14. [15] Assume that $(G,*)$ is a group. A F-subset A_{FS} is denotes as anti-fuzzy subgroup (FSG) of G. If the following point satisfied. For all $\dot{\tau}_1$ and $\dot{\tau}_2 \in G$:

i. $\Phi\left(\dot{\rho}_1\dot{\rho}_2\right) \leq max\left\{\Phi\left(\dot{\rho}_1\right), \Phi\left(\dot{\rho}_2\right)\right\}$.

ii. $\Phi(\dot{\rho}^{-1}) = \Phi(\dot{\rho})$.

Definition 2.15. [16] Assume that $(G,*)$ is a group. A BF-set B_{BFS} is denotes as a bipolar fuzzy subgroup (BFSG) of G. If the following point satisfied. For all $\dot{\rho}_1$ and $\dot{\rho}_2 \in G$:

i. $\Phi^+_{BFS}\left(\dot{\rho}_1\dot{\rho}_2\right) \geq min\left\{\Phi^+_{BFS}\left(\dot{\rho}_1\right), \Phi^+_{BFS}(\dot{\rho}_2)\right\}$.

ii. $\Phi^+_{BFS}(\dot{\rho}^{-1}) = \Phi^+_{BFS}(\dot{\rho})$.

iii. $\Phi^-_{BFS}\left(\dot{\rho}_1\dot{\rho}_2\right) \leq max\left\{\Phi^-_{BFS}\left(\dot{\rho}_1\right), \Phi^-_{BFS}(\dot{\rho}_2)\right\}$.

iv. $\Phi^-_{BFS}(\dot{\rho}^{-1}) = \Phi^-_{BFS}(\dot{\rho})$.

Definition 2.16. [17] Assume that $(G,*)$ is a group. A anti-BF-set B_{BFS} is denotes as a bipolar fuzzy subgroup (BFSG) of G. If the following point satisfied. For all $\dot{\rho}_1$ and $\dot{\rho}_2 \in G$:

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i. $\Phi^{+}_{BFS}\left(\dot{\rho}_1\dot{\rho}_2\right) \leq min\left\{\Phi^{+}_{BFS}\left(\dot{\rho}_1\right), \Phi^{+}_{BFS}(\dot{\rho}_2)\right\}$.

ii. $\Phi^{+}_{BFS}(\dot{\rho}^{-1}) = \Phi^{+}_{BFS}(\dot{\rho})$.

iii. $\Phi^{-}_{BFS}\left(\dot{\rho}_1\dot{\rho}_2\right) \geq max\left\{\Phi^{-}_{BFS}\left(\dot{\rho}_1\right), \Phi^{-}_{BFS}(\dot{\rho}_2)\right\}$.

iv. $\Phi^{-}_{BFS}(\dot{\rho}^{-1}) = \Phi^{-}_{BFS}(\dot{\rho})$.

3. Bipolar interval valued fuzzy subgroups

In this part, we introduce the concept of bipolar interval-valued fuzzy subgroups and interval-valued bipolar fuzzy normal subgroups and enquire into some related results.

Definition 3.1 Assume that a bipolar interval valued fuzzy set (BIVFS) B_{IVFS} on G given as a following form:

$$B_{BIVFS} = \{(\dot{\rho}, \Phi^{+}(\dot{\rho}), \Phi^{-}(\dot{\rho})), \forall \dot{\rho} \in \mathrm{G}\}$$

Where $\Phi^{+}(\dot{\rho}) = [\Phi^{L,+}(\dot{\rho}), \Phi^{U,+}(\dot{\rho})], \Phi^{-}(\dot{\rho}) = [\Phi^{L,-}(\dot{\rho}), \Phi^{U,-}(\dot{\rho})]$ Then, $(B_{IVFS})_\eta = \{\dot{\rho} \in \mathrm{G}, \Phi^{L,+}(\dot{\rho}) \geq \eta, \Phi^{U,+}(\dot{\rho}) \geq \eta, \eta \in [0,1]\}$ and $\{\dot{\rho} \in \mathrm{G}, \Phi^{L,-}(\dot{\rho}) \leq \eta, \Phi^{U,-}(\dot{\rho}) \leq \eta, \eta \in [-1,0]\}$ is called level BIVFS.

Definition 3.2 Assume that a bipolar interval valued fuzzy set (BIVFS) B_{IVFS} on G given as a following form:

$$B_{BIVFS} = \{(\dot{\rho}, \Phi^{+}(\dot{\rho}), \Phi^{-}(\dot{\rho})), \forall \dot{\rho} \in \mathrm{G}\}$$

Where $\Phi^{+}(\dot{\rho}) = [\Phi^{L,+}(\dot{\rho}), \Phi^{U,+}(\dot{\rho})], \Phi^{-}(\dot{\rho}) = [\Phi^{L,-}(\dot{\rho}), \Phi^{U,-}(\dot{\rho})]$ Then, $(B_{IVFS})_0 = \{\dot{\rho} \in \mathrm{G}, \Phi^{L,+}(\dot{\rho}) \geq 0, \Phi^{U,+}(\dot{\rho}) \geq 0, 0 \in [0,1]\}$ and $\{\dot{\rho} \in \mathrm{G}, \Phi^{L,-}(\dot{\rho}) \leq 0, \Phi^{U,-}(\dot{\rho}) \leq 0, 0 \in [-1,0]\}$ is called support BIVFS.

Definition 3.3 Assume that a bipolar interval valued fuzzy set (BIVFS) B_{IVFS} on G given as a following form:

$$B_{BIVFS} = \{(\dot{\rho}, \Phi^{+}(\dot{\rho}), \Phi^{-}(\dot{\rho})), \forall \dot{\rho} \in \mathrm{G}\}$$

Where $\Phi^{+}(\dot{\rho}) = [\Phi^{L,+}(\dot{\rho}), \Phi^{U,+}(\dot{\rho})], \Phi^{-}(\dot{\rho}) = [\Phi^{L,-}(\dot{\rho}), \Phi^{U,-}(\dot{\rho})]$ Then, $(B_{IVFS})_1 = \{\dot{\rho} \in \mathrm{G}, \Phi^{L,+}(\dot{\rho}) \geq 1, \Phi^{U,+}(\dot{\rho}) \geq 1, 1 \in [0,1]\}$ and $\{\dot{\rho} \in \mathrm{G}, \Phi^{L,-}(\dot{\rho}) \leq -1, \Phi^{U,-}(\dot{\rho}) \leq -1, -1 \in [-1,0]\}$ is called kernel BIVFS.

Definition 3.4 Assume that H be subset of G, then the Bipolar characteristic IVFS (BCIVFS) of $\Phi^{+}(\dot{\rho}) = [\Phi^{L,+}(\dot{\rho}), \Phi^{U,+}(\dot{\rho})], \Phi^{-}(\dot{\rho}) = [\Phi^{L,-}(\dot{\rho}), \Phi^{U,-}(\dot{\rho})]$ given as following:

$$\Phi^{+}(\dot{\rho}) = \begin{cases} \Phi^{L,+}(\dot{\rho}) = 1 \; if \; \dot{\rho} \in H, \\ \Phi^{L,+}(\dot{\rho}) = 0 \; if \; \dot{\rho} \notin H, \\ \Phi^{U,+}(\dot{\rho}) = 1 \; if \; \dot{\rho} \in H\,, \\ \Phi^{U,+}(\dot{\rho}) = 0 \; if \; \dot{\rho} \notin H, \end{cases}$$

And,

$$\Phi^{-}(\dot{\rho}) = \begin{cases} \Phi^{L,-}(\dot{\rho}) = -1 \; if \; \dot{\rho} \in H, \\ \Phi^{L,-}(\dot{\rho}) = 0 \; if \; \dot{\rho} \notin H, \\ \Phi^{U,-}(\dot{\rho}) = -1 \; if \; \dot{\rho} \in H\,, \\ \Phi^{U,-}(\dot{\rho}) = 0 \; if \; \dot{\rho} \notin H, \end{cases}$$

Definition 3.5 Assume that H be subset of G and $\dot{\rho}_1$ and $\dot{\rho}_2 \in G$, $\eta \in [1,-1]$ and BIVFS $\Phi^{+}(\dot{\rho}) = [\Phi^{L,+}(\dot{\rho}), \Phi^{U,+}(\dot{\rho})], \Phi^{-}(\dot{\rho}) = [\Phi^{L,-}(\dot{\rho}), \Phi^{U,-}(\dot{\rho})]$ given as following:

$$\Phi^{+}(\dot{\rho}) = \begin{cases} \Phi^{L,+}(\dot{\rho}) = \eta \; if \; \dot{\rho}_1 = \dot{\rho}_2, \\ \Phi^{L,+}(\dot{\rho}) = 0 \; if \; \dot{\rho}_1 \neq \dot{\rho}_2, \\ \Phi^{U,+}(\dot{\rho}) = \eta \; if \; \dot{\rho}_1 = \dot{\rho}_2\,, \\ \Phi^{U,+}(\dot{\rho}) = 0 \; if \; \dot{\rho}_1 \neq \dot{\rho}_2, \end{cases}$$

And,

$$\Phi^{-}(\dot{\rho}) = \begin{cases} \Phi^{L,-}(\dot{\rho}) = -\eta \; if \; \dot{\rho}_1 = \dot{\rho}_2, \\ \Phi^{L,-}(\dot{\rho}) = 0 \; if \; \dot{\rho}_1 \neq \dot{\rho}_2, \\ \Phi^{U,-}(\dot{\rho}) = -\eta \; if \; \dot{\rho}_1 = \dot{\rho}_2\,, \\ \Phi^{U,-}(\dot{\rho}) = 0 \; if \; \dot{\rho}_1 \neq \dot{\rho}_2, \end{cases}$$

Is called a BIVF point, η and support of of $\dot{\rho}_1$ and $\dot{\rho}_2$.

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Definition 3.6 Assume that $(G,*)$ is a group. A BIVF-set B_{BIVFS} is denotes as a bipolar interval valued fuzzy subgroup (BIVFSG) of G. If the following point satisfied. For all $\dot{\rho}_1$ and $\dot{\rho}_2 \in G$:

i. $\Phi^{L,+}_{BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{L,+}_{BIVFS}(\dot{\tau}_1), \Phi^{L,+}_{BIVFS}(\dot{\tau}_2)\}$ and $\Phi^{U,+}_{BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{U,+}_{BIVFS}(\dot{\tau}_1), \Phi^{U,+}_{BIVFS}(\dot{\tau}_2)\}$.
ii. $\Phi^{L,+}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{L,+}_{BIVFS}(\dot{\tau})$ and $\Phi^{U,+}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{U,+}_{BIVFS}(\dot{\tau})$.
iii. $\Phi^{L,-}_{BIVFS}(\tau_1\dot{\tau}_2) \leq max\{\Phi^{L,-}_{BIVFS}(\dot{\tau}_1), \Phi^{L,-}_{BIVFS}(\dot{\tau}_2)\}$ and $\Phi^{U,-}_{BIVFS}(\tau_1\dot{\tau}_2) \leq max\{\Phi^{U,-}_{BIVFS}(\dot{\tau}_1), \Phi^{U,-}_{BIVFS}(\dot{\tau}_2)\}$.
iv. $\Phi^{L,-}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{L,-}_{BIVFS}(\dot{\tau})$ and $\Phi^{U,-}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{U,-}_{BIVFS}(\dot{\tau})$.

Example 3.7. Let G be the Klein 4 −Group such that $G = \{e, a, b, ab\}$ and $a^2 = b^2 = e$ with $ab = ba$.

Here the fuzzy sub set of G Γ_{BIVFS} given as following:

$\Phi^{L,+}_{BIVFS}(e) = 0.8$, $\Phi^{U,+}_{BIVFS}(e)$=0,9

$\Phi^{L,+}_{BIVFS}(a) = 0.4$, $\Phi^{U,+}_{BIVFS}(a) = 0.6$,

$\Phi^{L,+}_{BIVFS}(b) = 0.4$ $\Phi^{L,+}_{BIVFS}(b) = 0.6$

and $\Phi^{L,+}_{BIVFS}(ab) = 0.6$, $\Phi^{U,+}_{BIVFS}(ab) = 0.6$.

Then, here based on **Definition 3.6** it is clearly Γ_{BIVFS} is fuzzy sub group of G.

Theorem 3.8 Let B_{BIVFS} is fuzzy sub group of G if and only if $(B_{BIVFS})^C$ is a fuzzy subgroup of G.
Proof.
Assume that B_{BIVFS} is interval-valued fuzzy sub group of G and for all $\dot{\rho}_1$ and $\dot{\rho}_2 \in G$.Then

$\Phi^{L,+}_{BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{L,+}_{BIVFS}(\dot{\tau}_1), \Phi^{L,+}_{BIVFS}(\dot{\tau}_2)\}$ and $\Phi^{U,+}_{BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{U,+}_{BIVFS}(\dot{\tau}_1), \Phi^{U,+}_{BIVFS}(\dot{\tau}_2)\}$.

$\Leftrightarrow \mathbf{1} - \left(\Phi^{L,+}_{BIVFS}\right)^c(\tau_1\dot{\tau}_2) \geq min\left\{\left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}_1), \left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}_2)\right\}$

and $\mathbf{1} - \left(\Phi^{U,+}_{BIVFS}\right)^c(\tau_1\dot{\tau}_2) \geq min\left\{\mathbf{1} - \left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}_1), \mathbf{1} - \left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}_2)\right\}$.

$\Leftrightarrow \left(\Phi^{L,+}_{BIVFS}\right)^c(\tau_1\dot{\tau}_2) \leq 1 - min\left\{1 - \left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}_1), 1 - \left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}_2)\right\}$

and $\left(\Phi^{U,+}_{BIVFS}\right)^c(\tau_1\dot{\tau}_2) \leq 1 - min\left\{1 - \left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}_1), 1 - \left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}_2)\right\}$.

$\Leftrightarrow \left(\Phi^{L,+}_{BIVFS}\right)^c(\tau_1\dot{\tau}_2) \leq max\left\{\left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}_1), \left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}_2)\right\}$

and $\left(\Phi^{U,+}_{BIVFS}\right)^c(\tau_1\dot{\tau}_2) \leq max\left\{\left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}_1), \left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}_2)\right\}$

A gain we have

$\Phi^{L,+}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{L,+}_{BIVFS}(\dot{\tau})$ and $\Phi^{U,+}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{U,+}_{BIVFS}(\dot{\tau})$ for all $\tau_{1,}\dot{\tau}_2 \in G$.
Then
$\Leftrightarrow \mathbf{1} - \left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau}^{-1}) = \mathbf{1} - \left(\Phi^{L,+}_{BIVFS}\right)^c(\dot{\tau})$ and $\left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau}^{-1}) = \left(\Phi^{U,+}_{BIVFS}\right)^c(\dot{\tau})$.

Definition 3.9. Assume that $(G,*)$ is a group. A anti-BIVF-set $B_{a-BIVFS}$ is denotes as a Anti-bipolar interval valued fuzzy subgroup (A-BIVFSG) of G. If the following point satisfied. For all $\dot{\tau}_1$ and $\dot{\tau}_2 \in G$:

i. $\Phi^{L,+}_{BIVFS}(\tau_1\dot{\tau}_2) \leq max\{\Phi^{L,+}_{a-BIVFS}(\dot{\tau}_1), \Phi^{L,+}_{BIVFS}(\dot{\tau}_2)\}$
and $\Phi^{U,+}_{a-BIVFS}(\tau_1\dot{\tau}_2) \leq max\{\Phi^{U,+}_{BIVFS}(\dot{\tau}_1), \Phi^{U,+}_{BIVFS}(\dot{\tau}_2)\}$.
ii. $\Phi^{L,+}_{a-BIVFS}(\dot{\tau}^{-1}) = \Phi^{L,+}_{a-BIVFS}(\dot{\tau})$ and $\Phi^{U,+}_{a-BIVFS}(\dot{\tau}^{-1}) = \Phi^{U,+}_{a-BIVFS}(\dot{\tau})$.
iii. $\Phi^{L,-}_{a-BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{L,-}_{a-BIVFS}(\dot{\tau}_1), \Phi^{L,-}_{a-BIVFS}(\dot{\tau}_2)\}$
and $\Phi^{U,-}_{a-BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{U,-}_{a-BIVFS}(\dot{\tau}_1), \Phi^{U,-}_{a-BIVFS}(\dot{\tau}_2)\}$.
iv. $\Phi^{L,-}_{a-BIVFS}(\dot{\tau}^{-1}) = \Phi^{L,-}_{a-BIVFS}(\dot{\tau})$ and $\Phi^{U,-}_{a-BIVFS}(\dot{\tau}^{-1}) = \Phi^{U,-}_{a-BIVFS}(\dot{\tau})$.

Theorem 3.10 Let B_{BIVFS} is fuzzy sub group of G if and only if $(B_{BIVFS})^C$ is a fuzzy subgroup of G.

Proof.
Assume that $B_{a-BIVFS}$ is an anti-BIVF sub group of G and for all $\tau_{1,}\dot{\tau}_2 \in G$. Then

$\Phi^{L,+}_{BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{L,+}_{BIVFS}(\dot{\tau}_1), \Phi^{L,+}_{BIVFS}(\dot{\tau}_2)\}$ and $\Phi^{U,+}_{BIVFS}(\tau_1\dot{\tau}_2) \geq min\{\Phi^{U,+}_{BIVFS}(\dot{\tau}_1), \Phi^{U,+}_{BIVFS}(\dot{\tau}_2)\}$.

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$\Leftrightarrow \mathbf{1} - \left(\Phi^{L,+}_{BIVFS}\right)^{\boldsymbol{c}}(\tau_1\dot{\tau}_2) \geq min\left\{\left(\Phi^{L,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}_1), \left(\Phi^{L,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}_2)\right\}$

and $\mathbf{1} - \left(\Phi^{U,+}_{BIVFS}\right)^{c}(\tau_1\dot{\tau}_2) \geq min\left\{\mathbf{1} - \left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}_1), \mathbf{1} - \left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}_2)\right\}$.

$\Leftrightarrow \left(\Phi^{L,+}_{BIVFS}\right)^{c}(\tau_1\dot{\tau}_2) \leq 1 - min\left\{1 - \left(\Phi^{L,+}_{BIVFS}\right)^{c}(\dot{\tau}_1), 1 - \left(\Phi^{L,+}_{BIVFS}\right)^{c}(\dot{\tau}_2)\right\}$

and $\left(\Phi^{U,+}_{BIVFS}\right)^{c}(\tau_1\dot{\tau}_2) \leq 1 - min\left\{1 - \left(\Phi^{U,+}_{BIVFS}\right)^{c}(\dot{\tau}_1), 1 - \left(\Phi^{U,+}_{BIVFS}\right)^{c}(\dot{\tau}_2)\right\}$.

$\Leftrightarrow \left(\Phi^{L,+}_{BIVFS}\right)^{c}(\tau_1\dot{\tau}_2) \leq max\left\{\left(\Phi^{L,+}_{BIVFS}\right)^{c}(\dot{\tau}_1), \left(\Phi^{L,+}_{BIVFS}\right)^{c}(\dot{\tau}_2)\right\}$

and $\left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\tau_1\dot{\tau}_2) \leq max\left\{\left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}_1), \left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}_2)\right\}$

A gain we have

$\Phi^{L,+}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{L,+}_{BIVFS}(\dot{\tau})$ and $\Phi^{U,+}_{BIVFS}(\dot{\tau}^{-1}) = \Phi^{U,+}_{BIVFS}(\dot{\tau})$ for all $\tau_{1,}\dot{\tau}_2 \in G$.
Then
$\Leftrightarrow \mathbf{1} - \left(\Phi^{L,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}^{-1}) = \mathbf{1} - \left(\Phi^{L,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau})$ and $\left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau}^{-1}) = \left(\Phi^{U,+}_{BIVFS}\right)^{\boldsymbol{c}}(\dot{\tau})$

Theorem 3.11. Let $H \neq \emptyset$ sub set of group G is called a subgroup of G iff its BCIVF is a BIVFsubgroup of group G.

Proof. Direct based on above definitions.

5. Conclusion

In this work, we studied a newly introduced concept, namely BIVFSs, which is an extension of previous concepts discussed in the previous studies section of this work. In this work, we establish and apply basic algebraic concepts applicable to this concept. We combined this concept with group theory, which has important properties and applications, generating important results, which are explained in the third section of this work. An important result of this work is BIVF-level set, support, BIVF-kernel and bipolar BIVF- characteristic function, and BCF point. Then, we interpret the BIVF-subgroup. Furthermore, we present the associated examples and theorems and prove these associated theorems. Through this study, it is possible to predict more future work that can be done and benefited by those interested in this field, as these tools can be linked with more algebraic concepts and discover the theory of rings based on BIVFSs. In topology, more results can also be obtained regarding topological structures like BIVF-Topology and more benefit can be gained. We recommend reviewing the following research works [15-19].

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