Neutrosophic of γ-BCK -Algebra

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Abstract

The most important applications of an algebra like BCK-Algebra. As a generalization of ring, we study γ-semi-ring and γ-ring in invarianent neutrosophic set. Neutrosophic concepts are widely used in the field of mathematics and other sciences, especially in studying the Algebra. In this paper, we present the concept of neutrosophic γ-BCK-Algebras as an example of this generalization. We also present neutrosophic sub-algebra, neutrosophic ideal and some other type structure algebraic. We proved that if $f : AI \to NI$ is a homomorphism of neutrosophic γ-BCK-algebras AI and NI, then f is injective if and only if neutrosophic $\ker(f) = \{0I\}$. Also, we presented, if NI be a normal neutrosophic subalgebra of neutrosophic γ-BCK-algebra AI, then " $\sim NI$ " is a congruence relation.

Kewords: BCK -Algebra; Semi-ring; Neutrosophic logic; Neutrosophic Set; Simple submodule

1 introduction

The concept of Neutrosophic logic in,[1] an original extended of fuzzy logic can find it in,[2] provides a more basic for uncertainty event, indeterminacy, and lose some data. Neutrosophic algebra present as an important study of this neutrosophic algebra,[3,4] where they standard algebra to consists neutrosophic theory.[5,6] Recent papers[7,13] have studied advances in our study of neutrosophic γ-BCK-algebra, showing novel properties and classical algebraic structures. These studies not only just theoretical, but also product new concepts for some results in more of fields, including algebra, concepts analysis, and coding algebra. By investigating these novel findings, we get more details about neutrosophic algebra. A refined neutrosophic structure[8] was presented by dividing the indeterminacy into two parts of sub-indeterminacies. nrefined neutrosophic structures was introduced as a generalization of refined neutrosophic,[9] and some properties of modules and fuzzy ideal by.[10] In[14] simple submodules and C-rings. Bipolar fuzzy hyper soft set and its application in decision making and Mapping on interval complex neutrosophic soft sets in.[14] Also, in[16] some properties of pythagorean neutrosophic Set. In this paper, we give some new results on Neutrosophic γ-BCK -Algebra, where we present new relations of neutrosophic algebra.

2 Preliminaries with some Tools.

In this section we recall some concepts and new results which are necessary for the paper. Let us define neutrosophic set as the following:

Definition 2.1. [11] Fuzzy Set means: $U_{FS} = \{a\,(T_x(x)),\ a \in U\}$, where $T_x : U \rightarrow PO[0,\ 1]$) is the membership degree of a , and $PO([0, 1])$ is the powerset of $[0, 1]$.

Definition 2.2. [12] Neutrosophic group means: (G, *) be any group and $\langle G \cup I \rangle = \{a + bI : a, b \in G\}. NE(G) = (< G \cup I \rangle, *)$ generated by G .

Definition 2.3. Let R be a ring. Then we define neutrosophic ring by: $(R, +, .) isany$ ring, $< R \cup I >= \{x + yI : x, y \in R\}$ is said to be neutrosophic ring.

Definition 2.4. [7] (AI, *,0I) is said to be neutrosophic BCK-algebra if:

1. $[(aIbI)(aIcI)](cIbI) = 0I,$
2. $(aI(aI * bI))bI = 0I,$
3. $aI * aIaI = 0I,$
4. $0I * aI == 0I,$
5. $aIbI = bIaL = oI \rightarrow aI = bI\ \ \forall\ \ aI, bI, cI \in AI.$

Note: We explain neutrosophic partial ordering $\leq$ on AI by aI $\leq bI \leftrightarrow aI * bI = 0I.$

Remark 2.5. The following statements are true in $(AI, *, 0I)$

1. $(aI^*bI) * (bI^*CI) \leq cI^*bI,$
2. $aI^*\,[aI^*\,(aI^*bI)] \leq cI * bI,$
3. $0I \leq aI,$
4. $aI * bI = 0I \leftrightarrow aI \leq bI,$
5. $aI * bI \rightarrow aI^*CI \leq bI^*cI$ and $cII{*}bI^2 \leq cI * aI,$
6. $(aI * bI) * cI = (aI * cI) * bI,$
7. $aI^*bI \leq cI \leftrightarrow aI^*cI \leq bI,$
8. $0I * (aI * bI) = (0I * aI) * (0I * bI),$
9. $(aI * bI) * aI = 0I,$
10. $(aI^*cI) * (bI * cI) \leq aI * bI\forall aI, bI, cI \in AI.$

Note: A neutrosophic BCK-algebra AI is called commutative if $bI * (bI^*aI) = aI\ (\ aI * bI\ ), \forall aI, bI \in AI.$

Example 2.6. Suppose that $A = \{0I, aI, bI$. So, * is defined by:

$*$	oI	aI	bI
oI	oI	oI	oI
aI	aI	oI	oI
bI	bI	oI	oI

Hence $(AI, *, 0I)$ is commutative.

Definition 2.7. Let $\varphi \neq I^*I$ neutrosophic subset of neutrosophic BCK-algebra AI. Then $I * I$ is said to be neutrosophic ideal of AI if

$$0I \in I^*I \ \text{ and }\ aII^*bI \in I^*I, bI \in I^*I \rightarrow aI \in I^*I$$

Definition 2.8. [7] Suppose that A_1I and A_2I be two neutrosophic BCK-algebra. Then $h : A_1I \to A_2I$ is called neutrosophic homomorphism if $h(aI * bI) = h(aI) * \ h(bI), \forall$ aI, bI $\in A_1I$.

Definition 2.9. [7] Let $\varphi \neq I \subseteq$ neutrosophic BCK-algebra AI is called neutrosophic subalgebra of AI, if $aI * bI \in I^*I, \ \forall \ aI, bI \in I^*$.

Definition 2.10. Let $\varphi \neq AI$ and $\varphi \neq \gamma I$ be two sets. Then AI is said to be a neutrosophic γ-semi-grroup if $\exists AI \times \gamma I \times AI \to AI;\ aI\alpha(bI\beta cI) = (aI\alpha bI)\beta cI, \forall aI, bI, cI \in AI, \alpha, \beta \in \gamma I$, where the image of ($aI\alpha bI$) denoted by aI $\alpha bI, aI, bI \in AI$.

Definition 2.11. Let $(AI, *)$ and $(\gamma I, +)$ be commutative neutrosophic semigroups. Then AI is called neutrosophic γ - Semi-ring if $\exists AI \times \gamma I \times AI$;

1. $aI\alpha(bI + cI) = aI\alpha bI + aI\alpha cI$,
2. $(aI + bI)\alpha cI = aI\alpha cI + bI\alpha cI$,
3. $aI(\alpha + \beta)bI = aI\alpha bI + aI\beta cI$.
4. $aI\alpha(bI\beta cI) = (aI\alpha bI)\beta cI \forall aI, bI, cI \in AI$ and $\alpha, \beta \in \gamma I$, where the image of (aI, α, bI) denoted by $aI\alpha bI \ \forall \ aI, bI, cI \in AI, \alpha \in \gamma I$.

Remark 2.12. Every neutrosophic γ-semi- ring is a neutrosophic $\gamma I = AI$

Definition 2.13. Any neutrosophic γ-Semi- ring AI has zero element if $\exists 0I \in AI;\ 0I + aI = aI = aI + 0I$ and $0I\alpha aI = 0I\alpha aI = 0I$.

On other hand, AI is called neutrosophic commutative γI - Semi- ring if aI $\alpha bI = bI\alpha aI, \ \ \forall \ \ aI, bI, \in AI, \alpha \in \gamma I$.

Definition 2.14. Suppose that AI is γI - Semi- ring. So $aI \in AI$ is called neutrosophic idempotent in AI if $\exists aI \in \gamma I; aI\alpha aI, aI$ is called α neutrosophic idempotent.

Note that $aI \in AI$ is called neutrosophic regular element of $AI, if \exists xI \in AI; aI = aI\alpha xI\beta aI$.

3 Neutrosophic γ-BCK-algebra

In this part, we present the concept neutrosophic γ-BCK-algebra with more new results.

But before that, we should introduce a definition of γ-BCK-algebra as a following:

Definition 3.1. Let AI be nutrosophic set with neutrosophic element) I and let γI be a neutrosophic set. If $\exists AI \times \gamma I \times AI \to AI$ satisfies the following:

i. $[(aI\alpha bI)\beta(aI\alpha cI)]\beta(cI\alpha bI) = oI$,

ii. $aI\alpha bI = bI\alpha aI = 0I \to aI = bI$,

iii. $aI\alpha aI = 0I$,

iv. $0I\alpha aL = 0I \forall \alpha, \beta \in \gamma I, aI, bI, cI \in AI$.

Then AI is called a neutrosophic γ - BCK- algebra.

α	0I	aI	bI	cI	dI	eI
0I	01	0I	0I	0I	0I	0I
aI	aI	0I	aI	aI	aI	aI
bI	bI	bI	0I	bI	bI	bI
cI	cI	cI	cI	0I	cI	cI
dI	dI	dI	dI	dI	0I	dI
eI	eI	eI	eI	eI	eI	oI

β	OI	aI	bI	cI	dI	eI
0I	0I	0I	0I	0I	0I	0I
aI	aI	aI	0I	bI	bI	bI
bI	bI	cI	0I	cI	cI	cI
cI	cI	dI	dI	0I	dI	dI
dI	dI	eI	eI	eI	0I	eI
eI	eI	aI	bI	aI	aI	0I

γ	OI	aI	bI	cI	dI	eI
OI	OI	0I	0I	OI	0I	0I
aI	aI	0I	cI	cI	cI	cI
bI	bI	dI	0I	dI	dI	dI
cI	cI	eI	eI	OI	eI	eI
dI	dI	aI	aI	aI	0I	aI
eI	eI	bI	bI	bI	bI	oI

σ	OI	aI	bI	cI	dI	eI
OI	OI	0I	0I	OI	OI	0 I
aI	aI	0I	0I	dI	dI	dI
bI	bI	eI	0 I	eI	eI	eI
cI	cI	aI	aI	OI	aI	aI
dI	dI	bI	bI	bI	OI	bI
eI	eI	cI	cI	cI	cI	$0I$

ψ	OI	aI	bI	cI	dI	eI
OI	OI	0I	0 I	OI	OI	0 I
aI	OI	eI	eI	eI	eI	eI
bI	bI	aI	0 I	aI	aI	aI
cI	cI	bI	bI	OI	bI	bI
dI	dI	cI	cI	cI	0I	cI
eI	eI	dI	dI	dI	dI	0 I

α	OI	aI	bI	cI
OI	OI	0I	OI	OI
aI	aI	0I	bI	bI
bI	bI	cI	OI	cI

β	OI	aI	bI	cI
0I	OI	0I	OI	0I
aI	aI	0I	aI	bI
bI	bI	bI	OI	bI

Remark 3.2. Let AI be a neutrosophic γI-BCK-algebra; $\alpha \in \gamma I$. Also $* : AI \times AI \to AI; aI * bI = aI\alpha bI \forall aI, bI \in AI$. So $(AI, *, 0I)$ is a neutrosophic BCKalgebra and it is denoted by $(AI)_\alpha$.

Example 3.3. $AI = \{0I, aI, bI, cI, dI, eI\}$ and $\gamma I = \{\alpha, \beta, \sigma, \delta, \psi\}$.
We have triple operations:

Note β that $NI = \{aI, 1I, 2I, 3I\}$ and $\gamma I = \{\alpha, \beta\}$ and we have ternary operation is defined by:

So, AI and NI are γI-BCK-algebras.

Example 3.4. Let $AI = \{0I, xI, yI\}$ and $\gamma I = \{\alpha, \beta\}$. The ternary operation is

α	OI	xI	yI
0I	0I	0I	0I
xI	xI	0I	0I
yI	yI	xI	0I

β	OI	aI	bI
0I	OI	0 I	0 OI
xI	xI	0 I	yI
yI	yI	0 I	OI

Then, AI is γI-BCK-algebras.

Example 3.5. Let us consider neutrosophic BCK-alagebra $(AI, *, 0I)$ as $\gamma I - BCK-$ algebra. If $\gamma I = \{0I\}$ and $xIOIyI$ is defined by $(xI, *, 0I) * yI,\ \forall\ xI, yI \in AI$.

If $AI = \{0I, b_1 I, b_2, b_3\}$ and

So, AI is a neutrosophic BCK-algebra. On the other hand, if $\gamma I = \{0I\}$, then

Hence, AI is a neutrosophic γI-BCK-algebra.

$*$	$0I$	b_1I	b_2I	b_3I
$0I$	$0I$	$0I$	$0I$	$0I$
b_1I	b_1I	$0I$	$0I$	$0I$
b_2I	b_2I	b_1I	$0I$	$0I$
b_3I	b_3I	b_3I	b_3I	$0I$

$0I$	$0I$	b_1I	b_2I	b_3I
$0I$	$0I$	$0I$	$0I$	$0I$
b_1I	b_1I	$0I$	$0I$	$0I$
b_2I	b_2I	b_1I	$0I$	$0I$
b_3I	b_3I	b_3I	b_3I	$0I$

Definition 3.6. Let $\varphi \neq AI \subseteq \gamma I$-BCK-algebra AI. Then, AI is calld neutrosophic subalgebra if $aI\alpha bI \in AI \forall aI, bI \in AI$.

Note that we can define normal neutrosophic subalgebra by the following:
$\varphi \neq NI \subseteq \gamma I$-BCK-algebra. AI is called normal neutrosophic subalgebra if:

$(xI\alpha aI)\alpha(yI\alpha bI) \in NI \ \ \forall xI\alpha yI, aI\alpha bI \in NI, \alpha \in \gamma I$, for example:

Let $AI = \{0I, 1I, 2I, 3I\}$ and $\gamma I = \alpha$. The triple operation is

α	0I	1I	2I	3 I
0I	0I	0I	0I	0I
1I	1I	0I	1I	1 I
2 I	2 I	2 I	0I	2 I
3 I	3 I	3 I	3 I	0 I

$I_1I = \{0I, 1I\}$ is a neutrosophic normal subalgebra.

Now we, introduce a definition of commutative neutrosophic γ I-BCK-algebra
γI-BCK-algebra AI is called commutative neutrosophic γI-BCK-algebra if:

$$yI\alpha(yI\beta xI) = xI\alpha(xI\beta yI), \forall xIyI, \in AI, \alpha, \beta \in \gamma I.$$

On the other hand, a γI-BCK-algebra AI is patially ordered by $xI \leq yI \leftrightarrow xI\alpha yI = 0I \forall\ \alpha \in \gamma I$, we called neutrosophic γI-BCK- ordering.

Example 3.7. $AI = \{0I, xI, yI, zI\}$. So, the ternary operation is

α	0I	xI	yI	zI
0I	0I	0I	0I	OI
xI	xI	0I	xI	xI
yI	yI	yI	01	yI
zI	zI	zI	zI	0I

β	0 I	xI	yI	zI
0I	0I	0I	0I	0 OI
xI	xI	0 I	xI	zI
yI	yI	xI	0 I	xI
zI	zI	yI	xI	0 I

Then the neutrosophic γI-BCK-algebra is commutative.

Example 3.8. $AI = \{0I, yI, zI\}, \gamma I = \{\alpha, \beta\}$. So ternary operation is:

Then, the neutrosophic γI-BCK-algebra AI is commutative.

In the next definition we explain neutrosophic homomorphism.

α	0I	yI	zI	β	OI	yI	zI
0I	0I	0I	0I	OI	OI	0I	OI
yI	yI	0I	0I	yI	yI	0I	xI
zI	zI	yI	0I	zI	zI	yI	OI

Definition 3.9. Suppose that AI and NI are neutrosophic $\gamma I - BCK$-algebra $f : AI \to NI$ is called neutrosophic homomorphism if:

$$f(xI\alpha yI) = f(xI)\alpha f(yI)\ \forall\ xI, yI \in AI, \alpha \in \gamma I.$$

And neutrosophic kernel of the mapping f can defined by the following:

Definition 3.10. Let AI, NI are neutrosophic γf-BCK-algebra and let $f : AI \to NI$ be nutrosophic homomorphism. So the following set : $\{xI \in AI; f(xI) = oI\}$ is called neutrosophic kernel of f and denoted by $\ker(f)$ and the set $\{f(xI); xI \in AI\}$ is called image of f and denoted by $\mathrm{Img}(f)$.

Corollary 3.11. *Let γI-BCK-algebra be a neutrosophic algebra. Then neutrosophic γI-BCK-algebra satisfies $xI\alpha(xI\beta yI)\alpha yI = 0I \forall xI, yI \in AI, \alpha, \beta \in \gamma I$.*

Proof. We have $(xI\beta yI)\alpha(xI\beta zI)\alpha(zI\beta yI) = 0I, xI, yI, zI \in AI, \alpha, \beta \in \gamma I$. Now put $yI = 0I, zI = yI$, in above equation. Thus, $xI\alpha(xI\beta yI) = \alpha yI = 0I$ □

Theorem 3.12. *Let γ I-BCK-algebra be a neutrosophic algebra. Then the following are equivalent*

1) *AI is commutative,*
2) *$xI \leq yI$, then $xI = yI\alpha(yI\beta xI), \forall xI, yI \in AI, \alpha, \beta \in \gamma I$.*

Proof. Suppose that AI is commutative. So, $xI\alpha(xI\beta yI) = yI\alpha(yI\beta xI)\forall xI, yI \in AI, \alpha, \beta \in \gamma I$. Hence xI $\alpha 0I = yI\alpha(yI\beta xI)$. Therefore, $xI = yI\alpha(yI\beta xI)$. Conversely, let $yI \leq xI$, so $Xi = yI\alpha(yI\beta xI)$. Also, $xI \leq yI$, so $xI\alpha yI = 0I$. Hence $xI = xI\alpha 0I = xI\alpha(xI\beta yI)$. Thus, $xI\alpha(xI\beta yI) = yI\alpha(yIOI\beta xI)$. □

Lemma 3.13. *Let $f : AI \to NI$ be a homomorphism of neutrosophic γ-BCK-algebra AI and NI. Then neutrosophic* $\ker(f)$ *is a subalgebra of M.*

Proof. Let $f : M \to N$ be a homomorphism of γ-BCK-algebra M and $x, y \in \ker(f)$. Then, $f(x) = f(y) = 0$ and so $f(x\alpha y) = f(x)\alpha f(y) = 0\alpha 0 = 0$. Hence xI $\alpha yI \in$ neutrosophic $\mathrm{Ker}(f)$. Therefore, neutrosophic $\mathrm{Ker}(f)$ is a neutrosophic subalgebra of AI. □

Lemma 3.14. *Let $f : AI \to NI$ be a homomorphism of neutrosophic γ-BCK-algebra AI and NI. Then*

(i) *$f(0) = 0$*
(ii) *if $xI\alpha yI = 0I$, then $f(xI)\alpha f(yI) = 0I$.*

Proof. (i) Now $f(0I) = f(0I\alpha 0I) = f(0I)\alpha f(0I) = 0I$.

(ii) Suppose xI $\alpha yI = 0I$. Then $f(xI\alpha yI) = f(0I)$ implies $f(xI)\alpha f(yI) = 0I$.

□

Theorem 3.15. *Let $f : AI \to NI$ be a homomorphism of neutrosophic γ-BCK-algebras AI and NI. Then f is injective if and only if neutrosophic* $\ker(f) = \{0I\}$.

α	0I	1I	2 I	3I
0I	0I	0I	0I	0I
1I	1I	0I	1I	1I
2I	2 I	2 I	0I	2 I
3I	3 I	3 I	3 I	0 I

β	0I	1I	2 I	3 I
0I	0I	0I	0I	0I
1I	1I	0I	2 I	2 I
2I	2 I	3 I	0I	3 I
3I	3 I	1 I	1I	0I

Proof. Suppose neutrosophic $\ker(f) = \{0I\}$ and $f(xI) = f(yI)$, for some $xI, yI \in$ AI. Then $f(xI\alpha yI) = f(xI)\alpha f(yI) = 0I$, that implies $xI\alpha yI \in$ neutrosophic $\ker(f) = \{0I\}$, implies xI $\alpha yI = 0$ I for all $\alpha \in \gamma$. Similarly, $yI\alpha xI = 0$ I for all $\alpha \in \gamma$. Therefore $xI = yI$.

Conversely, suppose f is injective and $xI \in$ neutrosophic $\ker(f)$. Then $f(xI) = 0I = f(0I)$ that implies $xI = 0I$, implies neutrosophic $\ker(f) = \{0I\}$. □

Lemma 3.16. *Let NI be a normal subalgebra neutrosophic of γ-BCK-algebra AI. If $xI\alpha yI \in NI$, for all $xI, yIxI\alpha yI \in AI$, then $yI\alpha xI \in NI, \alpha \in \gamma I$.*

Proof. Suppose that xI, yI and $xI\alpha yI \in NI$. Also, yI since NI is anormal neutrosophic subalgebra of AI. Therefore, $yI\alpha xI \in NI$. Let NI be a normal neutrosophic subalgebra of neutrosophic γ-BCK-algebra AI. Define a relation " $\sim NI$ " on AI by $xI \sim NI$ if and only if $xI\alpha yI \in N$, for any $xI, yI \in AI, \alpha \in \gamma$. □

Theorem 3.17. *Let NI be a normal neutrosophic subalgebra of neutrosophic γ-BCKalgebra AI. Then " $\sim NI$ " is a congruence relation.*

Proof. Let $xI \in AI, \alpha \in \gamma I$. Then the relation $\sim NI$ is reflexive, since $xI\alpha xI = 0I \in NI$.

The relation $\sim NI$ is symmetric. Suppose $xI \sim NIyI$ and $yI \sim NIzI \in NI$.Then $xI\alpha yI \in NI$ and $yI\alpha zI \in NI, yI\alpha zI \in NI$. Thus

$$(xI\alpha zI)\alpha(yI\alpha yI) = (xI\alpha zI)\alpha 0I = xI\alpha zI \in NI$$

Since NI is a normal neutrosophic sualgebra. Hence $xI \sim NIzI$. TheL $\sim NI$ is an equivalence relation. Let $xI \sim NIyI$ and $pI \sim NIqI$ for any xI, yI, pI and $qI \in AI$. Then $xI\alpha yI \in NI, pI\alpha II \in BI$, we have $(xI\alpha II)$ $\alpha(yI\alpha qI) \in NI$. Therefore $xI\alpha pI \sim NIy\alpha q$, since N is a normal neutrosophic sub-algebra. Hence $\sim NI$ congruence relation. □

Definition 3.18. Let NI congruence relation on neutrosophic γ-BCK-algebra M. Denoted $AI/_{NI} = \{[xI]_{NI}; xI \in AI\}$ where $[xI]_{NI} = \{yI \in AI; xI \sim NIyI\}$. Define $\{[xI]_{NI}\alpha[yI]_{NI} = [xI\alpha yI]_{NI}, \alpha \in \gamma I, AI/_{NI}$ is neutrosophic γ-BCK-algebra. Then neutrosophic γ-BCK-algebra AI/NI is called the quotient neutrosophic γ-BCKalgebra.

Example 3.19. Let $AI = \{0I, 1I, 2I, 3I\}$ and $\gamma I = \{\alpha, \beta\}$. Then ternary operation is defined by the following tables

Then, $NI = \{0I, 1I\}$ is a normal neutrosophic sub-algebra.

Denoted ${}^{AI}/NI = \{[xI]_{NI}; xI \in AI\}$. Define $[xI]_{NI}\alpha[yI]_{NI} = [xI\alpha yI]_{NI}; \alpha \in \gamma I$.
Then AI/NI is a quotient neutrosophic γ-BCK-algebra.

Theorem 3.20. *Let NI be a normal neutrosophic subagebra of a neutrosophic γ -BCK-algebra AI. Then the mapping $f : AI \to AI/_{NI}$ defined by $f(xI); [xI]_{NI}$ is a surjective homomorphism and neutrosophic* $\mathrm{Ker}(f) = NI$.

Proof. $f(xI\alpha yI) = [xI\alpha yI]_{NI} = f(xI)\alpha f(yI) = f(AI) = \{f(xI); xI \in AI\} = \{[xI]_{NI};$
$xI \in AI\} = AI/NI.$
Therefore, f is surjective neutrosophic $\mathrm{Ker}(f) = \{xI \in AI;\ f(xI) = NI\} = \{xI \in AI \quad ; \quad [xI]_{NI} = NI\} =\} =$
$\{xI \in AI \quad ; \quad [xI]_{NI} = [0]_N\} = xI \in AI;$
$xI \in NI\} = NI.$ □

4 Conclusions

The (Corollary 1.) indicates the neutrosophic γI-BCK-algebra which satisfies $xI\alpha(xI\beta yI)\alpha yI = 0I \forall xI, yI \in AI, \alpha, \beta \in \gamma I$ with some properties of this type of algebra. In (Lemma 1) and (Lemma 2), if $f : AI \rightarrow NI$ be a homomorphism of neutrosophic γ-BCK-algebra AI and NI, this means a neutrosophic $\ker(f)$ is a subalgebra of M and if we have $f : AI \rightarrow NI$ is a homomorphism of neutrosophic γ-BCK-algebra AI and NI, this imply that $f(0) = 0$ and if $xI\alpha yI = 0I$, so $f(xI)f(yI) = 0I$. Also, we studied if NI be a normal neutrosophic subalgebra of neutrosophic γ-BCK-algebra AI. Then " $\sim NI$ " is a congruence relation.

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