Discovering Novel Types of Irresolute and Contra Mappings for m-Polar Neutrosophic Topological Spaces

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Abstract

The present work explores the features of new kinds of neutrosophic continuous mappings, including neutrosophic irresolute β^*-continuous mapping (N$I\beta^*$CM) and neutrosophic continuous mappings, including neutrosophic contra β^*-continuous mapping (N$CO\beta^*$CM) and investigates some properties related them. Moreover, we study the relationships between these two concepts with the concept of irresolute α^* and contra α^*-continuous mapping. Finally, we introduced m-polar neutrosophic irresolute β^*-continuous mapping (MPN$I\beta^*$CM) and neutrosophic continuous mappings, including m-polar neutrosophic contra β^*-continuous mapping (MPN$CO\beta^*$CM) with investigates some properties related them.

Keywords: Neutrosophic irresolute β^*-continuous mapping; Neutrosophic contra β^*-continuous mapping; Contra m-polar mappings

1. Introduction

Since Zadeh [1] established the theory, the idea of fuzziness has affected nearly every area of mathematics. Many different disciplines use fuzzy sets. Chang [2] researched and created the fuzzy topological space theory-concept. Since then, some generic topological ideas have become better known.

Dontchev [3] examined the meaning of Contra continuity in 1998. Smarandache [5] introduced the concept of neutrosophic sets, and in 2014, the terms "neutrosophic closed set" and "neutrosophic continuous function" were introduced [6]. Numerous disciplines, including topology [3,4,7-9], algebra [10-12], and others [13,14] study the non-classical sets, it is one of the nonclassical sets, along with soft sets, fuzzy sets, nano sets, permutation sets, and so on.

We provide two kinds of neutrosophic mappings in this study, which we call neutrosophic irresolute β^*-continuous and antagonistic toward α mappings that are continuous. We then looked at and spoke about their fundamental characteristics.

2. Basic Concepts

Here, the sources [15-18] provide the fundamental concepts and notations utilized in this part.

Definition 2.1. Take $\mathcal{W}$ be a fixed set that is not empty. An object Ω of shape

$\Omega = \{\langle \hbar, \mu_\Omega(\hbar), \sigma_\Omega(\hbar), v_\Omega(\hbar)\rangle : \hbar \in \mathcal{W}\}$ is the neutrosophic set (NS) of Ω, for every $\hbar \in \mathcal{W}$, $\mu_\Omega(\hbar)$ represents a degree of membership, $\sigma_\Omega(\hbar)$ represents degree of indeterminacy and $v_\Omega(\hbar)$ represents a degree of non-membership.

DOI: https://doi.org/10.54216/IJNS.270101

Definition 2.2. We may have two neutrosophic sets, $\mathcal{M} = \{\langle \hbar, \mu_{\mathcal{M}}(\hbar), \sigma_{\mathcal{M}}(\hbar), v_{\mathcal{M}}(\hbar)\rangle : \hbar \in \mathcal{W}\}$ with $\mathcal{N} = \{\langle \hbar, \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{N}}(\hbar), v_{\mathcal{N}}(\hbar)\rangle : \hbar \in \mathcal{W}\}$

1. $\mathcal{M} \subseteq \mathcal{N} \Leftrightarrow \mu_{\mathcal{M}}(\hbar) \leq \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{M}}(\hbar) \leq \sigma_{\mathcal{N}}(\hbar)$ *and* $v_{\mathcal{M}}(\hbar) \geq v_{\mathcal{N}}(\hbar), \forall \hbar \in \mathcal{W}$.
2. $\mathcal{M} \subseteq \mathcal{N} \Leftrightarrow \mu_{\mathcal{M}}(\hbar) \leq \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{M}}(\hbar) \geq \sigma_{\mathcal{N}}(\hbar)$ *and* $v_{\mathcal{M}}(\hbar) \geq v_{\mathcal{N}}(\hbar), \forall \hbar \in \mathcal{W}$.
3. $\mathcal{M} \cap \mathcal{N} = \langle \hbar, \mu_{\mathcal{M}}(\hbar) \wedge \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{M}}(\hbar) \wedge \sigma_{\mathcal{N}}(\hbar), v_{\mathcal{M}}(\hbar) \vee v_{\mathcal{N}}(\hbar)\rangle$
4. $\mathcal{M} \cap \mathcal{N} = \langle \hbar, \mu_{\mathcal{M}}(\hbar) \wedge \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{M}}(\hbar) \vee \sigma_{\mathcal{N}}(\hbar), v_{\mathcal{M}}(\hbar) \vee v_{\mathcal{N}}(\hbar)\rangle$
5. $\mathcal{M} \cup \mathcal{N} = \langle \hbar, \mu_{\mathcal{M}}(\hbar) \vee \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{M}}(\hbar) \vee \sigma_{\mathcal{N}}(\hbar), v_{\mathcal{M}}(\hbar) \wedge v_{\mathcal{N}}(\hbar)\rangle$
6. $\mathcal{M} \cup \mathcal{N} = \langle \hbar, \mu_{\mathcal{M}}(\hbar) \vee \mu_{\mathcal{N}}(\hbar), \sigma_{\mathcal{M}}(\hbar) \wedge \sigma_{\mathcal{N}}(\hbar), v_{\mathcal{M}}(\hbar) \wedge v_{\mathcal{N}}(\hbar)\rangle$

Definition 2.3. Given an NS on $\mathcal{W}$, $\Omega = \langle \hbar, \mu_{\phi}(\hbar), \sigma_{\psi}(\hbar), v_{\psi}(\hbar)\rangle$, the complement $C(\Omega)$

gets stated as follows:

1. $C(\Omega) = \{\langle \hbar, v_{\psi}(\hbar), \sigma_{\psi}(\hbar), \mu_{\phi}(\hbar)\rangle : \hbar \in \mathcal{W}\}$,
2. $C(\Omega) = \{\langle \hbar, v_{\psi}(\hbar), 1 - \sigma_{\psi}(\hbar), \mu_{\phi}(\hbar)\rangle : \hbar \in \mathcal{W}\}$.,
3. $C(\Omega) = \{\langle \hbar, 1 - \mu_{\phi}(\hbar), 1 - v_{\psi}(\hbar)\rangle : \hbar \in \mathcal{W}\}$,

Notably, for any two sets $\mathcal{M}$ and $\mathcal{N}$ of neutrosophic then following

4. $C(\mathcal{M} \cap \mathcal{N}) = C(\mathcal{M}) \cup C(\mathcal{N})$,
5. $C(\mathcal{M} \cup \mathcal{N}) = C(\mathcal{M}) \cap C(\mathcal{N})$.

Definition 2.4. The following assumptions are satisfied by a Neutrosophic topology (NT) over a non-empty

set $\mathcal{W}$, that represents a family τ_N for neutrosophic subsets in $\mathcal{W}$:

i. $0_N, 1_N \in \tau_N$,
ii. $\mathcal{M} \cap \mathcal{N} \in \tau_N$ for any $\mathcal{M}, \mathcal{N} \in \tau_N$,
iii. $\cup \mathcal{M}_j \in \tau_N, \forall \{\mathcal{H}_j : j \in I\} \subseteq \tau_N$.

In this instance, each neutrosophic set in τ_N is referred to as a neutrosophic open set (NOS) in $\mathcal{W}$, and the pair $(\mathcal{W}, \tau_N)$ represents a neutrosophic topological space (NTS) if and only if its counterpart $C(\mathcal{M})$ represents a neutrosophic open set (NOS) in $\mathcal{W}$, then a neutrosophic set $\mathcal{M}$ becomes a neutrosophic close set (NCS).

Definition 2.5. Allow $\mathcal{M} = \{\langle \hbar, \mu_{\mathcal{M}}(\hbar), \sigma_{\mathcal{M}}(\hbar), v_{\mathcal{M}}(\hbar)\rangle : \hbar \in \mathcal{W}\}$ be an NS in $\mathcal{W}$ and let $(\mathcal{W}, \tau_N)$ be an NTS. The neutrosophic closure and the neutrosophic interior of $\mathcal{M}$ are then determined by:

$$NCl(\mathcal{M}) = \cap \{\mathcal{F} : \mathcal{F} \text{ is an NCS in } \mathcal{W} \text{ and } \mathcal{M} \subseteq \mathcal{F}\}$$

$$NInt(\mathcal{M}) = \cup \{\mathcal{S} : \mathcal{S} \text{ is an NOS in } \mathcal{W} \text{ and } \mathcal{S} \subseteq \mathcal{M}\}$$

It should be noted that $NCl(C(\mathcal{M})) = C(NInt(\mathcal{M}))$ *and* $NInt(C(\mathcal{M})) = C(NCl(\mathcal{M}))$ for every NS $\mathcal{M}$.

Proposition 2.6.

(1) Each NOS set represents NβOS.

(2) Each NαOS represents NβOS.

Definition 2.7. Consider g as a mapping from $(\mathcal{W}, \tau_N)$, a NTS, to $(\mathcal{U}, \sigma_N)$, another NTS. Whenever the inverse image of each neutrosophic closed set in $(\mathcal{U}, \sigma_N)$ corresponds to a NCS in $(\mathcal{W}, \tau_N)$, subsequently g is considered a neutrosophic irresolute mapping.

Definition 2.8. Let $\mathfrak{D}$ represent the non-empty reference set, and let mpn $(\mathfrak{D}^1)$ be the collection of all MPNSs defined over $\mathfrak{D}$. The set T_M is then the collection of subsets of mpn $(\mathfrak{D}^1)$, which form an m-polar neutrosophic topology (MPNT). For T_M to be considered a valid m-polar neutrosophic topology, the following conditions must hold:

(i) $\mathfrak{D}^0, \mathfrak{D}^1 \in T_M$.

(ii) If $\mathfrak{D}_1, \mathfrak{D}_2 \in T_M$, then $\mathfrak{D}_1 \cap \mathfrak{D}_2 \in T_M$

(iii) If $\{\mathfrak{D}_q\} \in T_M$, then $\cup_{q \in \Delta} \mathfrak{D}_q \in T_M$

The pair $(\mathfrak{D}, T_M)$ defines an m-polar neutrosophic topology. The elements of T_M are called MPNSs, and them complements are known as complementary MPNSs.

DOI: https://doi.org/10.54216/IJNS.270101

Example 2.9. Consider a set $\mathfrak{O} = \{s_1, s_2, s_3, s_4\}$ representing an arrangement of books. Let mpn ($\mathfrak{O}^1$) denote the collection of all m-polar neutrosophic sets (MPNSs) over $\mathfrak{O}$. Two 3-polar neutrophilic subgroups of mpn are defined ($\mathfrak{O}^1$), represented as:

$$\mathfrak{O}^1 = \{(s_1, \langle 0.725,0.514,0.514\rangle, \langle 0.523,0.501,0.533\rangle, \langle 0.325,0.389,0.219\rangle),$$
$$(s_2, \langle 0.611,0.525,0.672\rangle,$$
$$\langle 0.523,0.318,0.289\rangle, \langle 0.519,0.378,0.491\rangle), (s_3, \langle 0.701,0.518,0.429\rangle,$$
$$\langle 0.681,0.328,0.549\rangle, \langle 0.623,0.541,0.376\rangle),$$

$(s_4, \langle 0.619,0.359,0.734\rangle, \langle 0.435,0.511,0.627\rangle, \langle 0.806,0.522,0.489\rangle)\}$ and

$$\mathfrak{O}^2 = \{(s_1, \langle 0.725,0.548,0.429\rangle, \langle 0.501,0.533,0.618\rangle, \langle 0.381,0.444,0.419\rangle),$$
$$(s_2, \langle 0.672,0.427,0.541\rangle,$$
$$\langle 0.564,0.389,0.517\rangle, \langle 0.648,0.536,0.518\rangle), (s_3, \langle 0.563,0.429,0.438\rangle,$$
$$\langle 0.523,0.512,0.445\rangle, \langle 0.672,0.506,0.518\rangle),$$

$(s_4, \langle 0.325,0.556,0715\rangle, \langle 0.435,0.511,0.488\rangle, \langle 0.742,0.501,0.535\rangle)\}$. Clearly, the collection $T_M = \{\mathfrak{O}^0, \mathfrak{O}^1, \mathfrak{O}_1, \mathfrak{O}_2\}$ forms a 3-polar neutrosophic topological space.

Theorem 2.10. Let $(\mathfrak{O}, T_M)$ be an MPNT. The criteria that follow need to be met:

(i) $\emptyset$ and Q are considered open MPNSs.

(ii) It is still possible to combine any amount for open MPNSs.

(iii) A finite collection of closed MPNSs meet at a closed point.

Theorem 2.11. Let $(\mathfrak{O}, T_M)$ represent an MPNTS, and let $\mathfrak{O}_1$ be an element of mpn ($\mathfrak{O}^1$).

- The interior of $\mathfrak{O}_1$, denoted as $\mathfrak{O}_1^0$, is characterized as the sum of every open subsets for m-polar neutrosophic (MPNSs) that are contained within $\mathfrak{O}_1$. This interior is the largest open MPNS that fits entirely within the boundaries of $\mathfrak{O}_1$, effectively capturing the maximal open structure of $\mathfrak{O}_1$.
- The closure of $\mathfrak{O}_1$, denoted as $\overline{\mathfrak{O}_1}$, is defined as the intersection of all closed m-polar neutrosophic supersets (MPNSs) of $\mathfrak{O}_1$. It represents the smallest closed MPNS that contains $\mathfrak{O}_1$, capturing the minimal closed structure that fully encompasses $\mathfrak{O}_1$.

3. New class of neutrosophic β^*-mappings

The new forms of neutrosophic β^*-continuity, such as neutrosophic irresolute β^* (NIβ^*- CM), neutrosophic contra β^*-continuous mapping (NCβ^*- CM), and are provided in this study. Additionally, the relationships among these concepts are displayed.

Definition 3.1. Consider two neutrosophic topological spaces, $(\mathcal{W}, \tau_N)$, and $(\mathcal{U}, \sigma_N)$. Whenever $h^{-1}(\mathcal{M})$ represents a N$\beta-$ OS in $(\mathcal{W}, \tau_N)$ for each N$\beta-$ OS $\mathcal{M}$ of $(\mathcal{U}, \sigma_N)$, then $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ represents a NIβ^*- CM.

Example 3.2. Consider $\mathcal{W} = \{e, d\}$ and $\mathcal{U} = \{b, f\}$. Then $\tau_N = \{0_N, 1_N, \mathcal{Q}\}$ and $\sigma_N = \{0_N, 1_N, \mathcal{P}\}$ are NT on $\mathcal{W}$ and $\mathcal{U}$ respectively, where $\mathcal{Q} = \langle \hbar, (0.6,0.6), (0.4,0.4), (0.6,0.6)\rangle$and $\mathcal{P} = \langle g, (0.8,0.7), (0.3,0.3), (0.4,0.4)\rangle$ *where* $\hbar \in \mathcal{W}$ and $g \in \mathcal{U}$. Define a mapping $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ via $h(e) = b$ and $h(d) = f$. Here the NS $\mathcal{K} = \langle g, (0.6,0.5), (0.5,0.5), (0.2,0.4)\rangle$ is a Nβ^*–OS in $\mathcal{U}$. Thus, $h^{-1}(\mathcal{M}) = \langle x, (0.3,0.3), (0.3,0.3), (0.7,0.6)\rangle$ is a NOS in $(\mathcal{W}, \tau_N)$ as $h^{-1}(\mathcal{M}) \subseteq \mathcal{Q}$ then $NCl(NInt(NCl(h^{-1}(\mathcal{M})))) = 1_N \subseteq \mathcal{Q}$ where $\mathcal{Q}$ is a NOS in $\mathcal{W}$. Hence, h is a NIβ^*- CM.

Theorem 3.3. Every NIα^*- CM is NIβ^*- CM.

Proof: According to the concept of α-irresolute, we know that for every NCS $\mathcal{M}$ in $\mathcal{U}$

then $h^{-1}(\mathcal{M})$be an α-open in $\mathcal{W}$. But due to β-open is a weaker topology, any set of α-open is also β-open and $h^{-1}(\mathcal{M})$ must be an β -open in $\mathcal{W}$. For this reason, $h^{-1}(\mathcal{M})$ is N β -OS in $\mathcal{W}$. Therefore, NIα^*- CM is NIβ^*- CM.

DOI: https://doi.org/10.54216/IJNS.270101

Remark 3.4. The aforementioned reversals are untrue. Here is an example showing how this situation operates.

Example 3.5. Consider the $\mathcal{W} = \{e, d, c\}$ and $\mathcal{U} = \{b, f, w\}$ where the given neutrosophic topology is $\tau_N = \{0_N, 1_N, \mathcal{Q}\}$ and $\sigma_N = \{0_N, 1_N, \mathcal{P}\}$, respectively, also

$\mathcal{Q} = \langle h, (0.5,0.4,0.1), (0.5,0.5,0.5), (0.7,0.4,0.1) \rangle$ and

$\mathcal{P} = \langle g, (0.6,0.4,0.2), (0.5,0.5,0.5), (0.7,0.2,0.2) \rangle$ *where* $h \in \mathcal{W}$ and $g \in \mathcal{U}$. Define a mapping $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ via $h(e) = b$, $h(d) = f$ and $h(c) = w$. Hence, the NS $\mathcal{K} = \langle g, (0.3,0.4,0.2), (0.5,0.5,0.1), (0.8,0.4,0.4) \rangle$ is a Nβ^*−OS in $\mathcal{U}$. And so $h^{-1}(\mathcal{K})$ is a NOS in $(\mathcal{W}, \tau_N)$. But $\mathcal{K}$ is not a Nα^*−OS in $\mathcal{U}$.

Theorem 3.6. Suppose that $(\mathcal{W}, \tau_N)$, and $(\mathcal{U}, \sigma_N)$ are NTSs and $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$. When h represents N$I\beta^*$ − CM, thus $h|\mathcal{G}: \mathcal{G} \rightarrow \mathcal{U}$ is as well, where $\mathcal{G}$ is NOS of $\mathcal{W}$.

Proof. Consider that $\mathcal{G}$ is NOS in $\mathcal{W}$, and that $\mathcal{G}$ is a NOS in $\mathcal{U}$, because h is NIβ^*− CM and $h^{-1}(\mathcal{M})$ is NOS in $\mathcal{W}$. Therefore, $h^{-1}(\mathcal{M}) \cap \mathcal{G}$ is NOS in $\mathcal{W}$, but $(h|\mathcal{G})^{-1}(\mathcal{M}) = h^{-1}(\mathcal{M}) \cap \mathcal{G}$, hence $(h|\mathcal{G})^{-1}(\mathcal{M})$ is NOS in $\mathcal{G}$. Therefore, $h|\mathcal{G}$ represents N$I\beta^*$ − CM.

Theorem 3.7. Let $\mathcal{W} = \mathfrak{D} \cup \mathcal{C}$, where $\mathfrak{D}$ and $\mathcal{C}$ are disjoint NSs in $\mathcal{W}$, and let $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ be any mapping. Next, h represents NIβ^*− CM iff $h|\mathfrak{D}$ and $h|\mathcal{C}$ are, where $\mathfrak{D}$ and $\mathcal{C}$ are NOSs.

Proof: Assuming that $\mathcal{G}$ represents Nβ^*−OS in $\mathcal{U}$, $(h|\mathfrak{D})^{-1}(\mathcal{G})$ and $(h|\mathcal{C})^{-1}(\mathcal{G})$ are NOS in $\mathcal{W}$, whereas $h|\mathfrak{D}$ and $h|\mathcal{C}$ are NIβ^*− CM. Therefore, the other hand, $h^{-1}(\mathcal{G}) = ((h|\mathfrak{D})^{-1}(\mathcal{G}) \cup (h|\mathcal{C})^{-1}(\mathcal{G})$, meaning that $h^{-1}(\mathcal{G})$ represents NOS in $\mathcal{W}$. Consequently, h becomes NIβ^*− CM.

Sufficiency is then determined by applying Theorem 3.6.

Theorem 3.8. Assume that $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ be any mapping for each NS $\mathfrak{D}$ in $\mathcal{U}$ such that $h_{\mathfrak{D}}: h^{-1}(\mathfrak{D}) \rightarrow \mathfrak{D}$ can be expressed as $h_{\mathfrak{D}}(r) = h(r)$ where $r \in h^{-1}(\mathfrak{D})$, When $\mathfrak{D}$ is NCS in $\mathcal{U}$, $h_{\mathfrak{D}}$ is thus NIβ^*− CM.

Proof: Assume that $\mathcal{M}$ is Nβ^* − OS in $\mathfrak{D}$. Hence, $\mathcal{M}$ is Nβ^* − OS in $\mathcal{U}$, since $\mathfrak{D}$ is NCS in $\mathcal{U}$. Furthermore, $h^{-1}(\mathcal{M})$ is NOS in $\mathcal{W}$ because h is NIβ^*− CM. Consequently, $h^{-1}(\mathfrak{D})$ is NOS.

Theorem 3.9. If $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ and $g: (\mathcal{U}, \sigma_N) \rightarrow (\mathcal{S}, \beth_N)$ are any two NIβ^*− CM, then $g \circ h : (\mathcal{W}, \tau_N) \rightarrow (\mathcal{S}, \beth_N)$ is a NIβ^*− CM.

Proof: Suppose $\mathcal{M}$ be a Nβ^*− OS in $\mathcal{S}$. Then, by hypothesis, $g^{-1}(\mathcal{M})$ represents a NOS in $\mathcal{U}$. Hence, $h^{-1}(g^{-1}(\mathcal{M}))$ is a Nβ^*− OS in $\mathcal{W}$ because h is a NIβ^*− CM. Thus, $g \circ h$ is a NIβ^*− CM.

Theorem 3.10. Assume that $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ be an Nβ^*− CM and $g: (\mathcal{U}, \sigma_N) \rightarrow (\mathcal{S}, \beth_N)$ be a NIβ^*− CM , then $\mathfrak{T} = g \circ h : (\mathcal{W}, \tau_N) \rightarrow (\mathcal{S}, \beth_N)$ is a Nβ^*− CM.

Proof: To demonstrate this, we first demonstrate that the preimage $\mathfrak{T}^{-1}(\mathcal{M})$ is NOS in $\mathcal{W}$ for every Nβ^*− OS $\mathcal{M}$ in $\mathcal{S}$. First, look at $\mathcal{M}$ preimage beneath $\mathfrak{T}$ as $\mathfrak{T}^{-1}(\mathcal{M}) = \{h \in \mathcal{W}: \mathfrak{T}(h) = g(h(h)) \in \mathcal{M}\}$. The parts $h \in \mathcal{W}$ for which $g(h(h)) \in \mathcal{M}$ are in this collection. Because $g(h(h)) \in \mathcal{M}$, The preimage can be written as: $\mathfrak{T}^{-1}(\mathcal{M}) = \{h \in \mathcal{W}: h(h) \in g^{-1}(\mathcal{M})\}$.

Therefore, the set of parts $h \in \mathcal{W}$ which means $h(h)$ corresponds to the preimage of $\mathcal{M}$ under g, that is, $g^{-1}(\mathcal{M})$, represents the preimage of $\mathcal{M}$ under $\mathfrak{T}$.We know that the preimage of $g^{-1}(\mathcal{M})$, is NOS in $\mathcal{U}$ for every Nβ^*− OS $\mathcal{M}$ in $\mathcal{S}$ because g is NIβ^*− CM. Thus, in $\mathcal{U}$, $g^{-1}(\mathcal{M})$ is NOS in $\mathcal{U}$. Consequently, $h^{-1}(g^{-1}(\mathcal{M}))$ is NOS in $\mathcal{W}$ because $h^{-1}(\mathcal{M})$ is NOS in $\mathcal{W}$.

Hence, $\mathfrak{T}^{-1}(\mathcal{M}) = h^{-1}(g^{-1}(\mathcal{M}))$ is NOS in $\mathcal{W}$. Since $\mathfrak{T}^{-1}(\mathcal{M})$ is NOS in $\mathcal{W}$ for every Nβ^*− OS $\mathcal{M}$ in $\mathcal{S}$, we conclude that $\mathfrak{T} = g \circ h$ is Nβ^*− CM.

Definition 3.11. Assume two NTS $(\mathcal{W}, \tau_N)$ and $(\mathcal{U}, \sigma_N)$, and a mapping $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$, h is referred to as a neutrosophic contra β^*-continuous mapping (NCβ^* − CM) whenever $h^{-1}(\mathcal{N})$ is Nβ − CS in $\mathcal{W}$ for any Nβ − OS $\mathcal{N}$ in $\mathcal{U}$.

Proposition 3.12. Every NCα^*− CM is NCβ^*− CM.

Proof: Obvious.

Remark 3.13. The reversals described above are not accurate. Here is an illustration of how this situation works.

Example 3.14. Consider the $\mathcal{W} = \{e, d, c\}$ and $\mathcal{U} = \{b, f, w\}$ where the given neutrosophic topology is $\tau_N = \{0_N, 1_N, \mathcal{Q}, \mathcal{M}\}$ and $\sigma_N = \{0_N, 1_N, \mathcal{P}, \mathcal{N}\}$, respectively, also

DOI: https://doi.org/10.54216/IJNS.270101

$\mathcal{Q} = \langle \hbar, (0.6,0.5,0.2), (0.4,0.3,0.4), (0.3,0.6,0.4) \rangle$, $\mathcal{M} = \langle \hbar, (0.5,0.2,0.3), (0.5,0.5,0.5), (0.4,0.8,0.3) \rangle, \mathcal{P} = \langle g, (0.5,0.3,0.1), (0.4,0.3,0.3), (0.2,0.8,0.4) \rangle$

and $\mathcal{N} = \langle g, (0.9,0.5,0.1), (0.2,0.5,0.4), (0.2,0.7,0.8) \rangle$ *where* $\hbar \in \mathcal{W}$ and $g \in \mathcal{U}$. Define a mapping $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ via $h(e) = b$, $h(d) = f$ and $h(c) = w$. Thus, the NS $\mathcal{K} = \langle g, (0.5,0.3,0.3), (0.4,0.4,0.3), (0.6,0.5,0.3) \rangle$ is a Nβ^*−CS in $\mathcal{U}$. And so $h^{-1}(\mathcal{K})$ is a Nβ^* − CS in $(\mathcal{W}, \tau_N)$. But $\mathcal{K}$ is not a Nα^*−CS in $\mathcal{U}$.

Theorem 3.15. Assume a mapping $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$. The following claims are interchangeable:

(i) h is NCβ^*− CM,
(ii) For every i in $\mathcal{W}$ and every NCS $\mathcal{N}$ in $\mathcal{U}$ that contains $h(i)$, such that belong to $\mathcal{H}$, $h(\mathcal{C}) \subseteq \mathcal{N}$, there exists N$\beta^*$− OS $\mathcal{H}$ in $\mathcal{W}$.
(iii) $h^{-1}(\mathcal{N})$ is Nβ^*− OS of $\mathcal{W}$ for every NCS $\mathcal{N}$ of $\mathcal{U}$.

Proof. (i) → (ii) Assume that $\mathcal{N}$ is NCS in $\mathcal{U}$, then $C(\mathcal{N})$ is NOS in $\mathcal{U}$. Hence, $h^{-1}(C(\mathcal{N}))$ represents Nβ^*− CS in $\mathcal{W}$, but $h^{-1}(C(\mathcal{N})) = C[h^{-1}(\mathcal{N})]$. Thus, $h^{-1}(\mathcal{N})$ is Nβ^*− OS in $\mathcal{W}$, and $i \in h^{-1}(\mathcal{N})$. Set $\mathcal{H} = h^{-1}(\mathcal{N})$, so $h(\mathcal{H}) \subseteq \mathcal{N}$.

(ii)→(iii) Suppose that $\mathcal{N}$ is a NCS in $\mathcal{U}$ and $i \in h^{-1}(\mathcal{N})$, thus $h(i) \in \mathcal{N}$ and implies there exists Nβ^*− OS $\mathcal{H}$ containing $\mathcal{H}$, $h(\mathcal{H}) \subseteq \mathcal{N}$, thus $i \in \mathcal{H} = h^{-1}(\mathcal{N})$. Thus, $h^{-1}(\mathcal{N}) = \cup \{\mathcal{H}_i \mid i \in h^{-1}(\mathcal{N})\}$. Therefore, we obtain $h^{-1}(\mathcal{N})$ is Nβ^*− OS in $\mathcal{W}$.

(iii) → (i) Suppose that $\mathcal{N}$ is NCS in $\mathcal{U}$. Since, $h^{-1}(\mathcal{N})$ is Nβ^*− OS. Therefore, h is NCβ^*− CM.

Theorem 3.16. Assume that $(\mathcal{W}, \tau_N)$, and $(\mathcal{U}, \sigma_N)$ are NTSs and $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$. When h represents N$C\alpha^*$ − CM, thus $h|\mathcal{G}: \mathcal{G} \to \mathcal{U}$ is N$C\beta^*$ − CM, where $\mathcal{G}$ is NCS of $\mathcal{W}$.

Proof. Consider that $\mathcal{G}$ is NCS in $\mathcal{W}$, and that $\mathcal{G}$ is a NCS in $\mathcal{U}$, since h is NIα^*− CM, so h is NIβ^*− CM and $h^{-1}(\mathcal{M})$ is NCS in $\mathcal{W}$. Thus, $h^{-1}(\mathcal{M}) \cap \mathcal{G}$ is NCS in $\mathcal{W}$, but $(h|\mathcal{G})^{-1}(\mathcal{M}) = h^{-1}(\mathcal{M}) \cap \mathcal{G}$, hence $(h|\mathcal{G})^{-1}(\mathcal{M})$ is NCS in $\mathcal{G}$. Therefore, $h|\mathcal{G}$ represents N$C\beta^*$ − CM.

Theorem 3.17. Let $\mathcal{W} = \mathfrak{D} \cup \mathcal{C}$, where $\mathfrak{D}$ and $\mathcal{C}$ are disjoint NSs in $\mathcal{W}$, and let $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be any mapping. Next, h represents NCβ^*− CM iff $h|\mathfrak{D}$ and $h|\mathcal{C}$ are, where $\mathfrak{D}$ and $\mathcal{C}$ are NCSs.

Proof: Assuming that $\mathcal{G}$ represents Nβ^*−CS in $\mathcal{U}$, $(h|\mathfrak{D})^{-1}(\mathcal{G})$ and $(h|\mathcal{C})^{-1}(\mathcal{G})$ are NCS in $\mathcal{W}$, whereas $h|\mathfrak{D}$ and $h|\mathcal{C}$ are NCβ^*− CM. Therefore, the other hand, $h^{-1}(\mathcal{G}) = ((h|\mathfrak{D})^{-1}(\mathcal{G}) \cup (h|\mathcal{C})^{-1}(\mathcal{G})$, hence, $h^{-1}(\mathcal{G})$ represents NCS in $\mathcal{W}$. Therefore, h is NCβ^*− CM.

Theorem 3.16 is then used to assess sufficiency.

Theorem 3.18. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be any mapping for each NS $\mathfrak{D}$ in $\mathcal{U}$ such that $h_{\mathfrak{D}}: h^{-1}(\mathfrak{D}) \to \mathfrak{D}$ can be expressed as $h_{\mathfrak{D}}(r) = h(r)$ where $r \in h^{-1}(\mathfrak{D})$, When $\mathfrak{D}$ is NCS in $\mathcal{U}$, $h_{\mathfrak{D}}$ is thus NCβ^*− CM.

Proof: Assume that $\mathcal{M}$ is Nβ^* − CS in $\mathfrak{D}$. Hence, $\mathcal{M}$ is Nβ^* − CS in $\mathcal{U}$, since $\mathfrak{D}$ is NCS in $\mathcal{U}$. Furthermore, $h^{-1}(\mathcal{M})$ is NCS in $\mathcal{W}$ because h is NCβ^*− CM. Consequently, $h^{-1}(\mathfrak{D})$ is NCS.

Theorem 3.19. If $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ are any two NCβ^*− CM, then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a NCβ^*− CM.

Proof: Suppose $\mathcal{M}$ be a Nβ^*− CS in $\mathcal{S}$. Then, by hypothesis, $g^{-1}(\mathcal{M})$ represents a NCS in $\mathcal{U}$. Hence, $h^{-1}(g^{-1}(\mathcal{M}))$ is a Nβ^*− CS in $\mathcal{W}$ because h is a NCβ^*− CM. Thus, $g \circ h$ is a NCβ^*− CM.

Theorem 3.20. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be an Nβ^*− CM and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ be a NCβ^*− CM , then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a NCβ^*− CM.

Proof: To demonstrate this, we first verify that the preimage $(g \circ h)^{-1}(\mathcal{V})$ is a NCS in $\mathcal{W}$ for every NCS $\mathcal{V} \subseteq \mathcal{S}$. We must demonstrate that $(g \circ h)^{-1}(\mathcal{V})$ is NCS in $\mathcal{W}$. Let's begin by now examine the composition of $g \circ h$. Obtaining the preimage for a NCS $\mathcal{V} \subseteq \mathcal{S}$ under $g \circ h$ is our goal and $(g \circ h)^{-1}(\mathcal{V}) = h^{-1}(g^{-1}(\mathcal{V}))$. Given that g is NCβ^*− CM, we may conclude that $g^{-1}(\mathcal{V})$ is a NCS in $\mathcal{U}$ for any a NCS $\mathcal{V} \subseteq \mathcal{S}$. Because h is Nβ^*− CM, the preimage $h^{-1}(\mathcal{V})$ is NOS in $\mathcal{W}$ for any NOS $\mathcal{O} \subseteq \mathcal{U}$. Consequently, the preimage $h^{-1}(g^{-1}(\mathcal{V}))$ will additionally be NCS in $\mathcal{W}$ due to h. We infer that $(g \circ h)^{-1}(\mathcal{V})$ is NCS in $\mathcal{W}$, satisfying the requirement for being NCβ^*− CM, as $h^{-1}(g^{-1}(\mathcal{V}))$ is NCS in $\mathcal{W}$.

Theorem 3.21. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be an N$I\beta^*$− CM and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ be a NCβ^*− CM , then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a NCβ^*− CM.

Proof: Clearly holds.

DOI: https://doi.org/10.54216/IJNS.270101

Proposition 3.22. Every NIβ^*- CM is NCβ^*- CM.

Proof: Obvious.

Remark 3.23. The reversals described above are not accurate. Here is an illustration of how this situation works.

Example 3.24. Recall example 3.14. we see that h is NCβ^*- CM, but h does not NIβ^*- CM.

Remark 3.25. Figure 1 below shows how several classes relate to one another in the mapping $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$:

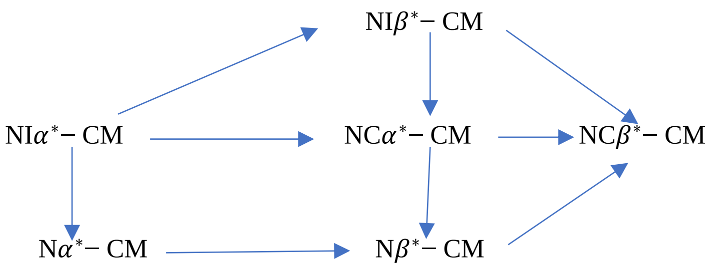

Figure 1. Relationships among different types of continuous mappings.

4. New class of neutrosophic β^*-mappings

The new forms of neutrosophic β^*-continuity, such as m-polar neutrosophic irresolute β^* (MPNIβ^*- CM), m-polar neutrosophic contra β^*-continuous mapping (MPNCβ^*- CM), and are provided in this study. Additionally, the relationships among these concepts are displayed.

Definition 4.1. Consider two m-polar neutrosophic topological spaces, $(\mathcal{W}, \tau_N)$, and $(\mathcal{U}, \sigma_N)$. Whenever $h^{-1}(\mathcal{M})$ represents a N β-OS in $(\mathcal{W}, \tau_N)$ with regard to its m-polar neutrosophic topology for each MPNβ^*- OS $\mathcal{M}$ of $(\mathcal{U}, \sigma_N)$, then $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$ represents a NIβ^*- CM.

Example.4.2. Think about two sets $\mathcal{W} = \{\mathscr{s}_1, \mathscr{s}_2, \mathscr{s}_3, \mathscr{s}_4\}$ and $\mathcal{U} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4\}$, each of which represents a group of items. Let MPNT on these sets be represented by τ_N and σ_N. Assume that the collections for every MPNSs across $\mathcal{W}$ and $\mathcal{U}$ are represented by mpn $(\mathcal{W}^1)$ and mpn $(\mathcal{U}^1)$, respectively. Give the following definitions for two 3-polar neutrosophic subsets:

$$\mathcal{W}^1 = \{(\mathscr{s}_1, \langle 0.762,0.412,0.351\rangle, \langle 0.241,0.332,0.413\rangle, \langle 0.324,0.213,0.321\rangle), (\mathscr{s}_2, \langle 0.543,0.231,0.451\rangle, \langle 0.634,0.543,0.412\rangle, \langle 0.432,0.332,0.342\rangle), (\mathscr{s}_3, \langle 0.412,0.532,0.341\rangle, \langle 0.325,0.451,0.312\rangle, \langle 0.523,0.324,0.312\rangle), (\mathscr{s}_4, \langle 0.621,0.334,0.517\rangle, \langle 0.519,0.343,0.232\rangle, \langle 0.221,0.411,0.319\rangle)\}$$

and

$$\mathcal{U}^1 = \{(\mathscr{b}_1, \langle 0.371,0.221,0.412\rangle, \langle 0.433,0.362,0.327\rangle, \langle 0.111,0.321,0.113\rangle), (\mathscr{b}_2, \langle 0.251,0.341,0.431\rangle, \langle 0.364,0.221,0.314\rangle, \langle 0.415,0.311,0.235\rangle), (\mathscr{b}_3, \langle 0.631,0.231,0.341\rangle, \langle 0.431,0.221,0.231\rangle, \langle 0.534,0.431,0.354\rangle)\}.$$

Examine the function $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$, which is defined by:

$h(\mathscr{s}_1) = \mathscr{b}_1, h(\mathscr{s}_2) = \mathscr{b}_2, h(\mathscr{s}_3) = \mathscr{b}_3$ and $h(\mathscr{s}_4) = \mathscr{b}_1$. Let $\tau_N = \{\mathcal{W}^0, \mathcal{W}^1, \mathcal{W}_1, \mathcal{W}_2\}$ and $\sigma_N = \{\mathcal{U}^0, \mathcal{U}^1, \mathcal{U}_1, \mathcal{U}_2\}$ be the collections of open sets in the MPNT for spaces $\mathcal{W}$and $\mathcal{U}$, respectively.

We analyze the preimages for open subsets in $\mathcal{U}$ under h in order to verify the continuation of the function h.If we choice $\mathcal{U}_1$, we see that $h^{-1}(\mathcal{U}_1) = \{\mathscr{s}_1, \mathscr{s}_4\}$. This is true since $\{\mathscr{s}_1, \mathscr{s}_4\}$ represents a semi open subset in $\mathcal{W}$. Now, if we choice $\mathcal{U}_2$, we see that $h^{-1}(\mathcal{U}_2) = \{\mathscr{s}_2\}$. This is true since $\{\mathscr{s}_2\}$ represents a semi open subset in $\mathcal{W}$. Finally, if we choice $\mathcal{W}^0$, we see that $h^{-1}(\mathcal{U}^0) = \mathcal{W}^0$. This is true since $\mathcal{W}^0$ represents a semi open subset in $\mathcal{W}$. Therefore, h represents MPNIβ^*- CM.

DOI: https://doi.org/10.54216/IJNS.270101

Theorem 4.3. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be any mapping for each MPNS $\mathfrak{D}$ in $\mathcal{U}$ such that $h_{\mathfrak{D}}: h^{-1}(\mathfrak{D}) \to \mathfrak{D}$ can be expressed as $h_{\mathfrak{D}}(r) = h(r)$ where $r \in h^{-1}(\mathfrak{D})$, When $\mathfrak{D}$ is NCS in $\mathcal{U}$, $h_{\mathfrak{D}}$ is thus MPNIβ^*- CM.

Proof: By using Theorem 3.8, we obtain the result.

Theorem 4.4. If $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ are any two MPNIβ^*- CM, then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a MPNIβ^*- CM.

Proof: From Theorem 3.9, we get the result.

Theorem 4.5. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be an Nβ^*- CM and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ be a MPNIβ^*- CM , then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a MPNβ^*- CM.

Proof: it is clear via using Theorem 3.10.

Definition 4.6. Assume two MPNTS $(\mathcal{W}, \tau_N)$ and $(\mathcal{U}, \sigma_N)$, and a mapping $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$, h is referred to as a m-polar neutrosophic contra β^*-continuous mapping (MPNC$\beta^* -$ CM) whenever $h^{-1}(\mathcal{N})$ is N$\beta^* -$ CS in $\mathcal{W}$ for any N β-OS $\mathcal{N}$ in $\mathcal{U}$.

Example. 4.7. Think about two sets $\mathcal{W} = \{s_1, s_2, s_3, s_4\}$ and $\mathcal{U} = \{b_1, b_2, b_3, b_4\}$, each of which represents a group of items. Let MPNT on these sets be represented by τ_N and σ_N. Assume that the collections for every MPNSs across $\mathcal{W}$ and $\mathcal{U}$ are represented by mpn $(\mathcal{W}^1)$ and mpn $(\mathcal{U}^1)$, respectively. Give the following definitions for two 3-polar neutrosophic subsets:

$$\mathcal{W}^1 = \{(s_1, \langle 0.744, 0.436, 0.411\rangle, \langle 0.312, 0.348, 0.519\rangle, \langle 0.289, 0.298, 0.308\rangle), (s_2, \langle 0.474, 0.356, 0.504\rangle, \langle 0.723, 0.634, 0.517\rangle, \langle 0.418, 0.393, 0.371\rangle), (s_3, \langle 0.488, 0.571, 0.261\rangle, \langle 0.428, 0.511, 0.277\rangle, \langle 0.618, 0.445, 0.268\rangle), (s_4, \langle 0.683, 0.341, 0.642\rangle, \langle 0.541, 0.378, 0.276\rangle, \langle 0.311, 0.331, 0.242\rangle)\}$$

and

$$\mathcal{U}^1 = \{(b_1, \langle 0.278, 0.354, 0.416\rangle, \langle 0.368, 0.375, 0.411\rangle, \langle 0.179, 0.222, 0.205\rangle), (b_2, \langle 0.199, 0.224, 0.315\rangle, \langle 0.287, 0.290, 0.399\rangle, \langle 0.372, 0.290, 0.199\rangle), (b_3, \langle 0.652, 0.245, 0.228\rangle, \langle 0.351, 0.320, 0.309\rangle, \langle 0.444, 0.499, 0.363\rangle), (b_4, \langle 0.525, 0.397, 0.428\rangle, \langle 0.433, 0.379, 0.322\rangle, \langle 0.299, 0.302, 0.119\rangle)\}.$$

Examine the function $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$, which is defined by:

$h(s_1) = b_1, h(s_2) = b_2, h(s_3) = b_3$ and $h(s_4) = b_4$. Let $\tau_N = \{\mathcal{W}^0, \mathcal{W}^1, \mathcal{W}_1, \mathcal{W}_2\}$ and $\sigma_N = \{\mathcal{U}^0, \mathcal{U}^1, \mathcal{U}_1, \mathcal{U}_2\}$ be the collections of open sets in the MPNT for spaces $\mathcal{W}$and $\mathcal{U}$, respectively. We analyze the preimages for open subsets in $\mathcal{U}$ under h in order to verify the continuation of the function h.If we choice $\mathcal{U}_1$, we see that $h^{-1}(\mathcal{U}_1) = \{s_1\}$. This is true since $\{s_1\}$ represents a closed subset in $\mathcal{W}$. Now, if we choice $\mathcal{U}_2$, we see that $h^{-1}(\mathcal{U}_2) = \{s_2\}$. This is true since $\{s_2\}$ represents a closed subset in $\mathcal{W}$. Finally, if we choice $\mathcal{W}^0$, we see that $h^{-1}(\mathcal{U}^0) = \mathcal{W}^0$. This is true since $\mathcal{W}^0$ represents a closed subset in $\mathcal{W}$. Therefore, h represents MPNC$\beta^* -$ CM.

Theorem 4.8. If $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ are any two MPNCβ^*- CM, then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a MPNCβ^*- CM.

Proof: it is clear via using Theorem 3.19.

Theorem 4.9. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be an MPNβ^*- CM and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ be a MPNCβ^*- CM , then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a MPNCβ^*- CM.

Proof: From Theorem 3.20, we get the result.

Theorem 4.10. Assume that $h: (\mathcal{W}, \tau_N) \to (\mathcal{U}, \sigma_N)$ be an MPN$I\beta^*-$ CM and $g: (\mathcal{U}, \sigma_N) \to (\mathcal{S}, \beth_N)$ be a MPNCβ^*- CM , then $g \circ h : (\mathcal{W}, \tau_N) \to (\mathcal{S}, \beth_N)$ is a MPNCβ^*- CM.

Proof: Clearly holds.

DOI: https://doi.org/10.54216/IJNS.270101

Remark 4.11. Figure 2 below shows how several classes relate to one another in the mapping $h: (\mathcal{W}, \tau_N) \rightarrow (\mathcal{U}, \sigma_N)$:

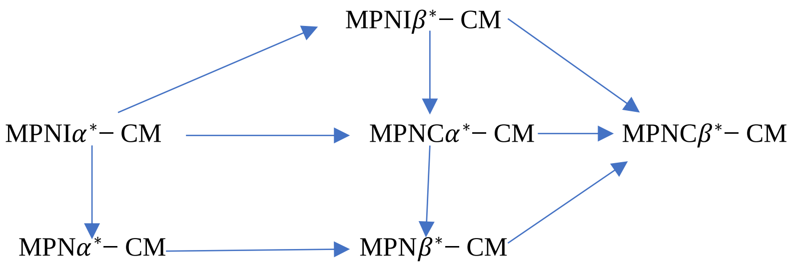

Figure 2. Relationships among various kinds of Contra mappings in MPNTSs.

5. Conclusion

The current work explores the properties of novel forms of neutrosophic continuous mappings. A thorough analysis of their characteristics is part of the investigation, emphasizing their use and significance in the larger framework of neutrosophic topology. Interestingly, the paper explores the relationship between these mappings and the ideas of NIβ^*- CM and NCβ^*- CM. Moreover, study the relationships among these concepts and other concepts such as NIα^*- CM, NCα^*- CM Nα^*- C and Nβ^*- CM. Through this comparison, the study offers important insights into the mathematical structure and possible applications of various kinds of neutrosophic continuous mappings, allowing for a clearer understanding of their behavior and relationships. Finally, the concept of MPNT was generalized to the irresolute and contra functions and some of the properties and relationships that were studied in the previous part were investigated. Further developments in neutrosophic analysis and its utilization in a variety of mathematical domains and allied disciplines are made possible by the relationships between the MPNIβ^*- CM and MPNCβ^*- CM, with other forms of continuous mappings, which improve the conceptual framework.

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DOI: https://doi.org/10.54216/IJNS.270101