Comparative Analysis of Fuzzy Time Series Methods for Predicting Indonesia's Export Performance

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Abstract

This study aims to forecast the export volumes of oil and gas and non-oil and gas sectors in Indonesia, as export volumes reflect the economic condition of a country. The research utilizes data from BPS, spanning from January 2018 to December 2023, and employs the Fuzzy Time Series (FTS) methodology. Six different methods are applied: First-Order FTS Chen, First-Order FTS Cheng, Second-Order FTS Chen, Second-Order FTS Cheng, Markov Chain FTS, and Time-Invariant FTS. FTS is a predictive technique based on fundamental logic and various concepts and rules within fuzzy sets. The prediction accuracy is evaluated using the Mean Absolute Percentage Error (MAPE). The MAPE values for these six methods are compared to determine the most suitable method for this case study. The findings reveal that First-Order FTS Chen achieves an accuracy of 4.07%, First-Order FTS Cheng 4%, Second-Order FTS Chen 1.61%, Second-Order FTS Cheng 1.58%, Markov Chain 3.96%, and Time-Invariant 8.88%. The results indicate that Second-Order FTS Cheng provides the highest accuracy and is effective for predicting the export volumes of oil and gas and non-oil and gas sectors in Indonesia.

Keywords: Fuzzy Time Series; Export energies; Forecasting; Markov Chain; MAPE

1. Introduction

A country's economy significantly affects its stability and sustainability. Economic growth serves as a key indicator for the sustainable development goals (SDGs) in the economic sector. To drive economic progress, international trade activities, including exports and imports, are essential. Enhanced export performance positively affects national sovereignty, contributing significantly to economic growth and the ongoing development process within the country. Furthermore, projections of Indonesia's export and import activities are crucial for setting national economic growth targets in the National Medium-Term Development Plan (RPJMN). The advantages of engaging in export and import activities include boosting the economy, fostering industrial growth, improving citizens' welfare, maintaining good international relations, fulfilling national needs, promoting domestic products, increasing foreign exchange reserves, and stabilizing product prices. Given the critical role of exports in the economic sector, numerous researchers have focused on predicting export trends in Indonesia.

Numerous researchers focus on predicting export prices and volumes. Alam [1] explored forecasting exports and imports using artificial neural networks and the autoregressive integrated moving average (ARIMA) method. Bustami et al. [2] forecast Indonesia's non-oil and gas export values using exponential smoothing methods—specifically Holt's double exponential smoothing and Holt-Winters triple exponential smoothing (additive and multiplicative)—and to identify the most accurate model based on MAPE for use in future policy decision-making. Meyliza et al. [3] studied the export of four-wheeled vehicles using artificial neural networks, identifying an optimal architectural model (4,4,1) with an 82% accuracy rate, 2261 epochs, and an MSE of 0.0081876. Ahmar et

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al. [4] forecast the value of Indonesia's oil and gas exports using the ARIMA method based on monthly export data from January 2010 to March 2022, and to determine the most suitable ARIMA model for accurate future predictions. Zhu et al. [5] forecast oil demand by incorporating both endogenous and exogenous economic factors using machine learning models, and to evaluate the predictive accuracy and stability of these models—particularly in the context of oil-exporting and oil-importing countries facing varying levels of uncertainty. Micocci & Rungi [6] applied machine-learning techniques to predict exports.

Another researcher, Goestjahjanti & Novitasari [7] to analyze the impact of fluctuations in credit interest rates and fuel prices on non-oil and gas exports, and how these factors influence the increase of Foreign Direct Investment (FDI) in Indonesia. Qu et al. [8] applied the Long Short-Term Memory method to predict monthly exports and imports in Shandong Province, obtaining an MSE score of 124.39. Guminta [9] To compare the forecasting performance of the ARIMA and SARIMA models in predicting the non-oil and gas export values of East Java using monthly data from January 2007 to January 2024, and to identify the most accurate model for supporting export planning and regional trade development. Sinaga et al. [10] employed the Box-Jenkins method to forecast the volume of oil and non-oil gas exports in Indonesia, resulting in a MAPE of 8.142%. Nairobi et al. [11] identified the best statistical model for describing and forecasting the export patterns of oil and gas as well as non-oil and gas products in Indonesia from 2008 to 2019, using ARMA-GARCH modeling techniques. Nooraeni et al. [12] develop AIS-based indicators for forecasting Indonesia's monthly export values and to enhance the predictive performance of the ANN model by integrating it with a GA, addressing the limitations of existing methods and data availability

The FTS is an effective method for making predictions, widely applied to various forecasting problems [13]-[15]. Historical data forms patterns that can be used to anticipate future events. Numerous researchers have developed different methods based on FTS. Aliyvev et al. [16] applied FTS to forecast hotel occupancy rates. Avazbeigi [17] explored n-factor FTS for predicting auto industry production. Duru [18] examined a fuzzy integrated logical forecasting model for predicting the dry bulk-shipping index. Gornatiuk et al. [19] applied FTS to predict an enterprise's net income level. Dinata et al. [20] forecast Indonesia's oil and gas, as well as non-oil and gas export and import values using the FTS method, enhanced by Box-Cox transformation to improve forecasting accuracy and support informed economic policy decisions.

Another researcher, Lee et al. [21], explored the use of grey system theory and FTS forecasting for the growth of green electronic materials. Rahman et al. [22] applied ANN and FTS to forecast air quality in monitoring systems. Sambas et al. [23] investigated chaotic behavior and an adaptive type-2 fuzzy controller approach for permanent magnet synchronous generator wind turbine systems. Sukono et al. [24] studied dynamic analysis and adaptive fuzzy control for the fractional-order financial risk chaotic system. Safitri [25] compare the forecasting accuracy of the FTS-MC and the Average Based FTS-MC methods in predicting the closing price of the Jakarta Composite Index, and to determine the more accurate method based on MAPE values. Johansyah et al. [26] analyze a novel hyperchaotic financial system with sinusoidal hyperbolic nonlinearities, applied to the modeling of average profit margin dynamics and designing a nonlinear feedback control strategy and an adaptive neural fuzzy controller to achieve synchronization between hyperchaotic systems. Tsaur & Kuo [27] apply an adaptive fuzzy time series model for forecasting Taiwan's tourism demand, aiming to improve prediction accuracy by incorporating weighted periods and enrollment forecasting values, and to validate the model's effectiveness through performance metrics such as MAPE and RMSE Bilal et al. [28] propose an improved FTS model for forecasting crude oil prices, and to evaluate its performance in comparison with existing FTS models, ARIMA, and machine learning. Several studies related to forecasting using machine learning can be seen in Refs [29]-[33].

This article will explore FTS methods developed by Chen and Cheng, as well as time-invariant and Markov Chain FTS. By leveraging various principles and logic within fuzzy sets, FTS forecasting can analyze historical data patterns to predict future events. Song and Chisom introduced the forecasting technique of FTS. The number of classes in time-invariant FTS is determined using Sturges' formula, while the number of fuzzy classes in Chen and Cheng's methods is determined using an average-based method. The key difference between Chen and Cheng's methods lies in the defuzzification process. This research employs six methods: First-Order FTS Chen, First-Order FTS Cheng, Second-Order FTS Chen, Second-Order FTS Cheng, Time-Invariant FTS, and Markov Chain FTS.

The main contribution and novelty of this work as follows:

a. This study introduces a systematic comparison of six different FTS techniques within a unified framework, enhancing the understanding of how methodological variations impact prediction accuracy.

b. The paper innovatively employs Second-Order FTS models, which consider two previous time points instead of one, significantly reducing prediction errors compared to traditional first-order models in the context of export forecasting.

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c. The study integrates Markov Chain transitions and Time-Invariant fuzzy logic structures into the FTS methodology, offering a hybrid perspective that expands the modeling capabilities for dynamic and fluctuating economic datasets.

The article is organized as follows: Section 1 introduces the importance of forecasting export values for Indonesia's economic development and reviews relevant literature on forecasting methods. Section 2 explains the six fuzzy time series methods used, including their data preparation, fuzzification, and forecasting processes. Section 3 presents the results, comparing the accuracy of each method using MAPE, and identifies the Second-Order FTS Cheng method as the most accurate. Section 4 concludes the study by highlighting the effectiveness of higher-order fuzzy models and their potential use in economic planning.

2. Proposed Methodology

Table 1 presents the historical data on the value of Indonesian exports from January 2018 to December 2023. This table includes the month and the corresponding export value in millions of dollars. The data shows a general trend of increasing export values over the observed period, with notable fluctuations. For example, the export value in January 2018 was 14,576.28 million dollars, and it rose to 22,392.15 million dollars by December 2023. This historical data serves as the basis for the study's application of six different FTS forecasting methods to predict future export values.

Table 1: Data on the value of export of Indonesia

No	Month	Historical Data
1	Jan-18	14576.28
2	Feb-18	14132.38
3	Mar-18	15510.61
…	…	…
70	Oct-23	22146.71
71	Nov-23	21998.25
72	Dec-23	22392.15

The Figure 1 illustrates the step-by-step process of forecasting export values using the FTS methods developed by Chen and Cheng, as well as the Markov Chain and Time-Invariant approaches. The first step involves collecting historical data on export values, followed by the determination of the universe of discourse, which sets the range of data. The universe is defined by calculating the minimum and maximum values (Xmin and Xmax) and adding arbitrary positive numbers (D1 and D2) to facilitate the calculation of the universe set. Once the universe is established, the length of the interval is determined using an average-based method for the Chen and Cheng methods, while the Sturges formula is used for the Time-Invariant method. This interval length is crucial for creating fuzzy sets.

After defining the intervals and fuzzy sets, the next step is to fuzzily the historical data, converting numerical values into fuzzy sets. This is followed by forming FLR and FLRG, which capture the transitions and relationships between different fuzzy sets over time. For the Chen method, this involves forming relationships without weighting, while the Cheng method includes weighting in the relationships. In the Markov Chain approach, the process involves calculating transition probabilities between fuzzy sets and forming a transition probability matrix. Finally, defuzzification is carried out to convert the fuzzy forecasts back into numerical values using specific rules for each method.

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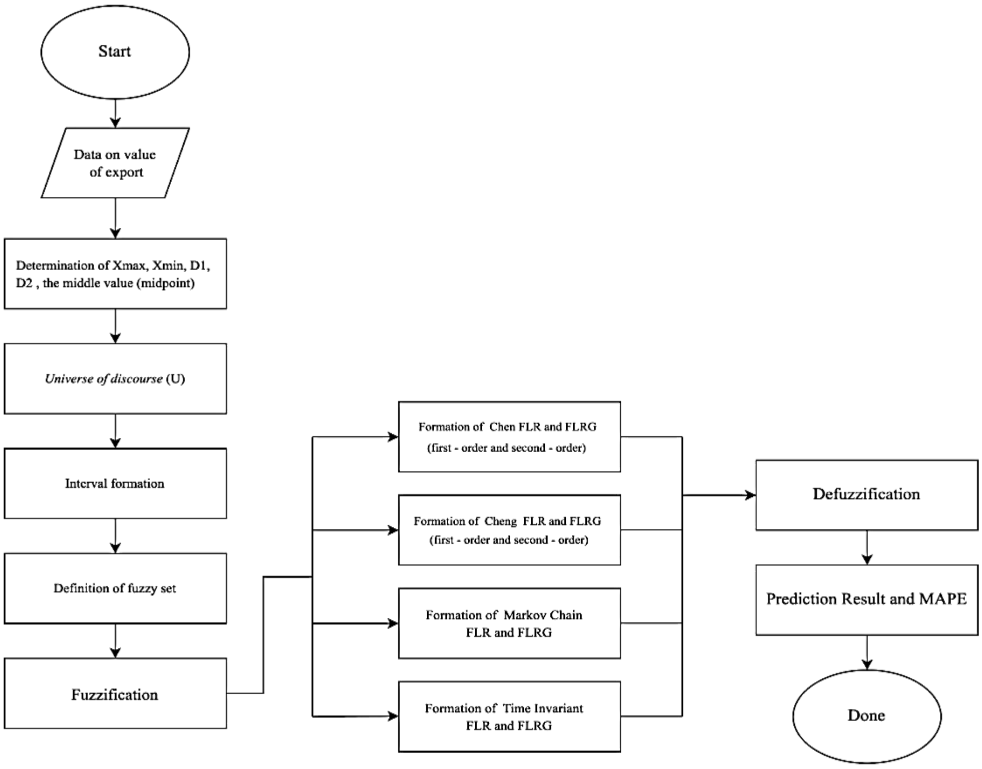

Figure 1. Chen, Cheng, Markov Chain and Time Invariant FTS Calculation Scheme

The controller receives these high-level routing decisions and converts them into low-level flow rules to be enforced on the destination switches. The design benefits from a modular design that has separated concerns between observing the network, making intelligent decisions, and implementing policy. Besides that, DeepBalance works without any modifications to the underlying SDN controller infrastructure, like ONOS or Ryu, with just plain northbound APIs, so no modification to the underlying infrastructure is required. The response loop between observation, learning, and action optimizes adaptation under dynamic network conditions without any human involvement.

The success of our DeepBalance solution heavily depends on a precise and informative description of the network state. We construct a multi-dimensional state vector that captures the evolving state of the network at each decision point. The primary features extracted are normalized ratios of utilization of the links for each link in the topology, switch interface queue depths, per-flow throughput statistics, and inter-switch latency measurements.

The explanation of the Chen and Cheng FTS calculation process:

1. Data on the fluctuation of export value is used as input for Chen and Cheng FTS method
2. Determine Xmax, Xmin, D1, D2 and the middle value (midpoint)

Xmax = maximum data
Xmin = minimum data
D1 and D2 are arbitrary positive numbers chosen by the researcher. Typically, these values are selected to simplify the calculation of the universe set from the historical data.

3. Establish U, the Universe of discourse, which represents the set of historical data ranges

$$U = \{XMin-D1, Xmax+D2\} \quad (1)$$

where $u_i (i = 1,2,3,..., p)$ is an element of the universal set u, and the number marked with the symbol "/" indicates the degree of membership of $\mu_{A_i}(u_i)$ to $A_i (i = 1,2,3,..., p)$, with possible values of 0, 0.5, or 1.

4. The interval length is determined using the average-based length method. Once this length is known, the total number of intervals can be calculated.

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5. With the number of intervals established, we then define the fuzzy sets according to the number of classes. For instance, a fuzzy set with a linguistic value is defined within the universe of discourse *U* as follows:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + ... + 0/u_p$$
$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + ... + 0/u_p \quad (2)$$

…

$$A_p = 0/u_1 + 0/u_2 + 0/u_3 + ... + 0.5/u_{p-1} + 1/u_p$$

Where $u_i (i = 1,2,3,...,p)$ is the element of the universal set *u* and the number marked with the symbol "/" represents the degree of membership $\mu_{A_i}(u_i)$ to $A_i (i = 1,2,3,...,p)$ which the value is 0, 0.5 or 1.

6. The historical data is categorized according to the fuzzy sets defined in step 5.
7. After defining the fuzzy sets, we create the FLR, which represents the relationship between data in year *i* and year *j*. If (*t*−1) = Ai (*t* - 1) = A_i(t−1) = Ai and *F*(t) = AjF(t) = A_jF(t) = Aj, then the FLR is established. Once all historical data is classified into FLRs, we form FLRG, which groups FLRs with the same left side. The key difference between the Chen and Cheng methods lies in the weighting of the FLRG: the Chen method does not apply weighting, while the Cheng method does.
8. Once FLRG is obtained, defuzzification is performed. In Chen and Cheng's Fuzzy Time Series, there are several forecasting rules, which include:

Rule 1. If the result of fuzzification in year t is A_i and there is no fuzzy logic relation, for example if $A_i \rightarrow \varnothing$, then the forecasting result F_{t+1} is m_i . m_i the middle value u_i .

Rule 2. If the fuzzification result in year t is A_i and A_j there is only one FLR in the FLRG, for example if $A_i \rightarrow A_j$, where A_i and A_j are fuzzy sets , then the forecasting result F_{t+1} is m_j

Rule 3. If the fuzzification result on day t is and has several FLRs on the FLRG, for example if $A_i \rightarrow A_{j1}, A_{j2}...A_{jk}$, where $A_i, A_{j1}, A_{j2},...A_{jk}$ is a fuzzy set and the maximum value of the membership function of is $A_{j1}, A_{j2},...A_{jk}$ in the interval $u_{j1}, u_{j2},...u_{jk}$ and the mean of is $u_{j1}, u_{j2},...u_{jk}$, then the forecasting result F_{t+1} is

$$F_{t+1} = \frac{m_{j1} + m_{j2} + ... + m_{jk}}{k} \quad (3)$$

Where *k* is the number of midpoints. To find the mean value in the fuzzy m_i set interval, the following equation can be used:

$$m_t = \frac{upper\ limit + lower\ limit}{2} \quad (4)$$

9. Determining the FLR for the second order involves using two or more historical data points, represented as *F* (*t*−*n*), …, *F*(*t*−2), *F*(*t*−1). For the second order, two historical data points are required to establish the FLR, specifically *F*(*t*−2) and *F*(*t*−1), forming FLRG groups based on observational data. For instance, if *F*(t−2) = Ai, *F*(*t*−1) = Aj and *F*(*t*) = Ak. Then the FLR formed is Ai, Aj→Ak, representing the second-order FLR notation.
10. The error rate of the prediction, or MAPE, can be calculated using the following formula:

$$MAPE = \sum_{i=1}^{n} \left| \frac{x_i - p_i}{x_i} \right| \times 100\% \quad (5)$$

Where X_i = actual data in month i

p_i = forecast value in month i

n = number of data

2.1 Explanation of Markov Chain FTS calculation scheme

1. Data on the fluctuation of export value is used as input for Markov Chain FTS method.
2. Determine D_{max}, D_{min}, D_1, D_2, the middle value (midpoint) X_{min} = Minimum value X_{max} = Maximum value D_1 and D_2 are arbitrary, positive numbers that the researcher chooses. Typically, D_1 and D_2 values are integers that facilitate the calculation of the universe set from the historical data generated.
3. Determine *U*, namely the Universe of discourse or the set of historical data universes, namely

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$$U = \{X_{min} - D_1, X_{max} + D_2\} \quad (6)$$

4. The length of the interval is calculated using Sturges formulation

$$K = 1+3.322 \log(n) \quad (7)$$

where k is the number of intervals and n is the number of observations in the data set.

5. Stage 5 is same with stage 5 of Chen and Cheng method. In step 5, the historical data is categorized in accordance with the fuzzy set.
6. We find FLR that used is $R(t, t-1)$, $F(t)$ depends on $F(t-1)$. We have relation $F(t-1) \rightarrow F(t)$. We calculate FLRG and set R_i for every i.
7. The FLRG calculated by every relation Ri for every fuzzy set i. For example, if there are two relations FLT, i.e. $A_2 \rightarrow A_1$ dan $A_2 \rightarrow A_3$. FLRG is $A_2 \rightarrow A_1, A_3$
8. The defuzzification process is carried out according to the rules of R.C Tsaur with the following steps:

a. Create a transition probability matrix of state R which contains probability information between states from the FLRG in the form

$$R = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}$$

b. From the probability matrix, the forecast value is calculated using the following rules

- If the fuzzy logic relation group of A_j is empty, then the forecast F(t) is m_j which is the midpoint of the interval u_j, then F(t) = mj
- If the fuzzy logic relation group of A_j is one to one ($A_i \rightarrow A_k$) with $p_{ij} = 0$ dan $p_n = 1$ $j \neq k$) then the forecast F(t) is m_k, midpoint u_k, with this equation : $F(t) = m_k P_{ik} = m_k$
- If the fuzzy logic relation group of A_j is one to many ($A_i \rightarrow A_1, A_2, \ldots, A_m$, j-1,2,…,m), if the data y(t-1) at t-1 in state Aj, then the forecast $F(t) = m_j p_{j1} + m_2 p_{j2} + \ldots + m_{j-1} p_{j(n-1)} + y_{(t-1)} p_{ij} + m_{j+1} p_{j(j+1)} + \ldots + m_n p_{jn}$ where $m_1, m_2, \ldots, m_n$ are midpoint u1,u2,…,un.

c. Ajust the trend of the forecast value using the following rules:

- If state Ai transitions from itself at time t−1 as F(t−1) = Ai and then makes an increasing transition to state Aj at time t (where $i < j$, the adjusting trend value Dt is defined as Dt1=(L/2).
- Rule 2. If state Ai transitions from itself at time t−1 as F(t−1) = Ai and then makes a decreasing transition to state Aj at time t (where i < j), the adjusting trend value Dt is defined as Dt1=−(L/2).
- Rule 3. If the current state is Ai at time t−1as F(t−1) = Ai , and then makes a forward jump transition to state Ai+s at time t (where 1 ≤ s ≤ n − i), the adjusting trend value Dt is defined as Dt2=(L/2)s (where 1 ≤ s ≤ n – i), with L being the length into which the universal discourse UUU is divided into n equal intervals.
- Rule 4. If the process is in state Ai at time t−1as F(t−1) = Ai , and then makes a backward jump transition to state Ai−v at time t (where 1 ≤ v ≤ i), the adjusting trend value Dt is defined as Dt2 = −(L/2)v (where 1 ≤ v ≤ i).
- Rule 5. Obtain adjusted forecasting result. If the fuzzy logical relationship group of Ai is one-to-many, and state Ai+1 is accessible from state Ai in which state Ai communicates with Ai , then adjusted forecasting result F0(t) can be obtained as F0(t) = F(t) ± Dt1 ± Dt2 = F(t) ± (L/2)v ± (L/2)v. L is average of differences between interval and v is jump transition.

2.2 Explanation of Time-Invariant FTS calculation scheme

1. Data on the fluctuation of export value is used as input for the Time-Invariant FTS method.
2. Determine D_{max}, D_{min}, D_1, D_2, the middle value (midpoint)

   D_{min} = Minimum difference

   D_{max} = Maximum difference

D_1 and D_2 are arbitrary, positive numbers that the researcher chooses. Typically, D_1 and D_2 values are integers that facilitate the calculation of the universe set from the historical data generated.

3. Determine U, namely the Universe of discourse or the set of historical data universes, namely.

$$U = \{D_{min} - D_1, D_{max} + D_2\}$$

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4. The length of the interval is calculated using Sturges formulation

$$K = 1+3.322 \log(n) \quad (8)$$

where k is the number of intervals and n is the number of observations in the data set.

5. We define the fuzzy set based on the number of classes after obtaining the number of intervals. For instance, if a set is fuzzy and has linguistic value, its definition in the world of speech U is as follows

$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + ... + 0/u_p$

$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + ... + 0/u_p$

…

$A_p = 0/u_1 + 0/u_2 + 0/u_3 + ...0.5/u_{p-1} + 1/u_p$

Where $u_i (i = 1,2,3,..., p)$ is the element of the universal set u and the number marked with the symbol "/" represents the degree of membership $\mu_{A_i}(u_i)$ to $A_i (i = 1,2,3,..., p)$ which the value is 0, 0.5 or 1. In step 5, the historical data is categorized in accordance with the fuzzy set.

6. We found Fuzzy Logic Relationship (FLR). FLR that is based $R(t, t-1)$, $F(t)$ depends on $F(t - 1)$. We have relation $F(t - 1) \rightarrow F(t)$. We calculate Fuzzy Logic Relationship Group (FLRG) and set R_i for every i. For example, if there $A_2 \rightarrow A_1$ dan $A_2 \rightarrow A_3$, union from this equation is $A_2 \rightarrow A_1, A_3$. If R_2 is fuzzy logic $A_2 \rightarrow A_1, A_3$, then $R_2 = A2^T \times A_1 \cup A2^T \times A_3$ (9)

where Ai^T is the transpose vector fuzzy A_i, for $i = 1, 2, \ldots, n$. The symbol $\cup$ is union operator on operation in fuzzy set

7. Fuzzy Logic Relationship Group (FLRG) calculated by every relation R_i for every fuzzy set i. For example, if there are two relations FLT, i.e. $A_2 \rightarrow A_1$ dan $A_2 \rightarrow A_3$. FLRG is $A_2 \rightarrow A_1, A_3$. If R_2 is union fuzzy relation on FLRG $A_2 \rightarrow A_1, A_3$, then $R_2 = A2^T \times A_1 \cup A2^T \times A_3$. where Ai^T is transpose vector fuzzy A_i, for $i = 1,2, \ldots, n$ and symbol "$\cup$" is union operator in fuzzy set.
8. To determine the predictive output, we first state the fuzzy logic relation group based on the known variation from the previous data, i.e. $A_{t-1} = A_j$ and $R_i = R_j$ for $j = 1,2,3, \ldots, n$ so that from the definition of the composition obtained $A_t = A_j \circ R_j$ After the output results are known, the defuzzification process is carried out as follows [11]-[12]:

a. If the membership degree value of the output is 0, then the predicted output value is 0.

b. The middle value of the period in which this value is reached is the projected output value if the membership degree value of the output only has one maximum value.
c. The middle value of the period in which this value is attained is the projected output value if the membership value of the output contains just two or more consecutive maximum values.
d. If the output value is other than the above, then the predicted output, value can be determined by the following formula: $y = \sum \mu A_i\, u_j \cdot m_i \sum \mu A_i\, u_j$. Where $\mu A_i\, u_j$ is the value of the degree of membership u_j to A_i and m_i is the middle value of the interval for $i = 1, 2, \ldots, n$

9. To determine the predictive value can be calculated using the following formula

$$K = 1+3.322 \log(n) \quad (10)$$

Note:
$F(t)$: prediction value at the data t
A_{t-1}: historical data at $t - 1$, y: defuzzification value

3. Results and Discussions

3.1 Calculation of Chen Fuzzy Time Series

Based on the value of export of Indonesia from January 2018 to December 2023 as shown in Table 1, we have Xmin = 10452.63 and Xmax = 27928.71. By using the average based, we have the length of interval is 727.66 and rounded to 730. We choose D1 = 2.63 and D2 =41.29 and we have the universe of discourse is [10450,27970]. With the interval length of 730, we have 24 intervals of fuzzy set, that is [10450,11180], [11180,11910],…, [26510,27240], [24240,27970]. Based on the interval that we created, the fuzzification is completed. The result of the fuzzing is described in Table 2.

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Table 2: Fuzzification

No	**Month**	**Value**	**Fuzzification**
1	Jan-18	14576.28	A6
2	Feb-18	14132.38	A6
3	Mar-18	15510.61	A7
…	…	…	…
…	…	…	…
70	Oct-23	22146.71	A17
71	Nov-23	21998.25	A16
72	Dec-23	22392.15	A17

The fuzzy logic relationship can be formed based on Table 3 and the first order Chen and Cheng FLRG can be seen in Table 4.

Table 3: Fuzzy Logic Relationship

No	Month to month	FLR
1	Jan → Feb-18	A6→A6
2	Feb → Mar-18	A6→A7
3	Mar → Apr-18	A7→A6
…	…	…
69	Sep → Oct-23	A15→A17
70	Oct → Nov-23	A17→A16
71	Nov → Dec-23	A16→A17

Table 4: First Order Chen and Cheng FLRG

No	Chen FLR	Cheng FLR
1	A1→A3	A1→A3
2	A2→A7	A2→A7
3	A3→A1, A5	A3→A1, A5
4	A4→A5, A6, A8	A4→A5, 2A6, A8
5	A5→A3, A4, A5, A6, A7	A5→A3, 2A4, 2A5, 2A6, A7
…	…	…
…	…	…

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No	Chen FLR	Cheng FLR
21	A22→A21	A22→A21
22	A23→A24	A23→A24
23	A24→A22	A24→A22

The second-order fuzzy logical relationship utilizes two consecutive historical data points to form the relationship. This method aims to reduce prediction errors. Referring to Table 5, we can identify the second-order FLR and FLRG as follows.

Table 5: Second-Order FLR

No	Month to month	FLR
1	Jan, Feb → Mar-18	A6, A6→A7
2	Feb, Mar → Apr-18	A6, A7→A6
3	Mar, Apr → May-18	A7, A6→A8
…	…	…
69	Aug, Sep → Oct-23	A16, A15→A17
70	Sept, Oct → Nov-23	A15, A17→A16
71	Oct, Nov → Dec-23	A17, A16→A17

Table 6: Second-Order Chen and Second-Order Cheng FLRG

No	Second Order Chen FLRG	Second Order Cheng FLRG
1	A1, A3→A5	A1, A3→A5
2	A2, A7→A6	A2, A7→A6
3	A3, A1→A3	A3, A1→A3
…	…	…
20	A7, A6→A5, A8	A7, A6→2A5, A8
…	…	…
56	A23, A24→A16	A23, A24→A16
57	A24, A16→A22	A24, A16→A22
58	A24, A20→A20	A24, A20→A20

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Defuzzification if forecasting value is done by identifying the middle value of each interval and then computing the forecast value using the previously described defuzzification criteria. Table 7 described the result of the defuzzification for the first order Chen method, and the first-order Cheng methods. In addition, Table 8 show the defuzzification for the Second-Order Chen method, and the Second-Order Cheng methods.

Table 7: Defuzzification result of the First-Order Chen and the First-Order Cheng method

No	Group	Prediction the First-Order Chen	Prediction the First-Order Cheng
1	A1	12275	12275
2	A2	15195	15195
3	A3	12275	12275
…	…	…	…
21	A22	25415	25415
22	A23	27605	27605
23	A24	23225	23225

Table 8: Defuzzification for the Second-Order Chen method, and the Second-Order Cheng methods.

No	Group	Prediction Second Order Chen	Prediction Second Order Cheng
1	A6, A6	15195	15195
2	A6, A7	15560	15560
3	A7, A6	14830	14465
…	…		
21	A16, A15	22495	22495
22	A15, A17	21765	21765
23	A17, A16	22495	22495

3.2 Calculation of Markov Chain Fuzzy Time Series

We have obtained X min = 10452.63, X max = 27928.71. Using Sturges formula, k = 1 + 3,32 log N, with N = 72, we have k = 7.166 and round to 7, we set D1 = 7026.08 and D2 = 21.29, so the value obtained is U = [10450, 27950]. We have an interval length is 2500 and the number of intervals is 7. Intervals: [10450,12950], [12950,15450], …, [22950, 25450], [25450,27950]. The Table 9 illustrates the transformation of historical export data into fuzzy sets and the subsequent formation of FLR between these sets over time using the Markov Chain FTS model. Each row represents a specific month, listing the historical export value, its corresponding fuzzified value (e.g., A1, A2, A3), and the FLR showing transitions from one fuzzy set to another (e.g., A2 → A3).

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Table 9: Fuzzification and FLR of Markov Chain FTS

No	Month	Data Historical	Fuzzification	FLR
1	Jan-18	14576.28	A2	
2	Feb-18	14132.38	A2	A2→A2
3	Mar-18	15510.61	A3	A2→A3
…	…	…	…	…
70	Oct-23	22146.71	A5	A5→A5
71	Nov-23	21998.25	A5	A5→A5
72	Dec-23	22392.15	A5	A5→A5

The Table 10 illustrates the transitions between different fuzzy sets in the Markov Chain FTS model. Each row represents a specific fuzzy set (e.g., A1, A2, A3) and lists the possible transitions to other sets based on historical data patterns. For example, if the row for fuzzy set A2 lists transitions to A1 and A3, it indicates that when data is categorized in A2, it tends to move to A1 or A3 in the subsequent time. This table is crucial for understanding how the model captures the likelihood of transitions between fuzzy sets, providing insights into the typical behavior and movement of data. By identifying these fuzzy logical relationships, the table helps in evaluating the accuracy and reliability of the Markov Chain FTS model, ensuring that the patterns identified in historical data can be effectively used for accurate future forecasts.

Table 10: Markov Chain FLRG

No	Markov Chain FLRG
1	A1→2A1, 3A2, A3
2	A2→3A1, 18A2, 4A3, A4
3	A3→A1, 4A2, A3, A4
4	A4→A3, 2A4, 3A5
5	A5→A4, 12A5, A6, 2A7
6	A6→A4, A5, 3A6
7	A7→A5, A6, 3A7

The transition probability matrixs is as follows

Pij	A1	A2	A3	A4	A5	A6	A7
A1	0.333333	0.5	0.166667				
A2	0.115385	0.692308	0.153846	0.038462			
A3	0.142857	0.571429	0.142857	0.142857			

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A4			0.166667	0.333333	0.5		
A5				0.0625	0.75	0.0625	0.125
A6				0.2	0.2	0.6	
A7					0.2	0.2	0.6

The Table 11 presents a detailed comparison between actual historical export data and the predicted values generated by the Markov Chain FTS model from January 2018 to December 2023. It includes columns for each month's actual export values, the initial forecasts, any necessary decreasing transition adjustments, and the final adjusted predictions. The early prediction column shows initial values based on historical data and transition probabilities, while the decreasing transition column accounts for significant data shifts, refining the initial forecasts. The final prediction column displays the adjusted forecasts, providing a more accurate representation by considering identified trends.

Table 11: Forecasting results of Markov Chain FTS

No	Month	Data Historical	Early Prediction	Decreasing Transition	Final Prediction
1	Jan-18	14576.28			
2	Feb-18	14132.38	14084.35	0	14084.35
3	Mar-18	15510.61	13777.03	1250	15027.03
…	…	…	…	…	…
49	Jan-22	19143.17	22818.29	-1250	21568.29
50	Feb-22	20489.07	20014.39	1250	21264.39
…	…	…	…	…	…
70	Oct-23	22146.71	21609.91	0	21609.91
71	Nov-23	21998.25	22660.03	0	22660.03
72	Dec-23	22392.15	22548.69	0	22548.69

3.3 Calculation of Time Invariant Fuzzy Time Series

Based on these data, maximum difference is 6097.66 and minimum difference is -5822.00. By using Sturges formula, we have k =1+3.322*log 72 = 7,166 and rounded to 7. Number of intervals for time -invariant method is 7. The length for time-invariant is 1703. The selected D_1 and D_2 are D_1 = 7.01 and D_2 = 35.34 so the value obtained is U = [-5830, 6133]. These intervals are [-5830, -4127], [-4127, -2417], …, [2713, 4423], [4423, 6133].

The Table 12 illustrates the process of converting historical export data into fuzzy sets and subsequently forming FLR. Fuzzification involves categorizing continuous numerical data into discrete intervals, each represented by a fuzzy set (e.g., A1, A2, A3, etc.). Each interval corresponds to a range of data values, and a membership function assigns a degree of membership to each data point within an interval. For example, if the historical export value in February 2018 is 14,132.38, it might fall within the interval for fuzzy set A2, thus being fuzzified into A2. This transformation is essential for converting raw data into a format that the fuzzy time series model can utilize for pattern recognition and forecasting.

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Table 12: Fuzzification and FLR of time-invariant fuzzy time series

No	Date	Historical Data	Difference	Fuzzification	FLR
1	Jan-18	14576.28			
2	Feb-18	14132.38	-443.9	A4	
3	Mar-18	15510.61	1378.23	A5	A4→A5
…	…	…	…	…	…
…	…	…	…	…	…
70	Oct-23	22146.71	1400.16	A5	A3→A5
71	Nov-23	21998.25	-148.46	A4	A5→A4
72	Dec-23	22392.15	393.9	A4	A4→A4

The formation of FLR follows fuzzification, showing how data points transition from one fuzzy set to another over consecutive time periods. An FLR is defined by examining the fuzzified data for consecutive months. For instance, if the export data in January 2018 is fuzzified as A2 and the data in February 2018 is fuzzified as A3, the resulting FLR is A2 → A3. This relationship indicates that when the data is in fuzzy set A2 at one time point, it transitions to fuzzy set A3 at the next time point. The table compiles multiple FLRs into FLRG for each fuzzy set. For example, if fuzzy set A2 transitions to A3, A4, and A5 over different time points, the FLRG for A2 would be A2 → {A3, A4, A5}. This structured representation helps in understanding the data's behavior and is crucial for accurate forecasting in the time-invariant FTS model.

Table 13: Time-invariant FLRG

No	FLRG
1	A1→ A5, A7
2	A2→A4, A5, 2A6
3	A3→2A3, 7A4, 8A5
4	A4→A1, A2, 8A3,10A4, 5A5, A6
5	A5→ A1, 3A2, 6A3, 4A4, A5, A7
6	A6→ A3, 2A4
7	A7→2A4

The Table 13 delineates the fuzzy logical relationships formed from historical data intervals in a time-invariant FTS model. Each row represents a specific fuzzy set (e.g., A1, A2, A3, etc.) and lists its transitions to other fuzzy sets, such as A1 → A5 and A1 → A7. This table provides valuable insights into data patterns by showing how data points transition from one fuzzy set to another over time. For instance, if A2 frequently transitions to A4 and A5, it indicates a strong relationship between these sets, suggesting that the values often move within these ranges. Understanding these transitions is crucial for making accurate predictions in the time-invariant FTS model, as it highlights the typical behavior of the data.

Table 14: Defuzzification value of time-invariant FTS

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No	Group Fuzzy	Defuzzification value
1	A1	2998
2	A2	1858
3	A3	148
4	A4	-706.417
5	A5	-114.538
6	A6	-707
7	A7	148

The Table 14 provides the crisp, defuzzied values for each fuzzy set used in the time-invariant FTS model. Each row represents a different fuzzy set class and provides its corresponding defuzzied value, typically the midpoint of the interval associated with that class. These defuzzied values are essential for understanding the forecast accuracy of the time-invariant FTS model. For example, if the defuzzied value for A1 is 2,998, this indicates that when data falls within the range defined by A1, the predicted value is 2,998. Accurate defuzzification values suggest that the model effectively captures the underlying patterns in the historical data and can make reliable predictions, illustrating the model's capability to translate fuzzy data into precise and actionable forecasts.

Table 15 presents the defuzzification results for the First-Order FTS Chen and First-Order FTS Cheng methods. This table lists the fuzzy set classes (e.g., A1, A2, A3, etc.) and their corresponding forecasted values for each method, allowing for a direct comparison of their defuzzification processes. For each fuzzy set class, the table provides the defuzzied value, which is the midpoint of the interval associated with that class. By examining these predicted values, one can assess the differences and similarities in how each method converts fuzzy values into crisp outputs. This comparison is essential for evaluating the reliability and accuracy of each forecasting method, as consistent and accurate defuzzied values suggest a more reliable approach.

Table 15: Defuzzification Results of the First-Order Chen and the First-Order Cheng Method

No	Class	First-Order Chen	First-Order Cheng
1	A1	12275	12275
2	A2	15195	15195
3	A3	12275	12275
4	A4	14708,33	14647,5
…	…	…	…
20	A21	27605	27605
21	A22	25415	25415
22	A23	27605	27605
23	A24	23225	23225

Table 16 details the defuzzification results for the Second-Order FTS Chen and Second-Order FTS Cheng methods, which consider two consecutive historical data points to enhance prediction accuracy. The table includes

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columns for the fuzzy set classes and the defuzzied forecast values for each class using both methods, with each row representing a different combination of fuzzy set classes (e.g., A1A3, A2A7, etc.). This structure allows for a comparative analysis of the second-order defuzzification processes, highlighting any differences in the predicted values. Accurate defuzzification across various class combinations indicates a robust forecasting approach, with the Second-Order Cheng method showing superior accuracy in predictions, as noted in the article.

Table 16: Defuzzification results of the Second-Order Chen and the Second-Order Cheng Method

No	Classes	Second-Order Chen	Second-Order Cheng
1	A1, A3	13735	13735
2	A2, A7	14465	14465
…	…	…	…
15	A6, A5	13978,33333	13978,33333
16	A6, A6	15195	15195
…	…	…	…
57	A24, A16	26145	26145
58	A24, A20	24685	24685

Table 17 presents a detailed comparison of the forecasted export values using six different FTS methods: First-Order FTS Chen, First-Order FTS Cheng, Second-Order FTS Chen, Second-Order FTS Cheng, Markov Chain FTS, and Time-Invariant FTS. The table includes columns for each month from January 2018 to December 2023, showing the actual historical export values alongside the predicted values from each of the six methods. This structure allows for a comprehensive examination of how well each forecasting method aligns with the actual export data. The Table 17 serves as a crucial tool for comparing the forecasting performance of the six methods. By examining the differences between the actual export values and the predicted values for each month, readers can assess which methods provide the most accurate predictions. For example, the table shows how the Second-Order FTS Cheng method consistently produces forecasts that are closest to the actual data, demonstrating its superior accuracy compared to the other methods. This side-by-side comparison is essential for identifying the most effective method for forecasting export values in Indonesia.

Table 17: Forecasting Results of First-Order Chen, First-Order Cheng, Second-Order Chen, Second-Order Cheng, Markov Chain and Time Invariant FTS

No	Date	Historical Data	First-Order Chen	First-Order Cheng	Second-Order Chen	Second-Order Cheng	Markov Chain	Time Invariant
1	Jan-18	14576,28						
2	Feb-18	14132,38	13978,3333	14059,4444			14084,34769	
3	Mar-18	15510,61	13978,3333	14059,4444	15195	15195	15027,03231	12982,06333
4	Apr-18	14496,24	15681,6666	15377,5	15560	15560	13494,37286	16774,30154
…	…	…	…	…	…	…	…	…
7	Jul-18	16284,72	14708,3333	14647,5	15925	15925	16697,24667	11543,14
…	…	…	…	…	…	…	…	…

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No	Date	Historical Data	First-Order Chen	First-Order Cheng	Second-Order Chen	Second-Order Cheng	Markov Chain	Time Invariant
70	Oct-23	22146,71	22495	22495	22495	22495	21609,9125	19643,12
71	Nov-23	21998,25	21035	21035	21765	21765	22660,0325	23432,33154
72	Dec-23	22392,15	22495	22130	22495	22495	22548,6875	21143,37333

Figure 2 illustrates the forecasting plots generated by six different FTS methods: First-Order FTS Chen, First-Order FTS Cheng, Second-Order FTS Chen, Second-Order FTS Cheng, Markov Chain FTS, and Time-Invariant FTS. Each plot within the figure compares the predicted export values against the actual historical export data from January 2018 to December 2023, providing a visual representation of the forecast accuracy for each method. This comparison helps to identify how closely each method's predictions align with the observed data trends.

The plots for First-Order FTS Chen and Cheng display the forecasted values using the first-order fuzzy logic approach, highlighting their respective prediction accuracies. Similarly, the Second-Order FTS Chen and Cheng plots show the forecasts produced by these more advanced models, which consider two consecutive historical data points to reduce errors and improve accuracy. The Markov Chain FTS plot represents the forecasts made using transition probabilities between different states derived from historical data, while the Time-Invariant FTS plot shows predictions by treating historical data intervals consistently over time.

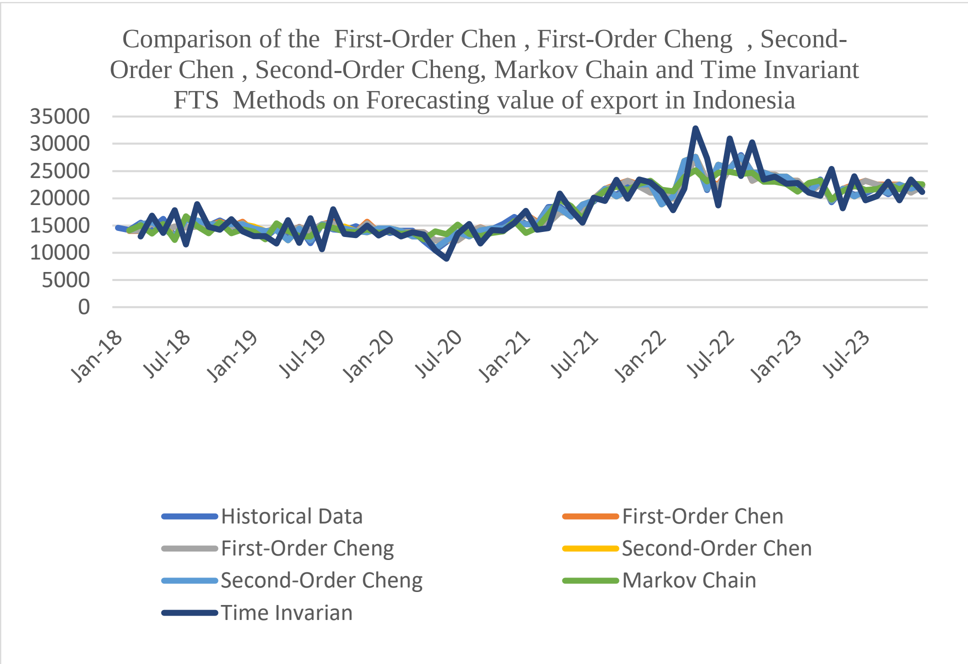

Figure 2. Forecasting plot using the first-order Chen, the first-order Cheng, the second-order Chen, the second-order Cheng, Markov Chain and Time Invariant FTS

In addition, Figure 2 provides a comparative visual representation of the six FTS methods' forecasting performance, illustrating their predictive capabilities and accuracy. The figure demonstrates how well each method captures the trends and patterns in the historical export data, with the Second-Order FTS Cheng method emerging as the most accurate based on the study's findings. This visual comparison is crucial for identifying the most suitable FTS method for forecasting Indonesia's export values, offering valuable insights for policymakers and stakeholders in economic planning.

4. Conclusion

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This study has explored various methods of FTS forecasting to predict the export volumes of oil and gas, as well as non-oil and gas sectors in Indonesia. By leveraging the principles and logic inherent in fuzzy sets, FTS effectively analyzed historical data patterns to anticipate future trends. The six methods applied in this research included First-Order FTS Chen, First-Order FTS Cheng, Second-Order FTS Chen, Second-Order FTS Cheng, Time-Invariant FTS, and Markov Chain FTS. The findings revealed that among these methods, Second-Order FTS Cheng demonstrated the highest accuracy with a MAPE of 1.58%. This was followed closely by Second-Order FTS Chen with a MAPE of 1.61%, indicating both methods' suitability for predicting export volumes in this context. In comparison, First-Order FTS Chen and Cheng yielded MAPEs of 4.07% and 4%, respectively. Markov Chain FTS and Time-Invariant FTS showed less accuracy, with MAPEs of 3.96% and 8.88%, respectively. These results suggest that higher-order FTS methods, particularly Second-Order FTS Cheng, are highly effective for forecasting export volumes in Indonesia. The study underscores the potential of fuzzy time series models in economic forecasting, offering valuable insights for policymakers and stakeholders in setting and achieving national economic growth targets.

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