Neutrosophic EWMA and DEWMA control chart on Exponential and Transformed Exponential Distributions

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Abstract

The sophisticated statistical methods known as Bayesian EWMA and DEWMA control charts are intended to track process performance and identify changes in data over time. They improve the capacity to monitor minute changes in the process by combining conventional smoothing methods with Bayesian inference. By integrating the idea of neutrosophic approaches into Bayesian EWMA and DEWMA models, the suggested approach seeks to address and get beyond this restriction. In this study, neutrosophic approaches are utilized to provide the manufacturing process with two tolerance limits instead of a set value for upper and lower control limits, particularly when all observations are uncertain, imprecise, or fuzzy. By combining the Exponential, Inverse Rayleigh, and Weibull distributions, five symmetric loss functions are examined while taking uniform prior into account. Additionally, for mean, variance, and control limits of the proposed work have been derived. Simulation studies were conducted and compared with previous work as well as all projected works. This study significantly advances the subject of control chart technique, especially when it comes to managing hard, vast, and complicated information.

Keywords: EWMA; DEWMA; Bayesian approach; Loss function; Monte Carlo Simulation; Average Run Length

1 Introduction

The Exponentially Weighted Moving Average (EWMA) control chart is a statistical process control tool tracking process variation across time by giving more weight to recent data points. Unlike traditional control charts, it is especially helpful in spotting minute variations in the mean of the process. Its enhanced sensitivity to minor changes makes it a valuable instrument for ensuring quality control and identifying process enhancements in many different fields.

EWMA control charts have been used from the outset[20] to help identify small process changes. Further developments improved tracking of slow process changes by means of the Double Exponential Weighted Moving Average (DEWMA) chart.[24] The Triple Exponentially Weighted Moving Average (TEWMA) chart, outperformed traditional control charts in spotting minor to moderate changes and maintaining process stability,[4] furthered these advances.

The Neutrosophic Exponential Weighted Moving Average (NEWMA) X-bar control chart was developed[6] to enhance process monitoring in erratic conditions. Using neutrosophic thinking helps this method effectively manage uncertain or imprecise data and improve the identification of small changes. Additional developments resulted in the development of neutrosophic double and triple exponentially weighted moving average (NDEWMA and NTEWMA) control charts, so enhancing shift detection in uncertain settings.[23] Moreover, using neutrosophic statistics—which characterize uncertainty and imprecision using neutrosophic numbers[9]—have been investigated the scale effect and anisotropy of rock joint roughness coefficients. This work gives a more realistic representation of rock joint behavior, so advancing engineering applications in rock mechanics.

Two approaches of statistical inference are classical and bayesian.While the Bayesian strategy depends on the prior information as well as the sample observations, the classical approach is based only on the observations. The Bayesian approach to estimate the unknown parameters in a quality control setting was first presented by Girshick and Rubin.[12] Very crucial for estimating the unknown parameter is the combination of the sample data and prior knowledge of the distribution using the Bayesian method. The Bayesian method now includes predictive distributions to enhance accuracy and decision-making in quality monitoring by means of control chart limits.[19] Using Bayesian inference to enhance quality control decision-making, studies examining the performance of the Bayesian Exponential Weighted Moving Average (BEWMA) chart have shown how well it can detect small process changes.

Bayesian estimation has been applied on many probability distributions in order to enhance uncertainty analysis and parameter estimate. Bayesian methods provide a structure for estimating the parameters of the exponential distribution,[2] so helping one to better grasp its behavior under uncertainty. The Weibull distribution has also been applied using bayesian inference, so enhancing parameter estimation in dependability[5] in life data modeling. Bayesian methods have also been applied for the inverse Rayleigh distribution with an emphasis on parameter estimation and dependability function analysis, which are fundamental for failure time modeling in dependability studies.[11] More study in this discipline led to the development of an X-bar control chart based on the inverse Rayleigh distribution under recurrent group sampling. This chart provides a good approach to find process modifications in systems defined by this distribution.[11]

Emphasizing process changes, a control chart especially meant to track quality attributes based on an exponential distribution was developed.[3] Particularly for systems with exponential distribution models, the paper underlines how well it performs for quality monitoring. Developed for the Weibull distribution,[13] a Moving Average Exponential Weighted Moving Average (MA-EWMA) chart helps to enhance the identification of changes in Weibull-distributed systems. This approach performs particularly effectively for identifying small process modifications in quality control. Further developments in process monitoring methods led to the proposal of a memory-type Max-EWMA control chart for Weibull processes under Bayesian theory.[17] This method offers a consistent instrument for dependability dataset monitoring and enhances shift detection by including Bayesian inference.

The study on Bayesian EWMA control charts for process monitoring based on exponential and transformed exponential distributions integrates Bayesian inference, so providing a more accurate and reliable method of quality control in scenarios involving these distributions.[16] This helps to improve the detection of changes in process behavior. The Weibull distribution is often used in engineering applications including microelectronics, where knowledge of component failure rate is crucial, to depict failure times and life statistics. Using Bayesian inference,[8] researchers have considered past knowledge on the behavior of the system and data ambiguity.

Simplicity and symmetric character of the Squared Error Loss Function (SELF) make it the most often used loss function. SELF penalizes deviations equally regardless of whether they indicate overestimations or underestimations; defined as $L(\theta, \hat{\theta}) = (\theta - \hat{\theta})$, where θ represents the true parameter and $\hat{\theta}$ is the estimate. This symmetric penalty structure makes SELF especially appropriate for circumstances when deviations on either side of the aim are equally unwelcome. SELF is frequently used in quality control in control chart design and process monitoring since it enables the computation of limits that reduce the average squared error between the estimated and real process parameters. SELF-driven control strategies guarantee that any departure from the target is promptly corrected to maintain quality standards in sectors including pharmaceuticals, where exact dosage concentrations are vital, or automotive manufacture, where torque precision is required for safety[1522]

SELF works well in many cases, but in others the expenses of mistakes are not symmetric. By letting various weights be assigned to overestimations and underestimations, the Precautionary Loss Function (PLF)[18] reflects the varying risk connected with each direction of inaccuracy. In sectors like aerospace, where overestimating a component's performance could cause catastrophic failures, PLF is especially beneficial when the repercussions of one type of error are more severe than the others[14] . In this regard, PLF assigns a larger penalty to overestimation mistakes therefore allowing a more cautious approach. This asymmetric method guarantees that although allowing minor underestimations that might be less dangerous, quality control systems are more sensitive to hazards that might threaten safety.

Introduced by Zellner (1988)[27] , the Weighted Balanced Loss Function (WBLF) expands the concept of asymmetry by including several goals inside a single framework. This loss function lets decision-makers give distinct values to several elements of the loss, so balancing conflicting objectives in quality control. WBLF[21] can help to customize decision rules that give particular elements of the process top priority over others, for instance in semiconductor manufacture where both process uniformity and the elimination of false alarms are vital.[15] Especially in sectors with several performance criteria that must be simultaneously optimized, WBLF allows a more flexible and operationally relevant method of process monitoring.

Derived from information theory, the General Entropy Loss Function (GELF) approaches problems relative rather than absolute. GELF[10] is especially helpful in settings like chemical production or financial risk management, where proportionate changes are more important than absolute differences and percentage deviations from goal values are more meaningful.[26] GELF penalizes mistakes depending on the estimated to true value ratio, therefore enabling increased sensitivity to undervalues—often more expensive than overestimations. In pharmaceutical stability testing, for example, underestimating the degradation rate of a medicine could lead to dangerous shelf-life recommendations whereas overestimating the rate would be less dangerous. Using GELF, quality managers can apply more cautious decision guidelines considering these asymmetric risk structures.[15]

Finally, although concentrating more on the information-theoretic side of the decision problem, the Entropy Loss Function (ELF) is intimately tied to GELF. Using logarithmic functions, ELF gauges the difference between estimated and actual values, so it is very sensitive to proportional errors and appropriate in high-stakes situations when relative performance counts more than absolute deviance. In nuclear power plant monitoring, for instance, underestimating important factors like radiation levels could have disastrous results whereas often overestimating them results in less damaging preventative measures. ELF makes it possible to include such risk asymmetries into the quality control process, therefore guaranteeing that decisions match the crucial character of the current tasks.[26] In Bayesian decision-making systems, where it enables to modify posterior decision rules depending on prior risk structures and observed data, this loss function has been progressively embraced.[7]

The proposed work deals with Neutrosophic Bayesian EWMA Control charts for Exponential distribution, Inverse Rayleigh distribution, Weibull distribution. The derivation of mean , variance and control limits were done for the above mentioned control charts . Random samples are drawn from the exponential, weibull and inverse rayleigh distributions and applied on the above mentioned control charts to detect the out of control.

2 Background

2.1 Neutrosophic $\bar{X}$ EWMA Control Chart

In a Neutrosophic $\bar{X}$ EWMA (NEWMA) control chart,[6] $\bar{Y}_{N,i}$ is known as the quality character of interest following neutrosophic normal distribution with a variance $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$ and a mean $\mu_N \in \{\mu_L, \mu_U\}$. The neutrosophic sample size is n_N. The NEWMA statistic is defined, per[6] is

$$Z_{N,i} = \lambda_N \bar{Y}_{N,i} + (1-\lambda_N) Z_{N,i-1}$$

Here, the neutrosophic smoothing constant is $\lambda_N \in \{\lambda_L, \lambda_U\}$, where $[0,0] \leq \lambda_N \leq [1,1]$.
The statistic $Z_{N,i} \in \{Z_{L,i}, Z_{U,i}\}$ is plotted against the neutrosophic upper and lower control limits, which are $LCL_N \in \{LCL_{N,L}, LCL_{N,U}\}$ and $UCL_N \in \{UCL_{N,L}, UCL_{N,U}\}$.

$$LCL_N/UCL_N = \mu_{N,0} \pm L_N \sigma_N \sqrt{\frac{\lambda_N}{n_N(2-\lambda_N)}}$$

$L_N \in \{L_{N,L}, L_{N,U}\}$ is the neutrosophic control limit coefficient.

2.2 Neutrosophic $\bar{X}$ Double EWMA Control Chart

Neutrosophic Double EWMA (NDEWMA) control chart[23] incorporate two neutrosophic EWMA control chart as a single chart. $\bar{Y}_{N,i}$ follows normal distribution mean $\mu_N \in \{\mu_L, \mu_U\}$ and variance $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$ where n_N is the neutrosophic sample size. The NDEWMA statistic as in[24] is defined as,

$$Z_{N,i} = \lambda_N \bar{Y}_{N,i} + (1-\lambda_N) Z_{N,i-1}$$
$$DZ_{N,i} = \lambda_N Z_{N,i} + (1-\lambda_N) DZ_{N,i-1}$$

Here $\lambda_N \in \{\lambda_L, \lambda_U\}$ is the neutrosophic constant where $[0,0] \leq \lambda_L \leq [1,1]$ and $[0,0] \leq \lambda_U \leq [1,1]$. The statistic $DZ_{N,i} \in \{DZ_{L,i}, DZ_{U,i}\}$ is plotted against the neutrosophic upper and lower control limits say,$LCL_N \in \{LCL_{N,L}, LCL_{N,U}\}$ and $UCL_N \in \{UCL_{N,L}, UCL_{N,U}\}$ respectively and is given by

$$LCL_N/UCL_N = \mu_{N,0} \pm L_N \sigma_N \sqrt{\frac{\lambda_N(2-2\lambda_N+\lambda_N^2)}{n_N(2-\lambda_N)^3}}$$

L_N is the neutrosophic control limit coefficient which helps in fixing the average run length value at a pre-specified level.

2.3 Bayesian EWMA control chart

A normal population with a mean of μ and variance of σ^2 is represented by a collection of random samples $x_1, x_2, ..., x_n$. As stated by the Exponentially Weighted Moving Average (EWMA) statistic,

$$Z_i = \lambda Y_i + (1-\lambda) Z_{i-1}$$

where λ is a smoothing parameter that lies within the range $0 \leq \lambda \leq 1$. The initial value Z_0 is typically set as either the target mean μ_0 or the mean of previous data points. Under different loss functions, the EWMA statistic can be generalized as:

$$Z_{LF,i} = \lambda \hat{\mu}_{LF} + (1-\lambda) Z_{LF,i-1}$$

where $\hat{\mu}_{LF}$ represents the mean estimator under various loss functions. Expression of the asymptotic control limits for the Bayesian EWMA control chart in a normal distribution environment is:

$$UCL/LCL = \mu_{LF} \pm L\sigma_{LF} \sqrt{\frac{\lambda}{(2-\lambda)}}$$

2.4 Bayesian Double EWMA control chart

Let $x_1, x_2, ..., x_n$ be a collection of random samples taken from a normal population with variance σ^2 and mean μ. The Double Exponentially Weighted Moving Average (DEWMA) statistic is:

$$Z_i = \lambda Y_i + (1-\lambda) Z_{i-1}$$
$$DZ_i = \lambda Z_i + (1-\lambda) DZ_{i-1}$$

The smoothing constant λ lies within the range $0 \leq \lambda \leq 1$. The initial value DZ_0 is typically chosen as either the target mean (μ_0) or the mean of past data points. Under various loss functions, the DEWMA statistic is expressed as:

$$Z_{LF,i} = \lambda \hat{\mu}_{LF} + (1-\lambda) Z_{LF,i-1}$$
$$DZ_{LF,i} = \lambda Z_{LF,i} + (1-\lambda) DZ_{LF,i-1}$$

where $\hat{\mu}_{LF}$ represents the mean estimator under different loss functions. Expression of the asymptotic control limits for the Bayesian DEWMA[1] control chart in a normal distribution environment is:

$$UCL/LCL = \mu_{LF} \pm L\sigma_{LF}\sqrt{\frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3}}$$

For the above mentioned two bayesian control charts, the Average Run Length (ARL) at a predetermined level is determined by the control limit coefficient, L. The estimated value and standard deviation of $\hat{\mu}_{LF}$ under various loss functions are shown by the expressions $\mu_{LF}\ and\ \sigma_{LF}$.

2.5 Prior and Posterior distribution

The original beliefs or understanding of a parameter prior to any data being seen are reflected in a prior distribution. The posterior distribution is obtained by combining the prior and the probability of the observed data in Bayesian inference. This allows parameter estimates to be continuously improved as new data becomes available. Priors can be classified as either non-informative, which conveys little to no prior information about the parameter, or informative, which incorporates past knowledge. A non-informative prior denotes a lack of prior information, whereas an informative prior offers particular insights about the unknown parameter. For an unknown parameter θ ,which is defined as $p(\theta) = \frac{1}{\theta}$, we employ a non-informative prior in this case, more precisely a uniform prior. Next, the matching posterior distribution is provided by:

$$p(\theta/x) = \frac{p(x/\theta)p(\theta)}{\int_0^\infty p(x/\theta)p(\theta)d\theta}$$

3 Proposed Neutrosophic Bayesian EWMA Control Chart

Let $Y_{N,i}$ the quality character of interest follows a neutrosophic normal distribution with an expected value $\mu_N \in \{\mu_L, \mu_U\}$ and variance $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$. The NEWMA statistic is expressed as,

$$Z_{N,i} = \lambda_N Y_{N,i} + (1-\lambda_N)Z_{N,i-1}$$

where $\lambda_N \in \{\lambda_{N,L}, \lambda_{N,U}\}$ represents the neutrosophic smoothing parameter. The Neutrosophic Bayesian EWMA statistic under a loss function is formulated as:

$$Z_{NLF,i} = \lambda_N \hat{\mu}_{NLF} + (1-\lambda_N)Z_{NLF,i-1}$$

where $\hat{\mu}_{NLF}$ denotes the mean estimator under a given loss function. The control limits for Neutrosophic Bayesian EWMA control chart's under various loss functions are as follows:

$$UCL_N/LCL_N = \mu_{NLF,0} \pm L_N\sigma_{NLF}\sqrt{\frac{\lambda_N}{(2-\lambda_N)}}$$

The neutrosophic control limit coefficient is represented by $L_N \in \{L_{N,L}, L_{N,U}\}$, and the expected value and standard deviation of $\hat{\mu}_{NLF}$ for different loss functions are represented by $\mu_{NLF,0}\ and\ \sigma_{NLF}$.

4 Proposed Neutrosophic Bayesian Double EWMA Control Chart

Let $Y_{N,i}$ follows a neutrosophic normal distribution with an expected value $\mu_N \in \{\mu_L, \mu_U\}$ and variance $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$. The NDEWMA statistic is defined as:

$$Z_{N,i} = \lambda_N Y_{N,i} + (1-\lambda_N)Z_{N,i-1}$$
$$DZ_{N,i} = \lambda_N Z_{N,i} + (1-\lambda_N)DZ_{N,i-1}$$

where $\lambda_N \in \{\lambda_{N,L}, \lambda_{N,U}\}$ is the neutrosophic smoothing parameter. The Neutrosophic Bayesian EWMA statistic under a loss function is given by:

$$Z_{NLF,i} = \lambda_N \hat{\mu}_{NLF} + (1-\lambda_N) Z_{NLF,i-1}$$
$$DZ_{NLF,i} = \lambda_N Z_{NLF,i} + (1-\lambda_N) DZ_{NLF,i-1}$$

$\hat{\mu}_{NLF}$ represents the mean estimator under the chosen loss function.

The control chart's control limits for various loss functions are as follows:

$$UCL_N/LCL_N = \mu_{NLF,0} \pm L_N \sigma_{NLF} \sqrt{\frac{\lambda_N(2-2\lambda_N+\lambda_N^2)}{(2-\lambda_N)^3}}$$

Here, $L_N \in \{L_{N,L}, L_{N,U}\}$ denotes the neutrosophic control limit coefficient, while $\mu_{NLF,0}\ and\ \sigma_{NLF}$ represent the mean and standard deviation of $\hat{\mu}_{NLF}$ under various loss functions.

5 Proposed Neutrosophic Bayesian EWMA Control Chart Using Various Loss Functions

5.1 For Exponential Distribution

This section looks at the exponential distribution for five different loss functions using various non-informative priors. An exponential distribution's probability density function (PDF) may be found using

$$f(x,\theta) = \frac{e^{-\frac{x}{\theta}}}{\theta}, \quad x > 0$$

where the exponential distribution mean is denoted by $\theta > 0$. For a given random sample $x_i (i = 1, 2, \ldots, n)$, the likelihood function is written as follows:

$$L(x \mid \theta) = \frac{e^{-\frac{s}{\theta}}}{\theta^n}$$

where $s = \sum_{i=1}^{n} x_i$ which follows a Gamma distribution. Using a uniform prior, the posterior distribution for the exponential distribution is obtained as,

$$f(\theta/s) = \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)}\ ;\ (n-1) > 0\ ,\ s > 0$$

The following is the format of the control limits for the Neutrosophic Bayesian EWMA control chart under an exponential distribution.

$$UCL_N/LCL_N = \mu_{NLF,0} \pm L_N \sigma_{NLF} \sqrt{\frac{\lambda_N}{(2-\lambda_N)}}$$

Likewise, for the exponential distribution, the control limits for the Neutrosophic Bayesian Double EWMA control chart are provided by,

$$UCL_N/LCL_N = \mu_{NLF,0} \pm L_N \sigma_{NLF} \sqrt{\frac{\lambda_N(2-2\lambda_N+\lambda_N^2)}{(2-\lambda_N)^3}}$$

where the neutrosophic smoothing constant, with limits, is represented by $\lambda_N \in \{\lambda_{N,L}, \lambda_{N,U}\}$and $[0,0] \le \lambda_{N,U} \le [1,1]$ $[0,0] \le \lambda_{N,L} \le [1,1]$. Table 1 displays the expected value and standard deviation of the exponential distribution under various loss functions, represented as $\hat{\mu}_{N,LF}\ and\ S_{N,LF}$. Plotting the calculated statistic $Z_{N,i} \in \{Z_{L,i}, Z_{U,i}\}$ against the neutrosophic control limits yields the lower and upper control limits as $LCL_N \in \{LCL_{N,L}, LCL_{N,U}\}$ and $UCL_N \in \{UCL_{N,L}, UCL_{N,U}\}$, respectively. Using the control limit coefficient $L_N \in \{L_{N,L}, L_{N,U}\}$, the average run length at a predetermined level is calculated.

Table 1: Estimator, Mean, and Standard Deviation for Exponential Distribution Under Various Loss Functions (LFs)

LF	Estimator under LF	Estimator of mean under LFs	μ_{NLF}	σ^2_{NLF}
SELF	$\frac{s}{n-2}$	$\frac{s}{n-2}$	$\frac{n\theta}{n-2}*(1+I_N)$	$\frac{n\theta^2}{(n-2)^2}*(1+I_N)^2$
PLF	$\frac{s}{\sqrt{(n-2)(n-3)}}$	$\frac{s}{\sqrt{(n-2)(n-3)}}$	$\frac{n\theta}{\sqrt{(n-2)(n-3)}}*(1+I_N)$	$\frac{n\theta^2}{(n-2)(n-3)}*(1+I_N)^2$
GELF	$s\left[\frac{\Gamma n-1}{\Gamma n+p-1}\right]^{(1/p)}$	$s\left[\frac{\Gamma n-1}{\Gamma n+p-1}\right]^{(1/p)}$	$n\theta\left[\frac{\Gamma n-1}{\Gamma n+p-1}\right]^{(1/p)}*(1+I_N)$	$n\theta^2\left[\frac{\Gamma n-1}{\Gamma n+p-1}\right]^{(2/p)}*(1+I_N)^2$
ELF	$\frac{s}{n-1}$	$\frac{s}{n-1}$	$\frac{n\theta}{n-1}*(1+I_N)$	$\frac{n\theta^2}{(n-1)^2}*(1+I_N)^2$
WBLF	$\frac{s}{n-3}$	$\frac{s}{n-3}$	$\frac{n\theta}{n-3}*(1+I_N)$	$\frac{n\theta^2}{(n-3)^2}*(1+I_N)^2$

5.2 For Inverse Rayleigh Distribution

The transformation $Y = \frac{1}{\sqrt{X}}$ yields an Inverse Rayleigh distribution with parameter $\sqrt{\frac{2}{\theta}}$ if X has an exponential distribution. The Inverse Rayleigh distribution's probability density function (PDF) may be found using

$$f(x,\theta) = \frac{2e^{-\frac{1}{\theta x^2}}}{\theta x^3}; \quad x>0,\, \theta>0$$

The probability function for a sample $x_1, x_2, \ldots, x_n$ is written as

$$L\left(x \mid \theta\right) = \frac{2^n}{\theta^n}\prod_{i=1}^{n}\frac{e^{\frac{-s}{\theta}}}{x_i^3}$$

where $s = \sum_{i=1}^{n} 1/x_i^2$, which follows a Gamma distribution. Assuming a uniform prior, the posterior distribution for the Inverse Rayleigh distribution is given by,

$$f(\theta/s) = \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n\Gamma(n-1)}\ ;\ (n-1)>0\ ,\ s>0$$

The Neutrosophic Bayesian EWMA control chart's control limits have the following structure:

$$UCL_N/LCL_N = \mu_{NLF,0} \pm L_N\sigma_{NLF}\sqrt{\frac{\lambda_N}{(2-\lambda_N)}}$$

Similarly, the Neutrosophic Bayesian Double EWMA control chart's control limits for the Inverse Rayleigh distribution are given by,

$$UCL_N/LCL_N = \mu_{NLF,0} \pm L_N\sigma_{NLF}\sqrt{\frac{\lambda_N(2-2\lambda_N+\lambda_N^2)}{(2-\lambda_N)^3}}$$

where the neutrosophic smoothing constant, with limits, is represented by $\lambda_N \in \{\lambda_{N,L}, \lambda_{N,U}\}$and $[0,0] \leq \lambda_{N,U} \leq [1,1]$ $[0,0] \leq \lambda_{N,L} \leq [1,1]$. Table 2 displays the expected value and standard deviation of the exponential distribution under various loss functions, represented as $\hat{\mu}_{N,LF}\ and\ S_{N,LF}$. Plotting the calculated statistic $Z_{N,i} \in \{Z_{L,i}, Z_{U,i}\}$ against the neutrosophic control limits yields the lower and upper control limits as $LCL_N \in \{LCL_{N,L}, LCL_{N,U}\}$ and $UCL_N \in \{UCL_{N,L}, UCL_{N,U}\}$, respectively. Using the control limit coefficient $L_N \in \{L_{N,L}, L_{N,U}\}$, the average run length at a predetermined level is calculated.

5.3 For Weibull Distribution

The transformation $Y = X^{\frac{1}{4}}$ yields a Weibull distribution with a scale parameter of $\theta^{0.25}$ and a shape parameter of 4 if a random variable X has an exponential distribution. The Weibull distribution's probability density

Table 2: Estimator, Mean, and Standard Deviation for Inverse Rayleigh Distribution Under Various Loss Functions (LFs)

LF	Estimator under LF	Estimator of mean under LFs	μ_{NLF}	σ^2_{NLF}
SELF	$\frac{s}{n-2}$	$\pi\sqrt{\frac{(n-2)}{s}}$	$\pi\sqrt{\frac{(n-2)}{\theta}}\frac{\Gamma n-\frac{1}{2}}{\Gamma n}*(1+I_N)$	$\pi^2\frac{(n-2)}{\theta\Gamma n}\left[\Gamma n-1-\frac{\Gamma n-\frac{1}{2}}{\Gamma n}\right]*(1+I_N)^2$
PLF	$\frac{s}{\sqrt{(n-2)(n-3)}}$	$\pi\sqrt{\frac{\sqrt{(n-2)(n-3)}}{s}}$	$\pi\sqrt{\frac{\sqrt{(n-2)(n-3)}}{\theta}}\frac{\Gamma n-\frac{1}{2}}{\Gamma n}*(1+I_N)$	$\pi^2\frac{\sqrt{(n-2)(n-3)}}{\theta\Gamma n}\left[\Gamma n-1-\frac{\Gamma n-\frac{1}{2}}{\Gamma n}\right]*(1+I_N)^2$
GELF	$s\left[\frac{\Gamma n-1}{\Gamma n+p-1}\right]^{(1/p)}$	$\frac{\pi}{\sqrt{s}}\sqrt{\left[\frac{\Gamma n+p-1}{\Gamma n-1}\right]^{(1/p)}}$	$\frac{\pi}{\sqrt{\theta}}\sqrt{\left[\frac{\Gamma n+p-1}{\Gamma n-1}\right]^{(1/p)}}\frac{\Gamma n-\frac{1}{2}}{\Gamma n}*(1+I_N)$	$\frac{\pi^2}{\theta\Gamma n}\left[\frac{\Gamma n+p-1}{\Gamma n-1}\right]^{(1/p)}\left[\Gamma n-1-\frac{\Gamma n-\frac{1}{2}}{\Gamma n}\right]*(1+I_N)^2$
ELF	$\frac{s}{n-1}$	$\pi\sqrt{\frac{n-1}{s}}$	$\pi\sqrt{\frac{n-1}{\theta}}\frac{\Gamma n-\frac{1}{2}}{\Gamma n}*(1+I_N)$	$\pi^2\frac{n-1}{\theta\Gamma n}\left[\Gamma n-1-\frac{\Gamma n-\frac{1}{2}}{\Gamma n}\right]*(1+I_N)^2$
WBLF	$\frac{s}{n-3}$	$\pi\sqrt{\frac{n-3}{s}}$	$\pi\sqrt{\frac{n-3}{\theta}}\frac{\Gamma n-\frac{1}{2}}{\Gamma n}*(1+I_N)$	$\pi^2\frac{n-3}{\theta\Gamma n}\left[\Gamma n-1-\frac{\Gamma n-\frac{1}{2}}{\Gamma n}\right]*(1+I_N)^2$

function (PDF) is

$$f(x,\theta,B)=\frac{B}{\theta}\left(\frac{x}{\theta}\right)^{B-1}\cdot e^{-\left(\frac{x}{\theta}\right)^B},\quad x>0,\quad \theta>0$$

The probability function for a given sample $x=x_1,x_2,\ldots,x_n$ is written as follows:

$$L(x|\theta)=\frac{B^n}{\theta^{nB}}\prod_{i=1}^{n}x_i^{B-1}e^{-\frac{s}{\theta^B}}$$

where $s=\sum_{i=1}^{n}x_i^B$, which follows a Gamma distribution. Assuming a uniform prior, the posterior distribution for the Weibull distribution is given by,

$$f\left(\frac{\theta}{s}\right)=\frac{B^n e^{\frac{-s}{\theta^B}}s^{\frac{nB-1}{B}}}{\theta^{nB}\Gamma\left(\frac{nB-1}{B}\right)}\quad ;\quad (nB-1)>0\quad ,\quad s>0$$

The Neutrosophic Bayesian EWMA control chart's control limits under the weibull distribution have been defined by

$$UCL_N/LCL_N=\mu_{NLF,0}\pm L_N\sigma_{NLF}\sqrt{\frac{\lambda_N}{(2-\lambda_N)}}$$

Similarly, the Neutrosophic Bayesian Double EWMA control chart's control limits for the Weibull distribution are provided by

$$UCL_N/LCL_N=\mu_{NLF,0}\pm L_N\sigma_{NLF}\sqrt{\frac{\lambda_N(2-2\lambda_N+\lambda_N^2)}{(2-\lambda_N)^3}}$$

where the neutrosophic smoothing constant, with limits, is represented by $\lambda_N\in\{\lambda_{N,L},\lambda_{N,U}\}$and $[0,0]\le\lambda_{N,U}\le[1,1]$ $[0,0]\le\lambda_{N,L}\le[1,1]$. Table 5.3 displays the expected value and standard deviation of the exponential distribution under various loss functions, represented as $\hat{\mu}_{N,LF}$ and $S_{N,LF}$. Plotting the calculated statistic $Z_{N,i}\in\{Z_{L,i},Z_{U,i}\}$ against the neutrosophic control limits yields the lower and upper control limits as $LCL_N\in\{LCL_{N,L},LCL_{N,U}\}$ and $UCL_N\in\{UCL_{N,L},UCL_{N,U}\}$, respectively. Using the control limit coefficient $L_N\in\{L_{N,L},L_{N,U}\}$, the average run length at a predetermined level is calculated.

LF	Estimator under LF	Estimator of mean under LFs	μ_{NLF}	σ^2_{NLF}
SELF	$s^{1/B}\frac{\Gamma\frac{nB-2}{B}}{\Gamma\frac{nB-1}{B}}$	$s^{1/B}\frac{\Gamma\frac{nB-2}{B}}{\Gamma\frac{nB-1}{B}}\Gamma 1+\frac{1}{B}$	$\theta^{1/B}\frac{\Gamma\frac{nB-2}{B}}{\Gamma\frac{nB-1}{B}}\Gamma 1+\frac{1}{B}\frac{\Gamma n+\frac{1}{B}}{\Gamma n}*(1+I_N)$	$\theta^{2/B}\frac{\Gamma\frac{nB-}{B}}{\Gamma\frac{nB-}{B}}$
PLF	$s^{1/B}\left[\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-1}{B}}\right]^{1/2}$	$s^{1/B}\left[\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-1}{B}}\right]^{1/2}\Gamma 1+\frac{1}{B}$	$\theta^{1/B}\left[\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-1}{B}}\right]^{1/2}\Gamma 1+\frac{1}{B}\frac{\Gamma n+\frac{1}{B}}{\Gamma n}*(1+I_N)$	$\theta^{2/B}\left[\frac{\Gamma nB}{\Gamma nB}\right.$
GELF	$s^{1/B}\left[\frac{\Gamma\frac{nB-1}{B}}{\Gamma\frac{nB+p-1}{B}}\right]^{1/p}$	$s^{1/B}\left[\frac{\Gamma\frac{nB-1}{B}}{\Gamma\frac{nB+p-1}{B}}\right]^{1/p}\Gamma 1+\frac{1}{B}$	$\theta^{1/B}\left[\frac{\Gamma\frac{nB-1}{B}}{\Gamma\frac{nB+p-1}{B}}\right]^{1/p}\Gamma 1+\frac{1}{B}\frac{\Gamma n+\frac{1}{B}}{\Gamma n}*(1+I_N)$	$\theta^{2/B}\left[\frac{\Gamma}{\Gamma nB}\right.$ ... $I_N)^2$
ELF	$s^{1/B}\left[\frac{\Gamma\frac{nB-1}{B}}{\Gamma n}\right]$	$s^{1/B}\left[\frac{\Gamma\frac{nB-1}{B}}{\Gamma n}\right]\Gamma 1+\frac{1}{B}$	$\theta^{1/B}\left[\frac{\Gamma\frac{nB-1}{B}}{\Gamma n}\right]\Gamma 1+\frac{1}{B}\frac{\Gamma n+\frac{1}{B}}{\Gamma n}*(1+I_N)$	$\theta^{2/B}\left[\frac{\Gamma nB}{\Gamma}\right.$
WBLF	$s^{1/B}\left[\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-2}{B}}\right]$	$s^{1/B}\left[\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-2}{B}}\right]\Gamma 1+\frac{1}{B}$	$\theta^{1/B}\left[\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-2}{B}}\right]\Gamma 1+\frac{1}{B}\frac{\Gamma n+\frac{1}{B}}{\Gamma n}*(1+I_N)$	$\theta^{2/B}\left[\frac{\Gamma nB}{\Gamma nB}\right.$

6 Simulation

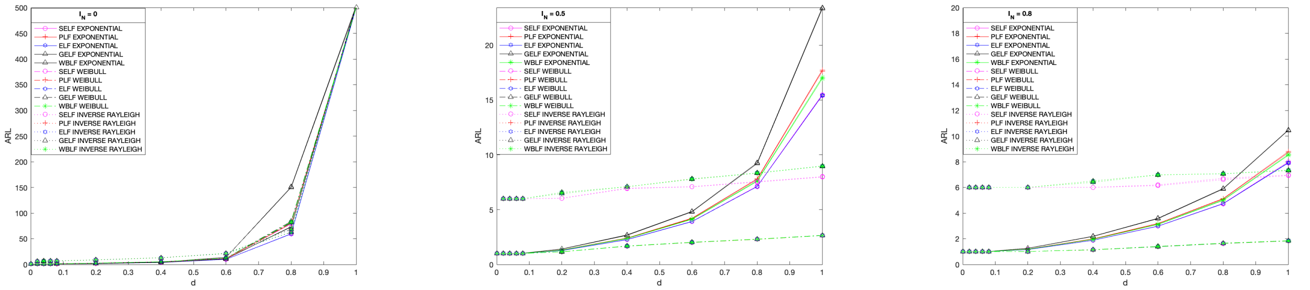
The suggested control charts' performance measurement tool is the ARL values. The suggested chart's performance is assessed over a range of mean shift values. The ARL values for in-control and out-of-control processes are ARL_0 and ARL_1, respectively. Changing the control limit coefficient L_N helps one to derive the value of ARL_0. The in-control process's ARL value is set at 500 (ARL_0 = 500). We may determine that the control chart is optimal if the value of ARL_1 for a given shift is modest. The effectiveness of the suggested control charts is assessed for varied I_N values, i.e., for $I_N = 0,\ 0.5\ , and\ 0.8$.

Monte Carlo simulation for the proposed Neutrosophic Bayesian EWMA control charts' in-control procedure:

1. Fix the value of the in control ARL say r_0 and the parameters λ_N (smoothing constant) and I_N (Indeterminacy) .
2. Employing the exponential, inverse Rayleigh, and Weibull distributions, generate random sample $Y_{N,i}$ of size n_N.
3. Determine the $Z_{N,i}\ and\ DZ_{N,i}$ testing statistics for the several suggested Neutrosophic Bayesian EWMA control charts.
4. Compute the control limits say $UCL_N\ and\ LCL_N$.
5. Set up $L_N$the control limit coefficient such that the in-control ARL of the control chart reaches the desired value of r_0.
6. If $LCL_N\ and\ UCL_N$ are within the test statistic, declare the process under control; if not, declare it out of control.
7. If the process is under control, do steps one through six again. If the process spirals out of control, mark the run length as the moment the first OOC is observed.
8. Compute the run length value (ARL).

In order to determine the in control ARL, repeat steps 1 through 7, 50,000 times. Proceed to method 2 and halt the procedure if the computed ARL $\geq r_0$. If not, repeat the previous procedures using a different value for the control limit coefficient, such as L_N.

Monte Carlo simulation for in control process of the proposed Neutrosophic bayesian EWMA control charts for shifted mean:



(a) (b) (c)

Figure 1: ARL comparison of the proposed Neutrosophic Bayesian EWMA control chart for $I_N = 0$, 0.5, and 0.8 on Neutrosophic data for $\lambda_N = \{0.1, 0.12\}$,$p = 2$ and $n_N = 5$.

1. Specify the value of λ_N (smoothing constant),I_N (Indeterminacy) and shift d .
2. With a mean shift $\theta_1 = d * \theta_0 * (1 + I_N)$, create a random sample $Y_{N,i}$ of size n_N from the Exponential, Inverse Rayleigh, and Weibull distributions.
3. Determine the $Z_{N,i}\ and\ DZ_{N,i}$ testing statistics for the several suggested Neutrosophic Bayesian EWMA control charts.
4. Compute the upper and lower control limits say $LCL_N\ and\ UCL_N$. Take the value of control limit coefficient L_N from the above mentioned algorithm.
5. If $LCL_N\ and\ UCL_N$ are within the test statistic, declare the process under control; if not, declare it out of control.
6. If the process is under control, do steps one through six again. If the process spirals out of control, mark the run length as the moment the first OOC is observed.
7. Compute the run length value (ARL).

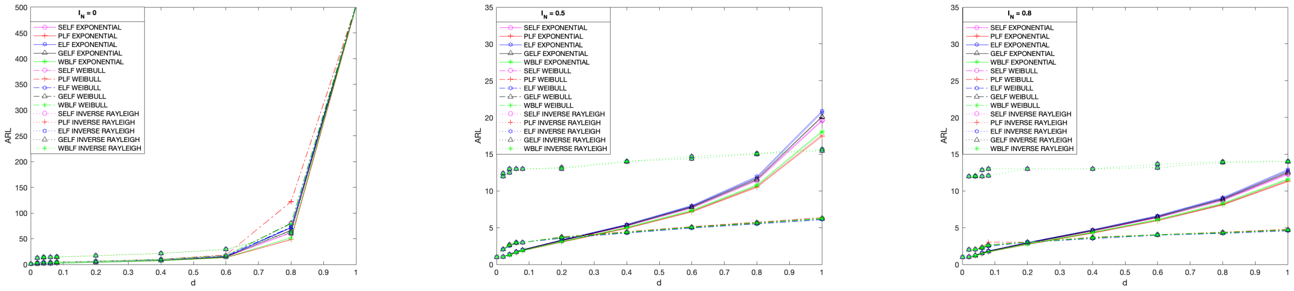
Repeat steps 1 to 7 for 50,000 times to calculate ARL1 for shifted process.

The Exponential, Inverse Rayleigh, and Weibull distributions with non-informative priors is done(i.e., uniform priors) to investigate the proposed Neutrosophic Bayesian EWMA control chart under many LFs. Five distinct LFs, including SELF, PLF, ELF, GELF, and WBLF, have been taken. Figure 1 illustrate the ARL graph under several LFs for the Exponential, Inverse Rayleigh, and Weibull distributions for $I_N = 0, 0.5,\ and\ 0.8$ for the proposed neutrosophic bayesian EWMA control chart and figure 2 illustrate the ARL graph under several LFs for the Exponential, Inverse Rayleigh, and Weibull distributions for $I_N = 0, 0.5,\ and\ 0.8$ for the proposed neutrosophic bayesian DEWMA control chart while considering $ARL_0 = 500$ and $n_N = 5$. It is presumed that the process is out-of-control for shift $d \leq 1$ and in-control for shift $d = 1$.

7 Comparitive Study

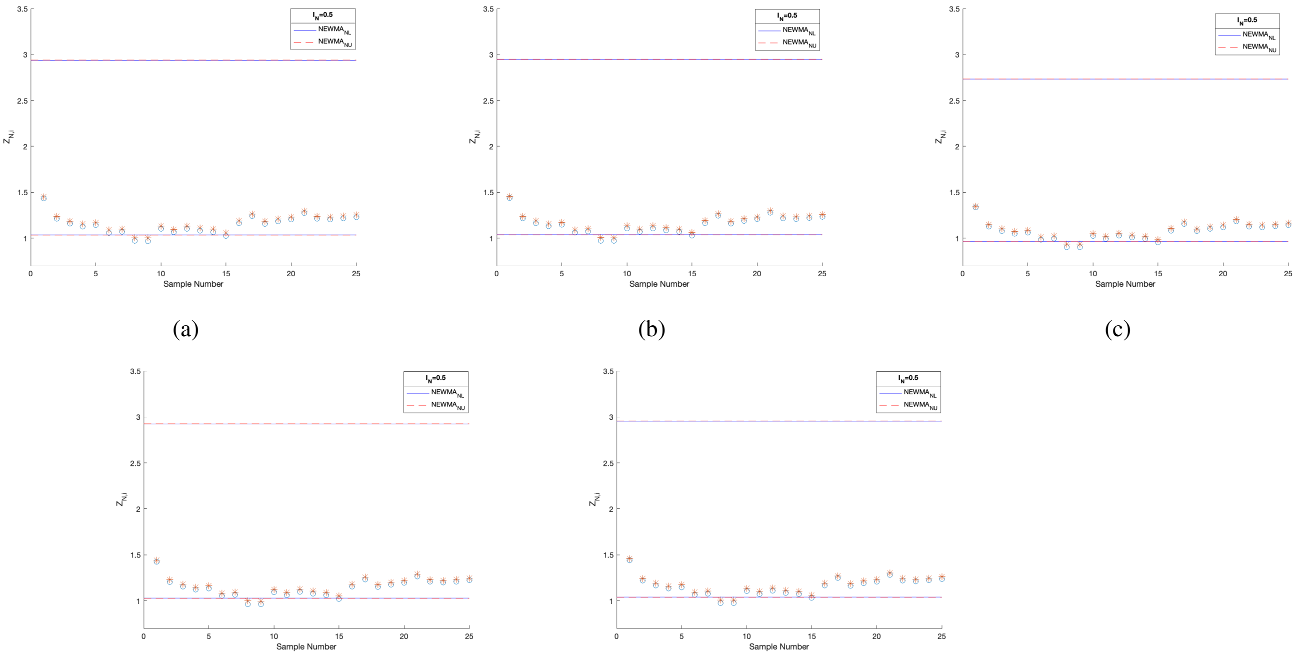
For EWMA control chart Fig 1 gives us an ARL comparison of Neutrosophic Bayesian location chart based on Exponential, Weibull and Inverse Rayleigh distribution under various LF's for I_N = 0, 0.5, and 0.8 . From ARL_1 values it is clear that Exponential distribution performs better than the Inverse Rayleigh distribution but from an overall comparison of Exponential, Weibull and Inverse Rayleigh distribution, it is clear that Weibull distribution gives smaller values of ARL_1 for smaller shifts.

For DEWMA control chart Fig 2 gives us an ARL comparison of Neutrosophic Bayesian location chart based on Exponential, Weibull and Inverse Rayleigh distribution under various LF's for I_N = 0, 0.5, and 0.8 . Fig 2 shows the comparison results for Exponential, Weibull and Inverse Rayleigh distribution under various LF's for uniform prior. It is clear that Exponential distribution shows smaller values of ARL_1 compaired to Inverse



(a) (b) (c)

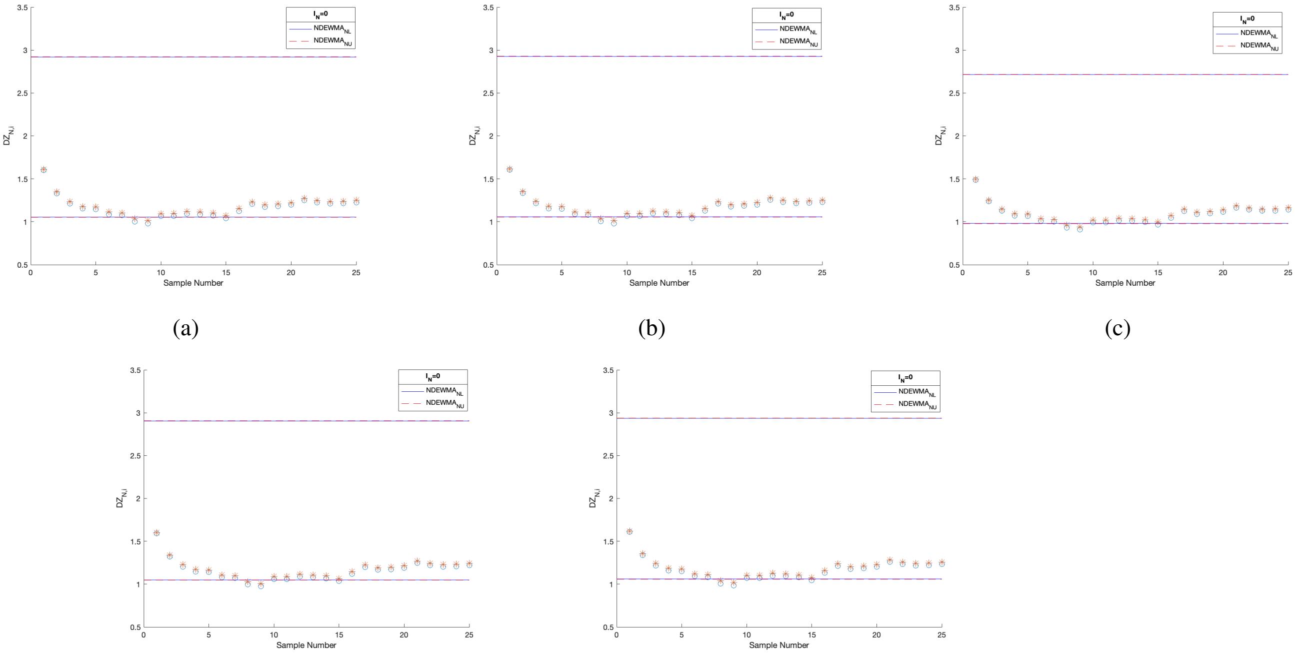
Figure 2: ARL comparison of the proposed Neutrosophic Bayesian DEWMA control chart for $I_N = 0, 0.5$, and 0.8 on Neutrosophic data for $\lambda_N = \{0.1, 0.12\}$,$p = 2$ and $n_N = 5$.



(a) (b) (c)

(d) (e)

Figure 3: Real Life Application of the proposed Neutrosophic Bayesian EWMA control chart for $I_N = 0.5$ on Neutrosophic data for $\lambda_N = \{0.7, 0.702\}$,$p = 30$ and $n_N = 5$.



(a) (b) (c)

(d) (e)

Figure 4: Real Life Application of the proposed Neutrosophic Bayesian DEWMA control chart for $I_N = 0.5$ on Neutrosophic data for $\lambda_N = \{0.7, 0.702\}$, $p = 30$ and $n_N = 5$.

Table 3: ARL values of Neutrosophic Bayesian EWMA control chart for Exponential, Inverse Rayleigh, and Weibull distributions under SELF for $\lambda_N = [0.1, 0.102]$ and $I_N = 0.5, 0.8$

d	Distributions	EWMA	$I_N = 0.5$	$I_N = 0.8$
	Exponential	[59.6013, 59.8684]	[7.1048, 7.1406]	[4.7240, 4.7485]
0.8	Inverse Rayleigh	[62.76402, 68.25648]	[7.5117, 7.56512]	[6.60254 ,6.7071]
	Weibull	[79.6224, 81.12062]	[2.29442, 2.3102]	[1.62948, 1.64964]
	Exponential	[10.12058, 10.15582]	[3.91082,, 3.93186]	[2.9795 ,2.99202]
0.6	Inverse Rayleigh	[21.35648, 21.66528]	[7.0793, 7.09874]	[6.13762 ,6.19756]
	Weibull	[12.36382, 12.40678]	[2.01064, 2.02428]	[1.3796, 1.39552]
	Exponential	[3.91782, 3.9364]	[2.2617, 2.2772]	[1.88358, 1.89032]
0.4	Inverse Rayleigh	[13.06082 ,13.07936]	[6.92148, 6.95772]	[6.00342 ,6.0056]
	Weibull	[5.2446, 5.27244]	[1.66598, 1.68496]	[1.13944 ,1.1451]
	Exponential	[1.02076, 1.02112]	[1.00354 ,1.00406]	[1.00182, 1.00188]
0.06	Inverse Rayleigh	[6.98812, 6.99954]	[6, 6]	[6 ,6]
	Weibull	[1.42442, 1.43918]	[1.00176, 1.00206]	[1.00004, 1.00012]
	Exponential	[1, 1]	[1 ,1]	[1, 1]
0	Inverse Rayleigh	[6.0017, 6.0032]	[6, 6]	[6 ,6]
	Weibull	[1, 1]	[1 ,1]	[1, 1]

Table 4: ARL values of Neutrosophic Bayesian EWMA control chart for Weibull distribution for various λ_N values for $I_N = 0.5$ and shift $(d) = 0.8$

λ_N	SELF	PLF	GELF	ELF	WBLF
$[0.1, 0.102]$	[2.30286, 2.31234]	[2.2983, 2.3163]	[2.30198, 2.3094]	[2.2976, 2.3123]	[2.29962, 2.31442]
$[0.5, 0.55]$	[1.30788, 1.33676]	[1.30498, 1.33742]	[1.30774, 1.32982]	[1.3068, 1.33664]	[1.30676, 1.33536]
$[0.72, 0.76]$	[1.24536, 1.2509]	[1.24592, 1.25144]	[1.24614, 1.25204]	[1.24688 ,1.25032]	[1.2459, 1.2542]
$[1, 1]$	[1.2436, 1.24384]	[1.24444, 1.24818]	[1.2449, 1.2479]	[1.24324, 1.24816]	[1.2448, 1.24822]

Rayleigh distribution. From an overall comparison it is clear that Weibull distribution gives smaller values of ARL_1 for smaller shifts than Exponential distribution and Inverse Rayleigh distribution. The results of the proposed Bayesian Neutrosophic EWMA control chart for Exponential and its transformation into Inverse Rayleigh and Weibull distributions under SELF are shown in Table 3.

In general amoung the three distributions Weibull distribution gives a much better result. Neutrosophic Bayesian EWMA control chart for Weibull distribution for various λ_N values considering $I_N = 0.5$ and shift $(d) = 0.8$ is depicted in Table 4. Its clear that as λ_N increases the ARL_1 decreases for the given loss functions.

8 Real Life Application

Many studies routinely show the usefulness and efficiency of suggested Bayesian Neutrosophic EMWA and DEWMA based on real-world data and simulated environments. In this setting, we investigate a real-world dataset to highlight the suggested Neutrosophic Bayesian control chart's strengths. In sectors guaranteeing material safety for aerospace and bridge building, tensile strength of fiber composites must be constantly monitored. To reach this, we investigate the real-life dataset[17] , which especially details the breaking strengths of carbon fibers employed in production of these composite materials. Research carried out at the U.S. Army Materials Technology Laboratory in Watertown, Massachusetts provides these realizations. Comprising 25 samples, each with a sample size of $n_N = 5$, the dataset follows the Weibull distribution with a scale parameter (B=2.9437) and shape parameter (θ=2.7929). Fifteen randomly selected, size $n_N = 5$ samples are first pulled without replacement and labelled as in-control samples. The dataset then is changed by adding one to every observation. After this change, ten randomly selected, size $n_N = 5$ samples without replacement are considered as out-of-control samples.

Designed for simultaneous monitoring of both process mean and dispersion, Fig. 3 for $I_N = 0.5$ offer a visual picture of the application of the given Neutrosophic Bayesian EWMA control chart. This monitoring uses SELF, PLF, ELF, GELF, and WBLF among several Loss functions techniques. Particularly considering a smoothing constant value of $\{0.7, 0.702\}$, a careful inspection of these charts displays unambiguous signs indicating that the process has gone out of control in the $8^{th}, 9^{th}\ and\ 15^{th}$ samples. In line with Fig. 4 for $I_N = 0.5$ provide the graphical depiction of Neutrosophic Bayesian DEWMA control chart for several loss functions including SELF, PLF, ELF, GELF and WBLF. Particularly considering a smoothing constant value of $\{0.7, 0.702\}$, a careful analysis of these charts reveals unambiguous indications indicating that the process has gone out of control in the $8^{th}, 9^{th}\ and\ 15^{th}$ samples.

9 Conclusion

Under five distinct LFs—SELF, PLF, ELF, GELF, and WBLF—the Neutrosophic Bayesian EWMA and DEWMA control chart has been examined for the Exponential, Inverse Rayleigh, and Weibull distributions for $I_N = 0, 0.5\ and\ 0.8$. By combining the Bayesian and Neutrosophic ideas along with the EWMA and DEWMA control charts the proposed work performs better than any other control chart now in use. The outperformance was explained by the Monte Carlo simulation. It can be concluded that the proposed Neutrosophic Bayesian EWMA and DEWMA control chart on Weibull distribution outperforms the traditional control charts. The simulation research produced ARLs for the Exponential distribution and transformed Exponential distributions with uniform priors. The results show that Weibull distribution produce smaller values

Symbol	Description
λ_N	Neutrosophic smoothing parameter
$Z_{N,i}$	Neutrosophic EWMA statistic
$DZ_{N,i}$	Neutrosophic DEWMA statistic
$UCL_{N,i}$	Neutrosophic Upper Control Limit
$LCL_{N,i}$	Neutrosophic Lower Control Limit
n_N	Neutrosophic sample size
λ	Smoothing parameter
Z_i	EWMA statistic
DZ_i	DEWMA statistic
$Z_{LF,i}$	Bayesian EWMA statistic for various Loss Functions
$DZ_{LF,i}$	Bayesian DEWMA statistic for various Loss Functions
$\hat{\mu}_{LF}$	Mean estimate of various Loss Functions
μ_{LF}	Estimated value of $\hat{\mu}_{LF}$ of various Loss Functions
σ_{LF}	SD of $\hat{\mu}_{LF}$ of various Loss Functions
L	Control limit coefficient
L_N	Neutrosophic Control limit coefficient
$\hat{\mu}_{NLF}$	Neutrosophic Mean estimate of various Loss Functions
μ_{NLF}	Neutrosophic Estimated value of $\hat{\mu}_{NLF}$ of various Loss Functions
σ_{NLF}	Neutrosophic SD of $\hat{\mu}_{NLF}$ of various Loss Functions

of ARL followed by Exponential distribution and Inverse Rayleigh distribution for both Neutrosophic EWMA and DEWMA control chart. The results obtained in the proposed work shows the significance of Neutrosophic and Bayesian control charts over existing control charts. Neutrosophic Bayesian control chart designed on Exponential, Inverse Rayleigh, and Weibull distributions under various loss functionsand provided excellent results when compaired with traditional control charts EWMA. For different loss functions, the proposed control charts perform almost the same under the converted exponential distribution in terms of their ability to detect shifts.

10 Appendix

Symbols used in this paper:

1. Exponential distribution:
The quality character of interest $x \sim Exp(\theta)$. The probability density function of x is defined as

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta},\ x \geq 0$$

The likelihood function of exponential distribution of a random sample of size n is

$$\begin{aligned} L(\frac{x}{\theta}) &= \prod_{i=1}^{n} \frac{1}{\theta} e^{-x_i/\theta} \\ &= \frac{1}{\theta^n} \exp\left(-\frac{s}{\theta}\right) \end{aligned}$$

where $s = \sum_{i=1}^{n} x_i \sim Gamma(\theta, n)$
The posterior distribution of exponential distribution with uniform prior is

$$\begin{aligned} f(\frac{\theta}{s}) &= \frac{L(\frac{x}{\theta})\pi(\theta)}{\int_0^\infty L(\frac{x}{\theta})\pi(\theta)d\theta} \\ &= \frac{e^{\overline{\theta}^{-s}} s^{n-1}}{\theta^n \Gamma(n-1)}\ ;\ (n-1) > 0\ ,\ s > 0 \end{aligned}$$

Squared Error Loss Function (SELF):

$$\begin{aligned}\hat{\theta}_{SELF} &= E_{\theta/x}[\theta] \\ &= \int_0^\infty \theta \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} d\theta \\ &= \int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n-1} \Gamma(n-1)} d\theta \\ &= \frac{s}{n-2}\end{aligned}$$

Precautionary Loss Function (PLF):

$$\begin{aligned}\hat{\theta}_{PLF} &= \sqrt{E_{\theta/x}[\theta^2]} \\ &= \sqrt{\int_0^\infty \theta^2 \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} d\theta} \\ &= \sqrt{\int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n-2} \Gamma(n-1)} d\theta} \\ &= \sqrt{\frac{s^2}{(n-2)(n-3)}}\end{aligned}$$

General Entropy Loss Function (GELF):

$$\begin{aligned}\hat{\theta}_{GELF} &= \Big[E_{\theta/x}[\theta^{-p}]\Big]^{-(1/p)} \\ &= \Big[\int_0^\infty \theta^{-p} \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} d\theta\Big]^{-(1/p)} \\ &= \Big[\int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n+p} \Gamma(n-1)} d\theta\Big]^{-(1/p)} \\ &= s\Big[\frac{\Gamma(n-1)}{\Gamma(n+p-1)}\Big]^{(1/p)}\end{aligned}$$

Entropy Loss Function (ELF):

$$\begin{aligned}\hat{\theta}_{ELF} &= \Big[E_{\theta/x}[\theta^{-1}]\Big]^{-1} \\ &= \Big[\int_0^\infty \theta^{-1} \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} d\theta\Big]^{-1} \\ &= \Big[\int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n+1} \Gamma(n-1)} d\theta\Big]^{-1} \\ &= \frac{s}{n-1}\end{aligned}$$

Weight Balance Loss Function (WBLF):

$$\begin{aligned}\hat{\theta}_{WBLF} &= \frac{E_{\theta/x}[\theta^2]}{E_{\theta/x}[\theta]} \\ &= \frac{\frac{s^2}{(n-2)(n-3)}}{\frac{s}{n-2}} \\ &= \frac{s}{n-3}\end{aligned}$$

2. Inverse Rayleigh Distribution:
The quality character of interest $x \sim IR(\sqrt{2/\theta})$. The probability density function of x is defined as

$$f(x,\theta) = \frac{2}{\theta x^3} e^{\left(-\frac{1}{\theta x^2}\right)}, x \geq 0$$

The likelihood function of Inverse Rayleigh distribution of a random sample of size n is

$$\begin{aligned}L(\frac{x}{\theta}) &= \prod_{i=1}^{n} \frac{2}{\theta x_i^3} e^{\left(-\frac{1}{\theta x_i^2}\right)} \\ &= \frac{2^n}{\theta^n} \prod_{i=1}^{n} x_i^3 e^{\left(-\frac{s}{\theta}\right)}\end{aligned}$$

where $s = \sum_{i=1}^{n} \frac{1}{x_i^2}$
The posterior distribution of Inverse Rayleigh Distribution with uniform prior is

$$\begin{aligned}f(\frac{\theta}{s}) &= \frac{L(\frac{x}{\theta})\pi(\theta)}{\int_0^\infty L(\frac{x}{\theta})\pi(\theta)d\theta} \\ &= \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} \; ; \; (n-1) > 0 \; , \; s > 0\end{aligned}$$

Squared Error Loss Function (SELF):

$$\begin{aligned}\hat{\theta}_{SELF} &= E_{\theta/x}[\theta] \\ &= \int_0^\infty \theta \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} d\theta \\ &= \int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n-1} \Gamma(n-1)} d\theta \\ &= \frac{s}{n-2}\end{aligned}$$

Precautionary Loss Function (PLF):

$$\begin{aligned}\hat{\theta}_{PLF} &= \sqrt{E_{\theta/x}[\theta^2]} \\ &= \sqrt{\int_0^\infty \theta^2 \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n \Gamma(n-1)} d\theta} \\ &= \sqrt{\int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n-2} \Gamma(n-1)} d\theta} \\ &= \sqrt{\frac{s^2}{(n-2)(n-3)}}\end{aligned}$$

General Entropy Loss Function (GELF):

$$\begin{aligned}
\hat{\theta}_{GELF} &= \Big[E_{\theta/x}[\theta^{-p}]\Big]^{-(1/p)} \\
&= \Bigg[\int_0^\infty \theta^{-p}\frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n\Gamma(n-1)}d\theta\Bigg]^{-(1/p)} \\
&= \Bigg[\int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n+p}\Gamma(n-1)}d\theta\Bigg]^{-(1/p)} \\
&= s\Bigg[\frac{\Gamma(n-1)}{\Gamma(n+p-1)}\Bigg]^{(1/p)}
\end{aligned}$$

Entropy Loss Function (ELF):

$$\begin{aligned}
\hat{\theta}_{ELF} &= \Big[E_{\theta/x}[\theta^{-1}]\Big]^{-1} \\
&= \Bigg[\int_0^\infty \theta^{-1}\frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^n\Gamma(n-1)}d\theta\Bigg]^{-(1/p)} \\
&= \Bigg[\int_0^\infty \frac{e^{\frac{-s}{\theta}} s^{n-1}}{\theta^{n+1}\Gamma(n-1)}d\theta\Bigg]^{-(1/p)} \\
&= \frac{s}{n-1}
\end{aligned}$$

Weight Balance Loss Function (WBLF):

$$\begin{aligned}
\hat{\theta}_{WBLF} &= \frac{E_{\theta/x}[\theta^2]}{E_{\theta/x}[\theta]} \\
&= \frac{\frac{s^2}{(n-2)(n-3)}}{\frac{s}{n-2}} \\
&= \frac{s}{n-3}
\end{aligned}$$

3. Weibull distribution:

The quality character of interest $x \sim Weibull(\theta^{1/4})$. The probability density function of x is defined as

$$f(x;\theta,B) = \frac{B}{\theta}\left(\frac{x}{\theta}\right)^{B-1} e^{\left(-\left(\frac{x}{\theta}\right)^B\right)}, \quad x \geq 0$$

The likelihood function of Weibull distribution of a random sample of size n is

$$\begin{aligned}
L(\theta,B) &= \prod_{i=1}^{n}\frac{B}{\theta}\left(\frac{x_i}{\theta}\right)^{B-1} e^{-\left(\frac{x_i}{\theta}\right)^B} \\
&= \frac{B^n}{\theta^{nB}}\prod_{i=1}^{n} x_i^{B-1} e^{\frac{s}{\theta^B}}
\end{aligned}$$

where $s = \sum_{i=1}^n x_i^B$

The posterior distribution of Weibull Distribution with uniform prior is

$$\begin{aligned}
f\left(\frac{\theta}{s}\right) &= \frac{L\left(\frac{x}{\theta}\right)\pi(\theta)}{\int_0^\infty L\left(\frac{x}{\theta}\right)\pi(\theta)\,d\theta} \\
&= \frac{B^n\, e^{-\frac{s}{\theta^B}}\, s^{\frac{nB-1}{B}}}{\theta^{nB}\,\Gamma\left(\frac{nB-1}{B}\right)}, \quad (nB-1) > 0,\ s > 0
\end{aligned}$$

Squared Error Loss Function (SELF):

$$\begin{aligned}\hat{\theta}_{SELF} &= E_{\theta/x}[\theta] \\ &= \int_0^\infty \theta \, \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-1}{B}}}{\theta^{nB} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta \\ &= \int_0^\infty \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-1}{B}}}{\theta^{nB-1} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta \\ &= s^{1/B} \frac{\Gamma \frac{nB-2}{B}}{\Gamma \frac{nB-1}{B}}\end{aligned}$$

Precautionary Loss Function (PLF):

$$\begin{aligned}\hat{\theta}_{PLF} &= \sqrt{E_{\theta/x}[\theta^2]} \\ &= \sqrt{\int_0^\infty \theta^2 \, \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-1}{B}}}{\theta^{nB} \, \Gamma\left(\frac{nB-2}{B}\right)} d\theta} \\ &= \sqrt{\int_0^\infty \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-2}{B}}}{\theta^{nB-1} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta} \\ &= s^{1/B} \sqrt{\frac{\Gamma \frac{nB-3}{B}}{\Gamma \frac{nB-1}{B}}}\end{aligned}$$

General Entropy Loss Function (GELF):

$$\begin{aligned}\hat{\theta}_{GELF} &= \left[E_{\theta/x}[\theta^{-p}]\right]^{-(1/p)} \\ &= \left[\int_0^\infty \theta^{-p} \, \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-1}{B}}}{\theta^{nB} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta\right]^{-(1/p)} \\ &= \left[\int_0^\infty \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-2}{B}}}{\theta^{nB+p} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta\right]^{-(1/p)} \\ &= s^{1/B} \left[\frac{\Gamma \frac{nB-1}{B}}{\Gamma \frac{nB+p-1}{B}}\right]^{-(1/p)}\end{aligned}$$

Entropy Loss Function (ELF):

$$\begin{aligned}\hat{\theta}_{GELF} &= \left[E_{\theta/x}[\theta^{-1}]\right]^{-1} \\ &= \left[\int_0^\infty \theta^{-1} \, \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-1}{B}}}{\theta^{nB} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta\right]^{-1} \\ &= \left[\int_0^\infty \frac{B^n \, e^{-\frac{s}{\theta^B}} \, s^{\frac{nB-2}{B}}}{\theta^{nB+1} \, \Gamma\left(\frac{nB-1}{B}\right)} d\theta\right]^{-1} \\ &= s^{1/B} \left[\frac{\Gamma \frac{nB-1}{B}}{\Gamma n}\right]\end{aligned}$$

Weight Balance Loss Function (WBLF):

$$\hat{\theta}_{WBLF} = \frac{E_{\theta/x}[\theta^2]}{E_{\theta/x}[\theta]} = \frac{s^{2/B}\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-1}{B}}}{s^{1/B}\frac{\Gamma\frac{nB-2}{B}}{\Gamma\frac{nB-1}{B}}} = s^{1/B}\frac{\Gamma\frac{nB-3}{B}}{\Gamma\frac{nB-2}{B}}$$

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