



Data-Driven Pricing Decisions for Ensuring the Success of Strategic Product Development

Ramazan Yasar^{1,*} Sergey Drominko²

¹ Ankara University, Engineering Faculty, Department of Artificial Intelligence and Data Engineering, Ankara, Turkey

² Faculty of Information Technology and Robotics, Vitebsk State Technological University, Belarus

Emails: yasarr@ankara.edu.tr · Serdrominko1996@vsu.by

Received: November 14, 2024 Revised: December 24, 2024 Accepted: January 27, 2025 ★ Corresponding author

ABSTRACT

By effectively including diffusion into the framework, this study further illustrates whether Optimal Control Theory may be used to identify and address the control of prices issue of technology items. There is a three-stage paradigm that it uses to describe the procedure of adoption process: consciousness, inspiration, and adopting itself. The process is described by the diffusion logistic functions; furthermore, the model takes into account price fluctuations' sensitivity. The selling price is a choice variable constrained such that the total profit over the relevant planning horizon is optimised. In this paper, the Hamiltonian function is used in obtaining necessary optimality conditions with spectral learning effects and costate equations complemented by the adoption rates by use of Pontryagin's Maximum Principle. As the problem is formulated as a continuous optimisation problem, it is discretised for its practical applications, and the model is solved with the help of LINGO 15.0 software. The data used to validate the model implemented was derived from historical sales records of the electronics and semiconductor industries to obtain a measure of realism. Analysed sensitivity studies show how variations in adoption parameters, including price elasticity and customer attrition, affect adoption rates and profitability. As such, the study offers managerial implications for the management of private sector schemes to focus on the application of dynamic pricing strategies as the optimal balance between consumers' perceived value and firm revenues. It provides managers with strong tools for the implementation of adoption into a new generation of technology-enabled markets, maximisation of revenues, and sustaining of competitive advantage. Outperforming all analysed models, the suggested technique employing Optimal Control Theory obtains an accuracy of 96%. This proves that the suggested strategy is the best at forecasting when a product will be adopted.

Keywords: LINGO ▪ SVM ▪ LR ▪ KNN ▪ RF ▪ Optimal Control Theory

1. INTRODUCTION

One of the most effective mathematics frameworks, Optimal Control Theory is used to ascertain the most effective method for directing an unstable system over a period [1]. From the Calculus of Modifications that concentrates on optimising functions by establishing the circumstances during which they acquire extremum standards, it arose as a component of the

Calculus of Variability. By handling changing systems that are controlled by mathematical formulas, Optimal Control Theory can optimise trajectories or choices that develop over a specific time range. This contrasts with static optimisation, which only manages static systems. Several fields, including economics, finance, biology, and business research, are all affected by its significance. Optimal Control Theory might be marked as originating approximately in the middle of the

twentieth century and recognised as an important constituent of modern theories of control. It was a race for space, in which scientists sought to provide optimal course charts for spaceships, that moved the theory forward. The theory mostly leaned on the elementary instruments provided by the publication of Richard Bellman on dynamism and Lev Pontryagin on Maximum Principles particularly [2]. Bellman's approach was to divide the problem into smaller related issues that can be solved easily, while Pontryagin's idea provided set standards that ensured that the control techniques developed were as optimal as possible.

Another feature of Optimal Control Theory that can be underlined is the ability to model systems that develop using state factors and control parameters. Control parameters are indications of force, strength, price of input, or choice, while state variables are indications of the growing conditions of the structure. Examples of state parameters include place, speed, and heat [3]. Optimal Control Theory provides means for constructing objective functions that characterise performance indicators such as expense, time, or profit. Such a situation is achieved by coupling differential equations expressing the dynamics of the whole system. It is generally believed that an optimum control issue is made up of three parts in its theoretical definition: a state equation, a cost function, and constraints. Analogous to conventional differential equations, the state equation sets out how the whole system behaves. The cost function is used to assess the efficiency of a definite method by integration over the time horizon. The system can be placed under equality and inequality conditions that represent material, financial, or technological constraints.

Optimal Control Theory has a great influence on technologies such as robotics, aircraft, and automobiles [4]. It is applied in robotics, for instance, to develop proper paths for robots that are supposed to perform activities such as welding or painting. An important function within the aerospace industry is to effectively control the flight paths of aeroplanes and spaceships and the amount of fuel they consume. Similarly, it contributes to new trends in automated automobiles by enhancing the scheduling of appropriate courses and uses of power. Among theoretical approaches, Optimal Control Theory is particularly significant in decisions about resource allocation, manufacturing schedules, and pricing strategies. It is used to maximise revenue or minimise expense, allowing businesses to determine how best to allocate resources such as labour and capital over a period. In macroeconomic guideline modelling, elements of the framework are applied to fiscal and quantitative measures aimed at preserving industrial growth and the inflation rate. The Hamiltonian function links the cost functional and the fluctuations of a given system [4]. Through assessment of Hamiltonian dynamics, it may be possible to build control techniques that either maximise the objective variable or keep it at the lowest level.

Moreover, the theory introduces costate variables, which can be described as Lagrange multiplier coefficients associated with shifts in requirements and which contain information about the implicit prices of state factors. With the appearance of Richard E. Bellman's Dynamic Programming, Optimal Control Theory was supplemented with another valuable tool [5]. This technique uses the optimisation concept according to which the smaller issues of an optimal approach

are themselves optimal. By solving backwards from the final state and determining function values at each step, Dynamic Programming develops improved control tactics for business challenges. It is especially useful for discrete-time systems and circumstances with complicated restrictions.

Optimal Control Theory has also seen increased use in health care and biological systems. This is particularly true for the transmission of viral diseases and the formulation of immunisation plans that place minimal pressure on public health-care systems. It also aids in enhancing the effectiveness of therapy for chronic conditions such as cancer or insulin resistance, ensuring positive effects while reducing detrimental impacts. More recent advances in computing techniques have greatly expanded the use of Optimal Control Theory. Examples of numerical solvers include Gradient Descent and Sequential Quadratic Programming. Software programs such as MATLAB, GAMS, and LINGO provide a strong foundation for carrying out and analysing optimum-control problems [6]. These technologies are useful in fields such as solar energy, where optimum control is applied to electricity storage operation and electrical-grid stability.

Optimal Control Theory also presents challenges. Whenever system dynamics are nonlinear or contain elements of uncertainty, mathematically handling multidimensional problems becomes a major concern [7]. In such cases, solutions can be approximated using approaches such as Model Predictive Control (MPC) or reinforcement learning. Immediate data permit continuous adjustments that maintain workability under unpredictable circumstances.

In the final analysis, Optimal Control Theory is an essential analytical framework for optimising systems in many fields. Its versatility and importance are indicated by applications spanning engineering, finance, medicine, and the environment [8]. Today, the theory remains a key factor in technological development because it allows decision-makers to achieve their desired ends efficiently. Continued advances in computational tools and interdisciplinary studies ensure that Optimal Control Theory will remain valuable for solving difficult problems in an increasingly dynamic environment.

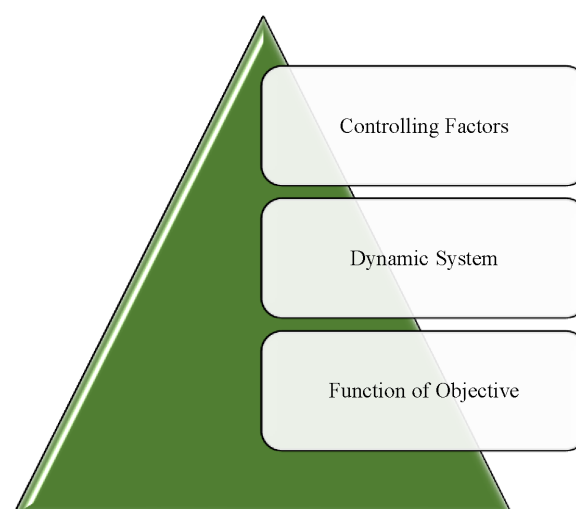


Figure 1. Dynamical Systems Optimisation Flow Diagram.

The optimal control theory is shown visually by the block schematic in Figure 1, which presents the fundamental ele-

ments and flow of the concept. The control-parameters block, located on the left side of the screen, serves as the starting point for the procedure [9]. Examples of decision parameters are force within a moving system and price within an economic simulation. These inputs are used to make decisions. In order to obtain the desired results, they are modified and have a direct impact on the present condition of the framework. This block is connected to the centre block by arrows that illustrate how these signals alter the functioning of the system.

The block that reflects the behavioural patterns exhibited by the system over time is called the system-dynamics block, and it is located in the middle of the diagram. This block is identified by state parameters such as location, speed, or energy state [10]. Differential equations guided by controlled inputs bring about the evolution of those parameters. The centre of the changing structure is reflected in this block, where changes are defined by the rules of the physical, economic, or technological world. Arrows pointing from the system-dynamics section to the opposite side indicate how changing circumstances affect the general outcome or objectives of the system.

The last step in the optimisation process is the objective-function block, located on the right side of the diagram. System performance is evaluated according to criteria such as minimising costs or maximising efficiency [11]. Other annotations, such as restrictions and optimisation strategies, draw attention to important aspects and approaches used to control inputs and arrive at the best possible solution. Optimisation methods such as Pontryagin's Maximum Principle direct decision-making so that it is both efficient and practicable. Constraints, on the other hand, restrict the plausible solutions. Combined, these components create an integrated structure through which optimum control theory can be understood and implemented.

2. EXISTING WORK DONE

The branch of mathematics called Optimal Control Theory focuses on defining a control procedure for a constantly changing environment in order to achieve an objective that fulfils specified criteria. Its mathematical models define options that vary over time. It is applicable across numerous domains, including finance, economics, and biology, because processes evolve over time under the influence of control factors [12]. Although rooted in classical optimisation approaches, the methodology has evolved dramatically since it was first conceived. Optimal Control Theory can be traced back to the Calculus of Variations, a branch of computational mathematics concerned mainly with the optimisation of functions. Researchers extended it to dynamic systems and expanded the range of possible applications. Among the important developments is Pontryagin's Maximum Principle (PMP). This concept, which provides the necessary conditions for an optimum-control problem, remains highly relevant. A fundamental component of the theory is a collection of differential equations and constraints that may be applied to determine the best possible solution to control problems.

Perhaps the first uses of optimal control were in the aviation sector, where researchers were tasked with designing the most efficient flight routes for spacecraft and aircraft. The broad

implementation of the concepts in engineering may be traced to the application of the Calculus of Variations and optimum management to such real-world systems. Optimal Control Theory continues to rely heavily on the Maximum Principle as one of its foundational principles [13]. The principle offers both a conceptual structure and practical tools for resolving control issues by providing requirements that must be met for a control approach to be considered optimum. To do this, a Hamiltonian function is defined that integrates the dynamics of the system with the objective function to be maximised or minimised.

The Hamiltonian is significant because it incorporates both control inputs and state parameters. This ensures that the variables governing each step are selected in a manner that maximises the overall objective. Costate factors are also included in the Maximum Principle. These costate variables are introduced as Lagrange multipliers to impose the limitations of the system. Dynamic Programming (DP) developed as another effective instrument for addressing optimum-control issues [14]. According to Bellman's Principle of Optimality, an optimum choice at any stage of an operation must take into consideration all possible choices for subsequent stages. This allows the problem to be broken down into more manageable subproblems that may be handled recursively. Dynamic Programming is especially useful when the time horizon is discrete and the state and control elements are subject to uncertainty.

Modern Dynamic Programming algorithms such as value iteration and policy iteration were developed from this work. These approaches handle optimum-control issues by solving Bellman equations. Until convergence is reached, the algorithms repeatedly update value functions [15]. In situations where a problem is too complicated for analytical approaches, such as probabilistic control structures and nonlinear systems, DP serves as an effective solution. After the presentation of the Maximum Principle, researchers sought to generalise the concept for a wider range of situations. The inclusion of stochastic mechanisms into Optimal Control Theory is one significant expansion. Because most real-world systems are not completely predictable, incorporating randomness enables the optimisation of processes subject to uncertainty.

Stochastic Optimal Control (SOC) extends the Maximum Principle to systems whose dynamics are affected by unplanned events. Such models are crucial in finance, where stock values fluctuate in response to random events, and in robotics, where modelling must account for variability in sensor data and actuation noise. One of the major obstacles to implementing Optimal Control Theory is the mathematical difficulty of solving multidimensional systems [16]. This is particularly true when system dynamics are nonlinear or involve many interacting state and control factors. Several computational methods have been developed to address this issue. Direct collocation, Sequential Quadratic Programming (SQP), and gradient-based optimisation have all made important contributions. Direct collocation techniques discretise the optimum-control problem over the time horizon and solve the resulting system of algebraic equations with nonlinear solvers. This strategy is useful for nonlinear challenges where other approaches may not be as successful.

Optimal Control Theory has important applicability in pro-

fessional domains where operations change over time and management choices must be implemented dynamically. In aerospace technology, the method was first used to optimise spacecraft trajectories [17]. By taking gravitational forces and fuel consumption into account, researchers demonstrated that the Maximum Principle could predict an efficient course for a spacecraft. Over time, the concept was extended to mechanics, electronics, and robotics. Motion planning, in which computers discover efficient paths while considering obstacles and energy consumption, has become an essential part of optimal control. Optimal Control Theory is also used in electrical-engineering systems to optimise load distribution and effectiveness. Models have been developed to optimise control choices involving switches, electrical lines, and converters [18], allowing demand to be met at the lowest feasible cost. The methods are also used in vehicle technology, particularly autonomous navigation, where safety, fuel efficiency, and travel time require real-time control decisions.

Some of the most notable applications of Optimal Control Theory are in finance and economics [19]. Researchers have investigated its use in modelling the global economy and in actions such as economic management and inflation regulation. The goal is to find a strategy that minimises the negative effects of unemployment, inflation, and national debt while maintaining marketplace stability. Governments may stabilise the financial sector over the long term by adjusting spending, taxation, and interest rates. Optimal Control Theory has also been applied to investment-portfolio management, especially for limiting risk and maximising returns under uncertainty. Researchers have constructed investment-optimising algorithms using these approaches [20]. These frameworks gradually reallocate assets such as stocks, bonds, or real estate to maximise return while mitigating risks associated with market fluctuations. Optimal Control Theory thus provides a framework for long-term investment, hedging, and risk management, making its application crucial in modern finance.

Biological systems and medical applications have also experienced major growth in research dedicated to Optimal Control Theory. Epidemic researchers use optimal control to model the emergence of infectious illnesses such as HIV, malaria, and tuberculosis and to determine effective vaccination schedules. Using optimal control and coefficients of variation, researchers can simulate disease progression as an ecosystem and determine suitable vaccination strategies [21]. Taking available resources and constraints into account can reduce infections and fatalities. Personalised medicine also uses Optimal Control Theory to schedule treatments such as radiation or insulin for managing diabetes and related complications. In such models, control inputs—for example, medicine dose or insulin injection—are modified over time to maximise treatment effectiveness while minimising adverse effects. These models are especially useful when a patient's condition changes during treatment and therapy must be proactively adapted to achieve the best long-term medical results.

Despite its broad range of applications, Optimal Control Theory faces several difficulties. The computational complexity of addressing large-scale dynamic systems is one of the most significant challenges, particularly when system functions are

highly nonlinear or involve many interacting variables [22]. In robotics and aerospace projects, for example, optimisation can be analytically demanding and expensive because systems may contain hundreds of state and control parameters. Approximation approaches and heuristic algorithms have been developed to overcome this issue. Model Predictive Control (MPC), which solves a succession of optimisation problems over a limited horizon, is used to estimate the best solution. Reinforcement Learning (RL) is another method that acquires control policies through trial and error. It has grown in popularity because it can optimise systems without requiring an exact model, enabling its use when modelling is difficult or unavailable [23]. Developments in machine learning, artificial intelligence, and real-time data processing are likely to have a significant impact on the future of Optimal Control Theory. They will enable more advanced and effective optimisation strategies. As the field develops, greater emphasis will also be placed on stochastic optimisation approaches for handling uncertainty in real-time applications [24, 25], especially in autonomous systems, banking, and medicine.

3. OBJECTIVE OF THE RESEARCH WORK

- Improving the long-term efficiency of complex structures requires the creation of computational models and methods.
- With certain limitations in mind, develop control techniques that aim to maximise or minimise a predetermined objective function, such as expenditure, effectiveness, or profitability.
- To tackle contemporary management-system problems, study and expand upon traditional ideas such as Dynamic Programming and Pontryagin's Minimum Theorem.

4. MOTIVATION FOR THE RESEARCH WORK

- When dealing with complicated dynamical optimisation issues, real-world systems often turn to Optimal Control Theory for a theoretical foundation.
- The significance of this study is driven by the need for effective decision-making in ever-changing domains including aerospace, robotics, and finance.
- To accomplish goals such as maximising performance or minimising costs, the methodology permits the modelling and optimisation of systems that evolve gradually.

5. THE PROJECTED METHOD

A framework for the spread of innovations and their impact on sales growth is established in this section. Incorporating a dynamic study of the influence of product pricing on customer purchase choices, the adoption procedure is conceptualised as a three-stage integrative process. The modelling approach also incorporates logistic learning curves for the product-acceptance rate. It is assumed that the corporation has already decided on its intended demographic and plans to use the best pricing method to maximise income. The effect of a cost-learning curve that varies with innovation sales is included. Through progressive implementation of Pontryagin's Maximum Principle in marketing, the optimal pricing approach is derived.

The diffusion process is modelled through a stage framework. A potential customer first becomes aware of the product, then becomes interested in it and develops a desire to purchase it

after becoming satisfied with its retail price. Recognition of the innovation is a critical factor in its acceptance. Therefore, the first stage of diffusion is dissemination of awareness, in which prospective buyers are informed about the existence and features of the innovation. The mathematical meaning of this transmission of information is

$$\frac{d}{dt}M_k(v) = c(n - M_k(v)). \quad (1)$$

Here, c is the percentage of people aware of the product. Equation (1) explains the instantaneous pace at which the prospective market becomes informed, which is proportional to the number of prospective consumers who remain unaware. Solving Equation (1) under the initial condition $M_k(0) = 0$ gives

$$M_k(v) = n(1 - f^{-cv}). \quad (2)$$

Equation (2) gives the aggregate number of aware customers at time v . There is a significant relationship between the retail price of a product and a customer's desire to purchase it. Pricing is one of the most important aspects of advertising. Once a person understands the product, they are more likely to accept it, but only after considering its cost. Because consumers prefer an affordable price to a higher one, the willingness consumers associate with price is clearly decreasing. Product value is therefore an important factor in determining whether an individual is motivated to acquire the innovation. The total number of prospective adopters motivated to make a purchase is stated as follows.

$$M_2(v) = M_2(v)\varphi(v). \quad (3)$$

The price function, denoted by $\varphi(v)$, is characterised by the following negatively exponential structure:

$$\varphi(v) = f^{-\theta Q(v)}, \quad (4)$$

where θ is the parameter representing the price component. The total number of enthusiastic adopters is calculated by substituting Equations (2) and (4) into Equation (3):

$$M_2(v) = n(1 - f^{-cv})f^{-\theta Q(v)}. \quad (5)$$

The inspired individual is encouraged to proceed with the purchase and ultimately becomes an adopter of the technology after becoming certain about the price of the item. As the diffusion process continues, the scope of the market expands together with the number of purchasers. Consequently, the adoption level is assumed to have a logistic form:

$$c(v) = \frac{c}{1 + \beta f^{-cv}}. \quad (6)$$

Consequently, the differential formulation of the demand function is

$$\frac{d}{dt}M_3(v) = \frac{c}{1 + \beta f^{-cv}}[M_2(v) - M_3(v)]. \quad (7)$$

Substituting Equations (5) and (6) into Equation (7) gives the selling-rate equation

$$M_3(v) = \frac{c}{1 + \beta f^{-cv}} \left[n(1 - f^{-cv})f^{-\theta Q(v)} - M_3(v) \right]. \quad (8)$$

Equation (8) represents the innovation's end users. The impact of pricing on product demand and on purchasing behaviour has been studied by many scholars. In the present investigation, the effect of price fluctuation on product popularity is taken into consideration. It is assumed that customer views concerning product acceptance may be changed to some degree by managing the product's cost. Based on this premise, an optimisation model is constructed from the producer's perspective to establish the best price, ultimately resulting in the greatest possible sales. Neither cost nor price is considered dynamic in this particular analysis. The cost-learning phenomenon is incorporated through a marginal price that depends on sales and decreases as adoption increases:

$$D(v) = D(M_3(v)), \quad \frac{\partial D(M_3(v))}{\partial M_3(v)} = D'(M_3(v)) \leq 0. \quad (9)$$

It is further assumed that the manufacturer wishes to maximise total profit by regulating the product's selling price over the specified planning period V at a discount rate s . The profit-maximisation problem is structured as an optimum-control problem. The selling price $Q(v)$ is treated as a control variable to be optimised in order to manage total product sales, $M_3(v)$, which is the system or state variable. The optimisation model maximises overall earnings subject to the sales-rate constraint. The optimum-control problem for maximising the benefit function can be organised as

$$y(v) = \dot{M}_3(v) = \frac{c}{1 + \beta f^{-cv}} \left\{ n(1 - f^{-cv})f^{-\theta Q(v)} - M_3(v) \right\}. \quad (10)$$

The relationship between the control parameter $Q(v)$ and the state factor $M_3(v)$ is characterised by the adoption-rate constraint in Equation (10). The primary function y is twice differentiable with respect to price and satisfies

$$y_q > 0, \quad y_{qq} < 0, \quad (11)$$

where

$$y_q = \frac{\partial y(v)}{\partial Q(v)}, \quad y_{qq} = \frac{\partial^2 y(v)}{\partial Q(v)^2}. \quad (12)$$

Pontryagin's Maximum Principle supplies the necessary optimality condition for the proposed management problem. First, the constrained problem is transformed into an unconstrained expression, producing the current-value Hamiltonian G :

$$G = \{ (Q(v) - D(v)) + \mu(v) \} M_3(v), \quad (13)$$

where $\mu(v)$ is the current-value costate variable representing the marginal variation in the objective value resulting from a small adjustment in the state variable, and $M_3(v)$ is the state variable. The costate relation is

$$\frac{d}{dt}\mu(v) = \dot{\mu}(v) = s\mu - \frac{\partial G}{\partial M_3(v)} = 0, \quad (14)$$

or

$$\dot{\mu}(v) = s\mu = \left\{ Q(v) - D(v) + \mu \right\} \frac{\partial y(v)}{\partial M_3(v)} + \frac{\partial D(v)}{\partial M_3(v)} y(v), \quad (15)$$

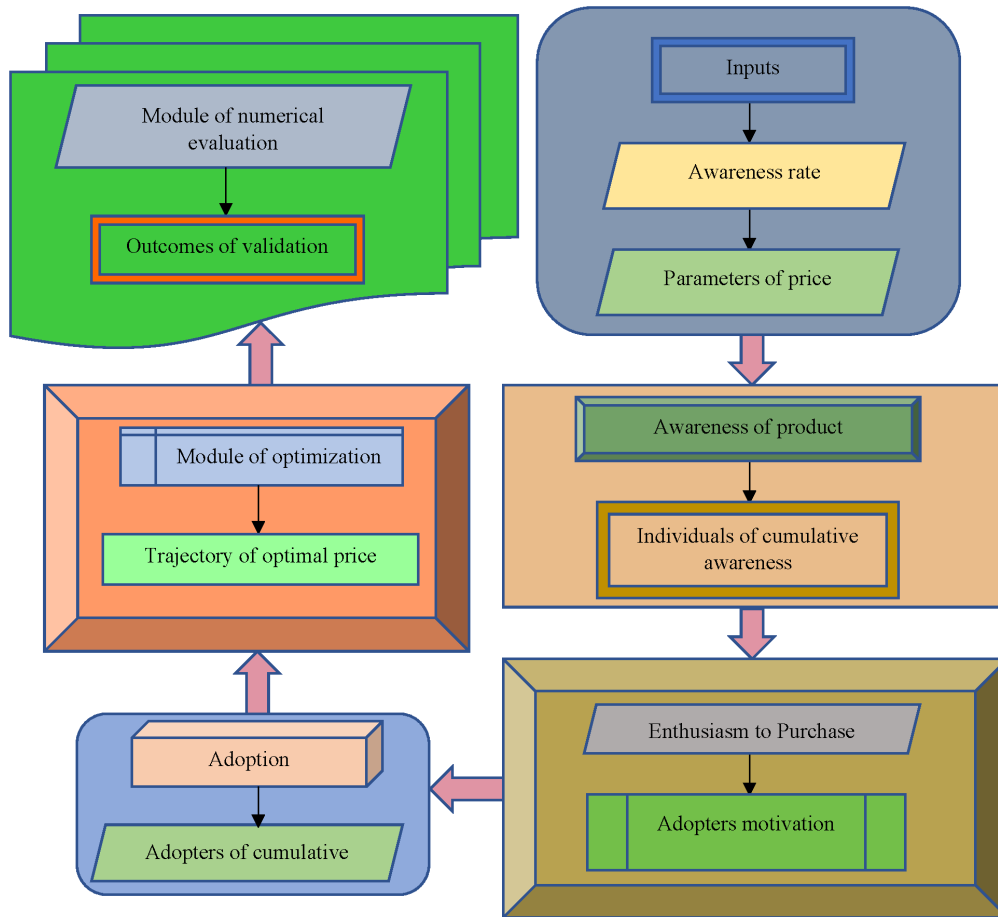


Figure 2. Architecture of proposed method.

with the transversality condition

$$\mu(V) = 0. \quad (16)$$

The Hamiltonian G represents total profit, including immediate benefits and future benefits. The necessary first-order condition for the best possible approach is

$$\frac{\partial G}{\partial Q(v)} = 0, \quad (17)$$

which gives

$$M_3(v) + \{Q(v) - D(v) + \mu(v)\}y_q = 0. \quad (18)$$

A numerical example demonstrates the practicality of the proposed technique. Actual sales and pricing data for consumer-durable items are considered in the statistical evaluation of the control problem. LINGO 15.0 is used to solve the numerical problem. The optimisation objective is to determine an optimum selling-price strategy for the new product that controls sales and maximises total company profit. Twelve equally spaced time points are assumed over the planning period. For the numerical demonstration, marginal cost is assumed constant and therefore independent of product sales.

The technology-diffusion process has long been an important topic for researchers and practitioners. Successful analysis of an innovative technology requires both reasoning and technical understanding. The findings based on real-life examples have several important managerial implications. The

proposed approach demonstrates the significance of product awareness and pricing strategy in final technology acceptance. Empirical investigation using real sales and pricing data for four distinct products can help management better understand adoption mechanisms. Product pricing is one of the most important factors accelerating diffusion and is therefore an essential element of product marketing. Selling price also directly influences company profit, so managers must thoroughly examine their pricing strategy to maximise profitability. The conclusions of the optimisation model can help managers stimulate sales by concentrating on dynamic pricing.

Most diffusion-modelling approaches assume that the potential market size remains constant over time and that all potential customers ultimately become end users. However, not every prospective buyer will complete a purchase. Potential customers receive product information that may support or oppose adoption, and they do not make a purchase until they are satisfied with that information. This study therefore analyses a three-stage innovation-diffusion model that investigates the role of dissatisfaction among potential customers. The first stage disseminates information about the innovation throughout the target market. The proposed model also accounts for unfavourable information and positive word-of-mouth effects after an innovation is accepted. In addition, a maximisation model produces a strategy for optimising the innovation's price decisions. The nonlinear optimisation problem is transformed into an optimal-control problem that maximises the total expected profit function. A discount rate

accommodates consumer time preferences for payment. The system or state elements—the potentially engaged population and overall sales—are optimised by controlling the price variable. The resulting model establishes an ideal adaptive pricing strategy for a new innovation to maximise overall company profit. Incremental adoption is a powerful metric of concern to virtually every business; consequently, the three-stage diffusion process is used to characterise total sales and determine pricing strategies. Figure 2 summarises the full procedure.

6. RESULTS AND ANALYSIS

To optimise pricing strategies for new-product acceptance, the suggested technique employs a three-stage diffusion model based on Optimal Control Theory. Product awareness, motivation to buy, and market adoption are the three stages of the diffusion process. During the awareness stage, dissemination and personal recommendations educate prospective customers. The motivational stage uses pricing strategies to gauge consumer interest, and the adoption stage uses a logistic growth function to predict purchase choices. The framework performs dynamic optimisation with price as the control variable to maximise profit over a defined planning horizon. Pontryagin's Maximum Principle determines an ideal price trajectory that adapts to market conditions and consumer behaviour. Validation with real-world data shows that the method increases adoption rates and profits.

6.1 Accuracy

Accuracy is the share of observations that are correctly predicted, including both positive and negative outcomes. It represents how well the model classifies users as adopters or non-adopters.

Table 1. Outcome of Statistical Measures of Accuracy

Models	Accuracy (%)
Optimal Control Theory (proposed method)	96
SVM	93
RF	90
KNN	86
LR	84

Compared with the other models, the suggested Optimal Control Theory approach achieves better accuracy than conventional methods. Its 96% accuracy shows that it is more effective at modelling and predicting adoption behaviour and pricing strategies. Support Vector Machine (SVM) comes second with 93% accuracy. Random Forest (RF) and K-Nearest Neighbours (KNN), with accuracy values of 90% and 86%, respectively, perform less effectively and may have difficulty representing the complex influence of pricing on consumer uptake. Logistic Regression (LR), with an accuracy of 84%, is least suited to nonlinear relationships arising during adoption. The proposed approach therefore stands above the competing methods for forecasting optimal pricing and product acceptance.

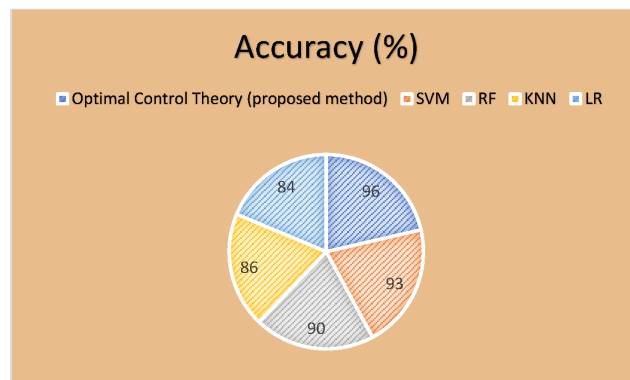


Figure 3. Evaluation of ML models in comparison to more traditional approaches.

6.2 F1-Score

The F1-score is the harmonic mean of precision and recall. It is particularly useful when the data are imbalanced because it provides a balanced assessment of a model's true-positive and false-negative prediction abilities.

Table 2. Results of F1-Score Statistical Measures

Models	F1-Score (%)
Optimal Control Theory (proposed method)	94
SVM	91
RF	88
KNN	84
LR	82

For forecasting product adoption, the suggested Optimal Control Theory strategy has the greatest F1-score, 94%, reflecting an outstanding balance between recall and precision. SVM is the next-best approach with an F1-score of 91%; it shows good performance but does not match the proposed method in capturing both true positives and true negatives. RF and KNN achieve lower F1-scores of 88% and 84%, respectively, indicating that they may not strike the ideal balance between recall and precision. LR is least effective, with an F1-score of 82%, showing difficulty with complicated dynamics, especially with imbalanced or nonlinear information. The proposed approach is thus the most effective for adoption forecasting according to the F1-score.

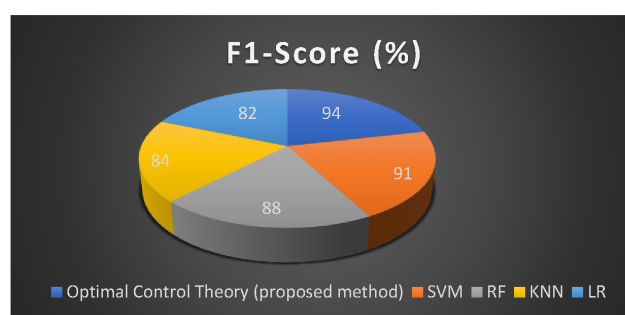


Figure 4. Comparing ML models to more conventional methods for evaluation.

6.3 Recall (Sensitivity)

Recall measures how well the model recognises actual adopters. It evaluates the model's capacity to include all prospective users in the specified market.

Table 3. Statistical Measures for Recall Outcomes

Models	Recall (%)
Optimal Control Theory (proposed method)	95
SVM	92
RF	89
KNN	85
LR	83

For accurate prediction of product adoption, the proposed Optimal Control Theory technique has an outstanding recall of 95%, showing that it can correctly recognise a majority of actual adopters. SVM comes second with a recall of 92%, indicating that it also detects true positives effectively but remains below the proposed technique. RF and KNN obtain recall values of 89% and 85%, respectively, indicating less effective capture of adopters. With a recall of only 83%, LR has the greatest difficulty identifying adopters. The proposed method's recall is substantially higher than competing methods, making it a more precise predictor of who will actually purchase the product.

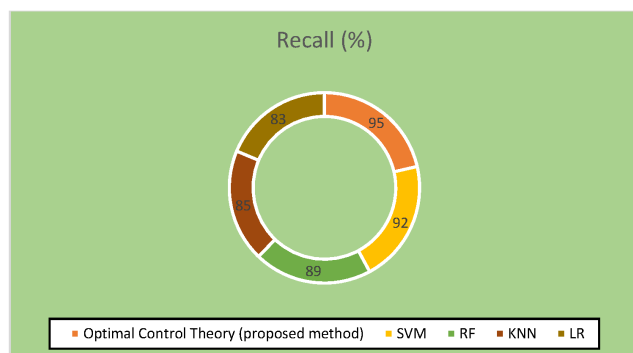


Figure 5. Efficacy of different systems.

6.4 Precision

Precision is the percentage of predicted adopters who actually adopt. It indicates whether the model can forecast adoption reliably without producing too many false positives.

Table 5. Exploratory DL Models in Relation to Proposed Methods

Models	Precision (%)
Optimal Control Theory (proposed method)	93
SVM	90
RF	87
KNN	84
LR	82

With a maximum precision of 93%, the proposed Optimal Control Theory approach shows that it can effectively detect genuine adopters with few false positives. SVM comes

second at 90%, which is excellent but does not match the proposed technique's precision in adopter prediction. RF and KNN achieve 87% and 84%, respectively, indicating more false-positive predictions than SVM and the proposed approach. LR has the lowest precision, 82%, suggesting that it is less effective at identifying who will actually adopt a product in this environment. Overall, the proposed method is the most accurate for making reliable probabilistic predictions about adoption.

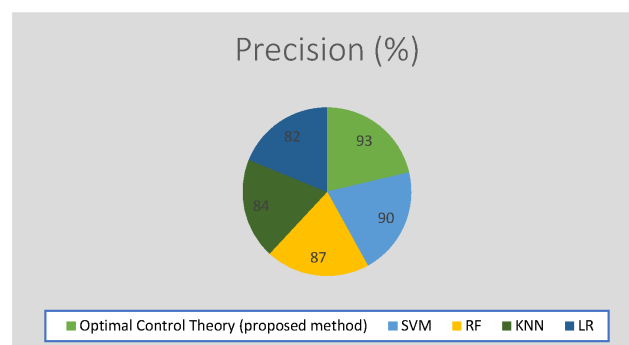


Figure 6. Effectiveness of different models.

7. CONCLUSION AND FUTURE SCOPE

Optimal Control Theory provides a conceptually sound foundation for identifying optimum control strategies for dynamic systems whose configuration and functionality change over time. It makes it possible to reduce cost, increase productivity, or enhance system performance through state parameters, control factors, and an objective function. The theory applies to aeronautical engineering, robotics, economics, medical care, and finance. Modern control systems are based on principles including Dynamic Programming and the Maximum Principle, which offer both theoretical support and real-life applicability. Difficulties nevertheless arise when Optimal Control Theory is applied to complex, high-dimensional, and stochastic systems. Advances in mathematical methods and connections with growing fields such as machine learning and artificial intelligence are moving the theory toward wider practical application. It is increasingly feasible in autonomous systems, solar-energy conversion, and personalised health care. Optimal Control Theory thus unifies efficient schemes for optimising complex systems while remaining flexible and robust. Its ability to adapt to variations in technical specifications demonstrates its continuing relevance to pressing global challenges.

Although Optimal Control Theory has developed considerably in dynamic optimisation, it retains substantial potential for further development. Future research may focus on multinode and nonlinear computing methods that model higher-dimensional systems. Open-loop problems arising from unclear or partially understood system dynamics may be addressed by combining Optimal Control Theory with reinforcement learning.

Funding: "This research received no external funding."

Conflicts of Interest: "The authors declare no conflict of interest."

REFERENCES

- [1] V. Mahajan and E. Muller, "Timing, diffusion and substitution of successive generations of technological innovations: The IBM mainframe case," *Technological Forecasting and Social Change*, vol. 51, pp. 109–132, 2020.
- [2] P. Renana, E. Muller, and V. Mahajan, "Innovation diffusion and new product growth models: A critical review and research directions," *International Journal of Research Marketing*, vol. 27, pp. 91–106, 2010.
- [3] N. Kim, D. R. Chang, and A. D. Shocker, "Modelling intercategory and generational dynamics for a growing information technology industry," *Management Science*, vol. 46, pp. 496–512, 2000.
- [4] U. Chanda and A. K. Bardhan, "Modelling innovation and imitation sales of products with multiple technological generations," *Journal of High Technology Management Research*, vol. 18, pp. 173–190, 2008.
- [5] C. H. Skiadas and A. N. Giovanis, "A stochastic bass innovation diffusion model for studying the growth of electricity consumption in greece," *Applied Stochastic Models and Data Analysis*, vol. 13, pp. 85–101, 2010.
- [6] Katsamaki and C. H. Skiadas, "Analytic solution and estimation of parameters on a stochastic exponential model for a technological diffusion process," *Applied Stochastic Models and Data Analysis*, vol. 11, pp. 59–75, 2010.
- [7] H. P. Boswijk and P. H. Franses, "On the econometrics of the bass diffusion model," *Journal of Business & Economic Statistics*, vol. 23, no. 3, pp. 255–268, 2005.
- [8] Oksendal, *Stochastic Differential Equations: An Introduction with Applications*. Springer, 2003.
- [9] P. I. Bass, "Generational diffusion model data: The IBM mainframe case," 2003, data set.
- [10] S. G. Armato *et al.*, "Lung image database consortium: Developing a resource for the medical imaging research community," *Radiology*, vol. 232, no. 3, pp. 739–748, 2004.
- [11] W. Gedara *et al.*, "DDPM-CD: Remote sensing change detection using denoising diffusion probabilistic models," 2022, arXiv preprint.
- [12] Baranchuk *et al.*, "Label-efficient semantic segmentation with diffusion models," 2021, arXiv preprint.
- [13] L. Barnett *et al.*, "Comparative accuracy of diagnosis by collective intelligence of multiple physicians vs individual physicians," *JAMA Network Open*, vol. 2, no. 3, pp. e190096–e190096, 2019.
- [14] F. Baumgartner *et al.*, "PHiSeg: Capturing uncertainty in medical image segmentation," in *International Conference on Medical Image Computing and Computer-Assisted Intervention*, 2019, pp. 119–127.
- [15] Kendall and Y. Gal, "What uncertainties do we need in bayesian deep learning for computer vision?" in *Advances in Neural Information Processing Systems*, 2017.
- [16] Kim *et al.*, "Diffusion adversarial representation learning for self-supervised vessel segmentation," 2022, arXiv preprint.
- [17] P. Kingma and M. Welling, "Auto-encoding variational bayes," 2013, arXiv preprint.
- [18] S. Kohl *et al.*, "A probabilistic U-Net for segmentation of ambiguous images," in *Advances in Neural Information Processing Systems*, 2018.
- [19] S. A. A. Kohl *et al.*, "A hierarchical probabilistic U-Net for modeling multi-scale ambiguities," 2019, arXiv preprint.
- [20] F. B. Hanson and J. J. Westman, "Optimal consumption and portfolio control for jump-diffusion stock process with log-normal jumps," in *Proceedings of the American Control Conference*, May 2002.
- [21] F. B. Hanson and L. L. Westman, "Portfolio optimization with jump-diffusions: Estimation of time-dependent parameters and application," in *Proceedings of the 41st IEEE Conference on Decision and Control*, vol. 1, Dec. 2002.
- [22] F. B. Hanson, J. J. Westman, and Z. Zhu, "Jump diffusion stock return models in finance: Stochastic process density with uniform-jump amplitude," *Journal of Financial Mathematics*, vol. 12, pp. 1–20, 2023.
- [23] Synowiec, "Jump-diffusion models with constant parameters for financial log-return processes," *Computers and Mathematics with Applications*, vol. 56, pp. 1–15, Oct. 2012.
- [24] M. S. Nikulin, "Advances in degradation modeling: Applications to reliability," in *Survival Analysis and Finance*. Birkhauser Boston, 2010.
- [25] W. E. J. Han, and J. Long, "Empowering optimal control with machine learning: A perspective from model predictive control," *IFAC-PapersOnLine*, vol. 55, no. 30, pp. 121–126, 2022.