



On Class of Bi-univalent functions involving Neutrosophic q -Poisson distribution Series

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Abstract

This paper introduces and investigates a new class of bi-univalent functions constructed through the Neutrosophic q -Poisson distribution series. The study focuses on estimating the upper bounds of the basic coefficients $|a_2|$ and $|a_3|$ in the Taylor series expansion of these functions.

Keywords: q -Poisson distribution; Bi-univalent functions; Unit disk; Analytic functions; Starlike functions

1. Introduction

Let \mathcal{A} denote the class of all analytic functions \mathcal{F} defined in the open unit disk \mathcal{U} and normalized by the conditions $\mathcal{F}(0) = 0$ and $\mathcal{F}'(0) - 1 = 0$. where $\mathcal{U} = \{z \in \mathbb{C} : |z| < 1\}$, Thus each $\mathcal{F} \in \mathcal{A}$ has a Taylor series expansion of the form :

$$\mathcal{F}(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathcal{U}. \quad (1.1)$$

According to this, every function $\mathcal{F} \in \mathcal{S}$ has an inverse map \mathcal{F}^{-1} that satisfies the following conditions:

$$\mathcal{F}^{-1}(\mathcal{F}(z)) = z \quad (z \in \mathcal{U}),$$

and

$$\mathcal{F}(\mathcal{F}^{-1}(w)) = w \quad (|w| < r_0(\mathcal{F}); r_0(\mathcal{F}) \geq \frac{1}{4}).$$

In fact, the inverse function is given by

$$\mathcal{F}^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots. \quad (1.2)$$

As the class $\mathcal{S}^*(\rho)$ of starlike functions of order ρ in the open unit disk \mathcal{U} has been extensively studied and is a subset of the function class \mathcal{S} . By definition, $\mathcal{S}^*(\rho)$ is given by the equation (1.1) see [8, 9, 16, 11, 17].

[7] introduced the class $\mathcal{q}(\rho)$, defined as:

$$\mathcal{q}(\rho) = \{ \mathcal{F} : \mathcal{F} \in \mathcal{S} \text{ and } \operatorname{Re}\{\mathcal{F}'(z)\} > \rho, (z \in \mathcal{U}; 0 \leq \rho < 1) \} \quad (1.3)$$

Additionally, [5] introduced the class $\mathcal{ST}(\rho)$, given by:

$$\mathcal{ST}(\rho) = \left\{ \mathcal{F} : \mathcal{F} \in \mathcal{S} \text{ and } \operatorname{Re}\left\{ \frac{\mathcal{F}(z)}{z} \right\} > \rho, (z \in \mathcal{U}; 0 \leq \rho < 1) \right\}. \quad (1.4)$$

A function $\mathcal{F} \in \mathcal{A}$ is said to be bi-univalent in \mathcal{U} if both $\mathcal{F}(z)$ and $\mathcal{F}^{-1}(z)$ are univalent in \mathcal{U} . Let Σ denote the class of bi-univalent functions in \mathcal{U} given by (1.1).

According to [3], a function $\mathcal{F} \in \mathcal{A}$ belongs to the class $\mathcal{S}_\Sigma^*(\rho)$ of strongly bi-starlike functions of order ρ ($0 < \rho \leq 1$) if it satisfies the following conditions:

$$\mathcal{F} \in \Sigma \text{ and } \left| \arg \left(\frac{z \mathcal{F}'(z)}{\mathcal{F}(z)} \right) \right| < \frac{\rho \pi}{2}, \quad (z \in \mathcal{U})$$

and

$$\left| \arg \left(\frac{w \mathcal{G}'(w)}{\mathcal{G}(w)} \right) \right| < \frac{\rho \pi}{2} \quad (w \in \mathcal{U}),$$

Where $\mathcal{G} = \mathcal{F}^{-1}$ and given by (1.2).

Furthermore, [2, 4] a function $\mathcal{F} \in \mathcal{A}$ falls into the class $\mathcal{S}_\Sigma^*(\rho)$ of bi-starlike function of order ρ ($0 \leq \rho < 1$) if it fulfills the following requirements:

$$f \in \Sigma \text{ and } \operatorname{Re} \left(\frac{z \mathcal{F}'(z)}{\mathcal{F}(z)} \right) > \rho \quad (z \in \mathcal{U})$$

and

$$\operatorname{Re} \left(\frac{w \mathcal{G}'(w)}{\mathcal{G}(w)} \right) > \rho \quad (w \in \mathcal{U}),$$

where $\mathcal{G} = \mathcal{F}^{-1}$ and given by (1.2).

[10] introduced the q -derivative operator $\frac{\partial}{\partial q}$ of a function \mathcal{F} given by equation (1.1) as follows:

$$\frac{\partial}{\partial q} \mathcal{F}(z) = \begin{cases} \frac{\mathcal{F}(z) - \mathcal{F}(qz)}{(1-q)z}, & \text{if } z \neq 0 \\ \mathcal{F}'(z), & \text{if } z = 0 \end{cases}, \quad (1.5)$$

for $q \in (0,1)$. It is clear that $\lim_{q \rightarrow 1^-} \frac{\partial}{\partial q} \mathcal{F}(z) = \mathcal{F}'(z)$ for \mathcal{F} is given by equation (1.1).

[12, 1] A discrete random variable θ is described as following a q -Poisson distribution if it takes the values 0, 1, 2, 3, ... with corresponding probabilities.

$$e_q^{-M}, \frac{M e_q^{-M}}{[1]_q!}, \frac{M^2 e_q^{-M}}{[2]_q!}, \frac{M^3 e_q^{-M}}{[3]_q!}, \dots \quad (1.6)$$

where M is the parameter, e_q^θ is the q -exponential function defined as

$$e_q^\theta = 1 + \frac{\theta}{[1]_q!} + \frac{\theta^2}{[2]_q!} + \frac{\theta^3}{[3]_q!} + \dots = \sum_{m=0}^{\infty} \frac{\theta^m}{[m]_q!} \quad (1.7)$$

$[m]_q! = [m]_q [m-1]_q \dots [2]_q [1]_q$ is the q -factorial function see [15].

For a q -Poisson distribution, the probability mass function is expressed as

$$\mathcal{P}_q(\theta = m) = \frac{e_q^{-M} M^m}{[m]_q!}, m = 0, 1, 2, \dots \quad (1.8)$$

The corresponding q -Poisson distribution series is defined by

$$\mathcal{P}_q(z) = z + \sum_{n=2}^{\infty} \frac{e_q^{-M} M^{n-1}}{[n-1]_q!} z^n, z \in \mathbb{U} \quad (1.9)$$

The ratio test, as referenced in [14], reveals that the series has an infinite radius of convergence. Building on this foundation, the neutrosophic theory, developed by Smarandache in 1995, offers a robust framework for addressing uncertain parameters. An example of this is the neutrosophic q -Poisson distribution, which extends the classical Poisson distribution by incorporating an interval-valued parameter $m\aleph$. The probability mass function of this modified distribution is

$$\aleph \mathcal{P}_q(\theta = k) = \frac{e_q^{-m\aleph} (m\aleph)^k}{[k]_q!} \quad (1.10)$$

where $\aleph = d + I$ represents a neutrosophic statistical number, and both the expectation and variance are $m\aleph$. The neutrosophic q -Poisson distribution series is given by

$$\mathbb{M}_q(m\aleph, z) = z + \sum_{n=2}^{\infty} \frac{e_q^{-m\aleph} (m\aleph)^{n-1}}{[n-1]_q!} z^n, z \in \mathbb{U} \tag{1.11}$$

As stated in [1, 20], the linear operator $\mathfrak{F}_{m\aleph}: \mathcal{A} \rightarrow \mathcal{A}$, constructed using convolution (Hadamard product), is expressed by

$$\mathfrak{F}_{m\aleph}(\mathcal{F}(z)) = \mathbb{M}_q(m\aleph, z) * \mathcal{F}(z) = z + \sum_{n=2}^{\infty} \frac{e_q^{-m\aleph} (m\aleph)^{n-1}}{[n-1]_q!} a_n z^n, z \in \mathbb{U} \tag{1.12}$$

Definition 1.1. A function $\mathcal{F}(z)$ given by (1.1) is said to be in the class $\mathbb{M}_{\Sigma}^{\rho}(\alpha, \lambda, m\aleph)$ if the following conditions are satisfied:

$$f \in \Sigma \text{ and } \left| \arg \left((1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{F}(z))}{z} \right)^{\alpha} + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{F}(z)) \right) \right| < \frac{\rho \pi}{2}$$

and

$$\left| \arg \left((1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{G}(w))}{w} \right)^{\alpha} + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{G}(w)) \right) \right| < \frac{\rho \pi}{2}$$

Where $(0 < \rho \leq 1, \lambda \geq 1, \alpha \geq 0, z, w \in \mathbb{U})$ and $\mathcal{G} = \mathcal{F}^{-1}$ and given by (1.2).

Definition 1.2. A function $\mathcal{F}(z)$ given by (1.1) is said to be in the class $\mathbb{M}_{\Sigma}^{\lambda}(\beta, \alpha, m\aleph)$ if the following conditions are satisfied.

$$f \in \Sigma \text{ and } \operatorname{Re} \left((1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{F}(z))}{z} \right)^{\alpha} + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{F}(z)) \right) > \beta$$

and

$$\operatorname{Re} \left((1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{G}(w))}{w} \right)^{\alpha} + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{G}(w)) \right) > \beta$$

Where $(0 \leq \beta < 1, \lambda \geq 1, \alpha \geq 0, w, z \in \mathbb{U})$ and $\mathcal{G} = \mathcal{F}^{-1}$ and given by (1.2).

To prove our theorem, we will make use of the following lemma:

Lemma 1.3 [6,13]. If $h \in H$, where H represents all analytic functions in \mathbb{U} and satisfy $\operatorname{Re}(h(z)) > 0$, where $h(z) = 1 + h_1z + h_2z^2 + \dots$, then $|h_i| \leq 2$ for each index i .

2. Coefficients bounds for classes $\mathbb{M}_{\Sigma}^{\rho}(\alpha, \lambda, m\aleph)$ and $\mathbb{M}_{\Sigma}^{\lambda}(\beta, \alpha, m\aleph)$

Theorem 2.1 Let $\mathcal{F}(z)$ given by (1.1) be in the class $\mathbb{M}_{\Sigma}^{\rho}(\alpha, \lambda, m\aleph)$, $0 < \rho \leq 1$ and $\lambda \geq 1$. Then

$$|a_2| \leq \frac{4 \sqrt{[2]_q!} \rho}{\sqrt{\mathcal{Q}_{\lambda, \alpha}^{\rho} - [2]_q! (\rho - 1) (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2}}$$

and

$$|a_3| \leq \frac{2\rho [2]_q!}{|e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]|} + \frac{4 \rho^2}{|(e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2|} \tag{2.1}$$

where

$$\mathcal{Q}_{\lambda, \alpha}^{\rho} = 2 e_q^{-m\aleph} (m\aleph)^2 \rho [\alpha (1 - \lambda) + [3]_q \lambda] + 2 [2]_q! \rho \left[(1 - \lambda) \frac{\alpha (\alpha - 1)}{2} (e_q^{-m\aleph} (m\aleph))^2 \right]$$

Proof To prove the theorem, The inequalities (2.3) and (2.4) are expressed in their equivalent forms.

$$(1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{F}(z))}{z} \right)^{\alpha} + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{F}(z)) = [\wp(z)]^{\rho} \tag{2.2}$$

$$(1 - \lambda) \left(\frac{\mathfrak{P}_{m\aleph}(\mathcal{G}(w))}{w} \right)^\alpha + \lambda \frac{\partial}{\partial q} \mathfrak{P}_{m\aleph}(\mathcal{G}(w)) = [q(w)]^\rho \tag{2.3}$$

Here, $p(z)$ and $q(w)$ are in the class H and satisfy the conditions defined in Definition 1.1. These functions can be written as:

$$p(z) = 1 + p_1z + p_2z^2 + p_3z^3 + \dots \text{ and } q(w) = 1 + q_1w + q_2w^2 + q_3w^3 + \dots$$

By equating coefficients in equations (2.2) and (2.3), The following relations are obtained.

$$e_q^{-m\aleph} (m\aleph) [\alpha (1 - \lambda) + \lambda[2]_q] a_2 = \rho p_1, \tag{2.17}$$

$$\frac{e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] a_3 + \left[(1 - \lambda) \frac{\alpha (\alpha - 1)}{2} (e_q^{-m\aleph} (m\aleph))^2 \right] a_2^2 = \rho p_2 + \frac{\rho(\rho - 1)}{2} p_1^2 \tag{2.18}$$

$$-e_q^{-m\aleph} (m\aleph) [\alpha (1 - \lambda) + \lambda[2]_q] a_2 = \rho q_1, \tag{2.19}$$

and

$$\frac{e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] (2a_2^2 - a_3) + \left[(1 - \lambda) \frac{\alpha (\alpha - 1)}{2} (e_q^{-m\aleph} (m\aleph))^2 \right] a_2^2 = \rho q_2 + \frac{\rho(\rho - 1)}{2} q_1^2. \tag{2.20}$$

Utilizing equations (2.4) and (2.19), The following is derived:

$$p_1 = -q_1 \tag{2.21}$$

and

$$2 (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda[2]_q)^2 a_2^2 = \rho^2 (p_1^2 + q_1^2). \tag{2.22}$$

Subsequently, combining equations (2.18), (2.20), and (2.22), The following is acquired:

$$\frac{2 e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] a_2^2 + 2 \left[(1 - \lambda) \frac{\alpha (\alpha - 1)}{2} (e_q^{-m\aleph} (m\aleph))^2 \right] a_2^2 = \rho(q_2 + p_2) + \frac{\rho(\rho - 1)}{2} (p_1^2 + q_1^2)$$

Thus, the conclusion is drawn:

$$a_2^2 = \frac{[2]_q! \rho^2 (q_2 + p_2)}{Q_{\lambda, \alpha}^\rho - [2]_q! (\rho - 1) (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda[2]_q)^2}$$

Consequently, we ascertain:

$$|a_2| \leq \frac{4 \sqrt{[2]_q!} \rho}{\sqrt{Q_{\lambda, \alpha}^\rho - [2]_q! (\rho - 1) (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda[2]_q)^2}}$$

This determines the upper bound for $|a_2|$ in accordance with (2.13).

Next, for the purpose of establishing the constraint on $|a_3|$, By subtracting (2.18) from (2.20), the following is obtained:

$$\frac{e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] (2a_3 - 2a_2^2) = \rho(p_2 - q_2) + \frac{\rho(\rho - 1)}{2} (p_1^2 - q_1^2)$$

By applying equation (2.17) and replacing it in equation (2.22) within equation (2.26) The following is obtained:

$$a_3 = \frac{\rho[2]_q! (p_2 - q_2)}{2e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]} + a_2^2 \tag{2.27}$$

Conversely, it can be stated as,

$$a_3 = \frac{\rho[2]_q! (p_2 - q_2)}{2e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]} + \frac{\rho^2 (p_1^2 + q_1^2)}{2 (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda[2]_q)^2} \tag{2.28}$$

We can establish the following upper bound for $|a_3|$:

$$|a_3| \leq \frac{2\rho[2]_q!}{|e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]|} + \frac{4\rho^2}{|(e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda[2]_q)^2|} \tag{2.29}$$

Which completes the proof.

Theorem 2.2 Let $\mathcal{F}(z)$ given by (1.1) be in the class $\mathbb{M}_{\Sigma}^{\lambda}(\beta, \alpha, m\aleph)$, $0 \leq \beta < 1$ and $\lambda \geq 1$. Then

$$|a_2| \leq \sqrt{\frac{4(1 - \beta)[2]_q!}{2 e_q^{-m\aleph} (m\aleph)^2 \Xi_{\alpha}^{\lambda} + [2]_q! [\alpha (\alpha - 1)(1 - \lambda) (e_q^{-m\aleph} (m\aleph))^2]}}$$

and

$$|a_3| \leq \frac{4(1 - \beta)^2}{(e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2} + \frac{2(1 - \beta)[2]_q!}{e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]} \tag{2.30}$$

where

$$\Xi_\alpha^\lambda = [\alpha (1 - \lambda) + [3]_q \lambda]$$

Proof It can be inferred from inequalities (2.5) and (2.6), there exist $\rho(z)$ and $q(w) \in H$ such that.

$$(1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{F}(z))}{z} \right)^\alpha + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{F}(z)) = \beta + (1 - \beta) \rho(z) \tag{2.31}$$

and

$$(1 - \lambda) \left(\frac{\mathfrak{F}_{m\aleph}(\mathcal{G}(w))}{w} \right)^\alpha + \lambda \frac{\partial}{\partial q} \mathfrak{F}_{m\aleph}(\mathcal{G}(w)) = \beta + (1 - \beta) q(w) \tag{2.32}$$

Equating coefficients in (2.31) and (2.32), this leads to:

$$e_q^{-m\aleph} (m\aleph) [\alpha (1 - \lambda) + \lambda [2]_q] a_2 = (1 - \beta) \rho_1, \tag{2.33}$$

$$\frac{e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] a_3 + \left[(1 - \lambda) \frac{\alpha (\alpha - 1)}{2} (e_q^{-m\aleph} (m\aleph))^2 \right] a_2^2 = (1 - \beta) \rho_2, \tag{2.34}$$

$$-e_q^{-m\aleph} (m\aleph) [\alpha (1 - \lambda) + \lambda [2]_q] a_2 = (1 - \beta) q_1, \tag{2.35}$$

and

$$\frac{e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] (2a_2^2 - a_3) + \left[(1 - \lambda) \frac{\alpha (\alpha - 1)}{2} (e_q^{-m\aleph} (m\aleph))^2 \right] a_2^2 = (1 - \beta) q_2. \tag{2.36}$$

Utilizing equations (2.33) and (2.35), The following is deduced:

$$\rho_1 = -q_1 \tag{2.37}$$

and

$$2 (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2 a_2^2 = (1 - \beta)^2 (\rho_1^2 + q_1^2) \tag{2.39}$$

From equations (2.33) and (2.35), it can be concluded that:

$$2 e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda] a_2^2 + [2]_q! [\alpha (\alpha - 1) (1 - \lambda) (e_q^{-m\aleph} (m\aleph))^2] a_2^2 = (1 - \beta) [2]_q! (\rho_2 + q_2)$$

Consequently, we obtain the following:

$$a_2 = \sqrt{\frac{(1 - \beta) [2]_q! (\rho_2 + q_2)}{2 e_q^{-m\aleph} (m\aleph)^2 \Xi_\alpha^\lambda + [2]_q! [\alpha (\alpha - 1) (1 - \lambda) (e_q^{-m\aleph} (m\aleph))^2]}} \tag{2.41}$$

This determines the upper bound for $|a_2|$, in accordance with (2.29):

$$|a_2| \leq \sqrt{\frac{4(1 - \beta) [2]_q!}{\left| 2 e_q^{-m\aleph} (m\aleph)^2 \Xi_\alpha^\lambda + [2]_q! [\alpha (\alpha - 1) (1 - \lambda) (e_q^{-m\aleph} (m\aleph))^2] \right|}} \tag{2.42}$$

Next, for the purpose of establishing the constraint on $|a_3|$, we subtract (2.34) from (2.36), we get

$$\frac{2e_q^{-m\aleph} (m\aleph)^2}{[2]_q!} [\alpha (1 - \lambda) + [3]_q \lambda] (a_3 - a_2^2) = (1 - \beta) (\rho_2 - q_2), \tag{2.43}$$

Alternatively, it can be expressed as:

$$a_3 = a_2^2 + \frac{(1 - \beta) [2]_q! (\rho_2 - q_2)}{2e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]}, \tag{2.44}$$

By using equation (2.38) in equation (2.43), It is found:

$$a_3 = \frac{(1 - \beta)^2 (\rho_1^2 + q_1^2)}{2 (e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2} + \frac{(1 - \beta) [2]_q! (\rho_2 - q_2)}{2e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]}, \tag{2.45}$$

We can establish the following upper bound for $|a_3|$:

$$|a_3| \leq \frac{4(1 - \beta)^2}{(e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2} + \frac{2(1 - \beta) [2]_q!}{e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]}, \tag{2.46}$$

Which completes the proof.

3. Corollaries and consequences

By substituting $\alpha = 1$ in Theorem (2.1), Theorem (2.2), we arrive at the following corollaries, respectively:

Corollary 3.1. Consider a function $\mathcal{F} \in \mathcal{A}$ of the form described in (1.1) that belongs to the function class $\mathbb{M}_{\Sigma}^{\rho}(1, \lambda, m\aleph)$, $0 < \rho \leq 1$ and $\lambda \geq 1$. Then

$$|a_2| \leq \frac{4 \sqrt{[2]_q!} \rho}{\sqrt{Q_{\lambda,1}^{\rho} - [2]_q! (\rho - 1) (e_q^{-m\aleph} (m\aleph))^2 ((1 - \lambda) + \lambda [2]_q)^2}}$$

and

$$|a_3| \leq \frac{2\rho [2]_q!}{|e_q^{-m\aleph} (m\aleph)^2 [(1 - \lambda) + [3]_q \lambda]|} + \frac{4 \rho^2}{|(e_q^{-m\aleph} (m\aleph))^2 ((1 - \lambda) + \lambda [2]_q)^2|}$$

Corollary 3.2 Let $\mathcal{F}(z)$ given by (1.1) be in the class $\mathbb{M}_{\Sigma}^{\lambda}(\beta, 1, m\aleph)$, $0 \leq \beta < 1$ and $\lambda \geq 1$. Then

$$|a_2| \leq \sqrt{\frac{4(1 - \beta)[2]_q!}{|2 e_q^{-m\aleph} (m\aleph)^2 \Xi_1^{\lambda}|}}$$

and

$$|a_3| \leq \frac{4(1 - \beta)^2}{(e_q^{-m\aleph} (m\aleph))^2 ((1 - \lambda) + \lambda [2]_q)^2} + \frac{2(1 - \beta)[2]_q!}{e_q^{-m\aleph} (m\aleph)^2 [(1 - \lambda) + [3]_q \lambda]}$$

By substituting $\lambda = 1, \beta = 0$ in Theorem (2.1), Theorem (2.2), we arrive at the following corollaries, respectively:

Corollary 3.3. Consider a function $\mathcal{F} \in \mathcal{A}$ of the form described in (1.1) that belongs to the function class $\mathbb{M}_{\Sigma}^{\rho}(\alpha, 1, m\aleph)$, $0 < \rho \leq 1$ and $\lambda \geq 1$. Then

$$|a_2| \leq \frac{4 \sqrt{[2]_q!} \rho}{\sqrt{Q_{1,\alpha}^{\rho} - [2]_q! (\rho - 1) (e_q^{-m\aleph} (m\aleph))^2 ([2]_q)^2}}$$

and

$$|a_3| \leq \frac{2\rho [2]_q!}{|e_q^{-m\aleph} (m\aleph)^2 [[3]_q]|} + \frac{4 \rho^2}{|(e_q^{-m\aleph} (m\aleph))^2 ([2]_q)^2|}$$

Corollary 3.4 Let $\mathcal{F}(z)$ given by (1.1) be in the class $\mathbb{M}_{\Sigma}^{\lambda}(0, \alpha, m\aleph)$, $0 \leq \beta < 1$ and $\lambda \geq 1$. Then

$$|a_2| \leq \sqrt{\frac{4[2]_q!}{|2 e_q^{-m\aleph} (m\aleph)^2 \Xi_{\alpha}^{\lambda} + [2]_q! [\alpha (\alpha - 1)(1 - \lambda)(e_q^{-m\aleph} (m\aleph))^2]|}}$$

and

$$|a_3| \leq \frac{4}{(e_q^{-m\aleph} (m\aleph))^2 (\alpha (1 - \lambda) + \lambda [2]_q)^2} + \frac{2[2]_q!}{e_q^{-m\aleph} (m\aleph)^2 [\alpha (1 - \lambda) + [3]_q \lambda]}$$

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