A Study on NCT −Filters in NCT −Topological Spaces

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Abstract

In this research, we introduce and develop new concepts in the field of Neutrosophic Topology (NCT). Particularly our study is focusing on the NCT −filter and its properties. Also, we present the properties of convergence of NCT −-filter, a specialized filter that incorporates neutrosophic values, providing a robust approach to handle uncertainty in topological spaces. Additionally, we explore the concept of adherent points in neutrosophic crisp triple topological spaces, offering a new perspective on the study of these spaces. Moreover, our findings contribute to expanding the understanding and application of neutrosophic theories in topology that will provide a solid foundation for future research in this area. Furthermore, this work opens new avenues for the study of topological spaces under uncertainty, with potential Applications in various domains, including data analysis, decision-making, and artificial intelligence, among others.

Keywords: NCT −Set; NCT −Point; NCT −Filter; NCT −Ultra Filter

1. Introduction

Neutrosophic Topology is a modern extension of traditional topology, originating from neutrosophic Logic, Which was introduced by Florentin Smarandache in the 1990s, as stated in [1]. This field aims to address uncertainty and contradictions in mathematical systems, providing a general framework that simultaneously handles true, indeterminate, and contradictory values [2]. The concept of Neutrosophy first emerged in logic and mathematics as a method for dealing with incomplete or undefined information, later expanding to Neutrosophic Set Theory in 1998 [1], and subsequently to topology in the early 21st century [2]. In 2014, the initial formal definition of neutrosophic topological spaces was presented, adapting concepts like open and closed sets, adherent points, and compactness, allowing for the accommodation of varying degrees of uncertainty, which makes it useful in areas like artificial intelligence, fuzzy data processing, and decision-making under uncertainty [3]. In this context, our research introduces several important concepts that contribute to the development of neutrosophic topology and offer deeper insights into the study of complex topological spaces. We developed the concept of the NCT −filter, a powerful tool that can be applied to expand the scope of neutrosophic set theory and related topological concepts. Additionally, we explore the Properties of Convergence of NCT −filter, a specific type of filter that takes into account neutrosophic values, enabling us to handle cases involving ambiguity and uncertainty.

Furthermore, we investigate adherent points in neutrosophic crisp triple topological spaces, a new concept introduced to the current literature in this field, providing additional tools for understanding the relationship between space sets and open sets in the context of neutrosophic theory.

This work builds upon the theoretical foundations established by several researchers in neutrosophic topology see for example [4], [5], [6], [7] and seeks to offer practical applications by introducing new concepts and techniques that contribute to the development of both academic and applied research in this promising field. The rest of the paper is organized as follows. In Section 2, some basic concepts about NCT −Set and NCT −Point are given. In Section 3, some results of NCT −Filter are established. NCT −Ultra Filter is presented in Section 4. Properties

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of Convergence of NCT −filter are presented and proved in Section 5. Section 6 states and proves adherent points in neutrosophic crisp triple topological space.In Section 7, some conclusions are given.

2. NCT −Set and NCT −Point

Definition 2.1. Let $X \neq \emptyset$. A neutrosophic crisp triple set NCT_A is an object having the form $NCT_A = \langle A_1, A_2, A_3 \rangle$. Where $A_1, A_2, A_3 \subseteq X$ satisfying $A_1 \subseteq A_2$ and $A_2 \cap A_3 = \emptyset$. And $NCT_X = \{NCT_A = \langle A_1, A_2, A_3 \rangle : A_1 \subseteq A_2$ and $A_2 \cap A_3 = \emptyset\}$ is the collection of all NCT −sets on X. From this definition we see that if $A_3 = X$, then $A_1 = A_2 = \emptyset$ and if $A_2 = X$, then $A_3 = \emptyset$, finally if $A_1 = X$, then $A_2 = X$ and $A_3 = \emptyset$.

Definition 2.2. Let $NCT_A = \langle A_1, A_2, A_3 \rangle$ and $NCT_B = \langle B_1 , B_2, B_3 \rangle$ are NCT −a non-empty set X over sets. Therefore:

1. NCT_A is a NCT −subset of NCT_B if $A_3 \supseteq B_3$ and $A_1 \subseteq B_1$, $A_2 \subseteq B_2$. We write $NCT_A \sqsubseteq NCT_B$
2. $NCT_A = NCT_B$ iff $NCT_A \sqsubseteq NCT_B$ and $NCT_B \sqsubseteq NCT_A$.
3. The NCT −complement of NCT −set NCT_A is $CNCT_A = \langle A_3, A_2^c, A_1 \rangle$.
4. $NCT_A \sqcup NCT_B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$ is a crisp triple set that is neutrosophic in union (NCT −union set).
5. $NCT_A \sqcap NCT_B = \langle A_1 \cap B_1 , A_2 \cap B_2, A_3 \cup B_3 \rangle$ is the intersection a crisp triple set that is neutrosophic (NCT −intersection sets).
6. $NCT_X = \langle X, X, \emptyset \rangle$ is NCT −universel set.
7. $NCT_\emptyset = \langle \emptyset, \emptyset, X \rangle$is NCT −null set. Clearly $CNCT_X = NCT_\emptyset$ and $CNCT_\emptyset = NCT_X$.

Definition 2.3. Let $X \neq \emptyset$ and $p \in X$. So NCT −point ($NCTP$) $NCT_p = \langle \{p\}, \{p\}, \{p^c\} \rangle$.

Definition 2.4. Let $X \neq \emptyset$ and $p \in X$ and $NCT_A = \langle A_1, A_2, A_3 \rangle$. Then the NCT −belong as follows:

$NCT_p \Subset NCT_A$ iff $p \in A_1$ and $NCT_p \not\Subset NCT_A$ iff $p \notin A_1$.

3. NCT −Filter

Definition 3.1. A NCT −filter (F_{NCT}^X for short) on a non-empty set X is a family of neutrosophic crisp triple sets in X satisfying the following axioms:

1. $NCT_\emptyset \notin F_{NCT}^X$,
2. If $NCT_A \in F_{NCT}^X$ and $NCT_A \sqsubseteq NCT_B$, then $NCT_B \in F_{NCT}^X$.
3. If $NCT_A \in F_{NCT}^X$ and $NCT_B \in F_{NCT}^X$, then $NCT_A \sqcap NCT_B \in F_{NCT}^X$.

The pair (NCT_X , F_{NCT}^X) is called a NCT −filter or neutrosophic crisp triple filter.

Example 3.2. Let $X = \{a, b\}$. Consider the family $F_{NCT}^X = \{NCT_X, NCT_A, NCT_B\}$, where $NCT_A = \langle \{a\}, \{a\}, \emptyset \rangle$ and $NCT_B = \langle \{a\}, X, \emptyset \rangle$. Then ($F_{NCT}^X$, NCT_X) is a F_{NCT}^X −filter on X.

Result. Let $\{F_{NCT}^X i : i \in I\}$ be a collection of NCT −filters on X . Then $\cap_{i \in I} F_{NCT}^X i$ forms an NC −filter on X.

Proof.

1. Since $NCT_X \in F_{NCT}^X i$ for all $i \in I$, then $NCT_X \in \cap_{i \in I} F_{NCT}^X i$, so $\cap_{i \in I} F_{NCT}^X i$ is non empty set.
2. Since $NCT_\emptyset \notin F_{NCT}^X i$ for all $i \in I$, then $NCT_\emptyset \notin \cap_{i \in I} F_{NCT}^X i$.
3. Let $NCT_A \in \cap_{i \in I} F_{NCT}^X i$ and $NCT_A \sqsubseteq NCT_B$, then $NCT_A \in F_{NCT}^X i$ for all $i \in I$, so $NCT_B \in F_{NCT}^X i$ for all $i \in I$ and we have $NCT_B \in \cap_{i \in I} F_{NCT}^X i$.
4. Let $NCT_A, NCT_B \in \cap_{i \in I} F_{NCT}^X i$, then $NCT_A, NCT_B \in F_{NCT}^X i$ for all $i \in I$, so $NCT_A \sqcap NCT_B \in F_{NCT}^X i$ for all $i \in I$ and we have $NCT_A \sqcap NCT_B \in \cap_{i \in I} F_{NCT}^X i$. Therefore $\cap_{i \in I} F_{NCT}^X i$ is NCT −filter on X.

Corollary 3.3. Union of a collection of NCT −filters need not be a NCT −filter and it is justified by the following example.

Example 3.4. Let $X = \{a, b\}$, $F_{NCT}^X 1 = \{\langle \{a\}, \{a\}, \emptyset \rangle, \langle \{a\}, X, \emptyset \rangle, \langle X, X, \emptyset \rangle\}$,

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$F^X_{NCT}2 = \{\langle\{b\},\{b\},\emptyset\rangle, \langle\{b\},X,\emptyset\rangle, \langle X,X,\emptyset\rangle\}$, now

$F^X_{NCT}1 \cup F^X_{NCT}2 = \{\langle\{a\},\{a\},\emptyset\rangle, \langle\{a\},X,\emptyset\rangle, \langle\{b\},\{b\},\emptyset\rangle, \langle\{b\},X,\emptyset\rangle, \langle X,X,\emptyset\rangle\}$, is not NCT −filter

Definition 3.5. Let X be a non-empty set. An "NCT −filter base" on X is a family F^X_{0NCT} of subsets of NCT satisfies:

1. $NCT_\emptyset \notin F^X_{0NCT}$

2. If $NCT_A \in F^X_{0NCT}$ and $NCT_B \in F^X_{0NCT}$, then there exists $NCT_C \in F^X_{0NCT}$ such that $NCT_C \sqsubseteq NCT_A \sqcap NCT_B$.

Observe that if $NCT_\emptyset \neq NCT_A \sqcap NCT_B \in F^X_{0NCT}$ for each NCT_A and NCT_B in F^X_{0NCT}, then F^X_{0NCT} is an NCT −filter base on X and so any NCT −filter on X is NCT −filter base.

Proposition 3.6. Let F^X_{0NCT} be an NCT −filter base on X, then

$$F^X_{NCT} = \{\, NCT_A : NCT_B \sqsubseteq NCT_A \; for\ some\ NCT_B \in F^X_{0NCT}\}$$

is an NCT −filter on X generated by F^X_{0NCT}.

Proof.

Let F^X_{0NCT} be an NCT −filter base on X.

1. Let $NCT_A \in F^X_{NCT}$ and $NCT_A \sqsubseteq NCT_B$. Then there exists $NCT_C \in F^X_{0NCT}$ such that $NCT_C \sqsubseteq NCT_A$, so $NCT_C \sqsubseteq NCT_B$. Thus $NCT_B \in F^X_{NCT}$.
2. Let $NCT_A \in F^X_{NCT}$ and $NCT_B \in F^X_{NCT}$. Then there exists $NCT_C \in F^X_{0NCT}$ and $NCT_D \in F^X_{0NCT}$ such that $NCT_C \sqsubseteq NCT_A$ and $NCT_D \sqsubseteq NCT_B$.

Since F^X_{0NCT} is an NCT −filter base, then there exists $NCT_E \in F^X_{0NCT}$ such that $NCT_E \sqsubseteq NCT_C \sqcap NCT_D$. So $NCT_E \sqsubseteq NCT_A \sqcap NCT_B$. Therefore $NCT_A \sqcap NCT_B \in F^X_{NCT}$.

From 1 and 2, F^X_{NCT} is an NCT −filter on X.

Definition 3.7. Let F^X_{NCT} be a NCT −filter on X and NCT_A be NCT −set of X. Then F^X_{NCT} is NCT −eventually of NCT_A if and only if $NCT_A \in F^X_{NCT}$.

Definition 3.8. Let F^X_{NCT} be a NCT −filter on a no empty set X and NCT_A be any neutrosophic crisp triple sets of X. Then F^X_{NCT} is said to be frequently in the neutrosophic crisp triple set NCT_A if and only if $NCT_A \sqcap NCT_F \neq NCT_\emptyset$ for all $NCT_F \in F^X_{NCT}$.

Remark 3.9. From the above definitions, it is clear that if F^X_{NCT} is eventually in NCT_A, then F^X_{NCT} is frequently in NCT_A because when F^X_{NCT} is eventually in NCT_A, then $NCT_A \in F^X_{NCT}$ and $NCT_A \sqcap NCT_F$ is the intersection of two members of NCT −filter F^X_{NCT} which is again in F^X_{NCT}. Since $NCT_\emptyset \notin F^X_{NCT}$, it follows that $NCT_A \sqcap NCT_F \neq NCT_\emptyset$ for all $NCT_F \in F^X_{NCT}$. That is NCT_A intersects every member of F^X_{NCT}. Hence F^X_{NCT} is frequently in NCT_A.

The converse of the above remark is not true as is clear from example given below.

$X = \{a,b,c\}$, $F^X_{NCT} = \{\langle X,X,\emptyset\rangle, \langle\{a,b\},\{a,b\},\emptyset\rangle, \langle\{a,b\},X,\emptyset\rangle\}$. Let $NCT_A = \langle\{a\},\{a\},\emptyset\rangle$.

F^X_{NCT} is a NCT −filter and $NCT_A \sqcap NCT_F \neq NCT_\emptyset$ for all $NCT_F \in F^X_{NCT}$, so that F^X_{NCT} is frequently in NCT_A. But as $NCT_A \notin F^X_{NCT}$, F^X_{NCT} is not eventually in NCT_A.

Definition 3.10. Let (NCT_X, τ^X_{NCT}) be a NCT −topological space and $p \in X$. Then NCT_A called a τ^X_{NCT} −neighbourhoods of $NCT_{\tilde{p}}$, if there exists NCT_G (NCT −open) such that $NCT_{\tilde{p}} \Subset NCT_G \sqsubseteq NCT_A$.

Definition 3.11. Let (NCT_X, τ^X_{NCT}) be a neutrosophic crisp triple topological space and $p \in X$. Then

$N^{\tilde{p}}_{NCT}$ be the collection of all τ^X_{NCT} −neighbourhoods of $NCT_{\tilde{p}} = \langle\{p\},\{p\},\{p^c\}\rangle$.

Theorem 3.12. Let (NCT_X, τ^X_{NCT}) be a neutrosophic crisp triple topological space and with each $p \in X$, let there be associated a family $N^{\tilde{p}}_{NCT}$ of NCT −sets of X called τ^X_{NCT} − neutrosophic crisp triple neighbourhoods satisfying the following conditions:

a) $N^{\tilde{p}}_{NCT} \neq NCT_\emptyset$.

b) $NCT_N \in N^{\tilde{p}}_{NCT}$ implies $NCT_{\tilde{p}} \Subset NCT_N$.

c) If $NCT_A \in N^{\tilde{p}}_{NCT}$, $NCT_M \sqsupseteq NCT_N$, then $NCT_M \in N^{\tilde{p}}_{NCT}$.

d) If $NCT_N \in N^{\tilde{p}}_{NCT}$, $NCT_M \in N^{\tilde{p}}_{NCT}$, then $NCT_M \sqcap NCT_N \in N^{\tilde{p}}_{NCT}$.

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e) $NCT_N \in N^{\tilde{p}}_{NCT}$ implies there exists $NCT_M \in N^{\tilde{p}}_{NCT}$ such that $NCT_M \sqsubseteq NCT_N$ and $NCT_M \in N^{\tilde{p}}_{NCT}$ for all $NCT_q \in NCT_M$.

Then there exists a unique neutrosophic crisp triple τ^X_{NCT} for X in such a way that if $N^{\tilde{p}^*}_{NCT}$ is the collection of $\tau^X_{NCT}-$ neutrosophic crisp triple of $NCT_{\tilde{p}}$ defined by the neutrosophic crisp triple topology τ^X_{NCT} , then $N^{\tilde{p}}_{NCT} = N^{\tilde{p}^*}_{NCT}$.

Proof.

Define $\tau^{*X}_{NCT} = \{NCT_A \in \tau^X_{NCT} : if\ NCT_A \in N^{\tilde{p}}_{NCT}\ for\ every\ NCT_{\tilde{p}} \in \mathrm{NCT_A}\}$.

Obviously $NCT_\emptyset \in \tau^{*X}_{NCT}$, since $NCT_\emptyset \in N^{\tilde{p}}_{NCT}$ for all $NCT_{\tilde{p}} \in NCT_\emptyset$ is true. Let $p \in X$. By (a), there exists some $NCT_A \in N^{\tilde{p}}_{NCT}$. By(c), $NCT_A \sqsubseteq NCT_X$. Hence $NCT_X \in N^{\tilde{p}}_{NCT}$. Therefore $NCT_X \in \tau^{*X}_{NCT} *$.

Let $NCT_A \in \tau^{*X}_{NCT}, NCT_B \in \tau^{*X}_{NCT}$, and choose $NCT_{\tilde{p}} \in NCT_A \sqcap NCT_B$, implies $NCT_{\tilde{p}} \in NCT_A$ and $NCT_{\tilde{p}} \in NCT_B$. So NCT_A and NCT_B are in $N^{\tilde{p}}_{NCT}$. Hence $NCT_A \sqcap NCT_B \in N^{\tilde{p}}_{NCT}$. Therefore $NCT_A \sqcap NCT_B \in \tau^{*X}_{NCT}$.

Let $NCT_{A\alpha}$ for all $\alpha \in \Lambda$. Let $N^{\tilde{p}}_{NCT} \in \sqcup\{NCT_{A\alpha} : \alpha \in \Lambda\}$. Thus $NCT_{\tilde{p}} \in NCT_{A\alpha}$ for some $\alpha \in \Lambda$ and $NCT_{A\alpha} \in \tau^X_{NCT}$ implies $NCT_{A\alpha} \in N^{\tilde{p}}_{NCT}$. Now $NCT_{A\alpha} \in N^{\tilde{p}}_{NCT}$ and $\sqcup\{NCT_{A\alpha} : \alpha \in \Lambda\} \sqsupseteq NCT_{A\alpha}$. Hence $\sqcup\{NCT_{A\alpha} : \alpha \in \Lambda\} \in N^{\tilde{p}}_{NCT}$ for every $NCT_{\tilde{p}} \in \sqcup\{NCT_{A\alpha} : \alpha \in \Lambda\}$. Therefore $\sqcup\{NCT_{A\alpha} : \alpha \in \Lambda\} \in \tau^{*X}_{NCT}$. Hence τ^{*X}_{NCT} is a $NCT-$topology of X.

Let $N^{\tilde{p}*}_{NCT}$ be the collection of all $\tau^X_{NCT}-$ neighbourhoods of $NCT_{\tilde{p}}$ and $N^{\tilde{p}}_{NCT}$ is the collection of all $NCT-$sets of X satisfying all the conditions (a), (b), (c), (d) and (e). Now $NCT_A \in N^{\tilde{p}}_{NCT}$ implies $NCT_{\tilde{p}} \in NCT_A$. Then there exists a $NCT_B \in N^{\tilde{p}}_{NCT}$ such that $NCT_B \sqsubseteq NCT_A$ and $NCT_B \in N^{\tilde{q}}_{NCT}$ for all $NCT_{\tilde{q}} \in NCT_B$. Thus NCT_B is a $\tau^X_{NCT}-$ neutrosophic crisp triple open set such that $NCT_{\tilde{p}} \in NCT_B \sqsubseteq NCT_A$. Hence NCT_A is a $\tau^X_{NCT}-$ neighbourhood of $NCT_{\tilde{p}}$. Hence $NCT_A \in N^{\tilde{p}*}_{NCT}$. Therefore $N^{\tilde{p}}_{NCT} \subseteq N^{\tilde{p}*}_{NCT}$.

Conversely, let $NCT_A \in N^{\tilde{p}*}_{NCT}$. Then NCT_A is a $\tau^X_{NCT}-$neighbourhood of $NCT_{\tilde{p}}$ so that by Definition 2.1.7 there exists a $\tau^X_{NCT}-$ neutrosophic crisp triple open set NCT_G such that $NCT_{\tilde{p}} \in NCT_G \sqsubseteq NCT\_A$. Now $NCT_G \in \tau^X_{NCT}$ implies $NCT_G \in N^{\tilde{p}}_{NCT}$ for every $NCT_{\tilde{p}} \in NCT_G$ so that $NCT_A \in N^{\tilde{p}}_{NCT}$. Therefore $N^{\tilde{p}}_{NCT} \subseteq N^{\tilde{p}*}_{NCT}$.

Theorem 3.13. Let (NCT_X, τ^X_{NCT}) be a neutrosophic crisp triple topological space and $NCT_{\tilde{p}}$ be a neutrosophic point in X. Then the $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ say $N^{\tilde{p}}_{NCT}$ is a $F^X_{NCT}-$filter on X.

Proof.

Let NCT_p be a neutrosophic point in X and NCT_X be a $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ that belongs to $N^{\tilde{p}}_{NCT}$ so that $N^{\tilde{p}}_{NCT}$ is non-empty. By Definition 4.1.5, each member of $N^{\tilde{p}}_{NCT}$, being a $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ must contain $NCT_{\tilde{p}}$ and as such no member of $N^{\tilde{p}}_{NCT}$ is empty so that $NCT_\emptyset \in N^{\tilde{p}}_{NCT}$. If NCT_A is a $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$, then a superset of NCT_A is also a $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. Hence $NCT_A \in N^{\tilde{p}}_{NCT}$ and $NCT_B \sqsupseteq NCT_A$ so that $NCT_B \in N^{\tilde{p}}_{NCT}$. Also it is known that if NCT_A and NCT_B are $NCT_{\tilde{p}}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$, then $NCT_A \sqcap NCT_B$ is also a $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ which belongs to $N^{\tilde{p}}_{NCT}$. Therefore $N^{\tilde{p}}_{NCT}$ is a $F^X_{NCT}-$filter on X.

Remark 3.14. $N^{\tilde{p}}_{NCT}$ is also called as the neighbourhood $F^X_{NCT}-$filter **of** $NCT_{\tilde{p}}$ with respect to τ^X_{NCT}.

Definition 3.15. Let (NCT_X, τ^X_{NCT}) be a $NCT-$topological space and F^X_{NCT} be a $NCT-$filter on X. Then F^X_{NCT} is said to $\tau^X_{NCT}-$ neutrosophic crisp triple converge to a neutrosophic crisp triple point $NCT_{\tilde{p}}$ if and only if F^X_{NCT} is eventually in every $\tau^X_{NCT}-$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. In that case, we write $F^X_{NCT} \to NCT_{\tilde{p}}$ and $NCT_{\tilde{p}}$ is a neutrosophic crisp triple limit point of F^X_{NCT}. The set of all neutrosophic crisp triple limit points of a $F^X_{NCT}-$filter denoted by $Lim(F^X_{NCT})$.

4. $NCT-$Ultra Filter

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Definition 4.1. A NCT −filter F_{NCT}^X on X is said to be NCT −ultrafilter on X if and only if F_{NCT}^X is not properly contained in any other NCT −filter on X. In other words there does not exist any other NCT −filter which is strictly finer than F_{NCT}^X. Here after it is denoted as UF_{NCT}^X.

Remark 4.2. From the above, an NCT-ultrafilter on X is a maximal element in the collection of all NCT-filters on X, ordered partially by the inclusion relation C.

Theorem 4.3. (The Ultrafilter theorem or The Existence Theorem for neutrosophic crisp triple sets) Every NCT −filter on a non-empty set X is contained in a NCT −ultrafilter on X.

Proof.

Let F_{NCT}^X be a NCT −filter on X and $C = \{Fi_{NCT}^X : F_{NCT}^X \subseteq Fi_{NCT}^X\}$, so that $C \neq \emptyset$, as at least $F_{NCT}^X \in C$. C is partially ordered by the inclusion relation. Let D be a collection of linearly ordered NCT −subset of C, so that for any two members $F1_{NCT}^X$ and $F2_{NCT}^X$ of C , either $F1_{NCT}^X \subseteq F2_{NCT}^X$ or $F2_{NCT}^X \subseteq F1_{NCT}^X$. Let $E = \cup \{F\alpha_{NCT}^X : F\alpha_{NCT}^X \in D\}$. Clearly, $E \neq \emptyset$. If $F\alpha_{NCT}^X$ is a NCT −filter on X, then $NCT_\emptyset \notin F\alpha_{NCT}^X$ for any $F\alpha_{NCT}^X \in D$. Hence, $NCT_\emptyset \notin E$. Let $NCT_F \in E$. Then $NCT_F \in F\alpha_{NCT}^X$ for at least one $F\alpha_{NCT}^X \in D$. Since each $F\alpha_{NCT}^X$ is a NCT −filter, if $NCT_H \sqsupseteq NCT_F$, then $NCT_H \in F\alpha_{NCT}^X$. Hence $NCT_H \in \cup \{F\alpha_{NCT}^X : F\alpha_{NCT}^X \in D\} = E$.

Let $NCT_{F_1}, NCT_{F_2} \in E$. Then $NCT_{F_1} \in F\alpha_{NCT}^X$ and $NCT_{F_2} \in F\beta_{NCT}^X$ for some $F\alpha_{NCT}^X$ and $F\beta_{NCT}^X$ respectively in D. As D is linearly ordered in NCT −sets, either $F\alpha_{NCT}^X \subseteq F\beta_{NCT}^X$ or $F\beta_{NCT}^X \subseteq F\alpha_{NCT}^X$. Hence, both NCT_{F_1} and NCT_{F_2} are contained in either $F\alpha_{NCT}^X$ or $F\beta_{NCT}^X$. Since $F\alpha_{NCT}^X$ and $F\beta_{NCT}^X$ are NCT −filters, $NCT_{F1} \sqcap NCT_{F_2}$ belongs to either $F\alpha_{NCT}^X$ or $F\beta_{NCT}^X$. Hence $NCT_{F_1} \sqcap NCT_{F_2} \in E$. Therefore E is a NCT −filter on X and E is finer than every member of D, so that E is an upper bound of D. Thus it is proved that in a partially ordered non-empty set C every linearly ordered neutrosophic crisp triple set has an upper bound. Hence C must contain a maximal element say NCT_{F_*} which implies, C is a NCT −ultra filter containing F_{NCT}^X.

Theorem 4.4. A NCT −filter F_{NCT}^X on a non-empty set X is a NCT −ultra filter on X if and only if F_{NCT}^X contains all those NCT −sets of X, which intersect any member of F_{NCT}^X.

Proof.

Let F_{NCT}^X be a NCT −filter on X such that F_{NCT}^X contains all those NCT −sets of X, which intersect any member of F_{NCT}^X. To prove that F_{NCT}^X is a NCT −ultra filter on X, it is enough to prove that there does not exist any other NCT −filter on X, which is strictly finer than F_{NCT}^X. If possible, let $F*_{NCT}^X$ be a NCT −filter on X, which is strictly finer than F_{NCT}^X. Now let $NCT_{F*} \in F*_{NCT}^X$ and being a NCT −filter on X, each member of $F*_{NCT}^X$ intersects every member of $F*_{NCT}^X$. Hence NCT_{F*} intersects every member of $F*_{NCT}^X$ and as $F_{NCT}^X \subseteq F*_{NCT}^X$ it is concluded that NCT_{F*} intersects every member of F_{NCT}^X and so $NCT_{F*} \in F_{NCT}^X$, which implies $F*_{NCT}^X \subseteq F_{NCT}^X$. But this is a contradiction to the assumption that $F*_{NCT}^X$ is a NCT −filter on X , which is strictly finer than F_{NCT}^X. Hence F_{NCT}^X is a NCT −ultra filter on X.

Conversely, assume F_{NCT}^X be a NCT −ultrafilter on X. Now choose an arbitrary neutrosophic crisp triple set NCT_A of X, which intersects every member of F_{NCT}^X.

Consider a collection $F*_{NCT}^X\ X = \{NCT_{F*} : NCT_{F*} \sqsupseteq NCT_A \sqcap NCT_F\ for\ some\ NCT_F \in F_{NCT}^X\}$. If $NCT_F \in F_{NCT}^X$, then $NCT_F \sqsupseteq NCT_A \sqcap NCT_F$. So $NCT_F \in F*_{NCT}^X$, which implies $F_{NCT}^X \subseteq F*_{NCT}^X$. As per the construction, A intersects every member of F_{NCT}^X. Hence $NCT_A \sqcap NCT_F \neq NCT_\emptyset$ for all $NCT_F \in F_{NCT}^X$ and $NCT_F *\sqsupseteq NCT_A \sqcap NCT_F$. Therefore $NCT_F \neq NCT_\emptyset$ for all $NCT_{F*} \in F*_{NCT}^X$. Thus $NCT_\emptyset \notin F*_{NCT}^X$.

Let $NCT_{F*} \in F*_{NCT}^X$. Then $NCT_{F*} \sqsupseteq NCT_A \sqcap NCT_F$ for some $NCT_F \in F_{NCT}^X$. If NCT_{G*} is a neutrosophic crisp triple set and $NCT_{G*} \sqsupseteq NCT_{F*}$, then obviously $NCT_{G*} \sqsupseteq NCT_A \sqcap NCT_F$ for some $NCT_F \in F_{NCT}^X$ which implies $NCT_{G*} \in F*_{NCT}^X$. That is all the supersets of $F*_{NCT}^X$ also lies in $F*_{NCT}^X$. Let NCT_{F1*} and NCT_{F2*} be both members of $F*_{NCT}^X$. Then $NCT_{F1*} \sqsupseteq NCT_A \sqcap NCT_{F1}$ and $NCT_{F2*} \sqsupseteq NCT_A \sqcap NCT_{F2}$ for some $NCT_{F1}, NCT_{F2*} \in F_{NCT}^X$. Hence $NCT_{F1*} \sqcap NCT_{F2*} \sqsupseteq (NCT_A \sqcap NCT_{F1}) \sqcap (NCT_A \sqcap NCT_{F2}) = NCT_A \sqcap (NCT_{F1} \sqcap NCT_{F2}) = NCT_A \sqcap NCT_F$, where $NCT_F NCT_{F1} \sqcap NCT_{F2} \in F_{NCT}^X$. Thus $NCT_{F1*} \sqcap NCT_{F2*} \sqsupseteq NCT_A \sqcap NCT_F$ for some $NCT_F \in F*_{NCT}^X$ and so $NCT_{F1*} \sqcap NCT_{F2*} \in F*_{NCT}^X$. Hence $F*_{NCT}^X$ is a NCT −filter on X such that $F*_{NCT}^X$ contains F_{NCT}^X. But it is given that F_{NCT}^X is a NCT −ultra filter on X. Therefore $F*_{NCT}^X \sqsupseteq F_{NCT}^X$ implies $F*_{NCT}^X = F_{NCT}^X$. Further $NCT_X \in F_{NCT}^X$, $NCT_A \sqcap NCT_X = NCT_A$ and $NCT_A \sqsupseteq NCT_A \sqcap NCT_X$ implies $NCT_A \in F*_{NCT}^X$ or $NCT_A \in F_{NCT}^X$. Thus F_{NCT}^X contains all those neutrosophic crisp triple sets of X, which intersect every member of F_{NCT}^X.

5. Properties of Convergence of NCT −filter

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Theorem 5.1. Let $X \neq \emptyset$ and τ_{NCT}^{X} be NCT −topological space on X. Also let $N_{NCT}^{\tilde{p}}$ be a τ_{NCT}^{X} −neighbourhood of $NCT_{\tilde{p}}$. Then

(a) Every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood F_{NCT}^{X} −filter $N_{NCT}^{\tilde{p}}$ converges to a unique limit.

(b) If τ_{NCT}^{X} is an indiscrete neutrosophic crisp triple topological space, then every F_{NCT}^{X} −filter on X converges to every neutrosophic crisp triple point of X.

(c) If $F_{NCT}^{X} \to NCT_{\tilde{p}}$, then $F_{NCT}^{X*} \to NCT_{\tilde{p}}$ where F_{NCT}^{X*} is finer than F_{NCT}^{X}.

(d) If $F_{NCT}^{X} \to NCT_{\tilde{p}}$ with respect to τ_{NCT}^{X}, then $F_{NCT}^{X} \to NCT_{\tilde{p}}$ with respect to τ_{NCT}^{X*} where τ_{NCT}^{X*} is a NCT −topology on X, which is coarser than τ_{NCT}^{X}.

Proof.

(a) Let $N_{NCT}^{\tilde{p}}$ be the collection of all τ_{NCT}^{X} −neighbourhoods of $NCT_{\tilde{p}}$ in a neutrosophic crisp triple topological space (NCT_X, τ_{NCT}^{X}). By Theorem 4.1.7, $N_{NCT}^{\tilde{p}}$ is a F_{NCT}^{X} −filter on X and $N_{NCT}^{\tilde{p}}$ is eventually in every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of neutrosophic crisp triple point $NCT_{\tilde{p}}$ so that $N_{NCT}^{\tilde{p}} \to NCT_{\tilde{p}}$. Further, this $NCT_{\tilde{p}}$ is unique because if $NCT_{\tilde{q}}$ is any other neutrosophic crisp triple point distinct from $NCT_{\tilde{p}}$, then $\langle\{q\},\{q\},\emptyset\rangle$ is a τ_{NCT}^{X} −neighbourhood of $NCT_{\tilde{q}}$ but it does not belong to $N_{NCT}^{\tilde{p}}$.

(b) Let F_{NCT}^{X} be a NCT −filter on X and $NCT_{\tilde{p}}$ be any arbitrary neutrosophic crisp triple point of X. Then only τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood F_{NCT}^{X} −filter of $NCT_{\tilde{p}}$ is $\{TCT_X\}$ and $TCT_X \in F_{NCT}^{X}$ so that $F_{NCT}^{X} \to NCT_{\tilde{p}}$. Since $NCT_{\tilde{p}}$ is arbitrary, every neutrosophic crisp triple filter on X converges to every neutrosophic crisp triple point of X.

(c) $F_{NCT}^{X} \to NCT_{\tilde{p}}$ if and only if every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ is contained in F_{NCT}^{X}. As F_{NCT}^{X*} is finer than F_{NCT}^{X}, F_{NCT}^{X*} is eventually in every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. Hence $F_{NCT}^{X*} \to NCT_{\tilde{p}}$.

(d) $F_{NCT}^{X} \to NCT_{\tilde{p}}$ with respect to τ_{NCT}^{X} if and only if every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ is contained in F_{NCT}^{X}. As τ_{NCT}^{X*} is coarser than τ_{NCT}^{X}, F_{NCT}^{X} is eventually in every τ_{NCT}^{X*} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. Then $F_{NCT}^{X} \to NCT_{\tilde{p}}$ with respect to τ_{NCT}^{X*}.

Theorem 5.2. A neutrosophic crisp triple filter F_{NCT}^{X} on a neutrosophic crisp triple topological space (NCT_X, τ_{NCT}^{X}) converges to a neutrosophic crisp triple point $NCT_{\tilde{p}}$ if and only if every neutrosophic crisp triple ultra filter on X containing F_{NCT}^{X} converges to $NCT_{\tilde{p}}$.

Proof.

Let $F_{NCT}^{X} \to NCT_{\tilde{p}}$ and F_{NCT}^{X*} be a neutrosophic crisp triple ultra filter containing F_{NCT}^{X}. Obviously F_{NCT}^{X*} is finer than F_{NCT}^{X}, so that $F_{NCT}^{X*} \to NCT_{\tilde{p}}$. Conversely, let every neutrosophic crisp triple ultra filter on X containing F_{NCT}^{X} converge to $NCT_{\tilde{p}}$. Therefore every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ is contained in every neutrosophic crisp triple ultra filter on X, which contains F_{NCT}^{X}. Since every τ_{NCT}^{X} −neighbourhood of $NCT_{\tilde{p}}$ is contained in the intersection of all the NCT −ultra filters on X which contain F_{NCT}^{X}. Thus every τ_{NCT}^{X} −neighbourhood of $NCT_{\tilde{p}}$ is contained in F_{NCT}^{X}. Hence $F_{NCT}^{X} \to NCT_{\tilde{p}}$.

Theorem 5.3. In a NCT −topological space (NCT_X, τ_{NCT}^{X}), a $NCT_G \neq NCT_{\emptyset}$ of X is τ_{NCT}^{X} −open if and only if NCT_G is contained in every NCT −filter, which converges to a neutrosophic crisp triple point of NCT_G.

Proof.

Let NCT_G be a τ_{NCT}^{X} −neutrosophic crisp triple open set and F_{NCT}^{X} be an arbitrary neutrosophic crisp triple filter on X, which converges to $NCT_{\tilde{p}} \in NCT_G$. Let $F_{NCT}^{X} \to NCT_{\tilde{p}} \in NCT_G$. Then every τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ is contained in F_{NCT}^{X}. Ultimately, τ_{NCT}^{X} −neutrosophic crisp triple open set NCT_G is contained in F_{NCT}^{X}.

Conversely, let NCT_G be contained in every neutrosophic crisp triple filter, which converges to a neutrosophic crisp triple point of NCT_G. Choose $NCT_{\tilde{p}} = \langle\{p\},\{p\},\{p^c\}\rangle$ to be any arbitrary neutrosophic crisp triple point of NCT_G, $N_{NCT}^{\tilde{p}}$ is the τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood filter of $NCT_{\tilde{p}}$ which converges to $NCT_{\tilde{p}}$ so that $NCT_G \sqsubseteq N_{NCT}^{\tilde{p}}$. In other words NCT_G is a τ_{NCT}^{X} −neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ and as $NCT_{\tilde{p}}$ is an arbitrary neutrosophic crisp triple point of NCT_G, we have NCT_G is a τ_{NCT}^{X} −neighbourhood of each of its neutrosophic crisp triple points. Hence NCT_G is τ_{NCT}^{X} −open.

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6. Adherent points in neutrosophic crisp triple topological space

Definition 6.1. Let (NCT_X, τ_{NCT}^X) be a NCT −topological space and NCT_A be a NCT −set of X. Then a NCT −point $NCT_{\tilde{p}}$ of X is called a neutrosophic crisp triple adherent point of NCT_A if and only if every τ_{NCT}^X −neighbourhood of $NCT_{\tilde{p}}$ contains at least one NCT −point NCT_A. That is $NCT_N \sqcap NCT_A \neq NCT_{\emptyset}$, where NCT_N is any $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. The set of all neutrosophic crisp triple adherent points of a given neutrosophic crisp triple set NCT_A of X is denoted by $dh(NCT_A)$.

Definition 6.2. Let (NCT_X, τ_{NCT}^X) be a NCT −topological space and F_{NCT}^X be a NCT −filter on X. Then a NCT −point $NCT_{\tilde{p}}$ of X is called a neutrosophic crisp triple adherent point of F_{NCT}^X if and only if F_{NCT}^X is frequently in each $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$.

Theorem 6.3. Let X be a given non-empty set and τ_{NCT}^X be a neutrosophic crisp triple topological space on X. Let NCT_N be any element of the $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of the neutrosophic crisp triple point $NCT_{\tilde{p}}$. Then

(a) If $F_{NCT}^X \to NCT_{\tilde{p}}$, then $NCT_{\tilde{p}}$ is a NCT −adherent point of F_{NCT}^X.

(b) $NCT_{\tilde{p}}$ is a NCT −adherent point of F_{NCT}^X if and only if $NCT_{\tilde{p}}$ is a NCT −adherent point of F_{NCT}^{X*}, where F_{NCT}^{X*} is a NCT −filter on X, which is coarser than F_{NCT}^X.

(c) $NCT_{\tilde{p}}$ is a $\tau_{NCT}^X - NCT$ −adherent point of F_{NCT}^X if and only if $NCT_{\tilde{p}}$ is a τ_{NCT}^{X*} neutrosophic crisp triple adherent point of F_{NCT}^X, where τ_{NCT}^{X*} is a NCT −topology on X, which is coarser than τ_{NCT}^X.

Proof.

(a) Let $F_{NCT}^X \to NCT_{\tilde{p}}$. Then F_{NCT}^X is eventually in every $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. As eventually in implies frequently in, F_{NCT}^X is frequently in every $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$. Hence every $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ intersects every neutrosophic crisp triple member of F_{NCT}^X.

(b) $NCT_{\tilde{p}}$ is a neutrosophic crisp triple adherent point of F_{NCT}^X if and only if every $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ intersects every neutrosophic crisp triple member of F_{NCT}^X . As F_{NCT}^{X*} is a neutrosophic crisp triple filter on X coarser than F_{NCT}^X , every $\tau_{NCT}^X -$ neutrosophic crisp triple neighbourhood of $NCT_{\tilde{p}}$ intersects every neutrosophic crisp triple member of F_{NCT}^{X*}. Then $NCT_{\tilde{p}}$ is a neutrosophic crisp triple adherent point of F_{NCT}^{X*}.

(c) Similar to that of (b).

7. Conclusion

In this study, we have successfully presented key advancements in neutrosophic topology by developing new concepts such as the NCT −filter and its convergence properties. These innovations lead to a novel approach to managing uncertainty within topological spaces. Moreover, the application of neutrosophic logic in mathematical frameworks is established. Also, investigating the properties of convergence of NCT −filters provided a robust methodology for addressing ambiguity in various topological structures. Additionally, the exploration of adherent points in neutrosophic crisp triple topological spaces has opened up fresh perspectives on the interaction between sets and open sets under uncertain conditions. Furthermore, our findings significantly expand the theoretical and practical understanding of neutrosophic topology, positioning this work as a foundation for future studies and applications in fields such as data analysis, decision-making, and artificial intelligence etc., where uncertainty plays a crucial role.

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