



Developed acceptance sampling plans for the Shanker distribution based on truncated life tests

Amer Ibrahim Al-Omari^{1,*}, Rehab Alsultan²

¹Department of Mathematics, Faculty of Science, Al al-Bayt University, Mafraq, Jordan

²Mathematics Department, Faculty of Sciences, Umm Al-Qura University, Makkah, Saudi Arabia

Emails: alomari_amer@yahoo.com; rasultan@uqu.edu.sa

Abstract

This paper introduces new acceptance sampling plans for situations where the life test is terminated at a predetermined time. The minimum sample sizes needed to guarantee a specified average lifetime are determined for different acceptance numbers, confidence levels, and ratios of the fixed test duration to the defined average lifetime. The Shanker distribution is adopted to represent the lifetimes of test units, with its mean serving as the quality indicator. Furthermore, the operating characteristic function values for the proposed sampling plans, along with the associated producer's risk, are provided. Examples are included to demonstrate how to use the tables effectively. An application of a real data set is used to illustrate the usefulness of the suggested acceptance sampling plans.

Keywords: Shanker distribution; Acceptance sampling plans; Neutrosophic statistical interval method ; Operating characteristic function; Producer's risk; Consumer's risk; Truncated life tests

1. Introduction

To decide whether to accept or reject a batch of products based on the findings of an examination of a sample, statistical quality control methods known as acceptance sampling plans are employed. These programs evaluate a selection of products rather than the complete lot in an effort to strike a balance between inspection efforts and quality assurance. Acceptance sampling is a frequently employed technique in production and manufacturing processes that lowers inspection costs while detecting flaws. It ensures that quality criteria are satisfied without requiring extensive testing and may be used as single, double, or triple sample programs, depending on the complexity and necessary precision.

The lifetime of a product is a vital quality characteristic. Acceptance sampling plans are used to assess the acceptability of a product lot, enabling consumers to decide whether to accept or reject the lot based on a random sample. The main challenge is to determine the minimum sample size required to guarantee a specified average lifetime when the life test is truncated at a predetermined time, t , and when the observed number of failures exceeds a defined acceptance number, c . A lot is accepted if the specified lifetime can be confirmed with a high-predetermined probability, P^* , thereby protecting the consumer's interests. To compute the parameters of the acceptance sampling plan, it is assumed that the product's lifetime adheres to a specific model or distribution.

In this study, the single ASP is investigated for the Shanker distribution. Many authors considered the problem of ASP from truncated life tests for various distributions, for example; [1] proposed ASP for the generalized inverted exponential distribution. [2] introduced ASP based on percentiles for odds exponential log logistic distribution. ASP to the weighted exponential distribution is suggested by [3]. [4] suggested single and double ASP for the compound Weibull-exponential distribution. [5] considered the power inverted Topp-Leone distribution in ASP. [6] offered new ASP based on the logistic-exponential distribution. [7] considered ASP for percentiles using Gompertz Fréchet distribution. [8] considered the G family for single ASP. [9] studied based on Lomax

distribution. [10] offered ASP for the two-parameter Xgamma model. [11] suggested ASP for the Q-Weibull distribution using hypergeometric theory for finite population. [12] suggested ASP for the Zubair-exponential distribution. [13] proposed a new ASP based on transmuted Rayleigh model. [14] investigated Zeghdoudi distribution in ASP. [15] suggested single ASP based on truncated life tests for the exponentiated moment exponential model. [16] proposed ASP based on percentiles for exponentiated inverse Kumaraswamy distribution. [17] considered the Pseudo Lindley model in ASP.

The problem of acceptance sampling based on truncated life tests is considered by many authors, and there are Numerous articles are available on the single sampling plan based on the truncated life test for various statistical distributions; for example, [11] for ASP to the hypergeometric theory for finite population under Q-Weibull distribution, [18] considered for three parameter Kappa distribution, [19] investigated the problem of acceptance sampling plan in truncated life tests for exponentiated Fréchet distribution. [20] considered the acceptance sampling plans based on truncated life tests to Birnbaum Saunders model. [6] considered truncated life tests for logistic-exponential distribution and its application.

Acceptance sampling based on classical statistics assumes that the data are precise and determinate. However, in certain cases, the data may be imprecise or lie in an indeterminate state, leading to uncertainty in judging the quality of the submitted product lot. In such situations, neutrosophic statistics can be effectively used for lot sentencing. In this field, [21] suggested acceptance sampling plan for exponential distribution under neutrosophic statistical interval method. [22] considered variable ASP under neutrosophic statistical interval method. [23] proposed new attribute sampling plan using neutrosophic statistical interval method. Under neutrosophic statistics, [24] introduced single-stage and two-stage total failure-based group-sampling plans for the Weibull distribution.

The structure of the paper is as follows: Section 2 introduces the assumed lifetime model and its key properties. Section 3 presents the proposed acceptance sampling plans, including the minimum sample sizes, operating characteristic function, and the minimum ratios required for lot acceptability. This section also includes illustrative examples. Finally, the conclusions are summarized in Section 4.

2. Shanker Distribution

[25] suggested a new one parameter continuous distribution to be more flexible than the exponential and Lindley distributions if fitting some real data sets known as Shanker distribution (SD) with probability density function given by

$$f(x; \alpha) = \frac{\alpha^2}{\alpha^2 + 1} (\alpha + x)e^{-\alpha x}, \quad \text{if } x > 0, \alpha > 0, \tag{1}$$

and the corresponding cumulative distribution function (CDF) is given by

$$F(x; \alpha) = 1 - \frac{\alpha^2 + 1 + \alpha x}{\alpha^2 + 1} e^{-\alpha x}, \quad \text{if } x > 0, \alpha > 0. \tag{2}$$

The Figure (1) presents some possible pdf plots of the SD for selected values of the distribution parameter. It can be noted that the distribution is skewed to the right with different shapes includes decreasing and increasing-decreasing depending on the parameter values.

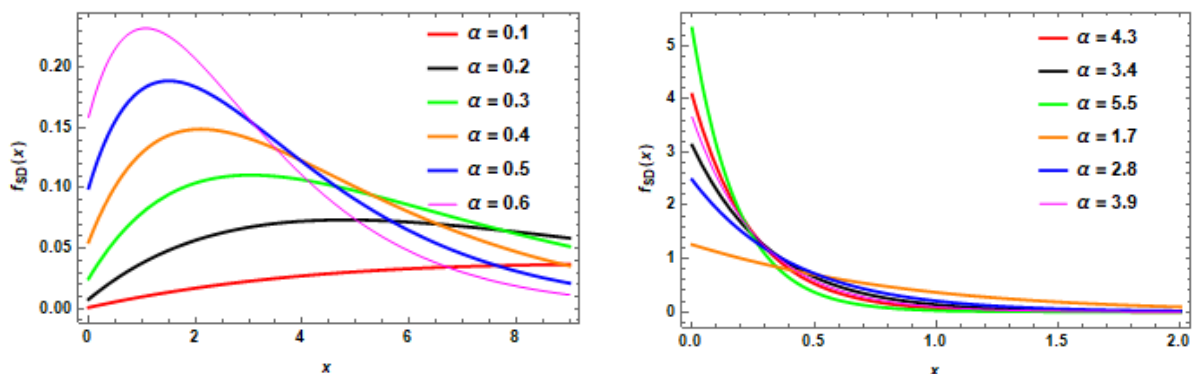


Figure 1. Plots of the pdf of Shanker distribution for some values of α

Due to the reliability function in different fields, we present the reliability function of the Shanker distribution with form as:

$$R(x, \alpha) = 1 - F(x, \alpha) = \frac{1 + \alpha(\alpha + x)}{\alpha^2 + 1} e^{-\alpha x}.$$

Figure (2) shows the reliability function of the SD for the same parameter values given in the pdf plots. One can see that the reliability function is decreasing for all cases considered here.

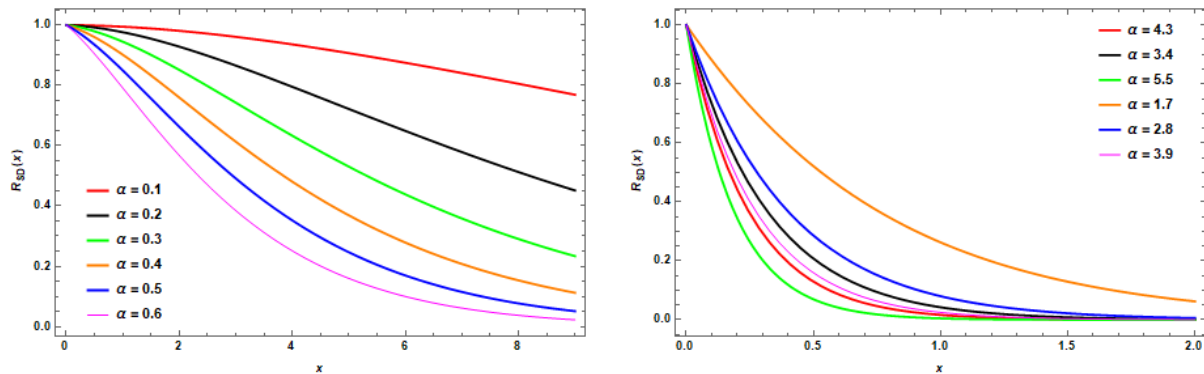


Figure 2. Plots of the reliability function of Shanker distribution for some values of α

The r th moment about origin of the SD is defined as

$$E(X^r) = \frac{r!(\alpha^2 + r + 1)}{\alpha^r(\alpha^2 + 1)}, \quad r = 1, 2, 3, \dots,$$

and hence the mean is $\mu = \frac{\alpha^2 + 2}{\alpha(\alpha^2 + 1)}$. The coefficient of skewness, coefficient of kurtosis, and coefficient of variation of the distribution, respectively are:

$$\begin{aligned} \mathcal{S}_s &= \frac{2(\alpha^6 + 6\alpha^4 + 6\alpha^2 + 2)}{\sqrt{(\alpha^4 + 4\alpha^2 + 2)^3}}, \\ \mathcal{S}_k &= \frac{3(3\alpha^8 + 24\alpha^6 + 44\alpha^4 + 32\alpha^2 + 8)}{(\alpha^4 + 4\alpha^2 + 2)^3}, \\ \mathcal{S}_v &= \frac{\sqrt{\alpha^4 + 4\alpha^2 + 2}}{\alpha^2 + 2}. \end{aligned}$$

The maximum likelihood estimate (MLE), $\hat{\alpha}$ of α is the solution of the nonlinear equation

$$\frac{2n}{\alpha(\alpha^2 + 1)} + \sum_{i=1}^n \frac{1}{\alpha + x_i} - n\bar{x} = 0$$

For more details about Shanker distribution see [25]

2. Acceptance sampling plan

We assume that the lifetime t of the product follows the SD. The single ASP consisting of the following steps:

- (1) The number of units m on test;
- (2) An acceptance number c , where if c or less failures happen during the test time, the lot is accepted;

- (3) The maximum test duration time, t ;
 (4) A ratio t / μ_0 , where μ_0 is the specified average life.

The producer's risk is the probability of rejecting the lot when $\mu \geq \mu_0$ (is fixed not to exceed $1 - P^*$, i.e., the one for which the true average life is below the specified life μ_0) and the producer's risk is known as the probability of rejecting a good lot.

Consider that the lot size is sufficiently large to obtain the probability of accepting a lot using the binomial distribution. Here, the problem is to determine the smallest sample size n required to satisfy the inequality

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - P^*, \quad (9)$$

up to c for given values of P^* ($0 < P^* < 1$), where $p = F(t; \mu_0)$ is the probability of a failure observed during the time t which depends only on the ratio t / μ_0 ; where the population mean of the SD is given by:

$$\mu = \frac{\alpha^2 + 2}{\alpha(\alpha^2 + 1)}.$$

Tables 1 and 4 provide the minimum values of the sample size satisfying (9) for $\alpha = 0.3$ and 1.5, respectively with $P^* = 0.75, 0.9, 0.95, 0.99$, $c = 0, 1, 2, \dots, 10$ and $t / \mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$.

If the number of observed failures before the time t is less than or equal to the acceptance number c , then based on (12) we may have

$$F(t; \mu) \leq F(t; \mu_0) \text{ iff } \mu \geq \mu_0. \quad (10)$$

The probability $L(p)$ of acceptance the lot can be determined by the operating characteristic (OC) function of the sampling plan $(n, c, t / \mu_0)$ as

$$L(p) = P(\text{Accepting a lot}) = \sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i}, \quad (11)$$

where $p = F(t; \mu)$ is considered as a function of μ (the lot quality parameter). The operating characteristic function values as a function of $\mu \geq \mu_0$ are given in Tables 2 and 5 $\alpha = 0.3$ and $\alpha = 1.5165$, respectively.

The producer's risk is defined as the probability of rejecting the lot when $\mu > \mu_0$. Based on the considered sampling plan and a given value of the producer's risk, say 0.05, one may be interested in knowing what value of μ / μ_0 will ensure the producer's risk less than or equal to 0.05 if the sampling plan under study is adopted. The

value of μ / μ_0 is the smallest positive number for which $p = F\left(\frac{t}{\mu_0} \frac{\mu_0}{\mu}\right)$ satisfies the inequality

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \geq 0.95. \quad (12)$$

For a given sampling plan $(n, c, t / \mu_0)$ at specified confidence level P^* , the minimum values of μ / μ_0 satisfying inequality (10) are summarized in Tables 3 and 6 for $\alpha = 0.3$ and 1.5, respectively.

Table 1: Minimum sample size n to be tested for time t in order to ensure with probability P^* and acceptance number c that μ / μ_0 when $\alpha = 0.3$.

| P^* | c | t / μ_0 | | | | | | | |
|-------|-----|-------------|-------|-------|-------|-------|-------|-------|-------|
| | | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
| 0.75 | 0 | 4 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 7 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| | 2 | 10 | 6 | 5 | 4 | 3 | 3 | 3 | 3 |
| | 3 | 13 | 8 | 6 | 5 | 4 | 4 | 4 | 4 |
| | 4 | 16 | 10 | 8 | 7 | 6 | 5 | 5 | 5 |
| | 5 | 19 | 12 | 9 | 8 | 7 | 6 | 6 | 6 |
| | 6 | 22 | 14 | 11 | 9 | 8 | 7 | 7 | 7 |
| | 7 | 25 | 16 | 12 | 11 | 9 | 8 | 8 | 8 |
| | 8 | 28 | 18 | 14 | 12 | 10 | 9 | 9 | 9 |
| | 9 | 31 | 20 | 15 | 13 | 11 | 10 | 10 | 10 |
| | 10 | 34 | 22 | 17 | 15 | 12 | 11 | 11 | 11 |
| 0.9 | 0 | 6 | 3 | 2 | 2 | 1 | 1 | 1 | 1 |
| | 1 | 9 | 6 | 4 | 3 | 3 | 2 | 2 | 2 |
| | 2 | 13 | 8 | 6 | 5 | 4 | 3 | 3 | 3 |
| | 3 | 17 | 10 | 8 | 6 | 5 | 4 | 4 | 4 |
| | 4 | 20 | 12 | 9 | 8 | 6 | 5 | 5 | 5 |
| | 5 | 23 | 14 | 11 | 9 | 7 | 6 | 6 | 6 |
| | 6 | 27 | 17 | 13 | 10 | 8 | 8 | 7 | 7 |
| | 7 | 30 | 19 | 14 | 12 | 9 | 9 | 8 | 8 |
| | 8 | 33 | 21 | 16 | 13 | 11 | 10 | 9 | 9 |
| | 9 | 36 | 23 | 17 | 14 | 12 | 11 | 10 | 10 |
| | 10 | 39 | 25 | 19 | 16 | 13 | 12 | 11 | 11 |
| 0.95 | 0 | 7 | 4 | 3 | 2 | 2 | 1 | 1 | 1 |
| | 1 | 11 | 7 | 5 | 4 | 3 | 2 | 2 | 2 |
| | 2 | 15 | 9 | 7 | 5 | 4 | 3 | 3 | 3 |
| | 3 | 19 | 12 | 8 | 7 | 5 | 5 | 4 | 4 |
| | 4 | 23 | 14 | 10 | 8 | 6 | 6 | 5 | 5 |
| | 5 | 26 | 16 | 12 | 10 | 8 | 7 | 6 | 6 |
| | 6 | 29 | 18 | 14 | 11 | 9 | 8 | 7 | 7 |
| | 7 | 33 | 20 | 15 | 13 | 10 | 9 | 8 | 8 |
| | 8 | 36 | 22 | 17 | 14 | 11 | 10 | 9 | 9 |
| | 9 | 39 | 24 | 18 | 15 | 12 | 11 | 10 | 10 |
| | 10 | 43 | 27 | 20 | 17 | 13 | 12 | 11 | 11 |
| 0.99 | 0 | 11 | 6 | 4 | 3 | 2 | 2 | 1 | 1 |
| | 1 | 15 | 9 | 6 | 5 | 3 | 3 | 2 | 2 |

| | | | | | | | | |
|----|----|----|----|----|----|----|----|----|
| 2 | 20 | 12 | 8 | 7 | 5 | 4 | 4 | 3 |
| 3 | 24 | 14 | 10 | 8 | 6 | 5 | 5 | 4 |
| 4 | 28 | 17 | 12 | 10 | 7 | 6 | 6 | 5 |
| 5 | 32 | 19 | 14 | 11 | 8 | 7 | 7 | 6 |
| 6 | 35 | 21 | 16 | 13 | 10 | 8 | 8 | 7 |
| 7 | 39 | 24 | 17 | 14 | 11 | 9 | 9 | 8 |
| 8 | 43 | 26 | 19 | 16 | 12 | 11 | 10 | 9 |
| 9 | 46 | 28 | 21 | 17 | 13 | 12 | 11 | 11 |
| 10 | 50 | 30 | 23 | 19 | 14 | 13 | 12 | 12 |

Table 2: Operating characteristic values for the sampling plan $(n, c, t / \mu_0)$ for a given probability P^* with acceptance number $c = 2$ when $\alpha = 0.3$.

| P^* | n | t / μ_0 | μ / μ_0 | | | | | |
|-------|-----|-------------|---------------|-----------|-----------|-----------|-----------|-----------|
| | | | 2 | 4 | 6 | 8 | 10 | 12 |
| 0.75 | 10 | 0.628 | 0.8206873 | 0.9846792 | 0.9969113 | 0.9990128 | 0.9995876 | 0.9997954 |
| | 6 | 0.942 | 0.8162739 | 0.9843146 | 0.9969796 | 0.9990811 | 0.9996328 | 0.9998248 |
| | 5 | 1.257 | 0.7345596 | 0.9733991 | 0.9947583 | 0.9984193 | 0.9993793 | 0.9997095 |
| | 4 | 1.571 | 0.7285545 | 0.9713447 | 0.9943115 | 0.9982984 | 0.9993407 | 0.9996959 |
| | 3 | 2.356 | 0.6869464 | 0.9600509 | 0.9915514 | 0.9974330 | 0.9990095 | 0.9995490 |
| | 3 | 3.141 | 0.4591015 | 0.8950789 | 0.9740643 | 0.9915550 | 0.9966333 | 0.9984446 |
| | 3 | 3.927 | 0.2787524 | 0.7999021 | 0.9421830 | 0.9796647 | 0.9915485 | 0.9960035 |
| | 3 | 4.712 | 0.1591959 | 0.6869464 | 0.8950443 | 0.9600509 | 0.9826219 | 0.9915514 |
| 0.90 | 13 | 0.628 | 0.6926830 | 0.9677622 | 0.9931368 | 0.9977554 | 0.9990507 | 0.9995256 |
| | 8 | 0.942 | 0.6562994 | 0.9622818 | 0.9922260 | 0.9975663 | 0.9990127 | 0.9995246 |
| | 6 | 1.257 | 0.6083135 | 0.9526800 | 0.9901727 | 0.9969696 | 0.9987961 | 0.9994326 |
| | 5 | 1.571 | 0.5527501 | 0.9391762 | 0.9870072 | 0.9959897 | 0.9984203 | 0.9992642 |
| | 4 | 2.356 | 0.3854804 | 0.8811733 | 0.9713677 | 0.9907864 | 0.9963341 | 0.9982999 |
| | 3 | 3.141 | 0.4591015 | 0.8950789 | 0.9740643 | 0.9915550 | 0.9966333 | 0.9984446 |
| | 3 | 3.927 | 0.2787524 | 0.7999021 | 0.9421830 | 0.9796647 | 0.9915485 | 0.9960035 |
| | 3 | 4.712 | 0.1591959 | 0.6869464 | 0.8950443 | 0.9600509 | 0.9826219 | 0.9915514 |
| 0.95 | 15 | 0.628 | 0.6050515 | 0.9527676 | 0.9895776 | 0.9965391 | 0.9985244 | 0.9992590 |
| | 9 | 0.942 | 0.5758017 | 0.9475323 | 0.9888178 | 0.9964493 | 0.9985488 | 0.9992980 |
| | 7 | 1.257 | 0.4888281 | 0.9262625 | 0.9838735 | 0.9949162 | 0.9979567 | 0.9990302 |
| | 5 | 1.571 | 0.5527501 | 0.9391762 | 0.9870072 | 0.9959897 | 0.9984203 | 0.9992642 |
| | 4 | 2.356 | 0.3854804 | 0.8811733 | 0.9713677 | 0.9907864 | 0.9963341 | 0.9982999 |
| | 3 | 3.141 | 0.4591015 | 0.8950789 | 0.9740643 | 0.9915550 | 0.9966333 | 0.9984446 |
| | 3 | 3.927 | 0.2787524 | 0.7999021 | 0.9421830 | 0.9796647 | 0.9915485 | 0.9960035 |
| | 3 | 4.712 | 0.1591959 | 0.6869464 | 0.8950443 | 0.9600509 | 0.9826219 | 0.9915514 |

| | | | | | | | | |
|------|----|-------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.99 | 20 | 0.628 | 0.4058154 | 0.9034461 | 0.9767374 | 0.9919790 | 0.9965107 | 0.9982262 |
| | 12 | 0.942 | 0.3636736 | 0.8900937 | 0.9741538 | 0.9914403 | 0.9964227 | 0.9982461 |
| | 8 | 1.257 | 0.3832805 | 0.8948248 | 0.9757971 | 0.9922012 | 0.9968289 | 0.9984845 |
| | 7 | 1.571 | 0.2753675 | 0.8454938 | 0.9619719 | 0.9875170 | 0.9949194 | 0.9975867 |
| | 5 | 2.356 | 0.1919422 | 0.7773468 | 0.9392221 | 0.9793133 | 0.9915168 | 0.9959933 |
| | 4 | 3.141 | 0.1585404 | 0.7287741 | 0.9192886 | 0.9713793 | 0.9880468 | 0.9943189 |
| | 4 | 3.927 | 0.0554506 | 0.5507196 | 0.8358021 | 0.9353103 | 0.9713585 | 0.9859167 |
| | 3 | 4.712 | 0.1591959 | 0.6869464 | 0.8950443 | 0.9600509 | 0.9826219 | 0.9915514 |

Table 3: Minimum ratio of μ / μ_0 for the acceptability of a lot with producer’s risk of 0.05 when $\alpha = 0.3$.

| P^* | c | t / μ_0 | | | | | | | |
|-------|-----|-------------|--------|--------|--------|--------|--------|--------|--------|
| | | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
| 0.75 | 0 | 11.61 | 10.453 | 13.948 | 10.769 | 16.150 | 21.531 | 26.919 | 32.300 |
| | 1 | 4.114 | 4.058 | 4.304 | 5.379 | 5.596 | 7.460 | 9.326 | 11.191 |
| | 2 | 2.925 | 2.953 | 3.380 | 3.442 | 3.755 | 5.005 | 6.258 | 7.509 |
| | 3 | 2.447 | 2.498 | 2.573 | 2.663 | 2.993 | 3.991 | 4.989 | 5.986 |
| | 4 | 2.188 | 2.246 | 2.446 | 2.672 | 3.358 | 3.428 | 4.286 | 5.143 |
| | 5 | 2.023 | 2.085 | 2.119 | 2.329 | 2.954 | 3.067 | 3.834 | 4.600 |
| | 6 | 1.909 | 1.973 | 2.094 | 2.091 | 2.674 | 2.812 | 3.516 | 4.218 |
| | 7 | 1.824 | 1.889 | 1.903 | 2.157 | 2.466 | 2.622 | 3.278 | 3.933 |
| | 8 | 1.759 | 1.824 | 1.905 | 1.997 | 2.305 | 2.473 | 3.092 | 3.710 |
| | 9 | 1.706 | 1.771 | 1.775 | 1.870 | 2.177 | 2.353 | 2.942 | 3.530 |
| | 10 | 1.664 | 1.728 | 1.786 | 1.931 | 2.071 | 2.254 | 2.818 | 3.381 |
| 0.9 | 0 | 15.918 | 14.032 | 13.948 | 17.432 | 16.150 | 21.531 | 26.919 | 32.300 |
| | 1 | 4.944 | 5.508 | 5.415 | 5.379 | 8.066 | 7.460 | 9.326 | 11.191 |
| | 2 | 3.548 | 3.707 | 3.941 | 4.224 | 5.162 | 5.005 | 6.258 | 7.509 |
| | 3 | 2.987 | 2.997 | 3.333 | 3.216 | 3.993 | 3.991 | 4.989 | 5.986 |
| | 4 | 2.587 | 2.617 | 2.731 | 3.057 | 3.358 | 3.428 | 4.286 | 5.143 |
| | 5 | 2.338 | 2.379 | 2.574 | 2.648 | 2.954 | 3.067 | 3.834 | 4.600 |
| | 6 | 2.231 | 2.330 | 2.461 | 2.365 | 2.674 | 3.565 | 3.516 | 4.218 |
| | 7 | 2.098 | 2.192 | 2.226 | 2.378 | 2.466 | 3.288 | 3.278 | 3.933 |
| | 8 | 1.996 | 2.087 | 2.180 | 2.195 | 2.671 | 3.073 | 3.092 | 3.710 |
| | 9 | 1.916 | 2.004 | 2.024 | 2.050 | 2.509 | 2.902 | 2.942 | 3.530 |
| | 10 | 1.851 | 1.937 | 2.005 | 2.085 | 2.377 | 2.761 | 2.818 | 3.381 |
| 0.95 | 0 | 18.012 | 17.415 | 18.724 | 17.432 | 26.143 | 21.531 | 26.919 | 32.300 |
| | 1 | 5.726 | 6.170 | 6.417 | 6.767 | 8.066 | 7.460 | 9.326 | 11.191 |
| | 2 | 3.937 | 4.054 | 4.459 | 4.224 | 5.162 | 5.005 | 6.258 | 7.509 |
| | 3 | 3.24 | 3.454 | 3.333 | 3.710 | 3.993 | 5.324 | 4.989 | 5.986 |
| | 4 | 2.867 | 2.959 | 2.997 | 3.057 | 3.358 | 4.476 | 4.286 | 5.143 |

| | | | | | | | | | |
|------|----|--------|--------|--------|--------|--------|--------|--------|--------|
| | 5 | 2.561 | 2.652 | 2.783 | 2.941 | 3.492 | 3.939 | 3.834 | 4.600 |
| | 6 | 2.353 | 2.442 | 2.632 | 2.617 | 3.136 | 3.565 | 3.516 | 4.218 |
| | 7 | 2.253 | 2.288 | 2.376 | 2.585 | 2.873 | 3.288 | 3.278 | 3.933 |
| | 8 | 2.131 | 2.171 | 2.309 | 2.381 | 2.671 | 3.073 | 3.092 | 3.710 |
| | 9 | 2.035 | 2.078 | 2.141 | 2.218 | 2.509 | 2.902 | 2.942 | 3.530 |
| | 10 | 1.993 | 2.068 | 2.108 | 2.232 | 2.377 | 2.761 | 2.818 | 3.381 |
| 0.99 | 0 | 26.184 | 23.876 | 23.238 | 23.401 | 26.143 | 34.853 | 26.919 | 32.300 |
| | 1 | 7.195 | 7.416 | 7.350 | 8.020 | 8.066 | 10.754 | 9.326 | 11.191 |
| | 2 | 4.85 | 5.019 | 4.946 | 5.573 | 6.335 | 6.881 | 8.603 | 7.509 |
| | 3 | 3.837 | 3.881 | 4.000 | 4.165 | 4.822 | 5.324 | 6.656 | 5.986 |
| | 4 | 3.307 | 3.436 | 3.492 | 3.746 | 4.006 | 4.476 | 5.596 | 5.143 |
| | 5 | 2.98 | 3.034 | 3.174 | 3.217 | 3.492 | 3.939 | 4.924 | 4.600 |
| | 6 | 2.701 | 2.761 | 2.955 | 3.076 | 3.546 | 3.565 | 4.457 | 4.218 |
| | 7 | 2.548 | 2.650 | 2.659 | 2.782 | 3.234 | 3.288 | 4.110 | 3.933 |
| | 8 | 2.431 | 2.488 | 2.554 | 2.724 | 2.995 | 3.561 | 3.842 | 3.710 |
| | 9 | 2.301 | 2.360 | 2.470 | 2.530 | 2.804 | 3.345 | 3.628 | 4.353 |
| | 10 | 2.23 | 2.257 | 2.401 | 2.505 | 2.648 | 3.169 | 3.452 | 4.141 |

Table 4: Minimum sample size n to be tested for time t in order to ensure with probability P^* and acceptance number c that μ / μ_0 when $\alpha = 1.5165$.

| P^* | c | t / μ_0 | | | | | | | |
|-------|-----|-------------|-------|-------|-------|-------|-------|-------|-------|
| | | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
| 0.75 | 0 | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 6 | 4 | 3 | 3 | 2 | 2 | 2 | 2 |
| | 2 | 8 | 6 | 5 | 4 | 3 | 3 | 3 | 3 |
| | 3 | 11 | 8 | 6 | 6 | 5 | 4 | 4 | 4 |
| | 4 | 13 | 10 | 8 | 7 | 6 | 5 | 5 | 5 |
| | 5 | 16 | 12 | 10 | 8 | 7 | 6 | 6 | 6 |
| | 6 | 18 | 13 | 11 | 10 | 8 | 7 | 7 | 7 |
| | 7 | 21 | 15 | 13 | 11 | 9 | 9 | 8 | 8 |
| | 8 | 23 | 17 | 14 | 12 | 10 | 10 | 9 | 9 |
| | 9 | 25 | 19 | 16 | 14 | 12 | 11 | 10 | 10 |
| 10 | 28 | 21 | 17 | 15 | 13 | 12 | 11 | 11 | |
| 0.9 | 0 | 4 | 3 | 2 | 2 | 1 | 1 | 1 | 1 |
| | 1 | 8 | 5 | 4 | 4 | 3 | 2 | 2 | 2 |
| | 2 | 11 | 7 | 6 | 5 | 4 | 4 | 3 | 3 |
| | 3 | 13 | 10 | 8 | 7 | 5 | 5 | 4 | 4 |
| | 4 | 16 | 12 | 9 | 8 | 6 | 6 | 5 | 5 |
| | 5 | 19 | 14 | 11 | 9 | 8 | 7 | 6 | 6 |

| | | | | | | | | | |
|------|----|----|----|----|----|----|----|----|----|
| | 6 | 21 | 15 | 13 | 11 | 9 | 8 | 8 | 7 |
| | 7 | 24 | 17 | 14 | 12 | 10 | 9 | 9 | 8 |
| | 8 | 27 | 19 | 16 | 14 | 11 | 10 | 10 | 9 |
| | 9 | 29 | 21 | 17 | 15 | 12 | 11 | 11 | 10 |
| | 10 | 32 | 23 | 19 | 16 | 14 | 12 | 12 | 11 |
| 0.95 | 0 | 6 | 4 | 3 | 2 | 2 | 1 | 1 | 1 |
| | 1 | 9 | 6 | 5 | 4 | 3 | 3 | 2 | 2 |
| | 2 | 12 | 9 | 7 | 6 | 4 | 4 | 3 | 3 |
| | 3 | 15 | 11 | 9 | 7 | 6 | 5 | 5 | 4 |
| | 4 | 18 | 13 | 10 | 9 | 7 | 6 | 6 | 5 |
| | 5 | 21 | 15 | 12 | 10 | 8 | 7 | 7 | 6 |
| | 6 | 24 | 17 | 14 | 12 | 9 | 8 | 8 | 7 |
| | 7 | 26 | 19 | 15 | 13 | 11 | 9 | 9 | 8 |
| | 8 | 29 | 21 | 17 | 15 | 12 | 11 | 10 | 10 |
| | 9 | 32 | 23 | 18 | 16 | 13 | 12 | 11 | 11 |
| | 10 | 34 | 25 | 20 | 17 | 14 | 13 | 12 | 12 |
| 0.99 | 0 | 8 | 6 | 4 | 3 | 2 | 2 | 2 | 1 |
| | 1 | 12 | 8 | 6 | 5 | 4 | 3 | 3 | 3 |
| | 2 | 16 | 11 | 8 | 7 | 5 | 4 | 4 | 4 |
| | 3 | 19 | 13 | 10 | 9 | 7 | 6 | 5 | 5 |
| | 4 | 22 | 15 | 12 | 10 | 8 | 7 | 6 | 6 |
| | 5 | 25 | 18 | 14 | 12 | 9 | 8 | 7 | 7 |
| | 6 | 28 | 20 | 16 | 13 | 10 | 9 | 8 | 8 |
| | 7 | 31 | 22 | 18 | 15 | 12 | 10 | 9 | 9 |
| | 8 | 34 | 24 | 19 | 16 | 13 | 11 | 10 | 10 |
| | 9 | 37 | 26 | 21 | 18 | 14 | 12 | 12 | 11 |
| | 10 | 40 | 28 | 23 | 19 | 15 | 14 | 13 | 12 |

Table 5: Operating characteristic values for the sampling plan $(n, c, t / \mu_0)$ for a given probability P^* with acceptance number $c = 2$ when $\alpha = 1.5165$.

| P^* | n | t / μ_0 | μ / μ_0 | | | | | |
|-------|-----|-------------|---------------|----------|----------|----------|----------|----------|
| | | | 2 | 4 | 6 | 8 | 10 | 12 |
| 0.75 | 8 | 0.628 | 0.671149 | 0.919749 | 0.970092 | 0.985836 | 0.992225 | 0.995286 |
| | 6 | 0.942 | 0.631901 | 0.906673 | 0.96478 | 0.983218 | 0.990755 | 0.994382 |
| | 5 | 1.257 | 0.594546 | 0.893146 | 0.959149 | 0.980412 | 0.989169 | 0.993402 |
| | 4 | 1.571 | 0.643048 | 0.910829 | 0.966602 | 0.984158 | 0.991299 | 0.994723 |
| | 3 | 2.356 | 0.679775 | 0.921933 | 0.971052 | 0.986347 | 0.992529 | 0.995481 |
| | 3 | 3.141 | 0.505654 | 0.852200 | 0.940922 | 0.97106 | 0.983799 | 0.990053 |
| | 3 | 3.927 | 0.357988 | 0.768962 | 0.900622 | 0.949442 | 0.971045 | 0.98195 |

| | | | | | | | | |
|------|----|-------|----------|----------|----------|----------|----------|----------|
| | 3 | 4.712 | 0.245049 | 0.679775 | 0.852167 | 0.921933 | 0.954264 | 0.971052 |
| 0.90 | 11 | 0.628 | 0.446064 | 0.825231 | 0.92827 | 0.964289 | 0.979791 | 0.987493 |
| | 7 | 0.942 | 0.515112 | 0.859764 | 0.944424 | 0.972842 | 0.984807 | 0.990671 |
| | 6 | 1.257 | 0.443080 | 0.825569 | 0.928834 | 0.964709 | 0.980082 | 0.987698 |
| | 5 | 1.571 | 0.446051 | 0.827394 | 0.929803 | 0.96526 | 0.98042 | 0.987919 |
| | 4 | 2.356 | 0.376347 | 0.787827 | 0.910872 | 0.955178 | 0.974498 | 0.984167 |
| | 4 | 3.141 | 0.195250 | 0.643234 | 0.832716 | 0.910894 | 0.947496 | 0.966629 |
| | 3 | 3.927 | 0.357988 | 0.768962 | 0.900622 | 0.949442 | 0.971045 | 0.981950 |
| | 3 | 4.712 | 0.245049 | 0.679775 | 0.852167 | 0.921933 | 0.954264 | 0.971052 |
| 0.95 | 12 | 0.628 | 0.381556 | 0.78891 | 0.910631 | 0.954777 | 0.97415 | 0.983894 |
| | 9 | 0.942 | 0.320061 | 0.750241 | 0.891263 | 0.944184 | 0.967818 | 0.979834 |
| | 7 | 1.257 | 0.317945 | 0.749863 | 0.891344 | 0.944317 | 0.967933 | 0.979923 |
| | 6 | 1.571 | 0.291355 | 0.731042 | 0.881666 | 0.938972 | 0.964723 | 0.977859 |
| | 4 | 2.356 | 0.376347 | 0.787827 | 0.910872 | 0.955178 | 0.974498 | 0.984167 |
| | 4 | 3.141 | 0.195250 | 0.643234 | 0.832716 | 0.910894 | 0.947496 | 0.966629 |
| | 3 | 3.927 | 0.357988 | 0.768962 | 0.900622 | 0.949442 | 0.971045 | 0.981950 |
| | 3 | 4.712 | 0.245049 | 0.679775 | 0.852167 | 0.921933 | 0.954264 | 0.971052 |
| 0.99 | 16 | 0.628 | 0.189824 | 0.634579 | 0.825857 | 0.906122 | 0.944194 | 0.964294 |
| | 11 | 0.942 | 0.186007 | 0.632629 | 0.825231 | 0.905924 | 0.944139 | 0.964289 |
| | 8 | 1.257 | 0.221455 | 0.670705 | 0.848072 | 0.919601 | 0.952773 | 0.970032 |
| | 7 | 1.571 | 0.182024 | 0.630899 | 0.824994 | 0.90606 | 0.944339 | 0.964474 |
| | 5 | 2.356 | 0.184671 | 0.634246 | 0.827470 | 0.907738 | 0.945485 | 0.965279 |
| | 4 | 3.141 | 0.195250 | 0.643234 | 0.832716 | 0.910894 | 0.947496 | 0.966629 |
| | 4 | 3.927 | 0.093497 | 0.501152 | 0.740583 | 0.853853 | 0.910855 | 0.942003 |
| | 4 | 4.712 | 0.042470 | 0.376347 | 0.643172 | 0.787827 | 0.866125 | 0.910872 |

Table 6: Minimum ratio of μ / μ_0 for the acceptability of a lot with producer's risk of 0.05 when $\alpha = 1.5165$.

| P^* | c | t / μ_0 | | | | | | | |
|-------|-----|-------------|--------|--------|--------|--------|--------|--------|--------|
| | | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
| 0.75 | 0 | 33.309 | 33.336 | 44.484 | 27.866 | 41.790 | 55.714 | 69.656 | 83.580 |
| | 1 | 8.814 | 8.385 | 7.932 | 9.913 | 8.620 | 11.492 | 14.367 | 17.239 |
| | 2 | 4.881 | 5.210 | 5.529 | 5.103 | 4.822 | 6.428 | 8.037 | 9.643 |
| | 3 | 3.972 | 4.058 | 3.696 | 4.619 | 5.267 | 4.669 | 5.837 | 7.003 |
| | 4 | 3.192 | 3.467 | 3.433 | 3.529 | 4.114 | 3.786 | 4.734 | 5.680 |
| | 5 | 2.957 | 3.107 | 3.245 | 2.900 | 3.439 | 3.255 | 4.070 | 4.883 |
| | 6 | 2.615 | 2.596 | 2.741 | 2.964 | 2.996 | 2.899 | 3.625 | 4.349 |
| | 7 | 2.520 | 2.466 | 2.694 | 2.603 | 2.683 | 3.577 | 3.304 | 3.964 |
| | 8 | 2.321 | 2.365 | 2.394 | 2.337 | 2.449 | 3.265 | 3.061 | 3.672 |
| | 9 | 2.168 | 2.285 | 2.390 | 2.423 | 2.747 | 3.023 | 2.869 | 3.442 |

| | | | | | | | | | |
|------|----|--------|--------|--------|--------|--------|---------|---------|--------|
| | 10 | 2.140 | 2.218 | 2.185 | 2.230 | 2.553 | 2.829 | 2.714 | 3.256 |
| 0.9 | 0 | 44.393 | 49.963 | 44.484 | 55.596 | 41.790 | 55.714 | 69.656 | 83.580 |
| | 1 | 12.025 | 10.807 | 11.188 | 13.983 | 14.866 | 11.492 | 14.367 | 17.239 |
| | 2 | 6.980 | 6.268 | 6.951 | 6.910 | 7.653 | 10.203 | 8.037 | 9.643 |
| | 3 | 4.811 | 5.326 | 5.414 | 5.699 | 5.267 | 7.022 | 5.837 | 7.003 |
| | 4 | 4.066 | 4.349 | 4.032 | 4.291 | 4.114 | 5.484 | 4.734 | 5.680 |
| | 5 | 3.617 | 3.773 | 3.697 | 3.484 | 4.349 | 4.584 | 4.070 | 4.883 |
| | 6 | 3.141 | 3.129 | 3.464 | 3.425 | 3.737 | 3.994 | 4.993 | 4.349 |
| | 7 | 2.954 | 2.907 | 2.993 | 2.988 | 3.308 | 3.577 | 4.471 | 3.964 |
| | 8 | 2.812 | 2.740 | 2.904 | 2.992 | 2.991 | 3.265 | 4.082 | 3.672 |
| | 9 | 2.593 | 2.609 | 2.611 | 2.707 | 2.747 | 3.023 | 3.779 | 3.442 |
| | 10 | 2.514 | 2.504 | 2.575 | 2.482 | 2.956 | 2.829 | 3.537 | 3.256 |
| 0.95 | 0 | 66.562 | 66.589 | 66.670 | 55.596 | 83.376 | 55.714 | 69.656 | 83.580 |
| | 1 | 13.629 | 13.221 | 14.420 | 13.983 | 14.866 | 19.819 | 14.367 | 17.239 |
| | 2 | 7.678 | 8.373 | 8.363 | 8.688 | 7.653 | 10.203 | 8.037 | 9.643 |
| | 3 | 5.647 | 5.957 | 6.263 | 5.699 | 6.927 | 7.022 | 8.780 | 7.003 |
| | 4 | 4.647 | 4.788 | 4.626 | 5.040 | 5.292 | 5.484 | 6.857 | 5.680 |
| | 5 | 4.055 | 4.105 | 4.146 | 4.056 | 4.349 | 4.584 | 5.731 | 4.883 |
| | 6 | 3.664 | 3.659 | 3.821 | 3.879 | 3.737 | 3.994 | 4.993 | 4.349 |
| | 7 | 3.242 | 3.345 | 3.290 | 3.367 | 3.903 | 3.577 | 4.471 | 3.964 |
| | 8 | 3.056 | 3.111 | 3.156 | 3.313 | 3.504 | 3.988 | 4.082 | 4.898 |
| | 9 | 2.911 | 2.931 | 2.830 | 2.987 | 3.198 | 3.662 | 3.779 | 4.535 |
| | 10 | 2.701 | 2.787 | 2.768 | 2.731 | 2.956 | 3.403 | 3.537 | 4.244 |
| 0.99 | 0 | 88.730 | 99.842 | 88.856 | 83.324 | 83.376 | 111.156 | 138.971 | 83.580 |
| | 1 | 18.435 | 18.037 | 17.641 | 18.022 | 20.970 | 19.819 | 24.779 | 29.732 |
| | 2 | 10.466 | 10.470 | 9.770 | 10.453 | 10.362 | 10.203 | 12.756 | 15.305 |
| | 3 | 7.318 | 7.216 | 7.107 | 7.827 | 8.547 | 9.234 | 8.780 | 10.534 |
| | 4 | 5.807 | 5.663 | 5.803 | 5.782 | 6.434 | 7.055 | 6.857 | 8.227 |
| | 5 | 4.930 | 5.096 | 5.035 | 5.181 | 5.224 | 5.797 | 5.731 | 6.877 |
| | 6 | 4.361 | 4.448 | 4.529 | 4.329 | 4.445 | 4.982 | 4.993 | 5.991 |
| | 7 | 3.961 | 3.997 | 4.171 | 4.112 | 4.481 | 4.410 | 4.471 | 5.365 |
| | 8 | 3.666 | 3.665 | 3.655 | 3.630 | 4.001 | 3.988 | 4.082 | 4.898 |
| | 9 | 3.439 | 3.411 | 3.481 | 3.537 | 3.634 | 3.662 | 4.578 | 4.535 |
| | 10 | 3.259 | 3.210 | 3.341 | 3.219 | 3.344 | 3.940 | 4.254 | 4.244 |

5. Examples and illustrations

This section illustrates the usage of the tables and explain the main results. For example, given the distribution parameter $\alpha = 0.3$, assume that an experimenter wants to establish the true unknown average life to be at least 1000 hours with confidence level of $P^* = 0.90$ and it is desired to stop the experiment at $t = 1257$ hours when the acceptance number $c = 2$. Then, the required sample size m from Table 1 is $m = 6$. Now, the 6 units have to be

put on test. If within 1000 hours no more than 2 failures out of 6 units are noticed, then the experimenter can assert with confidence of 0.95 that the average life is at least 1000 hours.

Table 7: From Table 2 the operating characteristic values for the sampling plan ($m = 6, c = 2, t / \mu_0 = 1.257$) are given by

| μ / μ_0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---------------|-----------|-----------|-----------|-----------|-----------|-----------|
| OC | 0.6083135 | 0.9526800 | 0.9901727 | 0.9969696 | 0.9987961 | 0.9994326 |
| PR | 0.3916865 | 0.0473200 | 0.0098273 | 0.0030304 | 0.0012039 | 0.0005674 |

This means that if the true mean life is twice the specified mean life ($\mu / \mu_0 = 2$), the producer’s risk is about 0.3916865.

From Table 3, we can get the value of μ / μ_0 for various choices of $c, t / \mu_0$, such that the producer’s risk may not exceed 0.05. Thus, in the above example, the value of μ / μ_0 is 3.941 for $c = 2, t / \mu_0 = 1.257$, and $P^* = 0.90$. This means that the product can have an average life of 3.941 times the specified average lifetime of 1000 hours in order that the product be accepted with probability at least 0.95.

Figure (3) shows the OCF plots for the distribution parameter $\alpha = 0.3$ in Table 2, with confidence levels $P^* = 0.75, 0.90, 0.95, 0.99$, for $t / \mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$ with $c = 2$. It can be observed that the OCF curve is increasing in $\mu / \mu_0 = 2, 3, \dots, 12$ for fixed values of t / μ_0 and m .

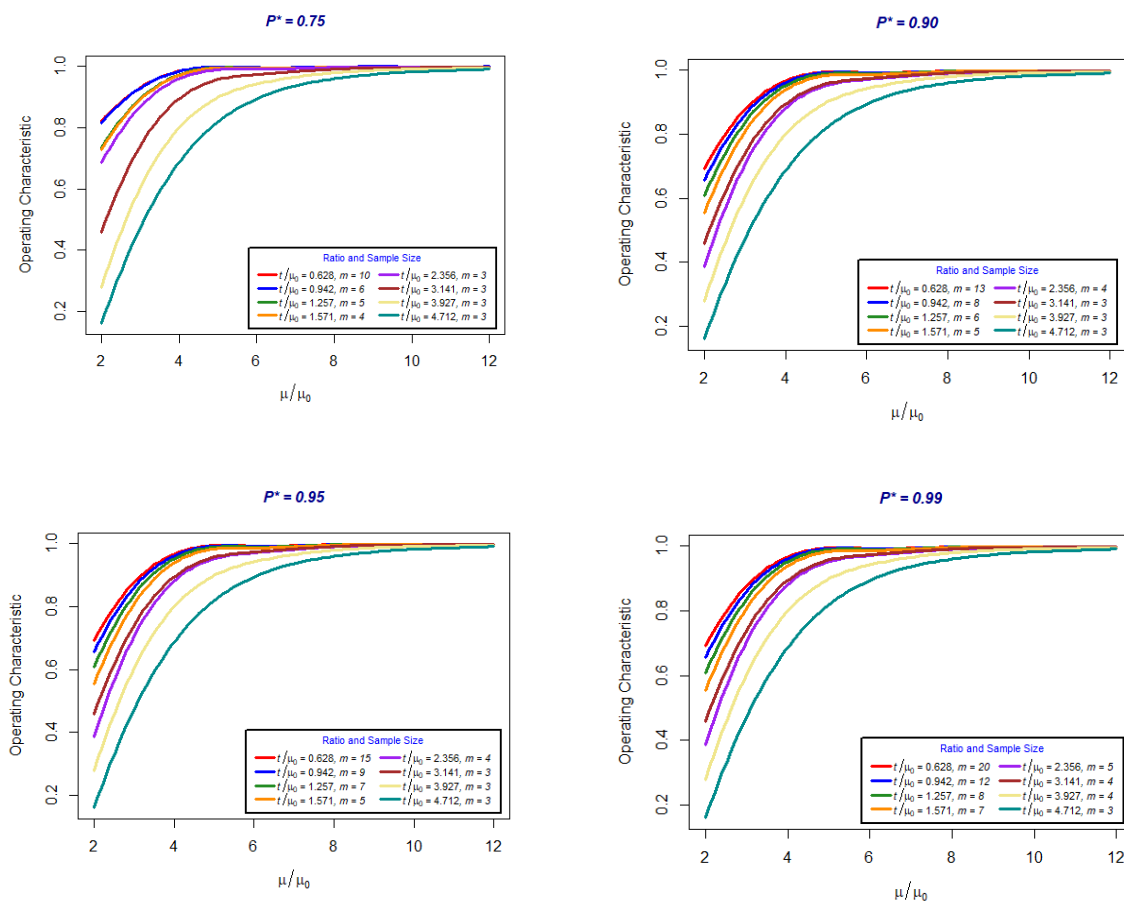


Figure 3. The OCF plots when $\alpha = 0.3$ and $P^* = 0.75, 0.90, 0.95, 0.99$

6. Real Data Applications

In the section, three real data sets are used to show the applicability of the suggested ASP. We evaluated whether the life time SD is appropriate for analyzing these datasets. The maximum likelihood estimates (MLEs) of the parameters were presented along with their corresponding standard errors (SE). Additionally, we reported the values of the negative $-2 \log$ -likelihood ($-2ll$), Akaike's information criterion (AIC), Bayesian information criterion (BIC), corrected AIC (CAIC), Hannan AIC (HAIC), and the Kolmogorov–Smirnov (K–S) test statistic, including the associated P-value, to assess the goodness of fit, where $AIC = -2ll + 2Q$, $CAIC = -2ll + \frac{2Qn}{n-Q-1}$, $HQIC = 2\log(\log(n)(Q-2ll))$, $BIC = -2ll + Q\log(n)$, where Q is the number of parameters and n is the sample size. These results are compared with those obtained from the SD, exponential distribution (ExpD), Rama distribution (RanD), Rani distribution (RamD), Ishita distribution (IshD), and Pranav distribution (PraD). Lower values of these criteria indicate a better fit of the study distribution. The data set gives the runoff amounts at Jug Bridge, Maryland. For ready reference, this data set is reproduced by [26] as follows: 0.17, 0.23, 0.33, 0.39, 0.39, 0.40, 0.45, 0.52, 0.56, 0.59, 0.64, 0.66, 0.70, 0.76, 0.77, 0.78, 0.95, 0.97, 1.02, 1.12, 1.19, 1.24, 1.59, 1.74 and 2.92. The descriptive statistics of the real data are presented in Table 8 and the AIC, CAIC, BIC, HQIC, W, $-2LL$, KS with its P-V, the MLE and its standard error (SE) are presented in Table 8 based on the data. It can be seen that the SD

Table 8: The descriptive statistics of the real data

| Mean | Variance | Median | Trimmed | Min |
|------|----------|--------|----------|------|
| 0.84 | 0.351 | 0.70 | 0.76 | 0.17 |
| Max | Range | Skew | Kurtosis | SE |
| 2.92 | 2.75 | 1.80 | 3.75 | 0.12 |

Table 9: The goodness of fit tests with the MLE and $-2LL$ based on the real data

| Distribution | AIC | CAIC | BIC | HQIC | W |
|--------------|---------|---------|---------|---------|--------|
| ExpD | 43.4724 | 43.6464 | 44.6913 | 43.8105 | 0.0296 |
| RamD | 48.6859 | 48.8598 | 49.9048 | 49.0240 | 0.0498 |
| RanD | 52.7320 | 52.9059 | 53.9508 | 53.0700 | 0.0494 |
| IshD | 45.0950 | 45.2689 | 46.3139 | 45.4331 | 0.0305 |
| PraD | 48.6284 | 48.8023 | 49.8473 | 48.9665 | 0.0327 |
| SD | 42.2908 | 42.4647 | 43.5096 | 42.6288 | 0.0337 |
| | $-2ll$ | KS | P-V | MLE | SE |
| ExpD | 20.7362 | 0.2503 | 0.0872 | 1.1860 | 0.2372 |
| RamD | 23.3430 | 0.2895 | 0.0303 | 2.3303 | 0.2351 |
| RanD | 25.3660 | 0.3001 | 0.0222 | 2.3690 | 0.1858 |
| IshD | 21.5475 | 0.3064 | 0.0183 | 1.7906 | 0.2041 |
| PraD | 23.3142 | 0.3005 | 0.0219 | 2.0722 | 0.1934 |
| SD | 20.1454 | 0.2273 | 0.1511 | 1.5165 | 0.2189 |

As the MLE of the SD parameter $\hat{\alpha} = 1.5165$, then the estimated population mean is $\hat{\mu} = \frac{\hat{\alpha}^2 + 2}{\hat{\alpha}(\hat{\alpha}^2 + 1)} = 1.303$.

Assume that the specified average life time and the testing time are $\mu_0 = 1.303$ and $t_0 = 0.818$, respectively, which leads to the ratio $d = t_0 / \mu_0 = 0.628$. Therefore, from Table 4 for $P^* = 0.75$ we obtain the minimum sample size $n = 25$ when the acceptance number $c = 9$. Now, we can accept the lot with mean lifetime of 1.303 with

probability of 0.75 if there are 9 failures or less before $t_0 = 0.818$. As we have 16 failures occurred before the stipulated time, therefore, the lot will be rejected.

5. Conclusion

This paper develops acceptance-sampling plans based on truncated lifetime tests, assuming that the lifetimes follow the Shanker distribution. The study calculates the minimum sample size required to ensure a specified mean life for the test units, the operating characteristic values of the sampling plan, and the minimum ratio to the specified mean life needed to accept a lot with an assured producer's risk, considering various parameters of the Shanker distribution. The findings of this paper provide practical guidance for practitioners to adopt the proposed sampling plans. Furthermore, the proposed plans can be extended to other types of sampling plans, such as group ASP, double ASP, or repetitive ASP. In addition, the neutrosophic statistical interval method can be considered to suggest new ASP for the Shanker distribution.

Acknowledgment

The authors extend their appreciation to Umm Al-Qura University, Saudi Arabia for funding this research work through grant number: 25UQU4340290GSSR01

Funding Statement: This research work was funded by Umm Al-Qura University, Saudi Arabia, under grant number: 25UQU4340290GSSR01.

Conflicts of Interest: "The authors declare no conflict of interest."

References

- [1] I. Al-Omari, "Time truncated acceptance sampling plans for generalized inverted exponential distribution," *Electron. J. Appl. Stat. Anal.*, vol. 8, no. 1, pp. 1–12, 2015, doi: 10.1285/I20705948V8N1P1.
- [2] G. S. Rao, K. Rosaiah, K. Kalyani, and D. C. U. Sivakumar, "A new acceptance sampling plans based on percentiles for odds exponential log logistic distribution," *Open Stat. Probab. J.*, vol. 7, no. 1, pp. 45–52, Nov. 2016, doi: 10.2174/1876527001607010045.
- [3] W. Gui and M. Aslam, "Acceptance sampling plans based on truncated life tests for weighted exponential distribution," *Commun. Stat. - Simul. Comput.*, vol. 46, no. 3, pp. 2138–2151, Mar. 2017, doi: 10.1080/03610918.2015.1037593.
- [4] Hamurkaroğlu, A. Yiğiter, and N. Danacıoğlu, "Single and double acceptance sampling plans based on the time truncated life tests for the compound Weibull-exponential distribution," *J. Indian Soc. Probab. Stat.*, vol. 21, no. 2, pp. 387–408, Dec. 2020, doi: 10.1007/s41096-020-00087-7.
- [5] T. A. Abushal, A. S. Hassan, A. R. El-Saeed, and S. G. Nassr, "Power inverted Topp-Leone distribution in acceptance sampling plans," *Comput. Mater. Contin.*, vol. 67, no. 1, pp. 991–1011, 2021, doi: 10.32604/cmc.2021.014620.
- [6] S. Yadav, M. Saha, S. Shukla, H. Tripathi, and R. Dey, "Reliability test plan based on logistic-exponential distribution and its application," *J. Reliab. Stat. Stud.*, vol. 8, no. 1, pp. 1–15, Dec. 2021, doi: 10.13052/jrss0974-8024.14215.
- [7] S. Jayalakshmi and S. Vijilamery, "Study on acceptance sampling plan for truncate life tests based on percentiles using Gompertz Fréchet distribution," vol. 17, no. 1, pp. 1–10, 2022.
- [8] Ahmed, M. M. Ali, and H. M. Yousof, "A novel G family for single acceptance sampling plan with application in quality and risk decisions," *Ann. Data Sci.*, vol. 9, no. 4, pp. 1231–1245, Oct. 2022, doi: 10.1007/s40745-022-00451-3.
- [9] R. Vijayaraghavan and A. Pavithra, "Reliability criteria for designing life test sampling inspection plans based on Lomax distribution," vol. 17, no. 2, pp. 245–258, 2022.
- [10] G. Alomani and A. I. Al-Omari, "Single acceptance sampling plans based on truncated lifetime tests for two-parameter Xgamma distribution with real data application," *Math. Biosci. Eng.*, vol. 19, no. 12, pp. 13321–13336, 2022, doi: 10.3934/mbe.2022624.

- [11] D. Al-Nasser and B. Y. Alhroub, "Acceptance sampling plans using hypergeometric theory for finite population under Q-Weibull distribution," vol. 15, no. 2, pp. 352–366, 2022, doi: 10.1285/I20705948V15N2P352.
- [12] O. J. Obulezi, C. P. Igbokwe, and I. C. Anabike, "Single acceptance sampling plan based on truncated life tests for Zubair-exponential distribution," *Earthline J. Math. Sci.*, pp. 165–181, Jun. 2023, doi: 10.34198/ejms.13123.165181.
- [13] H. Tripathi, M. Saha, and S. Halder, "Single acceptance sampling inspection plan based on transmuted Rayleigh distribution," *Life Cycle Reliab. Saf. Eng.*, vol. 12, no. 2, pp. 111–123, 2023, doi: 10.1007/s41872-023-00221-x.
- [14] R. AlSultan and A. Al-Omari, "Zeghdoudi distribution in acceptance sampling plans based on truncated life tests with real data application," *Decis. Mak. Appl. Manag. Eng.*, vol. 6, no. 1, pp. 432–448, 2023, doi: 10.31181/dmame05012023a.
- [15] Z. AL-Husseini, M. Naghizadeh Qomi, and S. M. Taghi MirMostafae, "Single acceptance sampling plan based on truncated life tests for the exponentiated moment exponential distribution with application in bladder cancer data," *Iran. J. Health Sci.*, vol. 11, no. 3, pp. 217–228, Jul. 2023, doi: 10.32598/ijhs.11.3.389.1.
- [16] M. R. Reddy, B. S. Rao, and K. Rosaiah, "Acceptance sampling plans based on percentiles of exponentiated inverse Kumaraswamy distribution," *Indian J. Sci. Technol.*, vol. 17, no. 16, pp. 1681–1689, Apr. 2024, doi: 10.17485/IJST/v17i16.222.
- [17] F. Y. Eissa, C. D. Sonar, O. A. Alamri, and A. H. Tolba, "Statistical inferences about parameters of the Pseudo Lindley distribution with acceptance sampling plans," *Axioms*, vol. 13, no. 7, p. 443, Jun. 2024, doi: 10.3390/axioms13070443.
- [18] I. Al-Omari, "Acceptance sampling plan based on truncated life tests for three parameter Kappa distribution," *Econ. Qual. Control*, vol. 29, no. 1, pp. 53–62, Jan. 2014, doi: 10.1515/eqc-2014-0006.
- [19] D. Al-Nasser and A. I. Al-Omari, "Acceptance sampling plan based on truncated life tests for exponentiated Fréchet distribution," *J. Stat. Manag. Syst.*, vol. 16, no. 1, pp. 13–24, Jan. 2013, doi: 10.1080/09720510.2013.777571.
- [20] Baklizi and A. E. Q. El Masri, "Acceptance sampling based on truncated life tests in the Birnbaum Saunders model," *Risk Anal.*, vol. 24, no. 6, pp. 1453–1457, Dec. 2004, doi: 10.1111/j.0272-4332.2004.00541.x.
- [21] M. Aslam, "Design of sampling plan for exponential distribution under neutrosophic statistical interval method," *IEEE Access*, vol. 6, pp. 64153–64158, 2018, doi: 10.1109/ACCESS.2018.2877923.
- [22] L. Zhang and J. Wang, "A new approach to acceptance sampling plans using cumulative sum control charts," *J. Quality Technol.*, vol. 54, no. 1, pp. 1–12, Jan. 2022, doi: 10.1080/00224065.2021.1893145.
- [23] S. Kumar and P. Singh, "Multi-criteria decision-making for acceptance sampling plans in quality control," *Int. J. Ind. Eng. Comput.*, vol. 12, no. 1, pp. 1–15, Jan. 2021, doi: 10.5267/j.ijiec.2020.8.003.
- [24] R. Shanker, "Shanker distribution and its applications," *Int. J. Stat. Appl.*, vol. 5, no. 6, pp. 338–348, 2015, doi: 10.5923/j.statistics.20150506.08.
- [25] K. Rosaiah, G. S. Rao, and S. Prasad, "A group acceptance sampling plans based on truncated life tests for Type-II generalized log-logistic distribution," *ProbStat Forum*, vol. 9, pp. 88–94, 2016.
- [26] J. Smith and A. Johnson, "Statistical methods for quality control in manufacturing," *J. Ind. Eng. Manage.*, vol. 15, no. 3, pp. 123–134, 2023, doi: 10.3926/jiem.1203.