Some Results on Neutrosophic Graphs in Neutrosophic Topological space

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Abstract

A tools and techniques of neutrosophic graph have found many applications in different areas such as topology, networks, computer of science, etc. In addition, neutrosophic graph is a generalization of intuitionistic fuzzy graph. Therefore, in this paper we study some characteristics of neutrosopheic graphs (NTCG) and some basic definitions. Moreover we investigate several kinds of arcs, η_μ -strong, σ_μ-strong, ρ_μ -arc, and η_ξ -strong, σ_ξ-strong, ρ_ξ –arc in neutrosopheic graphs (NTCG) , Finally we give neutrosophic μ-bridge and neutrosophic ξ-bridge (NTC ξ-bridge) and some interesting properties of neutrosophic bridge (NTCB), which is being taught for the first time, and obtain several important properties.

Keywords: Neutrosopheic set; η_μ -strong; σ_μ-strong; μ-bridge; ξ-bridge; μ_*-tree; ω_*-tree; ξ-tree

1. Introduction

The concept of fuzzy set was first introduced in 1965 by A. Zadeh[12], and was used afterwards by many other authors in various branches of Mathematics. In 1983 K. Atanassov [9,10] introduced a generalization of a fuzzy set is called an intuitionistic fuzzy set on a universe X, where besides on membership and nonmembership degree of each element resp. In addition, in 2002 Smarandache [7, 8] introduced the concept of neutrosophic sets as a generalization of intuitionistic fuzzy sets, where we have the degree of (membership, indeterminacy and nonmembership, resp.) of each element in X.

Fuzzy topology is a relatively new area of Mathematics, originating with Chang's article [4] of 1968. The field has developed in anything but an orderly linear pattern, with various fundamental definitions including those of fuzzy sets and of fuzzy topological space, being revised, sometimes several times. After the introduction of the neutrosophic sets, some researchers such as Smarandache [8], Lupianz [5,6], Saliama [2] and Wadei Al-Omeri [14,16] studied topology on neutrosophic sets.

In 1975 Rosenfeld [1] , R.T.Yeh and S.Y. Bang [13] and Wadei Al-Omeri [15,17] introduced the concepts of fuzzy graphs which are a powerful tools and techniques to deal with many combinational problems in sciences and technology such as algebra, topology, optimization, computer science, etc. In 1987, Bhattacharya [12] introduced some remarks and operation on fuzzy graphs.

In 2016 Broumi et al. [3] applied the defined of neutrosopheic set in graphical representation and they finally came up with neutrosophic graphs. Various developments and explorations have been done with the neutrosophic graph and its components. In this paper, we study some characteristics of neutrosopheic graphs (NTCG) and some basic definitions. Also we investigate several kinds of arcs, η_μ -strong, σ_μ-strong, ρ_μ -arc, and η_ξ -strong, σ_ξ-strong, ρ_ξ –arc in neutrosophic graphs (NTCG) , Also we give neutrosophic μ-bridge and neutrosophic ξ-bridge (NTC ξ-bridge) and some interesting properties of neutrosophic bridge (NTCB).

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2. Preliminaries

Definition 2.1. Let $\mathscr{H}$ be anon empty fixed set, a neutrosophic set (briefly NTCs) is an object having the form A $=\{(\mathfrak{H}, \mu_{*A}(\mathfrak{H}), \omega_{*A}(\mathfrak{H}), \xi_A(\mathfrak{H})) : \mathfrak{H} \in \mathscr{H}\}$ where $\mu_{*A}(\mathfrak{H}): \mathscr{H} \to [0,1]$, $\omega_{*A}(\mathfrak{H}) : \mathscr{H} \to [0,1]$ and

$\xi_A(\mathfrak{H}) : \mathscr{H} \to [0,1]$ where $(\mu_{*A}(\mathfrak{H}))$, $(\omega_{*A}(\mathfrak{H}))$ and $(\xi_A(\mathfrak{H}))$ which represent the degree of (membership, indeterminacy and nonmembership, resp.) of each element $\mathfrak{H} \in \mathscr{H}$ to the set A and satisfy the condition $0 \le \mu_{*A}(\mathfrak{H}) + \omega_{*A}(\mathfrak{H}) + \xi_A(\mathfrak{H}) \le 3$.

Definition 2.2. Let $\mathscr{H}$ be anon empty fixed set and let $\mathcal{Q}$ and $\mathcal{J}$ be an NTCss then

1- $\mathcal{Q} \subseteq$ iff $\mu_{*\mathcal{Q}}(\mathfrak{H}) \le \mu_{*\mathcal{J}}(\mathfrak{H})$, $\omega_{*\mathcal{Q}}(\mathfrak{H}) \le \omega_{*\mathcal{J}}(\mathfrak{H})$, $\xi_{\mathcal{Q}}(\mathfrak{H}) \ge \xi_{\mathcal{J}}(\mathfrak{H})$ of each element $\mathfrak{H} \in \mathscr{H}$

2- $\mathcal{Q}^c = \{(x, \xi_{\mathcal{Q}}(\mathfrak{H}), \omega_{*\mathcal{Q}}(\mathfrak{H}), \mu_{*\mathcal{Q}}(\mathfrak{H})) : \mathfrak{H} \in \mathscr{H}\}$ [complement of $\mathcal{Q}$]

3- $\mathcal{Q} \cap \mathcal{J} = \{ \mathfrak{H}, \min\{\mu_{*\mathcal{Q}}(\mathfrak{H}), \mu_{*\mathcal{J}}(\mathfrak{H})\}, \min\{\omega_{*\mathcal{Q}}(\mathfrak{H}), \epsilon_{\mathcal{J}}(\mathfrak{H})\}, \max\{\xi_{\mathcal{Q}}(\mathfrak{H}), \xi_{\mathcal{J}}(\mathfrak{H})\}$ for all : $\mathfrak{H} \in \mathscr{H}\}$

4- $\mathcal{Q} \cup \mathcal{J} = \{ \mathfrak{H}, \max\{\mu_{*\mathcal{Q}}(\mathfrak{H}), \mu_{*\mathcal{J}}(\mathfrak{H})\}, \max\{\omega_{*\mathcal{Q}}(\mathfrak{H}), \epsilon_{\mathcal{J}}(\mathfrak{H})\}, \min\{\xi_{\mathcal{Q}}(\mathfrak{H}), \xi_{\mathcal{J}}(\mathfrak{H})\}$ for all : $\mathfrak{H} \in \mathscr{H}\}$.

Definition 2.3. Let $\mathscr{H}$ be anon empty set, if (r, t, s) be subsets of $]0^-, 1^+[$, then the NTCss $x_{r,t,s}$ is called neutrosophic point in $\mathscr{H}$ given by

$x_{r,t,s}(x_p) = \begin{cases} (r,t,s) & if\ x = x_p \\ (0,0,1) & if\ x \neq x_p \end{cases}$ for $x_p \in \mathscr{H}$ is called the suport of $x_{r,t,s}$, where

(r , t, s) denotes the degree of (membership,indeterminacy,nonmembership, resp.) of $x_{r,t,s}$, and in this research we denote a neutrosophic singleton by $p=(\mu_p, \omega_{*p}, \xi_p)$.

Definition 2.4. Let $\mathscr{H} \neq \emptyset$, a neutrosophic singleton p defined on $\mathscr{H}$ is said to be belong to a neutrosophic set $A = \{(\mathfrak{H}, \mu_{*A}(\mathfrak{H}), \omega_{*A}(\mathfrak{H}), \xi_A(\mathfrak{H})) : \mathfrak{H} \in \mathscr{H}\}$ $(p \in A)$ if $\mu_p \le \mu_{*A}$, $\omega_{*p} \le \omega_{*A}$, and $\xi_A \le \xi_p$.

Definition 2.5. A neutrosophic topology (briefly NTCT) on $\mathscr{H} \neq \emptyset$ is a family $\mathcal{T}$ of NTCSS in $\mathscr{H}$ satisfied the following conditions

1- $0_N = \{(\mathfrak{H}, 0,0,1) : \mathfrak{H} \in \mathscr{H}\}$, $1_N = \{(\mathfrak{H}, 1,1,0) : \mathfrak{H} \in \mathscr{H}\}$ belong to $\mathcal{T}$

2- $\mathcal{Q} \cap \mathcal{J} \in \mathcal{T}$, for any $\mathcal{Q}, \mathcal{J} \in \mathcal{T}$

3- $\cup \mathcal{Q}_i \in \mathcal{T}$, for any family $\{\mathcal{Q}_i : i \in \Lambda\} \subseteq \mathcal{T}$. So the pair $(\mathscr{H}, \mathcal{T})$ is called neutrosophic topological space (NTCTS) and each neutrosophic set in $\mathcal{T}$ is called neutrosophic open set (NTCOs), The complement $\mathcal{Q}^c$ of an NTCOs is said to be neutrosophic closed set (NTCCs).

Example 2.6. Let $X = \{ q,, \mathfrak{z} \}$ and

$\mathfrak{A}$={(q,0.3,0.4,0.5),($\mathfrak{d}$,0.2,0.7,0.1),($\mathfrak{z}$,0.8,0.1,0.1)}

$\mathfrak{B}$={(q,0.5,0.3,0.7),($\mathfrak{d}$,0.2,0.3,0.4),($\mathfrak{z}$,0.9,0.1,0.5)}

$\mathfrak{C}$={(q,0.5,0.3,0.5),($\mathfrak{d}$,0.2,0.3,0.1),($\mathfrak{z}$,0.9,0.4,0.1)}

$\mathfrak{D}$={(q,0.3,0.3,0.7),($\mathfrak{d}$,0.2,0.3,0.4),($\mathfrak{z}$,0.8,0.1,0.5)}

$\mathfrak{F}$={(q,0.3,0.3,0.5),($\mathfrak{d}$,0.2,0.3,0.1),($\mathfrak{z}$,0.8,0.1,0.5)}

$\mathfrak{G}$={(q,0.5,0.4,0.5),($\mathfrak{d}$,0.2,0.7,0.1),($\mathfrak{z}$,0.9,0.1,0.1)}, the family

$\mathcal{T} = \{0_N, 1_N, \mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{E}, \mathfrak{F}\}$ of neutrosophic sets in X is neutrosophic topology on X.

Definition 2.7. A neutrosophic topological space (NTCTS) is said to be neutrosophic connected if there exists, A and B (NTCOs) such that $A \cup B \neq 1_N$ and $A \cap B = 0_N$.

Definition 2.8. A neutrosophic graph (NTCG) $G = (\mathcal{V}^*, E)$, where

1- $\mathcal{V}^* = \{v_0, v_1, v_2, \ldots, v_n\}$, such that $\mu_{*1}: \mathcal{V}^* \to [0,1]$, $\omega_{*1}: \mathcal{V}^* \to [0,1]$, $\xi_1: \mathcal{V}^* \to [0,1]$ denoted by the degree of (membership, indeteirminacy and nonmembership, resp.) of the element $v_i \in \mathcal{V}^*$, and

$0 \le \mu_{*1}(v_i) + \omega_{*1}(v_i) + \xi_1(v_i) \le 3$, $\forall v_i \in \mathcal{V}^*$ (i=1,2,…,n)

2- $E \subseteq \mathcal{V}^* \times \mathcal{V}^*$ where $\mu_{*2}: \mathcal{V}^* \times \mathcal{V}^* \to [0,1]$, $\omega_{*2} : \mathcal{V}^* \times \mathcal{V}^* \to [0,1]$, $\xi_2: \mathcal{V}^* \times \mathcal{V}^* \to [0,1]$ such that

(a) $\mu_{*2}(v_i, v_j) \le \min\{\mu_{*1}(v_i), \mu_{*1}(v_j)\}$,

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(b) $\omega_{*2}(v_i, v_j) \leq \min\{\omega_{*1}(v_i), \omega_{*1}(v_j)\}$,

(c) $\xi_2(v_i, v_j) \leq \max\{\xi_1(v_i), \xi_1(v_j)\}$

(d) $0 \leq \mu_{*2}(v_i, v_j) + \omega_{*2}(v_i, v_j) + \xi_2(v_i, v_j) \leq 3$, $\forall (v_i, v_j) \in E$.

Here $(v_i, \mu_{*1i}, \omega_{*1i}, \xi_{1i})$ denotes the degree of (membership, indeterminacy and nonmembership, resp.) of the vertex v_i, and $(e_{ij}, \mu_{*2ij}, \epsilon_{2ij}, \xi_{2ij})$ denotes the degree of (membership, indeterminacy and nonmembership, resp.) of the edge, $e_{ij}=(v_i, v_j)$ on $\mathcal{V}$, If the arcs of G with membership, then it's said to be μ_*- arcs, if the arcs of G indeterminacy, then it's said to be ω_*-arcs and, If the arcs of G with nonmembership, then it's said to be ξ-arcs.

Definition 2.9. Let $G = (\mathcal{V}^*, E)$ be an NTCG, an neutrosophic Subgraph (NTCSG) $(\grave{\mathcal{V}}^*, \grave{E})$ of G if $\grave{\mathcal{V}}^* \subseteq \mathcal{V}^*$, and $\grave{E} \subseteq E$, such that for $a \in \grave{\mathcal{V}}^*$, if $\grave{\mu}_{*1}(a) > 0$ or $\grave{\omega}_{*1}(a) > 0$ or $\grave{\xi}_1(a) > 0$, then $\grave{\mu}_{*1}(a) = \mu_{*1}(a)$, $\grave{\omega}_{*1}(a) = \omega_{*1}(a)$ and $\grave{\xi}_1(a) = \xi_1(a)$ and for $(a,b) \in \grave{E}$, if $\grave{\mu}_{*2}(a,b) > 0$ or $\grave{\omega}_{*2}(a,b) > 0$ or $\grave{\xi}_2(a,b) > 0$, then $\grave{\mu}_{*2}(a,b) = \mu_{*2}(a,b)$, $\grave{\omega}_{*2}(a,b) = \omega_{*2}(a,b)$ and $\grave{\xi}_2(a,b) = \xi_2(a,b)$. In addition, NTCSG is called a neutrosophic spanning Subgraph (NTCSSG) of G if $\grave{\mathcal{V}}^*=\mathcal{V}^*$.

Definition 2.10. A neutrosophic graph (NTCG) $G = (\mathcal{V}^*, E)$ is strong if for all $(v_i, v_j) \in E$, $\mu_{2ij} = \min\{\mu_{*1i}, \mu_{*1j}\}$, $\epsilon_{2ij} = \min\{\omega_{*1i}, \omega_{*1j}\}$ and $\xi_{2ij} = \max\{\xi_{1i}, \xi_{1j}\}$ and is complete if for all $v_i, v_j \in \mathcal{V}^*$,

$\mu_{2ij} = \min\{\mu_{*1i}, \mu_{*1j}\}$, $\epsilon_{2ij} = \min\{\omega_{*1i}, \omega_{*1j}\}$ and $\xi_{2ij} = \max\{\xi_{1i}, \xi_{1j}\}$.

Remark 2.11. If $\mu_{*2ij} = \omega_{*2ij} = \xi_{2ij} = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edg between v_i and v_j.

Definition 2.12. A neutrosophic graph (NTCG) , $G = (\mathcal{V}^*,E)$ is a μ_*-tree ($\mu_* tr$),

(ω_*-tree ($\omega_* tr$) , ξ-tree (ξtr), resp.) if G^*_μ (resp. $G^*_{\omega_*}$, G^*_ξ) are tree and G is a tree if is a $\mu_* tr$, (ω_*-tree ($\omega_* tr$) , ξ-tree (ξtr), resp.) and $G^*_{\mu_*} = G^*_{\omega_*} = G^*_\xi$. Also G is a μ_*-cycle($\mu_* cy$) ,(ω_*- cycle ($\omega_* cy$) , ξ- cycle(ξcy), resp.) if $G^*_{\mu_*}$ ($G^*_{\omega_*}$, G^*_ξ, resp.) are $\mu_* cy$, (($\omega_* cy$) , (ξcy), resp.) and G is a cycle if is $\mu_* cy$, (($\omega_* cy$) , (ξcy), resp.) and $G^*_{\mu_*} = G^*_{\omega_*} = G^*_\xi$.

Remark 2.13. If $G = (\mathcal{V}^*,E)$ is an NTCG, and G is a $\mu_* cy$, then the weakest μ_*- arc of G is the arc with minimum degree of membership. If G is a $\mu_* cy$, then the weakest ω_*- arc of G is the arc with minimum of membership. If G is a ξcy, then the weakest ξ- arc of G is the arc with maximum of membership.

Example 2.14 In Figure 1, let $G = (\mathcal{V}^*,E)$ be NTCG, such that $\mathcal{V}^* =\{a,b,c,d\}$, E=$\{(a,b),(b,c),(c,d),(a,d)\}$, then G is a μ_*-tree , ω_*-tree and ξ-tree but not a tree.

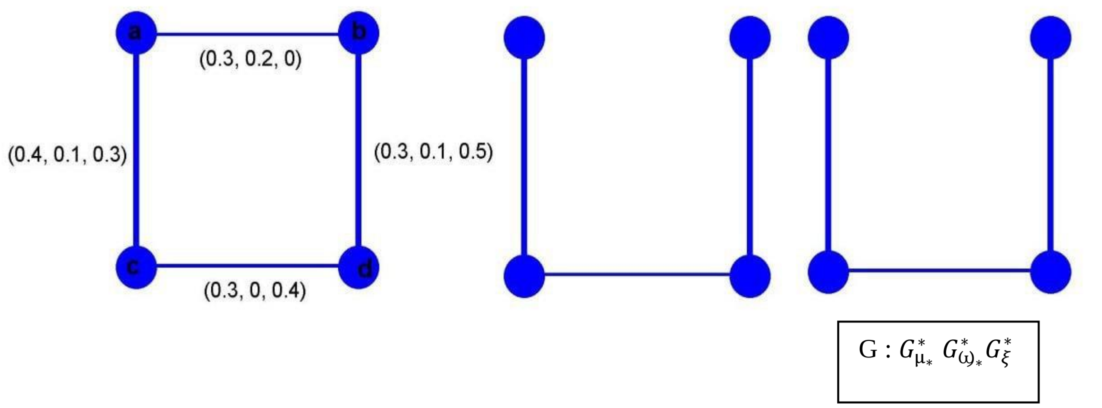

Figure 1. μ_*-tree, ω_*-tree and ξ-tree

Definition 2.15. Let $p : x = v_0 , y = v_1, \ldots, v_n$ be a sequience of distinst vertices in an (NTCG), then P is a μ_*-Path from q to r, if $\mu_{*2}(v_{i-1}, v_i) > 0$ and is ω_*-path from q to r, if $\omega_{*2}(v_{i-1}, v_i) > 0$ and is ξ-path

from q to r, if $\xi_2(v_{i-1}, v_i) > 0$, for i=1,…,n, p is Path if its μ_*-path, ω_*-path and ξ-path, a p its said a (q-r) path and length of p for n > 0 is n, If q=r and n>3, p its said μcy, μ_*- cy, ξ- cy, and cycle, resp.

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Definition 2.16. An (NTCG) $G = (\mathcal{V}^*, E)$ is μ_*- connected (μ_*-cond) { ω_*- connected (ω_*- cond), ξ- connected (ξ- cond), resp.} if exists a μ_*-path (μ_*-p) {ω_*-path (ω_*-p), ξ-path (ξ-p), resp.} among all pairs of verticeis in G. Also G is strong connected if found a path among all pairs of verteices in G.

Remark 2.17. In Figure 1, G is μ_*-cond {(ω_*- cond), (ξ- cond), resp.) but is not connected.

Definition 2.18. If $v_i, v_j \in \mathcal{V}^* \subseteq G$, the μ_*-strength (μ_*-Ş) of connected between v_i and v_j is $\mu_{*2}^{\infty}(v_i, v_j) = \sup\{\mu_{*2}^{w}(v_i, v_j) : w = 1,2,\ldots,n\}$, ω_*-strength (ω_*-Ş) of connected between v_i and v_j is $\omega_{*2}^{\infty}(v_i, v_j) = \sup\{\omega_{*2}^{w}(v_i, v_j) : w = 1,2,\ldots,n\}$, and ξ-strength (ξ-Ş) of connected between v_i and v_j is $\xi_2^{\infty}(v_i, v_j) = \inf\{\xi_2^{w}(v_i, v_j) : w = 1,2,\ldots,n\}$, if u, v are connected by (μ_*-p) of length L.

is defined as $\mu_{*2}^{L}(u, v) = sup\{\mu_{*2}(u, v_1) \cap \mu_{*2}(v_1, v_2) \cap \ldots \cap \mu_{*2}(v_{L-1}, v) : u, v, v_1, \ldots, v_{L-1} \in \mathcal{V}^*\}$, and if u, v are connected by ω_*-p of length L, is $\omega_{*2}^{L}(u, v) = \sup\{\omega_{*2}(u, v_1) \cap \omega_{*2}(v_1, v_2) \cap \ldots \cap \omega_{*2}(v_{L-1}, v) : u, v, v_1, \ldots, v_{L-1} \in V\}$, and if u, v are connected by ξ-p of leangth L, is defined as $\xi_2^{L}(u, v) = sup\{\xi_2(u, v_1) \cup \xi_2(v_1, v_2) \cup \ldots \cup \xi_2(v_{L-1}, v) : u, v, v_1, \ldots, v_{L-1} \in \mathcal{V}^*\}$, the μ_*-Ş and ω_*-Ş and ξ-Ş of connectedness between v_i and v_j in $G = (\mathcal{V}^*, E)$ is denoted by $\mu_{*G}^{\infty}(v_i, v_j)$ and $\omega_{*G}^{\infty}(v_i, v_j)$ and $\xi_G^{\infty}(v_i, v_j)$ resp. G- (v_i, v_j) obtain from G by delete the arc (v_i, v_j) , $\acute{\mu}_{*2}^{\infty}(v_i, v_j)$, $\acute{\omega}_{*2}^{\infty}(v_i, v_j)$, $\acute{\xi}_2^{\infty}(v_i, v_j)$ and denoted by $\mu_{*G-(v_i,v_j)}^{\infty}(v_i, v_j)$ and $\omega_{*G-(v_i,v_j)}^{\infty}(v_i, v_j)$ and $\xi_{G-(v_i,v_j)}^{\infty}(v_i, v_j)$ resp.

3. Some kinds of arcs and paths in neutrosophic space

Definition 3.1. An arc *(q,r)* in an NTCG $G = (\mathcal{V}^*, E)$ is μ_*-Ş, ω_* -Ş and ξ -Ş, if $\mu_{*2}(q, r) \geq \mu_{*2}'^{\infty}(q, r)$, $\omega_{*2}'^{\infty}(q, r) \geq \omega_{*2}'^{\infty}(q, r)$, and $\xi_2'^{\infty}(q, r) \leq \xi_2'^{\infty}(q, r)$ respectively. Also, *(q, r)* is strong, if it μ_*-Ş, ω_*-Ş and ξ -Ş.

Definition 3.2. An arc *(q, r)* in $G = (\mathcal{V}^*, E)$ is said to η_{μ_*} -Ş, σ_{μ_*}-Ş, ρ_{μ_*} -arc (ρ_{μ_*} -ar), , if $\mu_{*2}(q, r) > \mu_{*2}'^{\infty}(q, r)$, $\mu_{*2}(q, r) = \mu_{*2}'^{\infty}(q, r)$, $\mu_{*2}(q, r) < \mu_{*2}'^{\infty}(q, r)$, and is said to be η_{ω_*}-Ş, σ_{ω_*}-Ş, ρ_{ω_*} –arc(ρ_{ω_*} –ar) if $\omega_{*2}(q, r) > \omega_{*2}'^{\infty}(q, r)$, $\omega_{*2}(q, r) = \omega_{*2}'^{\infty}(q, r)$, $\omega_{*2}(q, r) < \omega_{*2}'^{\infty}(q, r)$, and is said to η_ξ -Ş, σ_ξ-Ş, ρ_ξ –arc (ρ_ξ –ar), if $\xi_2(q, r) < \xi_2'^{\infty}(q, r)$, $\xi_2(q, r) = \xi_2'^{\infty}(q, r)$ and $\xi_2(q, r) > \xi_2'^{\infty}(q, r)$.

Example 3.3. In Figure 2, let $G = (\mathcal{V}^*, E)$ be an NTCG, such that $\mathcal{V}^* = \{Å, \mathscr{B}, \mathscr{L}, w, z\}$, $E = \{(Å, \mathscr{B}), (Å, \mathscr{L}), (\mathscr{B}, \mathscr{L}), (\mathscr{B}, w), (\mathscr{L}, z), (w, z)\}$. Then the ar *(Å,$\mathscr{B}$)* is ρ_{μ_*}-arc and η_ξ-stroneg, the ar(Å, $\mathcal{L}$) is η_{μ_*}-Ş and ρ_ξ-arc, the ar$(\mathcal{B}, \mathcal{L})$ is η_{μ_*}-Ş and η_ξ-Ş and the ar $(\mathcal{B}, w)$, $(\mathcal{L}, z)$ and *(w, z)* are σ_{μ_*}-Ş and ρ_ξ-Ş.

Definition 3.4. Let $P: \mathscr{x} = v_0, \mathscr{y} = v_1, \ldots, v_n$ be a μ-path from $\mathscr{x}$ to $\mathscr{y}$ in $G = (\mathcal{V}^*, E)$. P is μ_*-Ş (η_{μ_*}-Ş), if for $i = 1, 2, \ldots, n$, the arcs (v_{i-1}, v_i) are μ-Ş (η_{μ_*}-Ş), let P be a ω_* –path, then P is ω_* -Ş (η_{ω_*}-Ş), if for $i = 1, 2, \ldots, n$, the arcs (v_{i-1}, v_i) are ω_* -Ş (η_{ω_*}-Ş), and Let P be a ξ-path, then P is ξ -Ş (η_ξ-Ş), if for $i = 1, 2, \ldots$, n, the arcs (v_{i-1}, v_i) are ξ-Ş (η_ξ-Ş). Let P be a path, then P is Strong (η -Ş), if P is μ_*-Ş, ω_*-Ş, or ξ -Ş (η_{μ_*}-Ş , η_{ω_*}-Ş or η_ξ-Ş).

Remark 3.5. In Figure 2, the path P: Å, $\mathcal{B}, \mathcal{L}$ is an η_{μ_*}-Şp and the path P': Å, $\mathcal{B}, \mathcal{L}$ is a η_ξ-Şp. Hence P and P' are η-Şp.

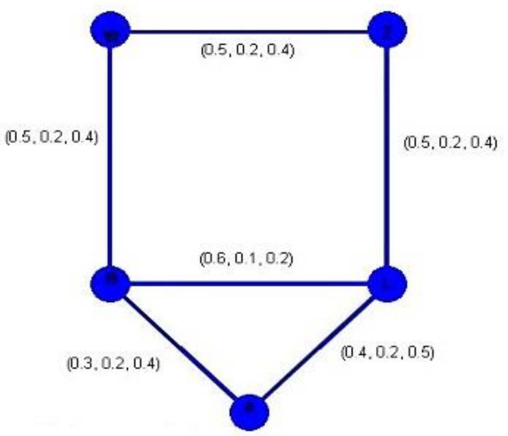

Figure 2. Neutrosophic graph G

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Proposition 3.6. If an NTCG $G = (\mathcal{V}^*, E)$ a ξ-cond, then exist a ξ-Şp between every vertices of G.

Proof: Let $G = (\mathcal{V}^*, E)$ be an (NTCG), and be a ξ-cond , hence there exist a ξ-p between every vertices. If (a, b) is not ξ-Ş arc, then we have ξ_2(a, b) > $\xi_2'^{\infty}$ (a, b). Hence there exist a ξ-p P from a to b, which ξ-Ş of it is less than ξ_2(a, b). So if some arcs of P are not ξ-Ş, this argument. Finally we will have a ξ-p from a to b, which is ξ-Ş.

Proposition 3.7. Let $G = (\mathcal{V}^*, E)$ be an NTCG, If a path P from x to y is a η_ξ-Ş, then P is an ξ-Ş $(x - y)$ path.

Proof: Let $P: x = v_0, y = v_1, \dots, v_n$ be a η_ξ-Ş path and let P is not a ξ-Ş $(\mathfrak{x} - \mathfrak{y})$ pa in G. Let $P': x = v_0', y = v_1', \dots, v_n'$ be a ξ-Ş $(x - y)$ path in G. Hence $\xi_2(v_{i-1}', v_i') < \xi_P^{\infty}(x, y)$ for $i = 1,2, \dots,n$. Also P and P' form a ξ_2-cy, is C. The weake ξ -ar of C is in P. Let (u, v) be a weakest ξ-arc in P. We consider P'' be the $(u - v)$ path in C not include (u, v). yield that $\xi_2(u, v) \geq \xi_P^{\infty}$ (u, v) $\geq \xi_P^{\infty}$ (u, v) which implies that (u, v) is not a η_ξ-Ş ar, this contradicts. Therefore, P is ξ-Ş $(x - y)$ path in G.

Definition 3.8. Let $G = (\mathcal{V}^*, E)$ be an (NTCG), An ar (x, y) is said to be an Neutrosophic μ_*-bridge (NTCμ_*-pg)), if delete (x, y) reuduces the μ_*- Ş of connected in some pair of vertices. Equivalent, there exist $u, v \in \mathcal{V}$ such that $(\mathfrak{x}, \mathfrak{y})$ is an arc of every μ_*-Ş $(u - v)$ path. An arc (x, y) is a neutrosophic ξ-bridg(NTC ξ-bg), if delete (x, y) increses the ξ-Ş of connectdness between some pair of vertices, equivalent there exist u, v $\in E\, \mathcal{V}$ such that (x, y) is an arc of every ξ-Ş $(u - v)$ path. An arc (x, y) is a neutrosophic bridge (NTC*B*), if it is an NTC μ_*-bg or NTC ξ-bg.

Definition 3.9. A vertex $x \in \mathcal{V}^*$ is an NTC$G = (\mathcal{V}^*, E)$ is a neutrosophic μ_*-cut vertix (NTC μ_*-cv), if delete it reduce the μ-Ş of connected of some two of vertices. Equivalent, there exist $u, v \in \mathcal{V}^*$ such that x is a vertex of every μ-Ş $(u - v)$ path. A vertex $x \in \mathcal{V}$ is a neutrosophic ξ -cut verteix (NT ξ-cv), if deleting it increase the ξ -Ş of connectied between some pair of vertices. Equivalent, there exist $u, v \in \mathcal{V}^*$ such that x is a vertex of every ξ-Ş t $(u - v)$ path. A verteix $x \in \mathcal{V}$ is a neutrosophic cut vertex (NTC $C\,\mathcal{V}^*$), if it is an NTC μ_*-cv or NTC ξ -cv.

Example 3.10. In Figure 3, NTCGG= $(\mathcal{V}^*, E)$, such that $\mathcal{V}^* = \{a, b, c, w, z\}$, $E = \{(a, b), (a, c), (b, c), (b, w), (v, z), (w, z)\}$. Then the ars (a, b) and (a, c) are σ_{μ_*}-Ş and η_ξ-Ş, the ars (b, c) and (b, w) are σ_{μ_*}-Ş and ρ_ξ-ar, the ars (c, z) and (w, z) are η_{μ_*}-Ş and η_ξ-Ş. all ars are stronge. And the ars (c, z) and (w, z) are NTC μ_* -bg, and the ars (a, b), (a, c), (c, z) and (w, z) are NTC ξ-bg. So arcs except (b, c) are NTC*B*. Also, z is an NTC μ_*-cv and a, z are NTC ξ-cv.

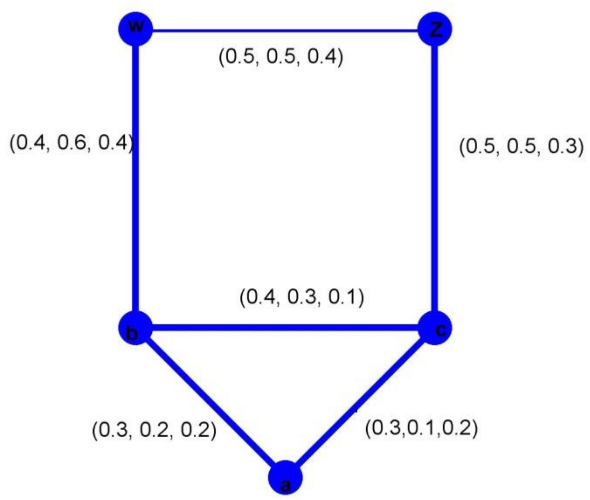

Figure 3. Neutrosophic graph G

Theorem 3.11. Let $(\mathfrak{h}, \mathfrak{W})$ be an arc in NTC$G = (\mathcal{V}^*, E)$, then

(i) $(\mathfrak{h}, \mathfrak{W})$ is an NTC μ_* -bridge *iff* $\mu_{*2}(, \mathfrak{W}) > \mu_{*2}'^{\infty}(\mathfrak{h}, \mathfrak{W})$.

(ii) $(\mathfrak{h}, \mathfrak{W})$ is an NTC ξ - bridge *iff* $\xi_2(\mathfrak{h}, \mathfrak{W}) < \xi_2'^{\infty}(\mathfrak{h}, \mathfrak{W})$.

(iii) $(\mathfrak{h}, \mathfrak{W})$ is an NTC bridge *iff* $\mu_{*2}(, \mathfrak{W}) > \mu_{*2}'^{\infty}(\mathfrak{h}, \mathfrak{W})$ or $\xi_2(\mathfrak{h}, \mathfrak{W}) < \xi_2'^{\infty}(\mathfrak{h}, \mathfrak{W})$.

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Proof: (i) It is obvious.

(*ii*) Assume that (𝔥, 𝔚) is an NTC ξ - bridge, hence there exist $u, v \in \mathcal{V}^*$ such that (𝔥, 𝔚) is an ars of ξ -Ş (u - v) path, which is said to be P. Now let P' be a ξ –path from u to v not include (𝔥, 𝔚) and the ξ-Ş of it be minimum of all the ξ-path from u to v not include (𝔥, 𝔚). Then P and P' form a cycle said C and $C -$ (𝔥, 𝔚) is a 𝔚-path said P". We have P" is the ξ -Ş path of 𝔥 and 𝔚. Let p' be a ξ-Ş paths 𝔥 and 𝔚, then delete (𝔥, 𝔚) not increase the ξ -Ş of u and v. This contradicts. a ξ_P^{∞} (𝔥, 𝔚) = $\xi_2'^{\infty}$ (𝔥, 𝔚), moreover the weak ξ -arc of C is on P', therefore ξ_2(𝔥, 𝔚) < $\xi_P'^{\infty}$ (𝔥, 𝔚) implies that ξ_2(𝔥, 𝔚) < $\xi_2'^{\infty}$(𝔥, 𝔚). Conversely, if ξ_2 (𝔥, 𝔚) < $\xi_2'^{\infty}$ (𝔥, 𝔚), then delete (𝔥, 𝔚) increase ξ-Ş of connected in 𝔥 and 𝔚, hence (𝔥, 𝔚) is an NTC ξ- bridge.

(*iii*) It fellows from (*i*) and (*ii*).

4. Conclusion

Graph theory is a fundamental mathematical tool with extensive applications across various scientific and technological fields. Neutrosophic graphs have proven their significance in addressing complex and uncertain problems, particularly in operations research, neural networks, artificial intelligence, and decision-making systems. In this study, the characteristics of Neutrosophic graphs, including different types of arcs and models such as NTCG, NTCpg, NTCB, and NTC NTC$\mathcal{CV}^*$, have been analyzed. These concepts have demonstrated their adaptability to the demands of modern applications. These concepts can be effectively applied in numerous practical and engineering domains, including database systems, expert systems, signal processing, pattern recognition, computer networks, robotics, and medical diagnosis. These findings open new avenues for developing innovative and efficient solutions to tackle complex technical challenges.

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