On Refined Neutrosophic R-module

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Abstract

Modules are one of the fundamental and rich algebraic structure concerning some binary operations in the study of algebra. In this paper, some basic structures of refined neutrosophic R-modules and refined neutrosophic submodules in algebra are generalized. Some properties of refined neutrosophic R-modules and refined neutrosophic submodules are presented. More precisely, classical modules and refined neutrosophic rings are utilized. Consequently, refined neutrosophic R- modules that are completely different from the classical modular in the structural properties are introduced. Also, neutrosophic R-module homomorphism is explained and some definitions and theorems are presented.

Keywords: Refined neutrosophic group, refined neutrosophic ring, refined neutrosophic R-module, weak refined neutrosophic R-module, strong refined neutrosophic R-module, refined neutrosophic R-module homomorphism.

1. Introduction

Neutrosophy is a new branch of philosophy that studies the nature, origin, and scope of neutralities as well as their interaction with ideational spectra. Neutrosophy is the base of neutrosophic logic, which is an extension of fuzzy logic in which indeterminacy is included. Florentin Smarandanche introduced the notion of neutrosophy as a new branch of philosophy in 1995. After that, he introduced the concept of neutrosophic logic and neutrosophic set where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic as well as an extension of intuitionistic fuzzy logic.

Neutrosophic set is the generalization of the classical set, neutrosophic group, and neutrosophic ring the generalization of classical group and ring, etc. The same way neutrosophic R-module is the generalization of the classical R-module. By utilizing the idea of neutrosophic theory, Vasantha Kanadasamy and Florentin Smarandache [11] studied neutrosophic algebraic structures by inserting an indeterminate element I in the algebraic structure and then combining 'I' with the corresponding binary operation for corresponding binary operation.

Agboola in [1], introduced the concept of refined neutrosophic algebraic structures and studied refined neutrosophic groups in particular. Since the introduction of refined neutrosophic algebraic structures, many neutrosophic researchers have established and studied more refined neutrosophic algebraic structures. Adeleke et al. [5] studied refined neutrosophic rings and refined neutrosophic subrings and presented their fundamental properties. Also in [6], Adeleke et al. studied refined neutrosophic ideals and refined neutrosophic homomorphisms and presented their basic

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properties. The present paper is devoted to the study of refined neutrosophic R-module. More properties of refined neutrosophic R-module will be presented. For more details about neutrosophy, refined neutrosophic logic, neutrosophic groups, and refined neutrosophic groups, the readers should see [2-4, 7-10, 12-25].

2. Preliminaries

In this section, we present the basic definitions that are useful in this research.

Definition 2.1. [1] Let $\left(X\left(I_1,I_2\right),+,\cdot\right)$ be any refined neutrosophic algebraic structure where + and $\cdot$ are ordinary addition and multiplication respectively. I_1 *and* I_2 are the split components of the indeterminacy factor I that is $I=\alpha I_1+\beta I_2$ with $\alpha,\beta\in R$ *or* C . Also, I_1 *and* I_2 **they** are taken to have the properties $I_1^2=I_1, I_2^2=I_2$ and $I_1 I_2=I_2 I_1=I_1$.

For any two elements, we define

1) $x+y=\left(a,bI_1,cI_2\right)+\left(d,eI_1,fI_2\right)=\left(a+d,\left(b+e\right)I_1,\left(c+f\right)I_2\right)$

2) $x\cdot y=\left(a,bI_1,cI_2\right)\cdot\left(d,eI_1,fI_2\right)=\begin{pmatrix} ad,\left(ae+bd+be+bf+ce\right)I_1, \\ \left(af+cd+cf\right)I_2 \end{pmatrix}$.

Definition 2.2. [1] Let $\left(G,*\right)$ be any group. The couple $\left(G\left(I_1,I_2\right),*\right)$ is called a refined neutrosophic group generated by G, I_1 *and* I_2. $\left(G\left(I_1,I_2\right),*\right)$ is said to be commutative if, for all $x,y\in\left(G\left(I_1,I_2\right),*\right)$ we have $x*y=y*x$ Otherwise, we $\left(G\left(I_1,I_2\right),*\right)$ are called a non-commutative refined neutrosophic group.

Example2.3. [1] Let $\mathbb{Z}_2\left(I_1,I_2\right)=\begin{Bmatrix} \left(0,0,0\right),\left(1,0,0\right),\left(0,I_1,0\right),\left(0,0,I_2\right) \\ \left(0,I_1,I_2\right),\left(1,I_1,0\right),\left(1,0,I_2\right),\left(1,I_1,I_2\right) \end{Bmatrix}$. Then $\left(Z_2\left(I_1,I_2\right),*\right)$ it is a commutative refined neutrosophic group of integers modulo2. Generally, for a positive integer $n\geq 2$, $\left(Z_2\left(I_1,I_2\right),*\right)$ it is a finite commutative refined neutrosophic group of integers modulo n.

Theorem2.4. [1]

(1) Every refined neutrosophic group is a semigroup but not a group.

(2) Every refined neutrosophic group contains a group.

Definition 2.5. [1] Let $\left(G\left(I_1,I_2\right),*\right)$ be a refined neutrosophic group and let $H\left(I_1,I_2\right)$, be a nonempty subset of $G\left(I_1,I_2\right)$. $H\left(I_1,I_2\right)$, is called a refined neutrosophic subgroup of $G\left(I_1,I_2\right)$ if $\left(H\left(I_1,I_2\right),*\right)$, it is a refined neutrosophic group. It must contain a proper subset which is a group. Otherwise, $H\left(I_1,I_2\right)$, it will be called a pseudo refined neutrosophic subgroup $G\left(I_1,I_2\right)$.

Definition 2.6. [1] Let $\left(G\left(I_1,I_2\right),*\right)$ and $\left(H\left(I_1,I_2\right),\circ\right)$, be two refined neutrosophic groups.

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Then the mapping: $\psi:(G(I_1,I_2),*)\to(H(I_1,I_2),\circ)$ is called a neutrosophic homomorphism if the following conditions hold:

$$\forall x,y\in(G(I_1,I_2),*)$$
$$1)\psi(x*y)=\psi(x)\circ\psi(y)$$
$$2)\psi(I_k)=I_k : k=1,2$$

(1) The kernel of ψ denoted by $\ker\psi$ is defined by the set $\{g\in G(I_1,I_2):\psi(g)=(0,0I_1,0I_2)\}$.

(2) The image of ψ denoted by $\operatorname{Im}\psi$ is defined by the set $\{h\in H(I_1,I_2),\exists g\in G(I_1,I_2):\psi(g)=h\}$.

3. On refined Neutrosophic R-module:

Definition 3.1 Let $(M,+,.)$ be any R-module over a commutative ring R . The triple $(M(I_1,I_2),+,\cdot)$ is called a weak refined neutrosophic R-module over a ring R generated by M , I_1 *and* I_2.

If $M(I_1,I_2)$ is a refined neutrosophic R-module over a refined neutrosophic ring $R(I_1,I_2)$, then $M(I_1,I_2)$ is called a refined strong neutrosophic R-module.

Theorem 3.2. Every strong refined neutrosophic R-module is a weak refined neutrosophic R-module.

Proof: Suppose that $M(I_1,I_2)$ is a strong refined neutrosophic R-module over a refined neutrosophic ring $R(I_1,I_2)$. $R\subseteq R(I_1,I_2)$ for every ring R , it follows that $M(I_1,I_2)$ is a weak refined neutrosophic R-module.

Theorem 3.3. Every weak (strong) refined neutrosophic R-module is an R-module.

Proof: If we have $m=(a,bI_1,cI_2),n=(d,eI_1,fI_2)\in M(I_1,I_2)$ where $a,b,c,d,e,f\in M$ and $\alpha=(p,qI_1,rI_2),\beta=(s,tI_1,uI_2)\in R(I_1,I_2):p,q,r,s,t,u\in R$ then:

$$1)\alpha(m+n)=\alpha(a+d,(b+e)I_1,(c+f)I_2)$$
$$=(p,qI_1,rI_2)(a+d,(b+e)I_1,(c+f)I_2)$$

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$$=\begin{pmatrix} p\left(a+b\right), \\ \left(p\left(b+e\right)+q\left(a+d\right)+q\left(b+e\right)+q\left(c+f\right)+r\left(b+e\right)\right)I_1, \\ \left(p\left(c+f\right)+r\left(a+d\right),r\left(c+f\right)\right)I_2 \end{pmatrix}$$

$$=\begin{pmatrix} pa+pb, \\ \left(pb+pe+qa+qd+qb+qe+qc+qf+rb+re\right)I_1 \\ \left(pc+pf+ra+rd+rc+rf\right)I_2 \end{pmatrix}$$

$$=\left(pa,\left(pb+qa+qb+rc+re\right)I_1,\left(pc+ra+rc\right)I_2\right)$$
$$+\left(sd,\left(se+td+te+tf+ue\right)I_1,\left(sf+ud+uf\right)I_2\right)$$
$$=\alpha m+\alpha n$$

2) $$\left(\alpha+\beta\right)m=\left(\left(p,qI_1,rI_2\right)+\left(s,tI_1,uI_2\right)\right)\left(a,bI_1,cI_2\right)$$
$$=\left(\left(p+s\right),\left(q+t\right)I_1,\left(r+u\right)I_2\right)\left(a,bI_1,cI_2\right)$$

$$=\begin{pmatrix} \left(p+s\right)a, \\ \left(\left(p+s\right)b+\left(q+t\right)a+\left(q+t\right)b+\left(q+t\right)c+\left(r+u\right)b\right)I_1, \\ \left(\left(p+s\right)c+\left(r+u\right)a+\left(r+u\right)c\right)I_2 \end{pmatrix}$$

$$=\begin{pmatrix} ap+as, \\ \left(pb+bs+qa+ta+qb+tb+qc+tc+rb+ub\right)I_1, \\ \left(pc+sc+ra+ua+rc+uc\right)I_2 \end{pmatrix}$$

$$=\left(p,qI_1,rI_2\right)\left(a,bI_1,cI_2\right)+\left(s,tI_1,uI_2\right)\left(a,bI_1,cI_2\right)$$
$$=\alpha m+\beta n$$

3) $$\left(\alpha\beta\right)m=\left(\left(p,qI_1,rI_2\right)\left(s,tI_1,uI_2\right)\right)\left(a,bI_1,cI_2\right)$$
$$=\left(\left(ps\right),\left(pt+qs+qt+qu+rt\right)I_1,\left(pu+rs+ru\right)I_2\right)\left(a,bI_1,cI_2\right)$$

$$=\begin{pmatrix} \left(\left(ps\right)a\right), \\ \begin{pmatrix} \left(ps\right)b+\left(pt+qs+qt+qu+rt\right)a+\left(pt+qs+qt+qu+rt\right)b \\ \left(pt+qs+qt+qu+rt\right)c+\left(pu+rs+ru\right)b \end{pmatrix}I_1, \\ \left(\left(ps\right)c+\left(pu+rs+ru\right)a+\left(pu+rs+ru\right)c\right)I_2 \end{pmatrix}$$

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$$= \begin{pmatrix} p(sa), \\ \begin{pmatrix} p(sb+ta+tc+tb+ub)+qsa+q(sb+ta+tc+tb+ub)+ \\ q(sc+ua+uc)+r(sb+ta+tc+tb+ub) \end{pmatrix} I_1, \\ (p(sc+ua+uc)+rsa+r(sc+ua+uc))I_2 \end{pmatrix}$$

4) For $(1,0,0) \in R(I_1,I_2)$ we have:

$$(1,0,0)\cdot m = (1,0I_1,0I_2)(a,bI_1,cI_2)$$
$$= (1a,(1b+0a+0b+0c+0b)I_1,(1c+0a+0c)I_2)$$
$$= (a,bI_1,cI_2) = m$$

Therefore, that $M(I_1,I_2)$ is an R-module.

Lemma 3.4: If we have $M(I_1,I_2)$ as a refined neutrosophic $R-$module over a refined neutrosophic ring $R(I_1,I_2)$ and if we take $m=(a,bI_1,cI_2), n=(d,eI_1,fI_2)$ *and* $s=(x,yI_1,zI_2) \in M(I_1,I_2)$ where $a,b,c,d,e,f,x,y,z \in M$ $\alpha=(p,qI_1,rI_2) \in R(I_1,I_2) : p,q,r \in R$ then:

1) $m+s=n+s \Rightarrow m=n$

2) $\alpha(0,0I_1,0I_2)=(0,0I_1,0I_2)$

3) $(0,0I_1,0I_2)m=(0,0I_1,0I_2)$

4) $(-\alpha)m=\alpha(-m)=-(\alpha m)$

Definition 3.5: Let $M(I_1,I_2)$ be a strong refined neutrosophic R- module over a refined neutrosophic ring $R(I_1,I_2)$ and let $N(I_1,I_2)$ be a nonempty subset of $M(I_1,I_2)$. $N(I_1,I_2)$ is called a strong refined neutrosophic submodule of $M(I_1,I_2)$ if $N(I_1,I_2)$ is itself a strong refined neutrosophic R- module over $R(I_1,I_2)$. It is essential that $N(I_1,I_2)$ contains a proper subset which is an R-module.

Definition 3.6: Let $M(I_1,I_2)$ be a weak refined neutrosophic R-module over a ring R and let N (I) be a nonempty subset of $M(I_1,I_2)$. $N(I_1,I_2)$ is called a weak refined neutrosophic submodule of $M(I_1,I_2)$, if $N(I_1,I_2)$ is itself a weak refined neutrosophic R-module over R. It is essential that $N(I_1,I_2)$ contains a proper subset, which is an R-module.

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Theorem 3.7: If we have $M(I_1,I_2)$ as a refined neutrosophic $R-$module over a ring $R(I_1,I_2)$ and if we take $N(I_1,I_2)$ as a subset of $M(I_1,I_2)$, $N(I_1,I_2)$ is a strong refined neutrosophic submodule of $M(I_1,I_2)$ if and only if the following conditions hold:

1) $n_1,n_2 \in N(I_1,I_2) \Rightarrow n_1+n_2 \in N(I_1,I_2)$

3) $for \quad all\ r=(\alpha,\beta I_1,\gamma I_2)\in R(I_1,I_2):\alpha,\beta,\gamma\in R$
$$n\in N(I_1,I_2)\Rightarrow \alpha n\in N(I_1,I_2)$$

3) $N(I_1,I_2)$ must have a proper subset which is a $R-$module.

Corollary 3.8: If we have $M(I_1,I_2)$ as a refined neutrosophic $R-$module over a refined neutrosophic ring $R(I_1,I_2)$ and if we take $N(I_1,I_2)$ as a subset of $M(I_1,I_2)$, then $N(I_1,I_2)$ is refined neutrosophic submodule of $M(I_1,I_2)$ if and only if the following conditions hold:

1) $for \quad all\ \alpha(p,qI_1,rI_2),\beta(s,tI_1,uI_2)\in R(I_1,I_2):p,q,r,s,t,u\in R$
$$n_1+n_2\in N(I_1,I_2) \quad implies \quad \alpha n_1+\beta n_2\in N(I_1,I_2)$$

2) $N(I_1,I_2)$ must have a proper subset which is a $R-$module.

Example 3.9. Let $M(I_1,I_2)$ be a weak (strong) refined neutrosophic R-module. $M(I_1,I_2)$ is a weak (strong) refined neutrosophic submodule called a trivial weak (strong) neutrosophic submodule.

Example 3.10. Let $M(I_1,I_2)=M_{m\times n}(I_1,I_2)=\left\{\left[a_{ij}\right]:a_{ij}\in R(I_1,I_2)\right\}$ be a strong refined neutrosophic R-module over the strong refined neutrosophic ring $R(I_1,I_2)$ and let

$$N(I_1,I_2)=N_{n\times m}(I_1,I_2)=\left\{\left[b_{ij}\right]:b_{ij}\in R(I_1,I_2),\ trace(N_{n\times m})=(0,0I_1,0I_2)\right\}$$

Then $N(I_1,I_2)$ is a strong refined neutrosophic submodule of $M(I_1,I_2)$.

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Theorem 3.11: Let $M(I_1,I_2)$ be a strong refined neutrosophic R-module over a refined neutrosophic ring $R(I_1,I_2)$ and let $\{N_n(I_1,I_2)\}_{n\in\lambda}$ be a family of strong refined neutrosophic submodules of $M(I_1,I_2)$. Then $\cap N_n(I_1,I_2)$ is a strong refined neutrosophic submodule of $M(I_1,I_2)$.

Proof: Clearly $(0,0I_1,0I_2)_{M(I_1,I_2)} \in \cap N(I_1,I_2)$ and $\cap N_n(I_1,I_2) \neq \emptyset$. Since for $\forall n\in\lambda, (0,0I_1,0I_2)_{M(I_1,I_2)} \in N_n(I_1,I_2)$ Let be $x=(a,bI_1,cI_2), y(d,eI_1,fI_2)\in \cap N_n(I_1,I_2)$ and let be $\alpha=(p,qI_1,rI_2)\in R(I_1,I_2)$. Then $x+y, \alpha x \in \cap N_n(I_1,I_2)$. Since, for $\forall n\in\lambda, x+y\in N_n(I_1,I_2)$ and $\alpha x\in \cap N_n(I_1,I_2)$ Hence $\cap N_n(I_1,I_2)$ is a strong refined neutrosophic submodule of $M(I_1,I_2)$.

Remark: Let $M(I_1,I_2)$ be a strong refined neutrosophic R-module over a refined neutrosophic ring $R(I_1,I_2)$ and let $N_1(I_1,I_2)$ and $N_2(I_1,I_2)$ be two distinct strong refined neutrosophic submodules of $M(I_1,I_2)$. Generally, $N_1(I_1,I_2)\cup N_2(I_1,I_2)$ is not a strong refined neutrosophic submodule $M(I_1,I_2)$.

However, if $N_1(I_1,I_2)\subseteq N_2(I_1,I_2)$ or $N_1(I_1,I_2)\supseteq N_2(I_1,I_2)$ then $N_1(I_1,I_2)\cup N_2(I_1,I_2)$ is a strong neutrosophic submodule of $M(I_1,I_2)$.

Definition 3.12. If we have $M(I_1,I_2)$ and $N(I_1,I_2)$ as two refined neutrosophic $R-$modules over a ring refined neutrosophic ring $R(I_1,I_2)$, a mapping $\varphi: M(I_1,I_2)\to N(I_1,I_2)$ is said to be a refined neutrosophic homomorphism $R-$module, precisely when:

1) $\varphi(rm+r'm')=r\varphi(m)+r'\varphi(m)$ for all $m,m'\in M(I_1,I_2)$ and $r,r'\in R(I_1,I_2)$

2) $\varphi(I_1)=I_1, \varphi(I_2)=I_2$.

Endomorphism, epimorphism, monomorphism, automorphism, and isomorphism of φ have the same definitions as those of the classical cases.

Definition 3.13. Let $M(I_1,I_2)$ and $N(I_1,I_2)$ be strong refined neutrosophic R-modules over a refined neutrosophic ring $R(I_1,I_2)$ and let $\psi: M(I_1,I_2)\to N(I_1,I_2)$ be a refined neutrosophic R-module homomorphism then:

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(1) The kernel of ψ denoted by $\ker\psi$ is defined by the set $\{m \in M(I_1,I_2) : \psi(m) = (0,0I_1,0I_2)\}$.

(2) The image of ψ denoted by $\operatorname{Im}\psi$ is defined by the set $\{n \in N(I_1,I_2) \exists m \in M(I_1,I_2) : \psi(m) = n\}$.

Example 3.14. Let $M(I_1,I_2)$ be a strong refined neutrosophic R-module over a refined neutrosophic ring $R(I_1,I_2)$. The mapping $\psi : M(I_1,I_2) \to M(I_1,I_2)$ defined by $\psi(m) = m$ for all $m \in M(I_1,I_2)$ is refined neutrosophic R-module homomorphism and $\ker\psi = (0,0I_1,0I_2)$.

Example 3.15. The mapping $\psi : M(I_1,I_2) \to M(I_1,I_2)$ defined by $\psi(m) = (0,0I_1,0I_2)$ for all $m \in M(I_1,I_2)$ is refined neutrosophic R-module homomorphism.

Definition 3.15. Let $M(I_1,I_2)$ and $N(I_1,I_2)$ be strong refined neutrosophic R-modules over a refined neutrosophic ring $R(I_1,I_2)$ and let $\psi : M(I_1,I_2) \to N(I_1,I_2)$ be a refined neutrosophic R-module homomorphism. Then:

(1) $\ker\psi$ is not a strong refined neutrosophic submodule of $M(I_1,I_2)$ but a submodule of M.

(2) $\operatorname{Im}\psi$ is a strong refined neutrosophic submodule of $N(I_1,I_2)$.

Proof: (1) Obviously, $I_1, I_2 \in M(I_1,I_2)$ but $\varphi(I_1) = I_1 \neq 0, \varphi(I_2) = I_2 \neq 0$. That $\ker\psi$ is a submodule of M is clear.

(2) Clear.

5. Conclusion

In this paper, the refined neutrosophic R-modules and refined neutrosophic submodules which are completely different from the classical modules and submodules in the structural properties were defined. It was shown that every refined neutrosophic R-module is an R-module. Finally, refined neutrosophic R-module homomorphism was explained and some definitions and theorems were given.

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