On Some Special Substructures of Refined Neutrosophic Rings

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Abstract

The objective of this article is to define and study the concepts of refined neurosophic AH-ideal and AHS-ideal in refined neutrosophic rings.We investigate the elementary properties of these concepts.

Keywords: Refined neutrosophic ring, Refined neutrosophic AH-ideal, Refined neutrosophicAHS-Ideal, Refined AHS-homomorphism.

1.Introduction

Neutrosophy as a new branch of philosophy can be applied into the algebraic systems, which leads to a better comprehension and evolution of these systems.The notion of neutrosophic groups and rings was defined by Kandasamy and Smarandache in [10], and studied widely in [4, 5, 7,8]. Studies were carried out on neutrosophic rings and neutrosophic hyperring. See[1, 3, 4,6].

Refined neutrosophic rings were defined and studied carefully in [2, 3], where special substructures such as refined neutrosophic subrings and refined neutrosophic ideals are defined. Many interesting results were proved. In [1] concepts as AH-ideal and AHS-ideal were defined and studied as interesting substructures of neutrosophic ring. Some related concepts such as weak principal, maximal, and prime AH-ideals were introduced. These concepts have many properties which are similar to classical case of rings.In this paper, we try to define concepts such as AH-ideal and AHS-ideal in refined neutrosophic ring with some related concepts such as weak prime, principal and maximal refined neutrosophic AH-ideals. Also, we introduce the notion of refined AHS-homomorphism in similar way to AHS-homomorphism defined in [1].

Motivation

This paper is the continuation of the work began in the paper entitled "On Some Special Substructures of Neutrosophic Rings and Their properties".

2. Preliminaries

Definition 2.1:[5]

Let (R,+,×) be a ring then R(I)={a+bI; a,b∈ R} is called the neutrosophic ring where I is a neutrosophic element with the condition $I^2 = I$.

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Remark 2.2: [2]

The element I can be splitinto two indeterminacies I_1 , I_2 with conditions:

$$\mathrm{I}_1^{\ 2} = \mathrm{I}_1 \,, I_2^{\ 2} = I_2 \,, I_1 I_2 = I_2 I_1 = I_1.$$

Definition 2.3: [2]

If X is a set then X$(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in X\}$ is called the refined neutrosophic set generated by X, I_1, I_2.

Definition 2.4: [2]

Let (R,+,×) be a ring, (R(I_1, I_2), +,×) is called a refined neutrosophic ring generated by R,I_1, I_2.

Definition 2.5: [2]

Let (R(I_1, I_2), +,×) be a refined neutrosophic ring; it is called commutative if

$$x \times y = y \times x, \forall x \,, y \in \mathrm{R}(I_1, I_2).$$

Theorem 2.6: [2]

Let (R(I_1, I_2), +,×) be a refined neutrosophic ring then it is a ring.

Definition 2.7: [3]

Let (R(I_1, I_2), +,×) be a refined neutrosophic ring and J be a nonempty subset of R(I_1, I_2) then J is called a neutrosophic refined ideal if:

(a) J is a refined neutrosophic subring of R(I_1, I_2).

(b) For every $x \in J$ and $r \in$ R(I_1, I_2) then $x \times r \in$ R(I_1, I_2).

Definition 2.8:[1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I = \{a_0 + a_1 I;\ a_0 \in P_0, a_1 \in P_1\}$.

(a)We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R.

(b)We say that P is an AHS-ideal if $P_0 = P_1$.

Definition 2.9:[1]

Let R(I), T(J) be two neutrosophic rings and the map $f: R(I) \to T(J)$; we say that f is a neutrosophic AHS-homomrphism :

Restriction of the map f on R is a ring homomorphism from R to T, i.e $f_R: R \to T$ is homomorphism and

$f(a + b\mathrm{I}) = f_R(a) + f_R(b)\, J$.

We say that R(I), T(J) are AHS-isomomrphic neutrosophic rings if there is a neutrosophic AHS-homomorphism

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$f: R(I) \to T(J)$ which is a bijective map; i.e (R $\cong T$), we say that f is a neutrosophic AHS-isomorphism.

Definition 2.10:[1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal, we define the AH-factor as:

$R(I)/P = R/P_0 + R/P_1\, I.$

Theorem 2.11:[1]

Let R(I) be a neutrosophic ring and $P = P_0 + P_1 I$ be an AH-ideal then $R(I)/P$ is a ring.

Theorem 2.12:[1]

Let R(I), T(J) be two neutrosophic rings and f: R(I)→T(J) is a neutrosophic ring AHS-homomorphism, let $P = P_0 + P_1 I$ be an AH-ideal of R(I) and $Q = Q_0 + Q_1 J$ be an AH-ideal of T(J), we have:

(a) f(P) is an AH-ideal of f(R(I)).

(b) $f^{-1}(Q)$ is an AH-ideal of R(I).

(c) If P ia an AHS-ideal of R(I),f(P) is an AHS-ideal of f(R(I)).

(d) $AH - kerf = kerf_R + kerf_R I$ is an AHS-ideal;f_R is the restriction of f on the ring R.

(e) The AH-factor $R(I)/kerf \;\; is\ AHS - isomorphic\ to\ f(R(I))$.

Definition 2.13: [1]

Let R(I) be a neutrosophic commutative ring and $P = P_0 + P_1 I$ be an AH-ideal, we say that:

(a) P is a weak prime AH-ideal if P_0, P_1 are prime ideals in R.

(b) P is a weak maximal AH-ideal if P_0, P_1 are maximal ideals in R.

(c) P is a weak principal AH-ideal if P_0, P_1 are principal ideals in R.

3. Main concepts and discussion

Definition 3.1:

Let (R(I_1, I_2), +,×) be a refined neutrosophic ring, and P_0, P_1, P_2 be three ideals in the ring R then the set

$P = (P_0, P_1 I_1, P_2 I_2) = \{(a, bI_1, cI_2); a \in P_0, b \in P_1, c \in P_2\}$is called a refined neutrosophic AH-ideal.

If $P_0 = P_1 = P_2$ then P is called a refined neutrosophic AHS-ideal.

Definition 3.2:

Let (R , +,×), (T, +, ×) be two rings and $f_R: R \to T$ is a homomorphism :

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The map $f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2); f(x, yI_1, zI_2) = (f_R(x), f_R(y)I_1, f_R(z)I_2)$ is called a refined AHS-homomorphism.

It is easy to see that for all$x, y \in \mathrm{R}(I_1, I_2)$, we have $f(x+y) = f(x) + f(y), f(x \times y) = f(x) \times f(y)$.

Example 3.3:

Suppose that $\mathrm{R} = (Z_6, +, \times), T = (Z_{10}, +, \times)$ are two rings and $f_R: R \to T$; $f(a) = 5a$ is homomorphism, the related refined AHS-homomorphism can be defined:

$f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2); f(x, yI_1, zI_2) = (5x, 5yI_1, 5zI_2)$.

The previous example shows that refined AH-homomorphism is not a refined neutrosophic homomorphism in general because:

$f(I_1) \neq I_1$

Definition 3.4:

(a) Let$f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism, we define refined AH-Kernel of f by:

$\mathrm{AH} - Kerf = \{(a, bI_1, cI_2); a, b, c \in Kerf_R\} = (Kerf_R, Kerf_R I_1, Kerf_R I_2)$.

(b) Let S=$(S_0, S_1 I_1, S_2 I_2)$ be a subset of $\mathrm{R}(I_1, I_2)$, then: $f(S) = (f_R(S_0), f_R(S_1)I_1, f_R(S_2)I_2) = \{(f_R(a_0), f_R(a_1)I_1, f_R(a_2)I_2); a_i \in S_i\}$.

(c) Let S=$(S_0, S_1 I_1, S_2 I_2)$ be a subset of $\mathrm{T}(I_1, I_2)$, then:

$$f^{-1}(S) = (f_T^{-1}(S_0), f_T^{-1}(S_1)I_1, f_T^{-1}(S_2)I_2).$$

Definition 3.5:

Let$f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism we say that f is a refined AHS-isomorphism if it is a bijective map, $\mathrm{R}(I_1, I_2), \mathrm{T}(I_1, I_2)$ are called AHS-isomorphic refined neutrosophic rings.

It is easy to see that restriction f_R will be an isomorphism between R, T .

Theorem 3.6:

Let$f: \mathrm{R}(I_1, I_2) \to \mathrm{T}(I_1, I_2)$ be a refined AHS-homomorphism we have:

(a) $AH - Ker\ f$is a refinedneutrosophic AHS-ideal of $\mathrm{R}(I_1, I_2)$.

(b) If P is a refined neutrosophic AH-ideal of $\mathrm{R}(I_1, I_2)$, $f(P)$ is a refined neutrosophic AH-ideal of $\mathrm{T}(I_1, I_2)$.

(c) If P is a refined neutrosophic AHS-ideal of $\mathrm{R}(I_1, I_2)$, $f(P)$ is a refined neutrosophic AHS-ideal of $\mathrm{T}(I_1, I_2)$.

Proof:

(a) Since $Kerf_R$ is an ideal of R,AH $- kerf = (Kerf_R, Kerf_R I_1, Kerf_R I_2)$ is a refined neutrosophic AHS-ideal of $\mathrm{R}(I_1, I_2)$.

(b)Suppose that $P = (P_0, P_1 I_1, P_2 I_2)$ is a refined neutrosophic AH-ideal of $\mathrm{R}(I_1, I_2)$. Since $f_R(P_i)$ is an ideal of T, $f(P) = (f_R(P_0), f_R(P_1)I_1, f_R(P_2)I_2)$ is a refined neutrosophic AH-ideal.

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(c) The proof is similar to (b).

Definition 3.7:

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring and P= (P_0, P_1I_1, P_2I_2) be a refined neutrosophic AH-ideal then:

(a) We say that P is a weak prime refined neutrosophic AH-ideal if $P_i; i \in \{0,1,2\}$ are prime ideals in R.

(b) We say that P is a weak maximal refined neutrosophic AH-ideal if $P_i; i \in \{0,1,2\}$ are maximal ideals in R.

(c) We say that P is a weak principal refined neutrosophic AH-ideal if $P_i; i \in \{0,1,2\}$ are principal ideals in R.

(d) We define the refined neutrosophic AH-factor as:

$$R(I_1, I_2)/P = (R/P_0, R/P_1I_1, R/P_2I_2) = \{([x_0 + P_0], [x_1 + P_1]I_1, [x_2 + P_2]I_2);\ x_0, x_1, x_2 \in R\}.$$

Theorem 3.8:

Let $f: R(I_1, I_2) \to T(I_1, I_2)$ be a refined AHS-homomorphism and P= (P_0, P_1I_1, P_2I_2) be a refined neutroosphic AH-ideal of $R(I_1, I_2)$, let Q=$(Q_0, Q_1I_1, Q_2I_2) \neq T(I_1, I_2)$ be a refined neutrosophic AH-ideal of $T(I_1, I_2)$, assume that $Kerf_R \leq P_i \neq R$ then:

(a) P is a weak prime refined neutrosophic AH-ideal of $R(I_1, I_2)$ if and only if $f(P)$ is a weak primerefinedneutrosophic AH-ideal in $f(R(I_1, I_2))$.

(b) P is a weak maximal AH-ideal of $R(I_1, I_2)$ if and only if $f(P)$ is a weak maximal in $f(R(I_1, I_2))$.

(c) Q is a weak prime AH-ideal of $T(I_1, I_2)$ if and only if $f^{-1}(Q)$ is a weak prime in $R(I_1, I_2)$.

(d) Q is a weak maximal AH-ideal of $T(I_1, I_2)$ if and only if $f^{-1}(Q)$ is a weak maximal in $R(I_1, I_2)$.

Proof:

The proof is similar to the Theorem 3.15 in [1].

It is easy to see that conditions (a), (b) are still true if P is an AHS-ideal and conditions (c), (d) are still true if Q is an AHS-ideal.

Theorem 3.9:

The refined neutrosophic AH-factor $R(I_1, I_2)/P$ is a ring with respect to the following operations:

Let $x = (x_0 + P_0, (x_1 + P_1)I_1, (x_2 + P_2) I_2)$, $y = (y_0 + P_0, (y_1 + P_1)I_1, (y_2 + P_2) I_2)$,be two arbitrary elements in $R(I_1, I_2)$ then:

$x + y = ([x_0 + y_0] + P_0, ([x_1 + y_1] + P_1)I_1, ([x_2 + y_2] + P_2) I_2)$,

$x \times y = ([x_0 \times y_0] + P_0, ([x_1 \times y_1] + P_1)I_1, ([x_2 \times y_2] + P_2) I_2)$.

Proof:

The proofis similar to the Theorem 3.9 in [1].

Example 3.10:

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Let R = $(Z_6, +,\times)$, T= $(Z_{10}, +,\times)$ be two rings, and $f$be the refined neutrosophic AHS-homomorphism defined in Example 3.3, we have the following:

(a) $P_0 = \{0,2,4\}, P_1 = \{0,3\}$are two ideals in Z_6 thus P=(P_0, P_0I_1, P_1I_2) is a refined neutrosophic AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$.

$(b) f(P) = (f(P_0), f(P_0)I_1, f(P_1)I_2)$= { (0,0, 0), $(0,0,5I_2)$} is a refined neutrosophic AH-ideal in $\mathrm{T}(\mathrm{I}_1, \mathrm{I}_2)$.

(c) $Q_0 = \{0,2,4,6,8\}$ is a maximal ideal in Z_{10} and $f_T^{-1}(Q_0) = \{0,2,4\}$, so $Q = (Q_0, Q_0I_1, Q_0I_2)$ is a weak maximal refined neutrosophic AHS-ideal in $\mathrm{T}(\mathrm{I}_1, \mathrm{I}_2)$, we have $f^{-1}(Q)$ =({0,2,4}, {0,2,4}I_1, {0,2,4}I_2) is a weak maximal refined neutrosophic AHS-ideal in $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$.

Example 3.11:

(a) In the ring (Z,+, ×), P= <3>, Q=<2> are two prime and maximal ideals, thus M=(P ,QI_1, QI_2) = {(3a, 2bI_1, 2cI_2); a, b, c$\in Z$ } is a weak maximal/prime refined neutrosophic AH-ideal of (Z(I_1, I_2), +,×).

(b)The map $f_Z : Z \to Z_6; f(a) = a \ mod\ 6$ is a homomorphism so the related refined neutrosophic AHS-homomorphism is

f: $\mathrm{Z}(\mathrm{I}_1, \mathrm{I}_2) \to \mathrm{Z}_6(\mathrm{I}_1, \mathrm{I}_2); f(a, bI_1, cI_2) = (a\ mod6, (b\ mod6)I_1, (c\ mod6)I_2), AH - kerf$ = (6Z,6ZI_1, 6ZI_2) $\leq M$ since 6Z$\leq P, Q$.

(c) $f(M) = (\{0,3\}, \{0,2,4\}I_1, \{0,2,4\}I_2)$ is a weak maximal/prime refined neutrosophic AH-ideal of $\mathrm{Z}_6(\mathrm{I}_1, \mathrm{I}_2)$.

Definition 3.12:

A refined neutrosophic ring $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$is called weak principal refined neutrosophic AH-ring if every refined neutrosophic AH-ideal is weak principal.

Theorem 3.13:

Let R be a principal ideal ring then $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$ is weak principal refined neutrosophic AH-ring.

Proof:

Let P= (P_0, P_1I_1, P_2I_2) be a refined neutrosophic AH-ideal of $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$. Since P_i are ideals in R and then principal this implies that P is a weak refined neutrosophic AH-ideal; thus $\mathrm{R}(\mathrm{I}_1, \mathrm{I}_2)$ must be weak principal refined neutrosophic AH-ring.

Example 3.14:

The ring (Z,+, ×) is principal ideals ring; thus $\mathrm{Z}(\mathrm{I}_1, \mathrm{I}_2)$ is weak principal refined neutrosophic AH-ring.

Definition 3.15:

Let $(\mathrm{R}(I_1, I_2), +,\times)$ be a refined neutrosophic ring and P= (P_0, P_1I_1, P_2I_2), Q = (Q_0, Q_1I_1, Q_2I_2) be two refined neutrosophic AH-ideals of $\mathrm{R}(I_1, I_2)$, then we define:

(a) $P \cap Q = (P_0 \cap Q_0, [P_1 \cap Q_1]I_1, [P_2 \cap Q_2]I_2)$.

$$(b) P + Q = (P_0 + Q_0, [P_1 + Q_1]I_1, [P_2 + Q_2]I_2).$$

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(c) $P \times Q = (P_0 \times Q_0, [P_1 \times Q_1]I_1, [P_2 \times Q_2]I_2)$.

Theorem 3.16:

Let (R$(I_1, I_2), +, \times$) be a refined neutrosophic ring and P= (P_0, P_1I_1, P_2I_2), Q = (Q_0, Q_1I_1, Q_2I_2) be two refined neutrosophic AH-ideals of R(I_1, I_2), then:

$P \cap Q, P + Q, P \times Q$are refined neutrosophic AH-ideals of R(I_1, I_2).

Proof:

As a result of Theorem 2.5 in[1], we have $P_i + Q_i, P_i \cap Q_i, P_i \times Q_i$are ideals of R, thus the proof is complete.

Remark 3.17:

Theorem 3.16 is still true if P and Q are refined neutrosophic AHS-ideals.

Example 3.18:

Let $R(I_1, I_2) = Z_8(I_1, I_2)$ and Q = {0, 4}, S = {0,2,4,6} be two principal ideals in R, then:

(a) P = (S, QI_1, SI_2) is a refined neutrosophic AH-ideal of $R(I_1, I_2)$, the related refined neutrosophic AH-factor is:

$R(I_1, I_2)/P$= $(R/S, R/Q\, I_1, R/S\, I_2)$ = ({S, 1+S}, {Q, 1+Q, 2+Q, 3+Q}I_1, {S, 1+S}I_2).

To clarify addition and multiplication on $R(I_1, I_2)/P$ we take:

$x = (1 + S, (1 + Q)I_1$, SI_2), $y = (S, (2 + Q)I_1, (1 + S)I_2)$, we have:

$x + y = ([1 + 0] + S, ([1 + 2] + Q)\, I_1, ([0 + 1] + S\,)I_2)$= (1+S, (3+Q)$I_1$, $(1 + S)I_2$).

$x \times y = ([1 \times 0] + S, ([1 \times 2] + Q)\, I_1, ([0 \times 1] + S\,)I_2)$= (S, (2+Q) I_1, SI_2).

Conclusion

In this article we defined concepts of refined neutrosophic AH-ideal/ AHS-ideal in a refined neutrosophic ring.We studied some of elementary properties of these concepts.Also, notions as weak maximal, prime and principal refined neutrosophic AH-ideal and refined AHS-homomorphisms were introduced and checked.

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