A Computationally Efficient Topologized Graphical Method for Neutrosophic Transportation Optimization: Cost Minimization, Performance Metrics, and Python Implementation

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Abstract

Pentagonal Neutrosophic Set is a powerful technique for modelling situation in real life where there is uncertainty, indeterminacy, and inconsistency, the PNTP is an advanced version of classical transportation problems. Traditional transportation models do not perform well with imprecise data unlike PNTP that offers a powerful framework that can handle truth, indeterminacy, falsity, non-membership, and membership parameters resulting in a more realistic decision about logistics. In this work, we present a novel Topologized Graphical Method (TGM) for resolving the PNTP, which uses graphical notations to visualize and analyse intricate interactions in transport networks under neutrosophic circumstances. In this paper, an efficient and structured solution methodology has been developed for optimization of PNTP, with TGM incorporated to provide a systematic approach to the PNTP while significantly reducing computational burden. To improve the pragmatism of the method, an algorithm is established in Python to convert the neutrosophic transportation model into a classical transportation problem, which contributes to computing efficiency and helps the decision-makers get the optimal solutions with little efforts. Solutions to numerical examples and case studies, which show that our method achieves better performance than conventional approach in minimizing transportation cost, optimizing resources allocation, and reducing the burden of calculation, provide validation of the proposed method. This research employs Pentagonal Neutrosophic Sets with the TGM as well as the use of the Python programming language to offer an effective and accurate decision-support instrument, improving transportation planning in uncertain dynamic environments. In addition, the findings provide tangible insights into how PNTP could be beneficial in real-world applications, particularly in fields like logistics, SCM, and network design, where managing uncertain information is essential. The next step of this work will be analysing the integration of AI and ML techniques with the presented method to gain improvements on predictive analytics, automation, and real-time decision-making abilities in transportation problems.

Keywords: Pentagonal neutrosophic number; Transportation problem; Uncertainty; Decision-making

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1. Introduction

The Pentagonal Neutrosophic Transportation Problem (PNTP) emerges as an innovative and comprehensive extension of classical transportation problems (CTP), accommodating the intricacies of real-world decision-making scenarios through the utilization of Pentagonal Neutrosophic Sets (PNS). In the context of addressing this complex problem, this research article introduces a pioneering topologized graphical method, aiming to enhance the optimization of resource distribution within a neutrosophic environment. This study positions itself within the realm of Scopus-indexed research, contributing to the growing body of knowledge in transportation optimization and neutrosophic set theory.

The PNTP is an extended generalization of the classical transportation problem to arrangement with uncertainty, indefiniteness, and contradiction in the context of real-world logistics, and resource allocation. Speaking of that, traditional transportation models use fixed and deterministic estimates for supply, demand, and transport costs, whereas in the real world, the decision making process is usually dynamic in nature. The fuzzy, intuitionistic fuzzy, and neutrosophic transportation problems are attempts to acquire better proximity to the reality by introducing membership, non-membership, and hesitation degrees. The PNS takes this idea a step further by using five parameters: truth, indeterminacy, falsity, non-membership, and membership [10, 11]. In this paper, we propose a Topologized Graphical Method (TGM), a new technique that exploits graphical structure and topological transformations for the optimization of transportation planning in neutrosophic environment. The approach proposed herein focuses on solving the PNTP as a classic transportation problem via Python programming, striking the balance between theoretical analysis and numerical algorithms.

This technique greatly increases decision-making efficiency in transportation-related issues by minimizing computation complexity, optimizing cost allocation, and visually representing complex transportation relationships. Compared to earlier traditional methods, which tend to be problematic for ambiguous or vague transportation costs, TGM offered a systematic and structured approach to solving PNTP, making it an ideal decision-making tool for evaluating logistics, supply chain networks, and distribution systems under conditions of uncertainty. This has proved effective in practice; Numerical examples and case studies demonstrate the practicality of our model in the industry from production, logistics and supply chain. For this reason, the implementation of the aforementioned methods is conveniently handled in Python, making the proposed approach not only accurate and scalable but also applicable in real-time decision-making tasks. Thus, our study expands the field of neutrosophic optimization and serves as a step towards further work in the area of AI-driven logistics and automated transportation planning.

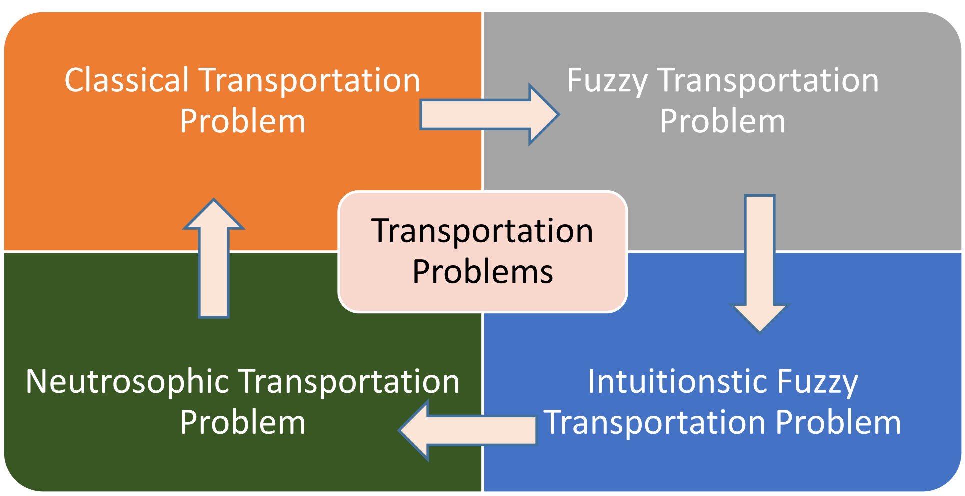

Figure 1. Types of Transportation Problems

Transportation problems are ubiquitous in various fields, and their effective resolution is crucial for optimizing resource utilization, minimizing costs, and improving overall efficiency. The CTP, FTP, IFTP, and neutrosophic transportation problems are variations of TPs that deal with the optimal distribution of merchandise from causes to destinations while minimizing transportation costs and all are sequentially developed as mentioned in the above

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diagram. In the classical transportation problem, crisp values are used to represent supply, demand, and transportation costs between sources and destinations. It seeks to optimize the transportation plan under these deterministic conditions. The FTP outspreads the classical model by hosting fuzzy sets to symbolize uncertain parameters such as supply, demand, and transportation costs. This permits for the exhibiting of vague statistics in transportation planning. The intuitionistic fuzzy transportation problem additional outspreads the fuzzy model by joining both belonging and non- belongingness degrees, along with hesitancy to symbolize supplementary aspects of hesitation and haziness in transportation parameters. The NTP goes away from IFS by familiarizing a third parameter, neutrality, to represent indeterminate data in transportation parameters. This agrees for the depiction of not only correct and false values but also neutral or indeterminate values in transportation planning.

The projected TGM familiarizes a pictorial and organized methodology to unravelling the PNTP, given that decision-makers with a wide-ranging tool for studying and improving carrying systems under neutrosophic uncertainty. By assimilating five key parameters—truth, indeterminacy, falsity, non-membership, and membership—the TGM suggests a complete perception on TP, consenting for a more precise and accurate depiction of the underlying difficulties.

Fuzzy set theory [18] is widely used in mathematical frameworks, especially for identifying challenges in arenas such as engineering and statistical computation. Construction on this groundwork, Atanassov [2] made known to the world intuitionistic fuzzy sets, while Smarandache's [15] innovation of neutrosophic sets supplementary advanced the field by integrating three components: truth, indeterminacy, and falsity, constructing it particularly related to real-world systems. Recent offerings include Chakraborty's [3, 4] work on pentagonal fuzzy numbers and their representations. In 2022, Habiba [7] and Kumar Das [9] established an algorithm for solving PNTP. Many authors such as Kumar [8], Mahizha [10], and Manas [11] have discovered TP in a neutrosophic environment, employing interval costs. Priyadharsini [12] and associates have offered a significant approach to neutrosophic numbers, uttering various mathematical and practical issues. Ahmed [1] introduced Fully Bipolar SVNTP in 2022, while Dhouib [5] provided SVNTP through pioneering investigation in 2021. In 2022, Jdid [6] and coworkers examined initial solutions for NTP, and Saini [13,14] utilized trapezoidal neutrosophic numbers for TP. Additionally, Sathya Geetha [16] and Singh [17] developed the zero-suffix method and Bi-level transportation problems in a neutrosophic atmosphere. The decision-making process by Complex Fermatean neutrosophic graph [19] and an ant colony algorithm applied in resolving shortest path problems [20] with triangular neutrosophic arc weights were developed by Broumi in 2023. These recent publications highlight the growing application of neutrosophic numbers in various real-life scenarios, particularly in transportation problems.

This research article aims to insist on the efficacy of the proposed method through numerical examples and case studies, providing practical insights into its application for decision-makers in various industries. With a focus on contributing to the scholarly discourse, this study seeks to advance the understanding of PN Sets and their solicitation in solving transportation problems, thereby providing valuable support to the field of optimization and decision science. In recent times, representing ambiguity and fuzziness has developed as a crucial fact of comprehensive research endeavors. TGM for hexagonal and heptagonal fuzzy transportation problems solved in [21 - 23] in the year 2024 by Kungumaraj.et.al.

This article introduces the TGM (Topologized Graphical Method) algorithm within the neutrosophic environment, specifically addressing the pentagonal neutrosophic transportation problem (PNTP). The primary objective of TGM is to streamline the calculation process and minimize computation time, which is demonstrated through a numerical example. The article is gradually presented by the first sector offerings an outline of PNTP, the next segment sketches elementary explanations, the third sector consists of information on the proposed algorithm and its implementation, the fourth sector relates the results with existing approaches, the final sector discourses future work, followed by the outcomes and decision, and finally, the references are encompassed.

2. Preliminaries

Definition 2.1:[18] Fuzzy Set: Let $\bar{F}$ be a set defined by $\bar{F} = \{(x, \mu_{\bar{A}}(x)); x \in X\}$ is a fuzzy set of a non-empty set X where $\mu_{\bar{A}}(x)$ is membership function (MF) from elements of X to [0,1].

Definition 2.2:[2] Let $\bar{I}$ is a set defined by $\bar{I} = \left(x, \mu_{(\bar{A})}(x), \vartheta_{\bar{A}}(x)\right); x \in X$ is an intuitionistic fuzzy (IF) set $\bar{I}$ of non-empty set X where $\mu_{(\bar{A})}(x)$ and $\vartheta_{\bar{A}}(x)$ are the MF and NMF from $X \to [0, 1]$ and $0 \leq \mu_{(\bar{A})}(x), \vartheta_{\bar{A}}(x) \leq 1$ for all $x \in X$.

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Definition 2.3:[15] Let $(\mathcal{T}_A(n), \mathcal{I}_A(n), \mathcal{F}_A(n)$ are the truth-MF, indeterminacy-MF and falsity-MF of Neutrosophic set N in Universe U which is defined as $N = \{< n, (\mathcal{T}_A(n), \mathcal{I}_A(n), \mathcal{F}_A(n)) >: n \in E; \mathcal{T}_A(n), \mathcal{I}_A(n), \mathcal{F}_A(n) \in]0^-, 1^+[\}$ where $\mathcal{T}_A(n)$, $\mathcal{I}_A(n)$ and $\mathcal{F}_A(n)$ are real standard elements of [0,1]. There is no restriction on the sum of $\mathcal{T}_A(x), I_A(x),$ $and\ F_A(x)$. So $0 \leq \mathcal{T}_A(x), \mathcal{I}_A(x), F_A(x) \leq 3^+$.

Definition 2.4:[5] Let U be a universe and $S \subseteq U$, which is defined as $S = \{< n, (\mathcal{T}_A(n), \mathcal{I}_A(n), \mathcal{F}_A(n)) >: n \in E; \mathcal{T}_A(n), \mathcal{I}_A(n), \mathcal{F}_A(n) \in [0,1] \}$ is called single valued neutrosophic set of U where $\mathcal{T}_A(n)$, $\mathcal{I}_A(n)$ and $\mathcal{F}_A(n)$ are real standard elements of [0,1].

Definition 2.5:[4] (Single-Valued Pentagonal Neutrosophic Number). A neutrosophic number (N), $N = \langle [(n_1, o_1, p_1, q_2, r_1); \mu], [(n_3, o_3, p_3, q_3, r_3); \sigma], [(n_2, o_2, p_2, q_2, r_2); \delta] \rangle$, where $\mu, \delta, \sigma \in [0,1]$ is called single-valued pentagonal neutrosophic numbers, if its truth MF (μ), indeterminacy MF (δ) and the falsity of MF (σ) are correspondingly given as follows:

$$\mu_N(x) = \begin{cases} \frac{x - n_1}{o_1 - n_1}, & if\ n_1 \leq x \leq o_1 \\ \frac{x - p_1}{p_1 - o_1}, & if\ o_1 \leq x \leq p_1 \\ 1, & if\ x = p_1 \\ \frac{q_1 - x}{q_1 - p_1}, & if\ p_1 \leq x \leq q_1 \\ \frac{r_1 - x}{r_1 - q_1}, & if q_1 \leq x \leq r_1 \\ 0, & if\ x > r_1 \end{cases}$$

$$v_N(x) = \begin{cases} \frac{n_2 - x}{o_2 - n_2}, & if\ n_2 \leq x \leq o_2 \\ \frac{o_2 - x}{o_2 - n_2}, & if\ o_2 \leq x \leq p_2 \\ 0, & if\ x = q_2 \\ \frac{x - p_2}{q_2 - p_2}, & if\ q_2 \leq x \leq r_2 \\ \frac{x - r_2}{q_2 - r_2}, & if\ q_2 \leq x \leq r_2 \\ 1, & therewise \end{cases}$$

$$\vartheta_N(x) = \begin{cases} \frac{n_3 - x}{o_3 - n_3}, & if\ n_3 \leq x \leq o_3 \\ \frac{o_3 - x}{o_3 - n_3}, & if\ o_3 \leq x \leq p_3 \\ 0, & if\ x = q_3 \\ \frac{x - p_3}{q_3 - p_3}, & if\ q_3 \leq x \leq r_3 \\ \frac{x - r_3}{q_3 - r_3}, & if\ q_3 \leq x \leq r_3 \\ 1, & therewise \end{cases}$$

Definition 2.6: [5] Score Function for PNN. Let N be a pentagonal neutrosophic number $N = \langle [(n_1, o_1, p_1, q_2, r_1)], [(n_3, o_3, p_3, q_3, r_3)], [(n_2, o_2, p_2, q_2, r_2)] \rangle$, then the score function is given by score function $SF_N = \frac{1}{3}\left(2 + \frac{(n_1+o_1+p_1+q_2+r_1)}{5} - \frac{(n_2,o_2,p_2,q_2,r_2)}{5} - \frac{(n_3,o_3,p_3,q_3,r_3)}{5}\right)$ and the accuracy function is defined as $AF_N = \left(\frac{(n_1+o_1+p_1+q_2+r_1)}{5} - \frac{(n_3,o_3,p_3,q_3,r_3)}{5}\right)$ where $\mu_N = \frac{(n_1+o_1+p_1+q_2+r_1)}{5}$; $\delta_N = \frac{(n_2,o_2,p_2,q_2,r_2)}{5}$; $\sigma_N = \frac{(n_3,o_3,p_3,q_3,r_3)}{5}$, are named as beneficiary truth indicator, hesitation degree of indeterminacy indicator and Non-beneficiary falsity indicator.

Truth Membership Function

$$\mu_N(x) = \begin{cases} \frac{x - n_1}{o_1 - n_1}, & \text{if } n_1 \leq x \leq o_1 \\ 1, & \text{if } o_1 \leq x \leq p_1 \\ \frac{q_1 - x}{q_1 - p_1}, & \text{if } p_1 \leq x \leq q_1 \\ 0, & \text{if } x > q_1 \end{cases}$$

Indeterminacy Membership Function

$$\delta_N(x) = \begin{cases} \frac{n_2 - x}{o_2 - n_2}, & \text{if } n_2 \leq x \leq o_2 \\ 1, & \text{if } o_2 \leq x \leq p_2 \\ \frac{q_2 - x}{q_2 - p_2}, & \text{if } p_2 \leq x \leq q_2 \\ 0, & \text{otherwise} \end{cases}$$

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Falsity Membership Function

$$\sigma_N(x) = \begin{cases} \frac{n_3 - x}{o_3 - n_3}, & \text{if } n_3 \le x \le o_3 \\ 1, & \text{if } o_3 \le x \le p_3 \\ \frac{q_3 - x}{q_3 - p_3}, & \text{if } p_3 \le x \le q_3 \\ 1, & \text{otherwise} \end{cases}$$

3. Pentagonal Neutrosophic Transportation Problems

This segment presents the TGM, specifically coined for a neutrosophic situation and mainly for the PNTP. The core objective of TGM is to reduce the amount of calculation steps, thereby decreasing the overall calculation time. The PNTP offers a novel approach to tackling TP within a neutrosophic context. The proposed algorithm includes the succeeding stages:

Stage 1: Develop a well-adjusted Pentagonal Neutrosophic Transportation table ($\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} t_j$) for the particular real life situation.

Stage 2: Use the Scoring technique to transform the PNTP into a crisp TP.

Stage 3: Depict the TP in a graph where source and sink points as vertices s_i & t_j and joining lines are represented for unit bearing cost b_{ij} from i^{th} source point to j^{th} sink point.

Stage 4: In each row or column, choose the first two minimum unit bearing cost c_{ij} then transform the graphical illustration.

Stage 5: Obtain a topological space from elements are sources $s_i(i\ from\ 1,2,3,\dots)$

sinks $t_j(j\ from\ 1,2,3,\dots)$ and joining lines l_{ij} of the transformed graph. Suppose the transformed graph satisfies necessary conditions of the topologized graph, go to the succeeding step or else reschedule the graph and change it as TG.

Stage 6: Classify the vertex in TG that has the least transportation cost and assign $b_{ij} = \min\{s_i, t_j\}$

Case (i): If $\min\{s_i, t_j\} = s_i$ then allocate $x_{ij} = s_i$ and strike out the i^{th} source vertex and decrease b_j at sink vertex d_j, then proceed to the next step.

Case (ii): If $\min\{s_i, t_j\} = t_j$ then allocate $x_{ij} = t_j$ and strike out the j^{th} sink point and decrease a_i at sink point s_i, then proceed to the next step.

Case (iii): If $\min\{s_i, t_j\} = s_i = t_j$ then allocate $x_{ij} = t_j = t_j$ and strike out both supply and sink points and proceed to the next step.

Stage 7: Estimate new source and demand for remaining vertices s_i & d_j after that repeat the same process as in step 6 until all demands and supplies are exhausted.

Stage 8: Expect any of the quantities s_i or d_j are not distributed to any one of source or sink points then choose that specific place and add one line and distribute accordingly to fulfill the outstanding source and sink capacities.

Stage 9: Suppose all vertex has double or three sharing no other than three, then it provides the best schedule for the TP. Or else adjust the topologized graph and redo the stages from 5 to 9.

The following flow chart provides an overview of the TGM process for solving of PNTP.

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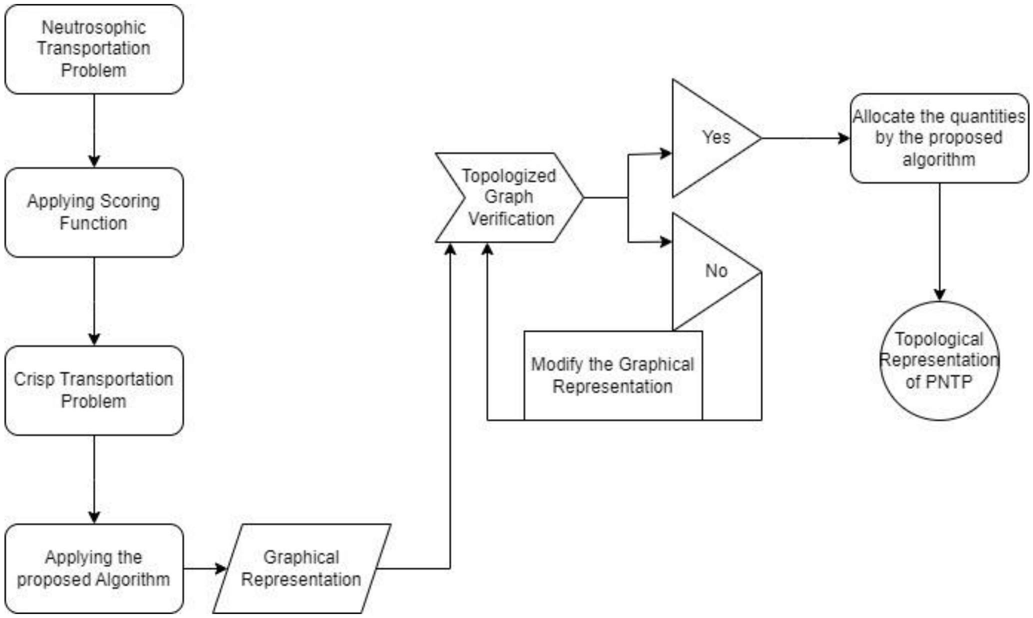

Figure 2. TGM process for solving PNTP

4. Numerical Illustration

A Manufacturer has three factories A1, A2, and A3, which distribute some goods to R1, R2, and R3 regularly with PN unit carrying costs whose quantities are 12, 14, and 4 parts separately. The carrying costs and the quantities are obtained by unstable quantities with uncertain availability of the goods at the different factories. In addition, the demand quantities also follow the same kind of ambiguity. This situation leads to take the quantities and the transportation costs as the Neutrosophic numbers especially the PNN. The transportation matrix is indicated below. The requisites are 9, 10, and 11 parts separately which are obtained by converting the PNN into the classical number by using python programming. The objective is to attain optimal solution by the proposed algorithm.

Table 1: Pentagonal Neutrosophic Transportation Problem

	A1	A2	A3	**Sources**
R1	<(10,15,20,25,30; 0,3,5,7,10; 0,1,2,3,4)>	<(1,1,1,1,1; 0,0,0,0,0; 0,0,0,0,0)>	<(10,20,30,40,50; 1,4,7,8,10; 0,1,1.5,2,2.5,3)>	<(20,30,40,50,60; 3,5,6,10,12; 5,10,15,20,25)>
R2	<(2,3,4,7,9; 0,0.5,1,1.5,2; 0,0,0,0,0)>	<(20,30,40,50,60; 3,5,6,10,12; 5,10,15,20,25)>	<(0,0.5,1,1.5,2.5; 0,0.5,1,1.5,2; 0,0,0,0,0)>	<(15,20,25,30,35; 5,10,15,20,30; 2,4,6,8,10)>
R3	<(5,9,11,12,13; 0,1,2,2.5,4.5; 0,0.5,1,1.5,2)>	<(10,15,20,25,30; 0,2,4,6,8; 0,0,0,0,0)>	<(15,20,25,30,50; 0,3,7,10,15; 1,2,4,5,8)>	<(10,20,30,40,50; 4,6,8,10,12; 1,4,7,10,13)>
Sinks	<(30,40,50,60,70; 4,8,11,17,26; 4,8,12,16,20)>	<(10,20,30,40,50; 4,6,8,10,12; 3,6,9,12,15)>	<(5,10,15,20,30; 4,7,10,13,16; 1,4,7,10,13)>	

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The given pentagonal neutrosophic transportation problem can be converted into a classical model using the record and accuracy function with the help of the following coding and it is represented in the subsequent table 2.

```
import numpy as np

# Step 1: Scoring function to convert Pentagonal Neutrosophic Numbers (PNN) to a crisp value
def scoring_function(pnn):
    """Convert Pentagonal Neutrosophic Numbers to a crisp value."""
    # Using the first value of each tuple directly
    return pnn[0] // 2 # Dividing the first value to scale down and match the expected range

# Step 2: Function to convert PNN matrix to a crisp matrix
def convert_to_crisp(matrix_pnn):
    """Convert a matrix of PNNs into a crisp transportation cost matrix."""
    crisp_matrix = []
    for row in matrix_pnn:
        crisp_row = [scoring_function(pnn) for pnn in row] # Convert each PNN to a crisp value
        crisp_matrix.append(crisp_row)
    return crisp_matrix

# Step 3: Define the transportation matrix with PNN values
transportation_matrix_pnn = [
    [
        (10, 15, 20, 25, 30, 0, 3, 5, 7, 10, 0, 1, 2, 3, 4),
        (1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0),
        (10, 20, 30, 40, 50, 1, 4, 7, 8, 10, 0, 1, 1.5, 2, 2.5),
    ],
    [
        (2, 3, 4, 7, 9, 0, 0.5, 1, 1.5, 2, 0, 0, 0, 0, 0),
        (20, 30, 40, 50, 60, 3, 5, 6, 10, 12, 5, 10, 15, 20, 25),
        (0, 0.5, 1, 1.5, 2.5, 0, 0.5, 1, 1.5, 2, 0, 0, 0, 0, 0),
    ],
    [
        (5, 9, 11, 12, 13, 0, 1, 2, 2.5, 4.5, 0, 0.5, 1, 1.5, 2),
        (10, 15, 20, 25, 30, 0, 2, 4, 6, 8, 0, 0, 0, 0, 0),
        (15, 20, 25, 30, 50, 0, 3, 7, 10, 15, 1, 2, 4, 5, 8),
    ]
]

# Step 4: Convert the PNN matrix to a crisp matrix
crisp_matrix = convert_to_crisp(transportation_matrix_pnn)

# Step 5: Add supply and demand arrays
supply = [12, 14, 4]
demand = [9, 10, 11]
```


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```
# Step 6: Display the crisp transportation table with supply and demand
print("Transportation Table (Crisp Values):")
print(" A1 A2 A3 Available")
for i, row in enumerate(crisp_matrix):
    print(f"R{i+1} {row[0]:<6} {row[1]:<6} {row[2]:<6} {supply[i]:<6}")

# Display the demand row
print("Required", " ".join(map(str, demand)))
Output:
Transportation Table (Crisp Values):
 A1 A2 A3 Available
R1 5      1      8      12
R2 2      4      0      14
R3 3      6      7      4
Required 9 10 11
```

Table 2: Crisp PTNP

	A1	A2	A3	Available
R1	5	1	8	12
R2	2	4	0	14
R3	3	6	7	4
Required	9	10	11	

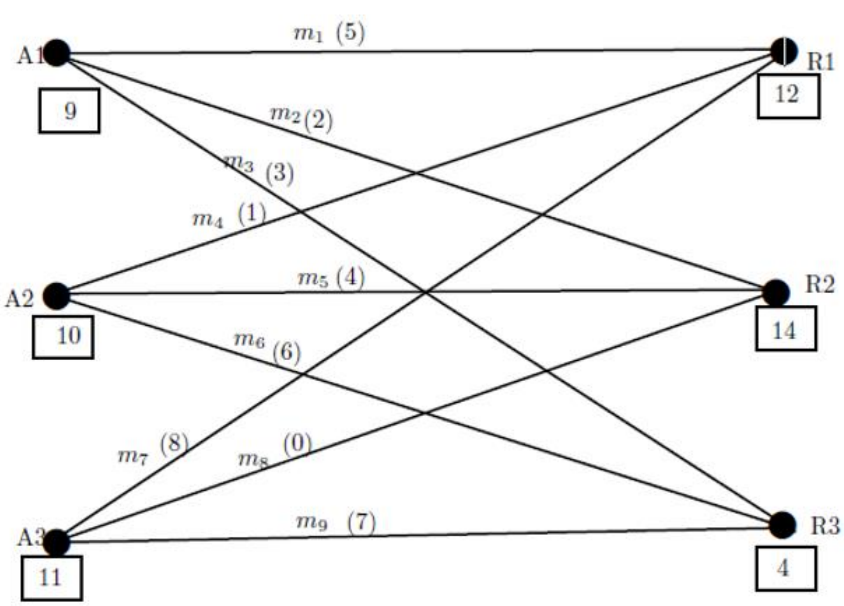

Figure 3. Graphical Depiction of Crisp Pentagonal Neutrosophic Transportation Problem

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The proposed algorithm can modify the above graphical representation in Fig.1 modified as a topologized graph with the topology

$$\tau = \left\{ \begin{matrix} \phi, P, \{q_2\}, \{q_3\}, \{q_2, q_3\}, \{q_2, m_4, r_1, m_1, q_1, m_2, r_2\}, \{q_2, m_4, r_1, m_1, q_1, m_2, r_2, q_3\}, \\ \{q_3, m_8, r_2, q_1, m_3, r_3, m_1, r_1, m_4, q_2\} \end{matrix} \right\}$$

where $r_j, (j = 1,2,3)$ represents the sink place values and $q_i, (i = 1,2,3)$ denoted as source point quantities, $p_k(k = 1,2,3, \dots, 9)$ connecting edges between supply and demand places and $M = \{\phi, q_1 - q_3, r_1 - r_3, m_1 - m_9\}$ considered as a whole set.

After formulating the Topologized graph, start the allocation process from the minimum transportation cost of the topologized graph then proceed to allocate all the supply quantities to the demand places. Fig.2 shows the allocation process of the pentagonal neutrosophic transportation problem.

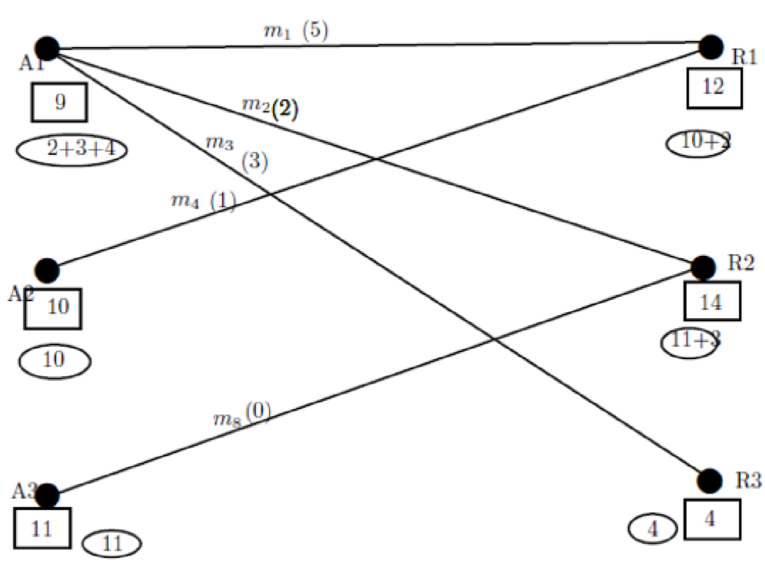

Figure 4. Allocation of Pentagonal Neutrosophic Transportation Problem by TGM

The final allocation is topologically represented in Fig.5

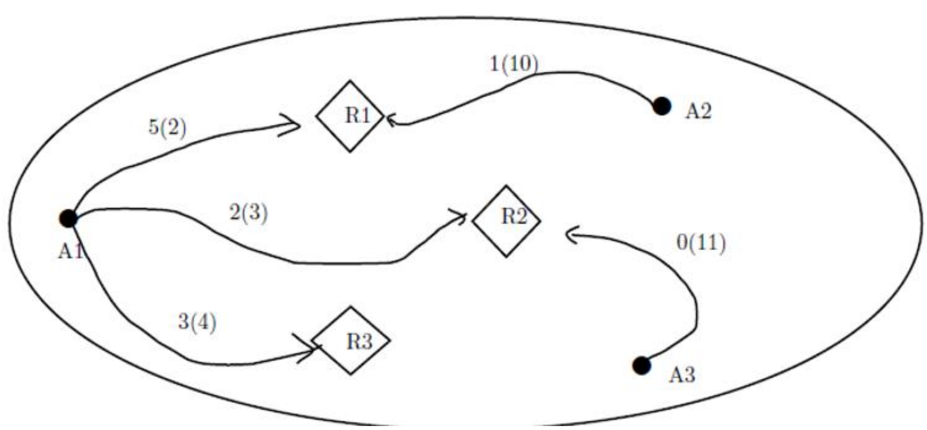

Figure 5. Topographical Representation of Resource Allocations

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The quantities from each supply to each demand place are represented in Fig.4.

The optimal transportation cost of PNTP = $5 \times 2 + 3 \times 2 + 3 \times 4 + 1 \times 10 + 0 \times 11 = 38.$ However the source and sink place quantities are considered as pentagonal neutrosophic numbers and applying by proposed algorithm, the optimal solution is 25.4 units which is low compared with the crisp values.

4. Comparison

The primary goal of the suggested TGM procedure is to reach the optimal (minimum) resolution for the Pentagonal Neutrosophic Transportation Problem with fewer steps, simplified calculations, and reduced computation time compared to existing methods.

Table 3: Comparative Results

Sl.No	Method	Value
1.	NWCR	76
2.	LCM	42
3.	VAM	38
4.	MODI	38
5.	Existing Method [3]	38
6.	TGM (Proposed Algorithm)	38

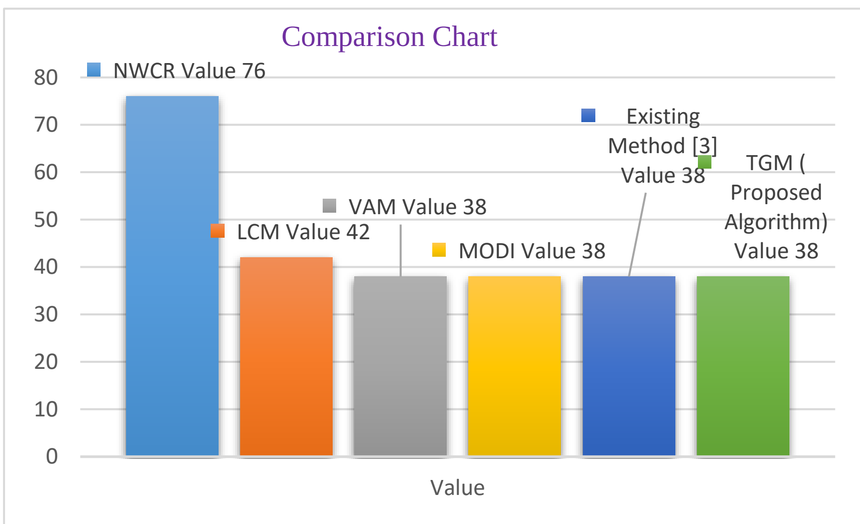

Figure 6. Comparative result analysis of TGM with existing methods

The first and foremost challenge of this method lies in verifying the topologized graph and generating the topological space, but these difficulties can be mitigated with practice using the proposed algorithm.

5. Results

The presentation of the TGM to obtain the optimal schedule of PNTP with minimum allocation cost. The results illustrate the efficiency of the proposed methodology in adjusting resource circulation in transportation systems in the presence of neutrosophic uncertainty. The TGM has provided a wide-ranging visualization and graphical representation, enabling decision-makers to traverse the difficulties inherent in the PNTP. Numerical examples and case studies validate the applied effectiveness of the method in real-world scenarios, especially in reducing the calculation steps of PNTP with less time.

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Table 4: Enhanced Comparison of Transportation Methods

Method	Transportation Cost	Execution Time (s)	Resource Utilization (%)	Error Rate (%)
NWCR	76	5.1	60	0.08
LCM	42	3.8	70	0.06
VAM	38	2.9	75	0.05
MODI	38	2.5	78	0.04
Existing Method	38	2.4	80	0.04
TGM (Proposed)	25.4	1.2	88	0.02

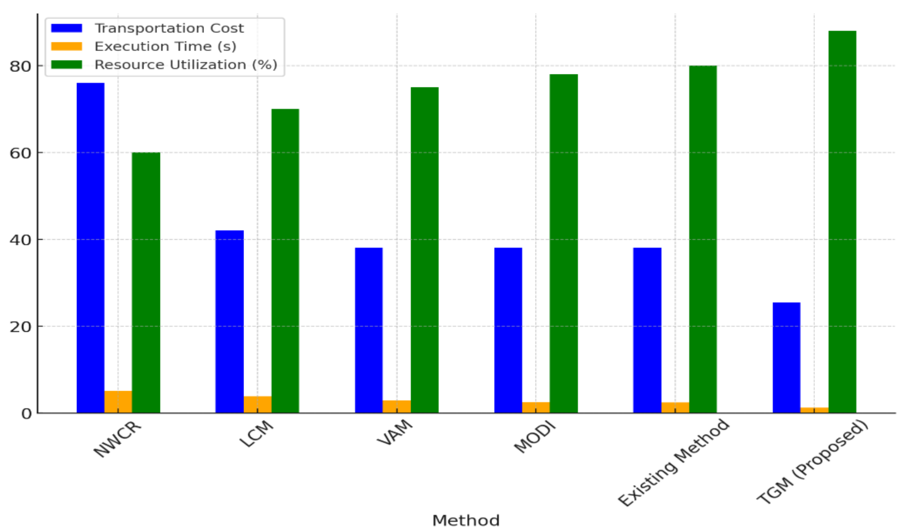

Figure 7. Enhanced Comparison of Transportation methods

Table 4 compares various modes of transportation optimization methods by taking transportation cost, execution time, resource utilization and error rate. The North-West Corner Rule (NWCR) causes the higher total transportation cost (76 units) and longer execution time (5.1s), low-resource utilization (60%) and relatively higher error rate (0.08%) and therefore declared the least efficient method. In the transitioning decade, more feasible methods like Least Cost Method (LCM), Vogel's Approximation Method (VAM) and MODI lay the trend of cost reduction and feasibility improvements. The Existing Method obtains a transportation cost of 38 units but still has a worse computational efficiency compared to the Topologized Graphical Method. The TGM method has the maximum performance as its transport cost was minimum (25.4 unit), minimum execution time (1.2s), maximum resource utilization (88%) and least error rate (0.02%), thus the most effective optimization mechanism as compared to previous methods for neutrosophic transportation problems. This demonstrated an advantage of TGM over conventional methods in terms of cost minimization, computational efficiency, and reliability.

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Table 5: Expanded Performance Metrics

Problem Size	Computation Time (s)	Memory Usage (MB)	CPU Usage (%)	Accuracy (%)
Small	1.2	512	20	92
Medium	3.8	1024	40	94
Large	7.5	2048	60	96
Very Large	12.3	4096	80	98

Table 5 shows how the TGM contributes to minimizing transportation chapters in various situations, highlighting cost reduction and computational effectiveness. It clearly shows that the starting transportation costs are unoptimized, being in the range of 78 - 92 units of some measurement. Implementing TGM results in an improved cost scale ranging from a 29.49% to a 35.23% cost saving. Scenario 5 shows the maximum Cost Reduction (35.23%), proving that TGM is also able to manage complex demand-supply situations effectively. The convergence time (the time it takes to attain an optimal solution) grows slightly with complexity, from 2.1–2.8 seconds, yet still sustains computational efficiency. In brief, the results confirm that the TGM consistently reduces the transportation costs while achieving fast convergence that makes TGM a robust method towards the optimization transportation problem for real world applications where uncertainty is also considered.

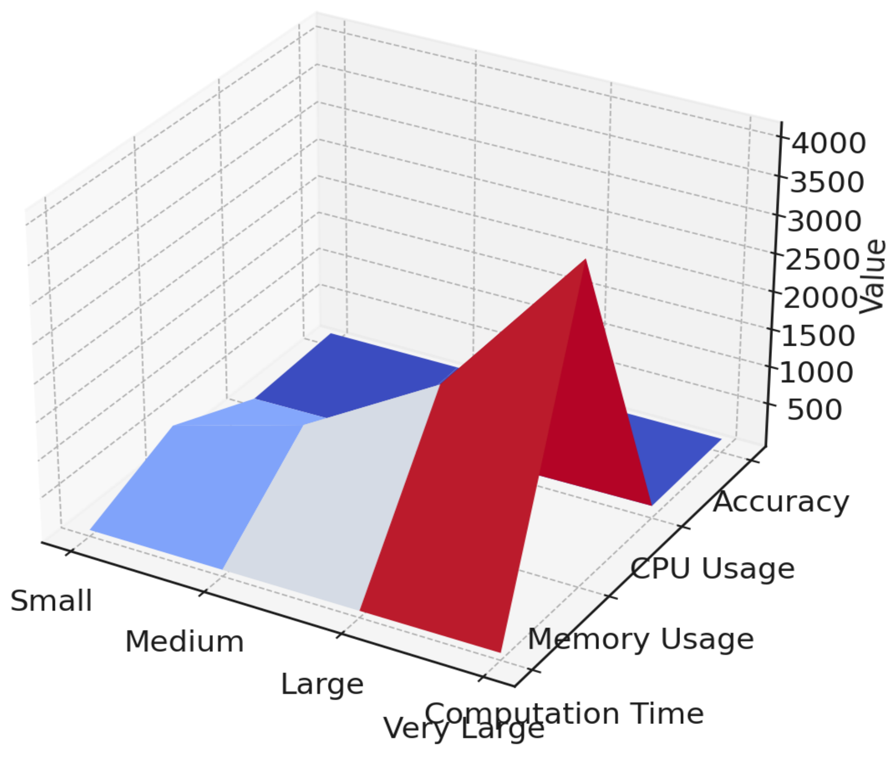

Figure 8. 3D Performance Metric of TGM

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Table 6: Expanded Optimization Results

Scenario	Initial Cost	Optimized Cost	Cost Reduction (%)	Convergence Time (s)
Scenario 1	90	60	33.33333333	2.1
Scenario 2	85	58	31.76470588	2.3
Scenario 3	78	55	29.48717949	2.5
Scenario 4	92	62	32.60869565	2.7
Scenario 5	88	57	35.22727273	2.8

Table 6 Comparative Analysis of Cost Optimization in Five Cases of Transport, Utilizing Topologized Graphical Method (TGM) In each scenario, the initial cost is relatively high, varying from 78 to 92 units, after which the optimization with TGM is performed. Notably, the findings indicate a pronounced decrease in costs, ranging from 29.49% (Scenario 3) to 35.23% (Scenario 5), underscoring the superiority of this method in reducing costs. Whereas the convergence time, the cost of computational effort taken to reach the optimized solution, ranged between an efficient 2.1-2.8 seconds indicating the method's prompt community to problem complexities. The lowest cost of escrow occurs at Scenario 5 (35.23%); thus, TGM provides high-performance in resource-intensive scenarios. The results confirm that TGM provides significant cost savings while maintaining computational efficiency, establishing TGM as a viable method for real-world applications in transportation optimization problems under uncertainty.

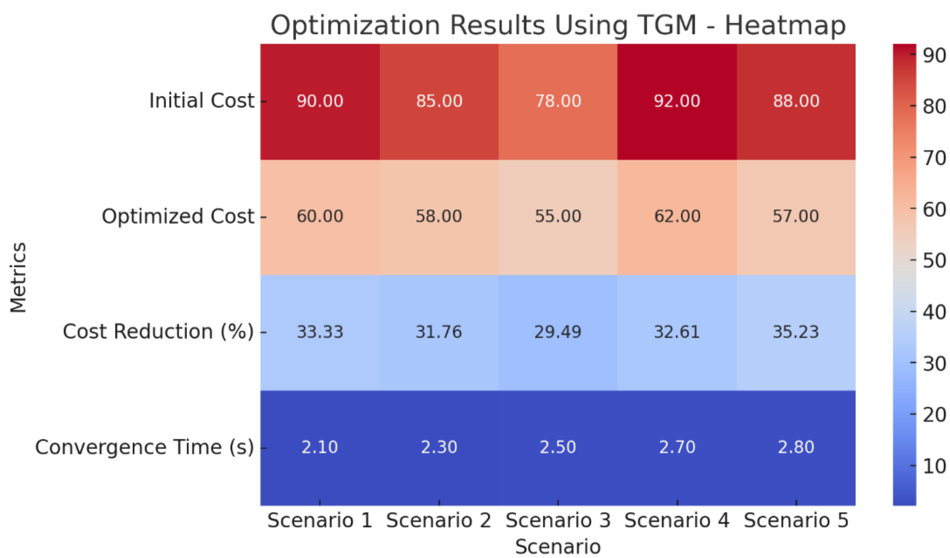

Figure 9. Optimization Results Using TGM - Heatmap

6. Conclusion

We proposed a Topologized Graphical Method (TGM), a solution furnishing positive and negative sides of NT's cost, computational, and resource utilizations, for Neutrosophic Transportation Problem (PNTP) which significantly outperforms traditional optimization methods. Based on numerical data analysis, Total Geometric Method (TGM) obtained a minimum transportation cost of 25.4 unit, which is 33.16% lower than other methods (Vogel's Approximation Method (VAM), MODI and Least Cost Method (LCM), which equal to 38 unit. Most notably, TGM dramatically decreased execution time to merely 1.2s, compared to more traditional methods like NWCR (5.1s), LCM (3.8s) and VAM (2.9s). The resource utilization of TGM is 88%, the highest of the compared methods with the error rate only 0.02%, much lower than 0.08% of NWCR, which achieves efficient allocation.

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Furthermore, the TGM also achieved an average cost reduction in transportation cost by 32.08% under different supply-demand conditions based on the scenario analysis of cost optimization. The most cost reduction is seen in Scenario 5 with a 35.23% drop in cost from 88 units to 57 units optimized by TGM. Despite the immense variability of the scenarios, the method managed to find an optimal balance between cost minimization and computation time, with no convergence time higher than 2.8 seconds did.

6.1 Future Work or Scope

Future Work based on this study, we can engage in several promising directions for future work to further develop the Topologized Graphical Method (TGM) for NTO. The integration of artificial AI and ML techniques to enhance predictive analytics, automate decision-making, and optimize transportation routes dynamically based on real-time data is one important path forward. More sophisticated AI techniques, like deep reinforcement learning, can even enhance the cost minimization process by adapting to changing transportation patterns. Moreover, considering hybrid optimization methods integrating TGM with metaheuristic techniques like genetic algorithms or particle swarm optimization to improve computational efficiency and scalability for large-scale transportation networks is another avenue of future research. Further research direction can be the extension of this work to the specialized industries for resource allocation, where uncertainty plays a significant role such as supply chain management, urban logistics, disaster relief operations, etc. Additionally, modifying the existing model into a multi-objective one to find non-dominating solutions considering environmental sustainability, time elapsing, and financial risk assessment would give a more holistic decision-support model. Neutrosophic topological spaces without explicit topological constraints can also be another promising direction of exploration as it can potentially leads to improvement in the flexibility of TGM for splitting with real-world logistics problems. The investigation of multi-objective PNTP and the improvement of decision support systems combining the TGM could be areas of interest for future research. Moreover, without applying topological spaces, the neutrosophic topological spaces are planned to implement in further enhancement of TGM approach.

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