Several Results on Some Kinds of Continuity via Fuzzy Neutrosophic $\beta^{\wedge m}$-Closed Sets

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Abstract

In this paper, we defined some new kinds of continuous functions in fuzzy neutrosophic topology and called fuzzy neutrosophic $\beta^{\wedge m}$ -continuous, fuzzy neutrosophic weakly $\beta^{\wedge m}$ continuous, fuzzy neutrosophic strongly $\beta^{\wedge m}$-continuous, fuzzy neutrosophic $\beta^{\wedge m}$-contra continuous, fuzzy neutrosophic weakly $\beta^{\wedge m}$-contra continuous and fuzzy neutrosophic strongly $\beta^{\wedge m}$-contra continuous functions. Then, we defined the relationship between the define functions with their comparative.

Keywords: Fuzzy neutrosophic $\beta^{\wedge m}$- continuous; Neutrosophic weakly $\beta^{\wedge m}$ continuous; Fuzzy neutrosophic strongly $\beta^{\wedge m}$- continuous; Fuzzy neutrosophic $\beta^{\wedge m}$-contra continuous; Fuzzy neutrosophic weakly $\beta^{\wedge m}$-contra continuous; Fuzzy neutrosophic strongly $\beta^{\wedge m}$-contra continuous functions

1. Introduction

Zadeh proposed the notion of fuzzy sets (FS) in 1965 [1]. Numerous scholars have extended fuzzy set theory. K. Atanassov introduced intuitionistic fuzzy sets (IFS) as an extension set in 1983 [2], where.fuzzy sets indicate an element's degree of membership. The intuitionistic fuzzy sets provide a degree of membership and non-membership functions. Subsequent research focused on expanding the concept of intuitionistic fuzzy sets. F. Smarandache introduced the terms neutrosophy, neutrosophic set, and neutrosophic component in 1999 [3]. A defines two concepts: neutrosophic set (NS) and neutrosophic topological space (NTS) by A. A. Salama and S.A. Alblowi (2012) [4]. In 2013, I. Arockiarani, I. R. Sumathi, and J. Martina Jency [5] defined the fuzzy neutrophilic set. In 2014, I. Arockiarani and J. Martina Jency [5] developed the fuzzy neutrosophic topology. Fuzzy neutrosophic sets were defined as membership, non-membership, and indeterminacy degrees. In 2017, Y. Veereswari [6] presented the fuzzy neutrosophic continuous function.

In this study, the term of some new continuity of functions called, fuzzy neutrosophic $\beta^{\wedge m}$ - continuous, fuzzy neutrosophic weakly $\beta^{\wedge m}$ - continuous, fuzzy neutrosophic strongly $\beta^{\wedge m}$- continuous, fuzzy neutrosophic $\beta^{\wedge m}$-contra continuous, fuzzy neutrosophic weakly $\beta^{\wedge m}$-contra continuous and fuzzy neutrosophic strongly $\beta^{\wedge m}$-contra continuous functions via fuzzy neutrosophic topological spaces were introduced as generalization of F. Mohammed [7-13].Also, we discussed some new relationships among the studied functions. Finally, there are many applications of neutrosophic sets in numerous fields, so we can enhance our work, we will try in the future to apply this work in different fields such as many authors applications [14].

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2. Preliminaries

Definition 2.1 [10]: " Let X_N be a non-empty fixed set. The $f$uzzy $n$eutrosophic set (FNS), Ƙ_N is an object having the form Ƙ_N= $\{< x, \mu_{\text{Ƙ}_N}(x), \sigma_{\text{Ƙ}_N}(x), v_{\text{Ƙ}_N}(x) >: x \in X_N\}$ where the functions $\mu_{\text{Ƙ}_N}, \sigma_{\text{Ƙ}_N}, v_{\text{Ƙ}_N}: X_N \rightarrow [0,1]$ denote the degree of membership function (namely $\mu_{\text{Ƙ}_N}(x)$), the degree of indeterminacy function (namely $\sigma_{\text{Ƙ}_N}(x)$) and the degree of non-membership function (namely $v_{\text{Ƙ}_N}$(x)) respectively of each element x$\in X_N$ to the set Ƙ_N and $0 \leq \mu_{\text{Ƙ}_N}(x) + \sigma_{\text{Ƙ}_N}(x) + v_{\text{Ƙ}_N}(x) \leq 3$, for each $x \in X_N$."

Remark 2.2 [11]: "FNS Ƙ_N= $\{< x, \mu_{\text{Ƙ}_N}(x), \sigma_{\text{Ƙ}_N}(x), v_{\text{Ƙ}_N}(x) >: x \in X_N\}$ can be identified to an ordered triple < x, $\mu_{\text{Ƙ}_N}$, $\sigma_{\text{Ƙ}_N}$, $v_{\text{Ƙ}_N}$ > in [0, 1] on X_N ."

Lemma 2.3 [10]: "Let X_N be a non-empty set and the FNSs Ƙ_N and Ƒ_N be in the form:

Ƙ_N= $\{< x, \mu_{\text{Ƙ}_N}(x), \sigma_{\text{Ƙ}_N}(x), v_{\text{Ƙ}_N}(x) >: x \in X_N\}$ and $\varpi_{FN} = \{< x, \mu_{\text{Ƒ}_N}(x), \sigma_{\text{Ƒ}_N}(x), v_{\text{Ƒ}_N}(x) >: x \in X_N : x \in X_N\}$ on X_N . Then,

i. $\text{Ƙ}_N \subseteq \text{Ƒ}_N$iff $\mu_{\text{Ƙ}_N}$(x) ≤ $\mu_{\text{Ƒ}_N}$ (x), $\sigma_{\text{Ƙ}_N}$(x) ≤ $\sigma_{\text{Ƒ}_N}(x)$ and $v_{\text{Ƙ}_N}$(x) ≥ $v_{\text{Ƒ}_N}$ (x) for all $x \in X_N$,

ii. $\text{Ƙ}_N = \text{Ƒ}_N$ iff $\text{Ƙ}_N \subseteq \text{Ƒ}_N$and $\text{Ƒ}_N \subseteq \text{Ƙ}_N$,

iii. $1_N - \text{Ƙ}_N = \{< x, v_{\text{Ƙ}_N}(x), 1 - \sigma_{\text{Ƙ}_N}(x), \mu_{\text{Ƙ}_N}(x) > : x \in X_N\}$,

iv. $\text{Ƙ}_N \cup \text{Ƒ}_N = \{< x, Max(\mu_{\text{Ƙ}_N}(x), \mu_{\text{Ƒ}_N}(x)), Max(\sigma_{\text{Ƙ}_N}(x), \sigma_{\text{Ƒ}_N}(x)), Min(v_{\text{Ƙ}_N}(x), v_{\text{Ƒ}_N}(x)) > : x \in X_N\}$,

v. $\text{Ƙ}_N \cap \text{Ƒ}_N = \{< x, Min(\mu_{\text{Ƙ}_N}(x), \mu_{\text{Ƒ}_N}(x)), Min(\sigma_{\text{Ƙ}_N}(x), \sigma_{\text{Ƒ}_N}(x)), Max(v_{\text{Ƙ}_N}(x), v_{\text{Ƒ}_N}(x)) > : x \in X_N\}$,

vi. $0_N = < x, 0, 0, 1 >$ and $1_N = < x, 1, 1, 0 >$."

Definition 2.4 [10]: " Fuzzy $neutrosophic$ $t$opology (FNT) on a non-empty set X_N is a family τ of fuzzy neutrosophic subsets in X_N satisfying the following axioms.

i. 0_N, $1_N \in \tau_N$,

ii. $\text{Ƙ}_{N_1} \cap \text{Ƙ}_{N_2} \in \tau_N$ for any Ƙ_{N_1}, $\text{Ƙ}_{N_2} \in \tau_N$,

iii. $\cup\, \text{Ƙ}_{N_j} \in \tau_N$, $\forall$ $\{\text{Ƙ}_{N_j}$: j$\in$ J$\} \subseteq \tau_N$.

The pair (X_N , τ_N) is called $f$uzzy $neutrosophic$ $t$opological space ($FNTS$). Every elements of τ are called $f$uzzy $neutrosophic$-open sets (FN-open set). The complement of FN-open set in the FNTS (X_N , τ_N) is called $f$uzzy $n$eutrosophic -closed set (FN-closed set)."

Definition 2.5 [10]: "Let (X_N , τ_N) is FNTS and Ƙ_N= {< x, $\mu_{\text{Ƙ}_N}$(x), $\sigma_{\text{Ƙ}_N}$(x), $v_{\text{Ƙ}_N}$ (x) >: $x \in X_N$ } is FNS in X_N . Then the $f$uzzy $neutrosophic$ -closure ($FNcl$) and the $f$uzzy $n$eutrosophic -interior ($FNint$) of $\text{Ƙ}_N$are defined by:

$FNcl(\text{Ƙ}_N)$ = ∩ {G_N: $G_N$is FN-closed set in X_N and $\text{Ƙ}_N \subseteq G_N$},

$FNint$ (Ƙ_N) = ∪ {G_N: $G_N$is FN-$open$ set in X_N and $G_N \subseteq \text{Ƙ}_N$}.

Now, the $FNcl(\text{Ƙ}_N)$ is FN-closed set and $FNint(\text{Ƙ}_N)$ is FN-open set in X_N . Further,

i. Ƙ_N is FN-closed set in X_N iff $FNcl(\text{Ƙ}_N) = \text{Ƙ}_N$,

ii. Ƙ_N is FN-open set in X_N iff $FNint(\text{Ƙ}_N) = \text{Ƙ}_N$."

Proposition 2.6 [12]: " Let (X_N , τ_N) is $FNTS$ and Ƙ_N, Ƒ_N are $FNSs$ in X_N . Then, the following properties hold:

i. $FNint(\text{Ƙ}_N) \subseteq \text{Ƙ}_N$ and $\text{Ƙ}_N \subseteq FNcl(\text{Ƙ}_N)$,

ii. $\text{Ƙ}_N \subseteq \text{Ƒ}_N \Longrightarrow FNint(\text{Ƙ}_N) \subseteq FNint(\text{Ƒ}_N)$ and $\text{Ƙ}_N \subseteq \text{Ƒ}_N \Longrightarrow FNcl(\text{Ƙ}_N) \subseteq FNcl(\text{Ƒ}_N)$,

iii. $FNint(FNint(\text{Ƙ}_N))$ = $FNint(\text{Ƙ}_N)$ and $FNcl(FNcl(\text{Ƙ}_N))$ = $FNcl(\text{Ƙ}_N)$,

iv. $FNint(\text{Ƙ}_N \cap \text{Ƒ}_N)$ = $FNint(\text{Ƙ}_N) \cap FNint(\text{Ƒ}_N)$ and $FNcl(\text{Ƙ}_N \cup \text{Ƒ}_N)$ = $FNcl(\text{Ƙ}_N) \cup FNcl$ (Ƒ_N),

v. $FNint(1_N)$ = 1_N and $FNcl(0_N)$ = 0_N."

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Definition 2.7 [13]:" Let (X_N, τ_N) be $FNTS$ for each $Ƙ_N, Ƒ_N \in X_N$. A $fuzzy$ set $Ƙ_N$ is called $fuzzy\ neutrosophic\ \beta^{\wedge m} - closed\ set$ $(FN\beta^{\wedge m} - closed\ set)$. If $FNcl\ (FNint\ (Ƙ_N)) \subseteq Ƒ_N$ where $Ƙ_N \subseteq Ƒ_N$ and $Ƒ_N$ is an $FN\beta - open\ set\ in\ X_N$. A ς_{FN} is called an fuzzy neutrosophic $\beta^{\wedge m}\ open$ (FN$\beta^{\wedge m}$-open set) if and only if $1_N - Ƙ_N$ is an $FN\beta^{\wedge m} - closed\ set$."

Definition 2.8[13]:" Let $(\mathrm{X_N}, \tau_\mathrm{N})$ is $FNTS$ and $Ƙ_N = \{< x, \mu_{Ƙ_N}(x), \sigma_{Ƙ_N}(x), \nu_{Ƙ_N}(x) >: x \in X_N\}$ is FNS in $\mathrm{X_N}$ .Then, the $fuzzy\ neutrosophic\ \beta^{\wedge m}$- $closure$ $(FN\beta^{\wedge m}cl)$ and the $fuzzy\ neutrosophic\ \beta^{\wedge m}$-interior $(FN\beta^{\wedge m}int)$ of $Ƙ_N$ are defined by:

i. $FN\beta^{\wedge m}cl\ (Ƙ_N) = \cap\ \{G_N: G_N$ is FN$\beta^{\wedge m} - closed\ set\ in\ \mathrm{X_N}$ and $Ƙ_N \subseteq G_N\}$,

ii. $FN\beta^{\wedge m}int(Ƙ_N) = \cup\ \{$ K$_{\mathrm{FN}}$: G_N is FN$\beta^{\wedge m} - open\ set\ in\ X_N$ and $G_N \subseteq Ƙ_N\}$.

Now, $FN\beta^{\wedge m}cl(Ƙ_N)$ is FN$\beta^{\wedge m} - closed\ set$ and FN$\beta^{\wedge m}int(Ƙ_N)$ is FN$\beta^{\wedge m} - open\ set\ in\ \mathrm{X_N}$. Further,

i- $Ƙ_N$ is $FN\beta^{\wedge m} - closed\ set\ in\ \mathrm{X_N}$ if and only if FN$\beta^{\wedge m}cl(Ƙ_N) = (Ƙ_N)$,
ii- $Ƙ_N$ is FN$\beta^{\wedge m} - open\ set\ in\ \mathrm{X_N}$ if and only if FN$\beta^{\wedge m}int(Ƙ_N) = (Ƙ_N)$."

Definition 2.9[14]:" Let $(\mathrm{X_N}, \tau_\mathrm{N})$ is $FNTS$. $Fuzzy\ neutrosophic$ set $Ƙ_N$ is called $fuzzy\ neutrosophic$ generalized-closed set $(FNG - closed\ set)$ if $FNcl\ (Ƙ_N) \subseteq Ƒ_N$ wherever, $Ƙ_N \subseteq Ƒ_N$ and $Ƒ_N$ is $FN - open\ set\ in\ \mathrm{X_N}$. $Ƙ_N$ is said to be $fuzzy\ neutrosophic\ generalized$-open set ($FNG$-$open\ set$) in $(\mathrm{X_N}, \tau_\mathrm{N})$ if the complement $1_\mathrm{N} - Ƙ_N$ is FNG$-closed\ set\ in$ $(\mathrm{X_N}, \tau_\mathrm{N})$."

Proposition 2.10[13]: "For any FNS in FNTS. We have,

i- $Every\ FN -\ open\ set\ is\ FN\beta - open\ set$
ii- $Every\ FN -\ closed\ set\ is\ FN\beta^{\wedge m} - closed\ set,$

Definition 2.11 [10]: "Let $(\mathrm{X_N}, \tau_\mathrm{x})$ and **(**$\mathrm{Y_N}, \tau_\mathrm{y})$ are two $FNTSs$. Then, a function $\vartheta: (\mathrm{X_N}, \tau_\mathrm{x}) \to (\mathrm{Y_N}, \tau_\mathrm{y})$ is called $fuzzy\ neutrosophic - continuous$ $(FN - cont$, for short) if the inverses image of every $FNOS$ $(FNCS)$ in $(\mathrm{Y_N}, \tau_\mathrm{y})$ is $FNOS$ $(FNCS)$ in $(\mathrm{X_N}, \tau_\mathrm{x})$."

Definition 2.12 [10]:" Let $(\mathrm{X_N}, \tau_\mathrm{x})$ and $(\mathrm{Y_N}, \tau_\mathrm{y})$ are two $FNTSs$. Then, a function $\vartheta: (\mathrm{X_N}, \tau_\mathrm{x}) \to (\mathrm{Y_N}, \tau_\mathrm{y})$ is called fuzzy $neutrosophic - contra\ continuous$ $(FN - ccont$, for short) if the inverses image of every $FNOS$ $(FNCS)$ in $(\mathrm{Y_N}, \tau_\mathrm{y})$ is $FNCS$ $(FNOS)$ in $(\mathrm{X_N}, \tau_\mathrm{x})$. "

Proposition 2.13 [10]: "Let $Ƙ_N$ is FNS in $\mathrm{X_N}$, $Ƒ_N$ is FNS in $\mathrm{Y_N}$and $\vartheta: \mathrm{X_N} \to \mathrm{Y_N}$is a function. Then,

i. $\vartheta^{-1}(1_\mathrm{N}) = 1_\mathrm{N}$, $\vartheta^{-1}(0_\mathrm{N}) = 0_\mathrm{N}$,

ii. $\vartheta^{-1}(\mathrm{y}) = x$,

iii. $Ƙ_{N_1} \subseteq Ƙ_{N_2} \Rightarrow \vartheta(Ƙ_{N_1}) \subseteq \vartheta(Ƙ_{N_2})$ and $Ƒ_{N_1} \subseteq Ƒ_{N_2} \Rightarrow \vartheta^{-1}(Ƒ_{N_1}) \subseteq \vartheta^{-1}(Ƒ_{N_2})$."

§3: Some New Functions in Fuzzy Neutrosophic Topology Functions

States a new concepts in fuzzy neutrosophic topological space and called (fuzzy neutrosophic $\beta^{\wedge m}$-continuous, fuzzy neutrosophic weakly $\beta^{\wedge m}$-continuous fuzzy neutrosophic strongly $\beta^{\wedge m}$-continuous and fuzzy neutrosophic irresolute $\beta^{\wedge m}$-continuous) functions.

Definition 3.1: Let $(\mathrm{X_N}, \tau_\mathrm{N})$ be $FNTS$ for each $Ƙ_N, Ƒ_N \in \mathrm{X_N}$. A fuzzy neutrosophic set $Ƙ_N$ is called:

i- Fuzzy neutrosophic weakly $\beta^{\wedge m}$- closed set ($FNW\beta^{\wedge m}$-$closed$ set) If $FN\beta^{\wedge m}cl(Ƙ_N) \subseteq Ƒ_N$ where $Ƙ_N \subseteq Ƒ_N$ and $Ƒ_N$ is a FNSemi$-open\ set\ in\ \mathrm{X_N}$. A $Ƙ_N$ is called an fuzzy neutrosophic weakly $\beta^{\wedge m} - open\ set$ (FNW$\beta^{\wedge m} - open\ set$) if and only if $1_\mathrm{N} - Ƙ_N$ is an $FNW\beta^{\wedge m} - closed\ set$.

ii- Fuzzy neutrosophic weakly generalized $\beta^{\wedge m} - closed\ set$ $(FNWG\beta^{\wedge m} - closed\ set)$ If $FNcl(FNint(\varsigma_{FN})) \subseteq Ƒ_N$ where $Ƙ_N \subseteq Ƒ_N$ and $Ƒ_N$ is an FN$\beta^{\wedge m} - open\ set\ in\ \mathrm{X_N}$. A $Ƙ_N$ is called an fuzzy neutrosophic weakly generalized $\beta^{\wedge m} - open\ set$ ($FNWG\beta^{\wedge m} - open\ set$) if and only if $1_\mathrm{N} - Ƙ_N$ is an FNWG$\beta^{\wedge m} - closed\ set$.

iii- Fuzzy neutrosophic strongly generalized $\beta^{\wedge m}$- closed set (FNSG$\beta^{\wedge m}$-closed set). If FN$\beta^{\wedge m}cl(Ƙ_N) \subseteq Ƒ_N$ where $Ƙ_N \subseteq Ƒ_N$ and $Ƒ_N$ is an FNG$\beta^{\wedge m}$-open set in $\mathrm{X_N}$. A $Ƙ_N$ is called an fuzzy neutrosophic strongly generalized $\beta^{\wedge m}$- open set (FNSG$\beta^{\wedge m}$-open set) if and only if $1_\mathrm{N} - Ƙ_N$ is FNSG$\beta^{\wedge m}$-$closed$ set.

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Proposition 3.2:

i. $Every\ FN-closed\ set\ is\ FNWG\beta^{\wedge m}-closed\ set.$

ii. $Every\ FN-closed$ set is FNSG$\beta^{\wedge m}$-closed set.

Proof:-

i- Let Ƙ_N is $FN-closed\ set$ in $FNTS(X_N,\tau_N)$. Then by Definition (2.5) (i).

We have, $\text{Ƙ}_N = FNcl(S_N)$ …..(1). And by Proposition (2.6) (i) we get $FNint\ (\text{Ƙ}_N) \subseteq \text{Ƙ}_N$ ……(2).

But, $FNcl\ (FNint\ (\text{Ƙ}_N)) \subseteq FNcl(\ \text{Ƙ}_N)$. By (1) we get, $FNcl(FNint\ (\text{Ƙ}_N)) \subseteq \text{Ƙ}_N$.

Now, let ƒ_N be FN$\beta^{\wedge m}-open\ set$ such that $\text{Ƙ}_N \subseteq \text{ƒ}_N$.

Then, $FNcl(FNint\ (\text{Ƙ}_N)) \subseteq \text{Ƙ}_N \subseteq \text{ƒ}_N$.

Therefore, $FNcl(FNint\ (\text{Ƙ}_N)) \subseteq \text{ƒ}_N$.

Hence Ƙ_N is $FNWG\beta^{\wedge m}-closed\ set\ in\ (X_N,\tau_N)$.

ii. Similar to proof **i.**

Definition 3.3: Let (X_N,τ_x) and (Y_N,τ_y) be any two FNTS. A function $\vartheta:(X_N,\tau_x)\rightarrow(Y_N,\tau_y)$ is called *fuzzy neutrosophic* $\beta^{\wedge m}$*-continuous* ($FN\beta^{\wedge m}$-cont.) if the inverse image of every FN-closed (FN-open) set in (Y_N,τ_y) is FN$\beta^{\wedge m}$-closed (FNβ^{m}-open) set in (X_N,τ_x).

Example 3.4: Let X =Y = $\{a_1\}$ define FNSs ω_N and δ_N, in X_N as follows:

$\omega_N = <x,a_1(0.2,0.5,0.4)>$, $\delta_N = <x,a_1(0.1,0.7,0.4)>$,

The family, $\tau_x = \{0_N, 1_N, \omega_N, \delta_N\}$ is FNT.

And, $\lambda_N = <y,a_1(0.3,0.7,0.6)>$. With the family, $\tau_y = \{0_N, 1_N, \lambda_N\}$ is FNT.

Define $\vartheta:(X_N,\tau_x)\rightarrow(Y_N,\tau_y)$ as follows $\vartheta(a_1) = a_1$.

If, $\lambda_N = <y,a_1(0.3,0.7,0.6)>$ is FN-open set in τ_y.

Then,$\vartheta^{-1}(1_N - \lambda_N) = <x,a_1(0.6,0.3,0.3)> = \text{Ƙ}_N$.

Now, if $\text{ƒ}_N = \delta_N = <x,a(0.1,0.7,0.4)>$ where, ƒ_N is FN-open set in τ_x such that, $\text{Ƙ}_N \subseteq \text{ƒ}_N$. By **Proposition (2.10)(i) .** If ƒ_N is FNβ-open set. Then, $FNint\ (\text{Ƙ}_N) = 0_N$, $FNcl(FNint\ (\text{Ƙ}_N)) = 0_N$.

And, $FNcl(FNint\ (\text{Ƙ}_N)) \subseteq \text{ƒ}_N$.

Since, $<x,a_1(0,0,1)> \subseteq <x,a_1(0.1,0.7,0.4)$.

Therefore, Ƙ_N is FN$\beta^{\wedge m}$-closed set in τ_x. Hence ϑ is (FN$\beta^{\wedge m}$cont.) function.

Theorem 3.5: Let (X_N,τ_x) and (Y_N,τ_y) be any two FNTS. If the function $\vartheta:(X_N,\tau_x)\rightarrow(Y_N,\tau_y)$ is (FN-cont.). Then, ϑ is (FN$\beta^{\wedge m}$-cont.) function.

Proof:- Suppose that, $\vartheta:(X_N,\tau_x)\rightarrow(Y_N,\tau_y)$ is (FN-cont.) function.

If, λ_N is FN-open set in τ_y so, $1_N - \lambda_N$ is FN-closed set in τ_y.

Then, by **Definition 2.11.** $\vartheta^{-1}(1_N - \lambda_N)$ is FN-closed set in τ_x.

By, **Proposition 2.10 (ii)**. We have every FN-closed set is $FN\beta^{\wedge m}$-closed set.

So, $\vartheta^{-1}(1_N - \lambda_N)$ is $FN\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is $(FN\beta^{\wedge m}$-cont.) function.

Remark 3.6: The following example demonstrates that the converse of Theorem 3.5 is not true in general.

Example 3.7: Take, **Example 3.4.** Then ϑ is (FN$\beta^{\wedge m}$cont.) function. But ϑ is not (FN-cont.) function. Because $\vartheta^{-1}(1_N - \lambda_N) = <x,a_1(0.6,0.3,0.3)> \notin (1_N - \tau_x)$.

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Definition 3.8: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A function $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is called fuzzy neutrosophic $\beta^{\wedge m}$- contra continuous (FN$\beta^{\wedge m}$-ccont.) if the inverse image of every FN-closed (FN-open) set in (Y_N, τ_y) is $FN\beta^{\wedge m}$- open ($FN\beta^{\wedge m}$- closed) set in (X_N, τ_x).

Example 3.9: Take, **Example 3.2**. If, $\lambda_N = < x, a_1(0.6, 0.3, 0.3)>$.

The family, $\tau_y = \{0_N, 1_N, \lambda_N\}$ is FNT.

Define $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ as follows ϑ(a) = a.

If $\lambda_N = < y, a_1(0.6, 0.3, 0.3)>$ is FN-open set in τ_y.

Then,$\vartheta^{-1}(\lambda_N) = < x, a_1(0.6, 0.3, 0.3)> = Ꜧ_N$.

Now, if $Ƒ_N = \delta_N = < x, a_1(0.1, 0.7, 0.4) >$ where, $Ƒ_N$ is FN-open set in τ_x such that, $Ꜧ_N \subseteq Ƒ_N$.

By **Proposition (2.10) (i).** We have, M_N is FNβ-open set.

Then, $FNint\ (Ꜧ_N) = 0_N$, $FNcl(FNint\ (Ꜧ_N)) = 0_N$.

And, $FNcl(FNint\ (Ꜧ_N)) \subseteq Ƒ_N$.

Since, $< x, a_1(0, 0, 1) > \subseteq < x, a_1(0.1, 0.7, 0.4)>$

Therefore, $Ꜧ_N$ is $FN\beta^{\wedge m}$-closed set inτ_x. Hence ϑ is ($FN\beta^{\wedge m}$ccont.)function.

Theorem 3.10: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. If function $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is (FN-ccont.) function. Then, ϑ is (FN$\beta^{\wedge m}$-ccont.) function.

Proof:- Suppose that, $\vartheta : (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is (FN-ccont.) function.

If, λ_N is FN-open set in τ_y. Then, by **Definition 2.12**. $\vartheta^{-1}(\lambda_N)$ is FN-closed set in τ_x.

By, **Proposition (2.10)(ii).** Every FN-closed set is $FN\beta^{\wedge m}$-closed set.

So, $\vartheta^{-1}(\lambda_N)$ is $FN\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is (FN$\beta^{\wedge m}$-ccont)function.

Remark 3.11: The convers of ***Theorem* 3.10** is not true in general as shown by the following example:

Example 3.12: Take, the **Example 3.9.** Then ϑ is ($FN\beta^{\wedge m}$ccont.) function. But, ϑ is not (FN-ccont.) function. Because $\vartheta^{-1}(\lambda_N) = < x, a_1(0.6, 0.3, 0.3)> \notin (1_N - \tau_x)$.

Remark 3.13: The relation between ((FN$\beta^{\wedge m}$-cont.) and (FN$\beta^{\wedge m}$-ccont.)) functions is independent, as demonstrated by the following example:

Example 3.14:

1- Take, **Example 3.4**. Then, ϑ is (FN$\beta^{\wedge m}$-cont) function.

But, ϑ is *not* (FN$\beta^{\wedge m}$-ccont) function. Since, $\vartheta^{-1}(\lambda_N) \nsubseteq Ƒ_N$.

2- Take, **Example 3.9.** Then, ϑ is (FN$\beta^{\wedge m}$-ccont) function.

But, ϑ is not (FN$\beta^{\wedge m}$-cont) function. Since, $\vartheta^{-1}(1_N - \lambda_N) \nsubseteq Ƒ_N$.

The next theorem shows the condition gave to the **Remark 3.15** to give new relations between ((FN$\beta^{\wedge m}$-cont.) and (FN$\beta^{\wedge m}$-ccont.)) functions.

Theorem 3.15: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. And $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is a function. ϑ is (FN$\beta^{\wedge m}$-ccont.) iff ϑ is (FN$\beta^{\wedge m}$-cont.) whenever, every set in τ_y is FN- clopen set.

Proof:-Let ϑ is (FN$\beta^{\wedge m}$-ccont.) function. If, β_N is FN-open set in τ_y.

Then, by **Definition 3.8**. $\vartheta^{-1}(\lambda_N)$ is FN$\beta^{\wedge m}$-closed set in τ_x.

But, $\lambda_N$is FN-clopen set in τ_y we get, $\lambda_N = 1_N - \lambda_N$ is FN-closed set in τ_y. Then, $\vartheta^{-1}(1_N - \lambda_N)$ is FN$\beta^{\wedge m}$-closed set in τ_x. Hence, by **Definition 3.5**. ϑ is (FN$\beta^{\wedge m}$-cont.) function.

Conversely, the proof is direct.

Definition 3.16: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A function $\vartheta: (X_N, \tau_x) \to (Y_N, \tau_y)$ is called fuzzy neutrosophic weakly $\beta^{\wedge m}$-continuous (FNW$\beta^{\wedge m}$-cont.) if the inverse image of every FN-open (FN-closed) set in (Y_N, τ_y) is FNWGβ^{m}-open (FNWG$\beta^{\wedge m}$-closed) set in (X_N, τ_x).

Example 3.17: Let $\mathrm{X_N} = \mathrm{Y_N} = \{a_1, b_1\}$ define FNS ω_N in $\mathrm{X_N}$ and λ_N in $\mathrm{Y_N}$ as follows:

$\omega_\mathrm{N} = < x, a_1\ (0.5, 0.5, 0.4), b_1(0.5, 0.5, 0.5) >$.

And, the family, $\tau_\mathrm{x} = \{0_\mathrm{N}, 1_\mathrm{N}, \lambda_\mathrm{N}\}$ be FNT where, $\lambda_\mathrm{N} = < y, a_1\ (0.5, 0.5, 0.6), b_1(0.4, 0.5, 0.5) >$.

And define the family, $\tau_\mathrm{y} = \{0_\mathrm{N}, 1_\mathrm{N}, \lambda_\mathrm{N}\}$ to be FNT in $\mathrm{Y_N}$.

Define $\vartheta: (X_N, \tau_x) \to (Y_N, \tau_y)$ as follows $\vartheta(a_1) = b_1$ and $\vartheta(b_1) = a_1$.

If, $\lambda_\mathrm{N} = < y, a_1\ (0.5, 0.5, 0.3), b_1(0.4, 0.5, 0.5) >$ is FN-open set in τ_y.

Then, $\vartheta^{-1}(1_\mathrm{N} - \lambda_\mathrm{N}) = < x, a_1\ (0.5, 0.5, 0.4), b_1(0.3, 0, 5, 0.5) > = Ꞗ_N$.

Now, if $Ꞙ_N = \omega_\mathrm{N}$ is $FN\beta^\mathrm{m}$-open set such that, $Ꞗ_N \subseteq Ꞙ_N$.

Then, $FNint(Ꞗ_N) = 0_\mathrm{N}$ and $FNcl(FNint(Ꞗ_N)) = 0_\mathrm{N}$. Therefore, $FNcl(FNint\ (Ꞗ_N)) \subseteq Ꞙ_N$.

Since, $< x, a_1(0,0,1), b_1\ (0,0,1) > \subseteq < x, a_1\ (0.5, 0.5, 0.4), b_1(0.5, 0.5, 0.5) >$.

So, $Ꞗ_N$ is FNWG$\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is (FNW$\beta^{\wedge m}$ cont.) function.

Theorem 3.18: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. If function $\vartheta: (X_N, \tau_x) \to (Y_N, \tau_y)$ is (FN-cont.) function. Then, ϑ is (FNW$\beta^{\wedge m}$-cont.) function.

Proof:- Suppose that, $\vartheta : (X_N, \tau_x) \to (Y_N, \tau_y)$ is (FN-cont.) function.

If, λ_N is FN-open set in τ_y so, $1_N - \lambda_N$ is FN-closed set in τ_y.

Then, by **Definition 2.9**. $\vartheta^{-1}(1_N - \lambda_N)$ is FN-closed set in τ_x.

By, **Proposition 3.2.** We have every FN-closed set is FNWGβ^{m}-closed set.

So, $\vartheta^{-1}\ (1_N - \lambda_N)$ is FNWG$\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is (FNW$\beta^{\wedge m}$-cont.) function.

Remark 3.19: The convers of ***Theorem* 3.18** is not true in general as shown by the following example:

Example 3.20: Take, **Example 3.17**. Then ϑ is (FNW$\beta^{\wedge m}$cont.) function. But ϑ is not (FN-cont.) function. Because $\vartheta^{-1}(1_\mathrm{N} - \lambda_\mathrm{N}) = < x, a_1\ (0.5, 0.5, 0.4), b_1(0.3, 0, 5, 0.5) > \notin (1_\mathrm{N} - \tau_x)$.

Definition 3.21: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A function $\vartheta: (X_N, \tau_x) \to (Y_N, \tau_y)$ is called fuzzy neutrosophic weakly $\beta^{\wedge m}$-contra continuous (FNW$\beta^{\wedge m}$-ccont) if the inverse image of every FN-open (FN-closed) set in (Y_N, τ_y) is FNWG$\beta^{\wedge m}$- closed (FNWG$\beta^{\wedge m}$- open) set in (X_N, τ_x).

Example 3.22: Let $\mathrm{X_N} = \mathrm{Y_N} = \{a_1, b_1\}$ define FNS ω_N in $\mathrm{X_N}$ and λ_N in $\mathrm{Y_N}$ as follows:

$\omega_\mathrm{N} = < x, a_1(0.4, 0.5, 0.5), b_1(0.5, 0.5, 0.4) >$.

The family, $\tau_x = \{0_\mathrm{N}, 1_\mathrm{N}, \lambda_\mathrm{N}\}$ be FNT.

And, $\lambda_\mathrm{N} = < y, a_1(0.5, 0.5, 0.6), b_1(0.4, 0.5, 0.5) >$ with the family, $\tau_\mathrm{y} = \{0_\mathrm{N}, 1_\mathrm{N}, \lambda_\mathrm{N}\}$ to be FNT.

Define $\vartheta: (X_N, \tau_x) \to (Y_N, \tau_y)$ as follows $\vartheta(a_1) = b_1$ and $\vartheta(b_1) = a_1$.

If, $\lambda_\mathrm{N} = < y, a_1\ (0.5, 0.5, 0.6), b_1(0.4, 0.5, 0.5) >$ is FN-open set in τ_y.

Then, $\vartheta^{-1}(\lambda_\mathrm{N}) = < x, a_1\ (0.4, 0.5, 0.5), b_1(0.5, 0, 5, 0.6) > = Ꞗ_N$.

Now, if $Ꞙ_N = \omega_\mathrm{N}$ is $FN\beta^{\wedge m}$-open set such that, $Ꞗ_N \subseteq Ꞙ_N$.

So, $FNint(Ꞗ_N) = 0_\mathrm{N}$ and $FNcl(FNint(Ꞗ_N)) = 0_\mathrm{N}$. Therefore, $FNcl(FNint\ (Ꞗ_N)) \subseteq Ꞙ_N$.

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Since,$< \ x, a_1 \ (0,0,1), b_1 \ (0,0,1) > \subseteq < \ x, a_1 \ (0.4, 0.5, 0.5), b_1 (0.5, 0.5, 0.4)$.

$So, Ƙ_N \ is \ FNWG\beta^m - closed \ set \ in \ \tau_x. Hence, \vartheta \ is \ (FNW\beta^{\wedge m}$ccont.) function.

Theorem 3.23: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. If $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is (FN-ccont.) function. Then, ϑ is (FNW$\beta^{\wedge m}$-ccont.) function.

Proof:- Suppose that, $\vartheta : (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is (FN-ccont.) function.

If, λ_N is FN-open set in τ_y . Then, by ***Definition* 2.12**. $\vartheta^{-1}(\lambda_N)$ is FN-closed set in τ_x.

And, by, **Proposition 3.2.** We have every FN-closed set is $FNWG\beta^{\wedge m}$-closed set. So,ϑ^{-1} (λ_N) is

$FNWG\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is $(FNW\beta^{\wedge m}$-cont.) function.

Remark 3.24: The convers of ***Theorem* 3.26** is not true in general as shown by the following example:

Example 3.25: Take, **Example 3.22**. Then ϑ is $(FNW\beta^{\wedge m}$-ccont.) function. But ϑ is not (FN-ccont.) function. Because $\vartheta^{-1}(\lambda_N) = < \ x, a_1$ (0.4 ,0.5, 0.5), b_1 (0.5,0,5,0.6) $> \notin (1_N - \tau_x)$.

Remark 3.26: The relation between ((FNW$\beta^{\wedge m}$-cont.) and (FNW$\beta^{\wedge m}$-ccont.)) functions is independent, as shown by the following example:

Example 3.27:

1- Take, **Example 3.17**. Then, ϑ is (FNW$\beta^{\wedge m}$-cont) function. But, ϑ is not (FNW$\beta^{\wedge m}$-ccont) function. Since, $\vartheta^{-1}(\lambda_N) \nsubseteq ƒ_N$.
2- Take, **Example 3.22**. Then, ϑ is (FNW$\beta^{\wedge m}$-ccont) function. But, ϑ is not (FNW$\beta^{\wedge m}$-cont) function. Since, $\vartheta^{-1}(1_N - \lambda_N) \nsubseteq ƒ_N$.

The next theorem shows the relation between ((FNW$\beta^{\wedge m}$-cont.)and (FNW$\beta^{\wedge m}$-ccont.)) functions.

Theorem 3.28: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is a function. ϑ is (FNW$\beta^{\wedge m}$-ccont.) if and only if ϑ is (FNW$\beta^{\wedge m}$-cont.) whenever, every FNS in τ_y is clopen set.

Proof:- Let ϑ is (FNW$\beta^{\wedge m}$-ccont.) $function$. If, β_N is FN-open set in τ_y.

Then, by ***Definition* 3.21** $\vartheta^{-1}(\lambda_N)$ is FNW$\beta^{\wedge m}$G-closed set in τ_x.

But, $\lambda_N$is FN-clopen set in τ_y we get, $\lambda_N = 1_N - \lambda_N$.

So, $1_N - \lambda_N$ is FN-closed set in τ_y. Then, ϑ^{-1} $(1_N - \lambda_N)$ is FNWG$\beta^{\wedge m}$-closed set in τ_x. Hence, by **Definition 3.18**. ϑ is (FN$\beta^{\wedge m}$-cont.) function.

Conversely; the proof is direct.

Definition 3.29: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A function $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ is called fuzzy neutrosophic strongly $\beta^{\wedge m}$continuous (FNS$\beta^{\wedge m}$-cont) if $\vartheta^{-1}(\lambda_N)$ is FN-closed (FN-open) set in (X_N, τ_x) for every FNSG$\beta^{\wedge m}$- closed (FNSG$\beta^{\wedge m}$- open) set λ_N in (Y_N, τ_y) .

Example 3.30: Let X = $\{a_1\}$ define FNSs ω_N and λ_N in X_N as follows:

$\omega_N = < x, a_1(0.3 ,0.4, 0.1)>$, $\psi_N = < x, a_1(0.5 ,0.7, 0) >$,

The family, $\tau_x = \{0_N, 1_N, \omega_N, \psi_N \}$ is FNTS.

And, $\lambda_N = < y, a$ (0.6 ,0.8, 0.1) > be FNS with the family, $\tau_y = \{0_N, 1_N, \lambda_N\}$ is FNT.

Define $\vartheta: (X_N, \tau_x) \rightarrow (Y_N, \tau_y)$ as follows $\vartheta(a_1) = a_1$.

If, $\lambda_N = < y, a_1(0.6 ,0.8, 0.1 \ ) >$ is FN-open set in τ_y then,

$\vartheta^{-1}(1_N - \lambda_N) = < x, a_1(0.1 ,0.2, 0.6)> = Ƙ_N$. Now if, $G_N = < x, 0.2 ,0.3, 0.5 >$.

If, $ƒ_N = \omega_N$ where, $ƒ_N$is $FNG\beta^{\wedge m}$-open set such that, $Ƙ_N \subseteq ƒ_N$. Then, $FN\beta^{\wedge m}cl$ $(Ƙ_N)$ =G_N and $FN\beta^{\wedge m}cl$ $(Ƙ_N) \subseteq ƒ_N$.

Since, $< x, 0.2\ ,0.3, 0.5 >\ \subseteq\ < x, 0.3\ ,0.4, 0.1>$. So, $Ƙ_N$ is $FNSG\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is (FNS$\beta^{\wedge m}$-cont.) function.

Theorem 3.31: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. If ϑ: $(X_N, \tau_x) \to (Y_N, \tau_y)$ is (FN-cont.) function. Then, ϑ is (FNS$\beta^{\wedge m}$-cont.) function.

Proof:- Similar to proof **Theorem 3.18**

Remark 3.32: The convers of ***Theorem* 3.31** is not true in general as shown by the following example:

Example 3.33: Take, **Example 3.30.** Then ϑ is (FNS$\beta^{\wedge m}$-cont.) function. But ϑ is not (FN-cont.) function. Because $\vartheta^{-1}(1_N - \lambda_N) = < x, a(0.1\ ,0.2, 0.6\ )> \notin (1_N - \tau_x)$.

Definition 3.34: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A function ϑ: $(X_N, \tau_x) \to (Y_N, \tau_y)$ is called fuzzy neutrosophic strongly $\beta^{\wedge m}$ –contra continuous (FNS$\beta^{\wedge m}$-ccont) if $\vartheta^{-1}(\lambda_N)$ is FN-closed (FN-open) set in (X_N, τ_x) for every FNSG$\beta^{\wedge m}$- open (FNSG$\beta^{\wedge m}$- closed) λ_N in (Y_N, τ_y) .

Example 3.35: Let X = $\{a_1\}$ define FNSs ω_N and λ_N in X_N as follows:

$\omega_N = < x, a_1(0.3\ ,0.4, 0.1)>$, $\psi_N = < x, a_1(0.5\ ,0.7, 0) >$,

The family, $\tau_x = \{0_N, 1_N, \omega_N, \psi_N\}$ is FNTS.

Take, $\lambda_N = < y, a_1(\ 0.1\ ,0.2, 0.6) >$ with the family, $\tau_y = \{0_N, 1_N, \lambda_N\}$ to be FNT.

Define ϑ: $(X_N, \tau_x) \to (Y_N, \tau_y)$ as follows $\vartheta(a_1) = a_1$.

If, $\lambda_N = < y, a_1(0.1\ ,0.2, 0.6\ \ ) >$ is FN-open set in τ_y.Then,$\vartheta^{-1}(\lambda_N) = < x, a_1(0.1\ ,0.2, 0.6)> = Ƙ_N$. Now if, $Ǥ_N = < x, 0.2\ ,0.3, 0.5 >$ and $ƒ_N = \omega_N$ where, $ƒ_N$ is FNG$\beta^{\wedge m}$-open set such that,

$Ƙ_N \subseteq ƒ_N$. Then, $FN\beta^{\wedge m}cl\ (Ƙ_N) = Ǥ_N$ and $FN\beta^{\wedge m}cl\ (Ƙ_N) \subseteq ƒ_N$.

Since, $< x, 0.2\ ,0.3, 0.5 >\ \subseteq < x, 0.3\ ,0.4, 0.1>$. So, $Ƙ_N$ is $FNSG\beta^{\wedge m}$-closed set in τ_x.

Hence, ϑ is (FNS$\beta^{\wedge m}$-ccont.) function.

Theorem 3.36: Let (X_N, τ_x) and (Y_N, τ_y) be any two FNTS. A map ϑ: $(X_N, \tau_x) \to (Y_N, \tau_y)$ is (FN-ccont.) function. Then, ϑ is (FNS$\beta^{\wedge m}$-ccont.) function.

Proof:- Similar to proof **Theorem 3.23**

Remark 3.37: The convers of **Theorem 3.36** is not true in general as shown by the following example:

Example 3.38: Take, E**xample 3.35**. Then ϑ is (FNW$\beta^{\wedge m}$-ccont.)function. But ϑ is not (FN-ccont.) function. Because $\vartheta^{-1}(\lambda_N) = < x$, a $(0.1\ ,0.2,\ 0.6\ )> \notin (1_N - \tau_x)$.

Remark 3.39: The relation between ((FNS$\beta^{\wedge m}$-cont.) and (FNS$\beta^{\wedge m}$-ccont.)) functions is independent, as shown by the following example.

Example 3.40:

1- Take, ***Example* 3.30.** Then, ϑ is (FNS$\beta^{\wedge m}$-cont)function. But, ϑ is not (FNS$\beta^{\wedge m}$-ccont) function. Since, $\vartheta^{-1}(\lambda_N) \nsubseteq ƒ_N$.
2- Take, **Example 3.35.** Then, ϑ is (FNS$\beta^{\wedge m}$-ccont) function. But, ϑ is not (FNS$\beta^{\wedge m}$-cont) function. Since, $\vartheta^{-1}(1_N - \lambda_N) \nsubseteq ƒ_N$.

The next theorem shows the relation between ((FNS$\beta^{\wedge m}$-cont.) and (FNS$\beta^{\wedge m}$-ccont.)) functions.

Theorem 3.41: Let (X_N, τ_x) and (Y_N, τ_y) be any two $FNTS$. If ϑ: $(X_N, \tau_x) \to (Y_N, \tau_y)$ is a function. ϑ is ($FNS\beta^{\wedge m}$-ccont.) iff ϑ is ($FNS\beta^{\wedge m}$-cont.) whenever, every FNS in τ_y is clopen set.

Proof:- Similar to proof **Theorem 3.28.**

Remark 3.42:

i. The relation between (FNW$\beta^{\wedge m}$-cont.) and (FN$\beta^{\wedge m}$-cont.) function is dependent,

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ii. The relation between (FNW$\beta^{\wedge m}$-ccont.) and (FN$\beta^{\wedge m}$-ccont.) function is dependent.

Remark 3.43:

i- Every (FNS$\beta^{\wedge m}$-cont.) is (FN$\beta^{\wedge m}$-cont.) function but the converse is not true,
ii- Every (FNS$\beta^{\wedge m}$-ccont.) is (FN$\beta^{\wedge m}$-ccont.) function but the converse be.

Example 3.44:

i- Take, ***Example*** **3.4**. Then, ϑ is (FN$\beta^{\wedge m}$-cont.) function.

But, FN$\beta^{\wedge m}$cl (S_N) = < x, 0.6, 0.3, 0.3>. Then, FN$\beta^{\wedge m}$cl ($Ƙ_N$)$\not\subseteq Ƒ_N$.

So, $Ƙ_N$ is not FNSG$\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is (FNS$\beta^{\wedge m}$-cont.) function,

ii- Take, **Example 3.9**. Then, ϑ is (FN$\beta^{\wedge m}$-cont.) function.

But, FN$\beta^{\wedge m}$cl ($Ƙ_N$) = < x, 0.6, 0.3, 0.3>. Then, FN$\beta^{\wedge m}$cl ($Ƙ_N$)$\not\subseteq Ƒ_N$.

So, $Ƙ_N$ is not FNSG$\beta^{\wedge m}$-closed set in τ_x. Hence, ϑ is (FNS$\beta^{\wedge m}$-ccont.) function.

Remark 3.45: The next diagram shows the relationship between different functions (FN-cont.),(FN-ccont.),(FN$\beta^{\wedge m}$-cont.),(FNW$\beta^{\wedge m}$-cont.),(FNS$\beta^{\wedge m}$-cont.),(FN$\beta^{\wedge m}$-ccont.), (FNW$\beta^{\wedge m}$-ccont.) and (FNS$\beta^{\wedge m}$-ccont.). But the convers be.

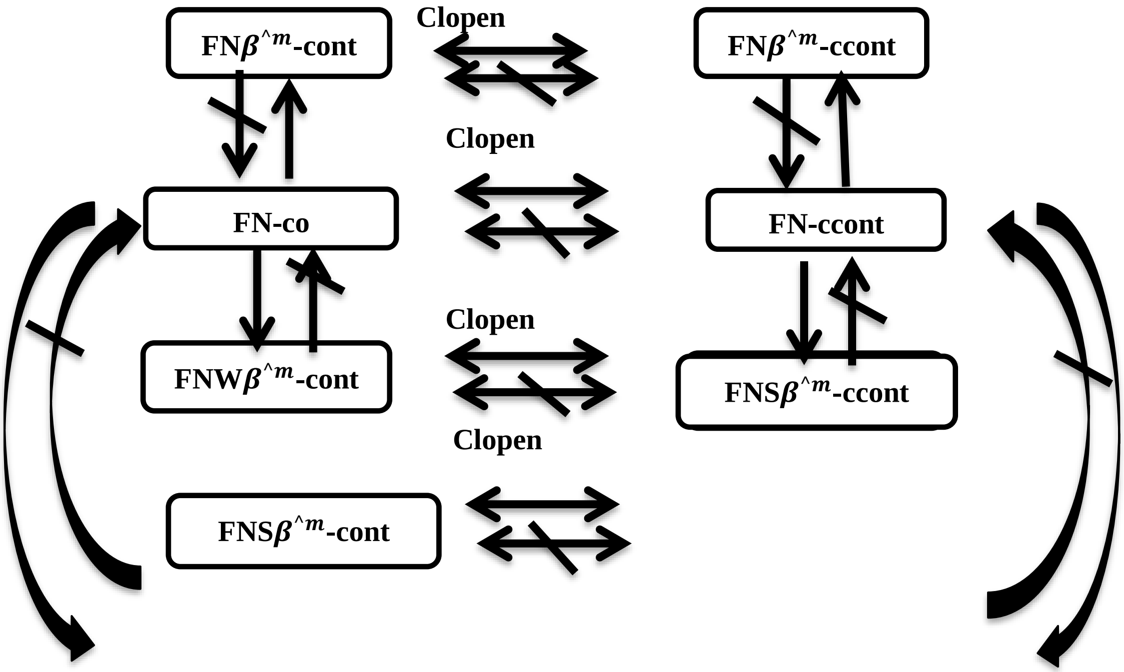

Diagram 1. The relationship between different functions (FN-cont.), (FN-ccont.), (FN$\beta^{\wedge m}$-cont.), (FNW$\beta^{\wedge m}$-cont.),(FNS$\beta^{\wedge m}$-cont.),(FN$\beta^{\wedge m}$-ccont.), (FNW$\beta^{\wedge m}$-ccont.) and (FNS$\beta^{\wedge m}$-ccont.).

3. Conclusion

This manuscript introduces a new types of continuity of functions, including fuzzy neutrosophic $\beta^{\wedge m}$-continuous, fuzzy *neutrosophic* $\beta^{\wedge m}$-contra continuous, fuzzy neutrosophic $\beta^{\wedge m}$-continuous, fuzzy *neutrosophic* $\beta^{\wedge m}$-contra continuous, fuzzy neutrosophic strongly $\beta^{\wedge m}$-continuous, and fuzzy *neutrosophic* strongly $\beta^{\wedge m}$-contra continuous. In addition, properties are obtained, and their correlations are compared and evaluated.

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