Beta Special Function of Symbolic 2-Plithogenic and 3-Plithogenic Real Numbers

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Abstract

The main goal of this paper is to define and study the concept of beta special function defined over the ring of symbolic 2-plithogenic numbers and symbolic 3-plithogenic numbers. Besides, we prove some of the elementary properties of these two versions of beta function by using the isomorphism that connect plithogenic numbers with the classical real numbers. In addition, we represent the relationships between plithogenic beta functions and classical beta functions using the same proposed technique.

Keywords: Special functions; Beta special function; Symbolic 2- plithogenic number; Symbolic 3- plithogenic number

1. Introduction

Special functions are considered one of the most important examples of mathematics applications in many fields of sciences, specifically physics [8-9]. Many well-known real problems a be solved and represented using special functions, and many of such applications in different scientific fields can be founded in these references [11-12].

In this study, a new expression for the special functions has been developed to outperform the previously proposed models by many researchers, over rapid periods. Plithogenic sets have been studied by many researchers and have been developed to be used in the study of algebraic structures and mathematical analysis [1-3,7-10], as well as in various branches of mathematics [2,4-6].

In this work, we aim to study the concept of beta special function defined over the ring of symbolic 2-plithogenic numbers and symbolic 3-plithogenic numbers as two main ways of expressing the special functions, where we prove some of the elementary properties of these two versions of beta function by using the isomorphism that connect plithogenic numbers with the classical real numbers. In addition, we represented the relationships between plithogenic beta functions and classical beta functions using the same technique.

2. Main Discussion

Definition:

The ring of Symbolic 2-Plithogenic number is:

$$2-SP_{\mathbb{R}}=\{x+yp_1+zp_2\,,x,y,z\in\mathbb{R},p_1^2=p_1,p_2^2=p_2,p_1p_2=p_2p_1=p_2\}$$

Remark:

$2-SP_{\mathbb{R}}\cong R\times R\times R$, with:

$f\colon 2-SP_{\mathbb{R}}\to R\times R\times R$:

$$f(x+yp_1+zp_2)=(x,x+y,x+y+z)$$

Remark:

If $g\colon 2-SP_R\to 2-SP_R$is a function in one variable $X=x_0+x_1p_1+x_2p_2$, then (g) can be represented by three classical functions $g_1,g_2,g_3\colon R\to R$ with:$f\circ g(x)\colon 2-SP_R\to R\times R\times R$

$f\circ g(x_0+x_1p_1+x_2p_2)=(g_1(x_0),g_2(x_0+x_1),g_3(x_0+x_1+x_2))$.

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Definition:

The beta Symbolic 2-Plithogenic function is defined as follows:

For $m = m_0 + m_1p_1 + m_2p_2, n = n_0 + n_1p_1 + n_2p_2$:

$$\beta_{2-SP}(m,n) = \int_0^1 (x_0 + x_1p_1 + x_2p_2)^{m-1}\,(1 - x_0 - x_1p_1 - x_2p_2)^{n-1} d(x_0 + x_1p_1 + x_2p_2)$$

With: $x_0, x_1, x_2 \in \mathbb{R}, m_0, m_0 + m_1, m_0 + m_1 + m_2 > 0, n_0, n_0 + n_1, n_0 + n_1 + n_2 > 0$.

Theorem:

1] $\beta_{2-SP}(m,n) = \beta_{2-SP}(n,m)$

2] $f \circ \beta_{2-SP}(m,n) = \big(\beta(m_0,n_0), \beta(m_0 + m_1, n_0 + n_1), \beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2)\big)$

Proof:

1] $\beta_{2-SP}(n,m) = \int_0^1 (x_0 + x_1p_1 + x_2p_2)^{n-1}\,(1 - x_0 - x_1p_1 - x_2p_2)^{m-1} d(x_0 + x_1p_1 + x_2p_2) = \int_0^1 (1 - x_0 - x_1p_1 - x_2p_2)^{n-1}. [1 - (1 - x_0 - x_1p_1 - x_2p_2)]^{m-1} d(x_0 + x_1p_1 + x_2p_2) = \int_0^1 t^{m-1}\,(1-t)^{n-1} dt = \beta_{2-SP}(m,n)\,; t = 1 - x_0 - x_1p_1 - x_2p_2$.

2] we have: $f(x_0 + x_1p_1 + x_2p_2)^{m-1} = \big(x_0^{m_0-1}, (x_0 + x_1)^{m_0+m_1-1}, (x_0 + x_1 + x_2)^{m_0+m_1+m_2-1}\big)$

$f(1 - x_0 - x_1p_1 - x_2p_2)^{n-1} = ((1 - x_0)^{n_0-1}, (1 - x_0 - x_1)^{n_0+n_1-1}, (1 - x_0 - x_1 - x_2)^{n_0+n_1+n_2-1})$

Thus: $f \circ \beta_{2-SP}(m,n) = (L_1, L_2, L_3)$; where:

$$L_1 = \int_0^1 x_0^{m_0-1}. (1 - x_0)^{n_0-1} dx_0 = \beta(m_0, n_0),$$

$$L_2 = \int_0^1 (x_0 + x_1)^{m_0+m_1-1}. (1 - x_0)^{n_0+n_1-1} d(x_0 + x_1) = \beta(m_0 + m_1, n_0 + n_1),$$

$L_3 = \beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2)$ by a similar argument.

Theorem:

1] $\beta_{2-SP}(m,n) = \int_0^\infty \frac{(x_0+x_1p_1+x_2p_2)^{n-1}}{(1+x_0+x_1p_1+x_2p_2)^{m+n}}. d(x_0 + x_1p_1 + x_2p_2) = \int_0^\infty \frac{(x_0+x_1p_1+x_2p_2)^{m-1}}{(1+x_0+x_1p_1+x_2p_2)^{m+n}}. d(x_0 + x_1p_1 + x_2p_2)$.

2] $\beta_{2-SP}(m,n) = 2\int_0^{\frac{\pi}{2}} (\sin\theta)^{2m-1}\,(\cos\theta)^{2n-1}. d\theta\ ;\ \theta = \theta_0 + \theta_1p_1 + \theta_2p_2$.

Proof:

1] According to the Properties of classical beta function, we have:

$\beta(m_0, n_0) = \int_0^\infty \frac{{x_0}^{n-1}}{(1+x_0)^{m+n}}. dx_0 = \int_0^\infty \frac{{x_0}^{m-1}}{(1+x_0)^{m+n}}. dx_0$,

$\beta(m_0 + m_1, n_0 + n_1) = \int_0^\infty \frac{(x_0+x_1)^{n-1}}{(1+x_0+x_1)^{n+m}}. d(x_0 + x_1) = \int_0^\infty \frac{(x_0+x_1)^{m-1}}{(1+x_0+x_1)^{m+n}}. d(x_0 + x_1)$,

$\beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2) = \int_0^\infty \frac{(x_0+x_1+x_2)^{n-1}}{(1+x_0+x_1+x_2)^{n+m}}. d(x_0 + x_1 + x_2) = \int_0^\infty \frac{(x_0+x_1+x_2)^{m-1}}{(1+x_0+x_1+x_2)^{n+m}}. d(x_0 + x_1 + x_2)$,

$\beta_{2-SP}(m,n) = f^{-1} \circ [f \circ \beta_{2-SP}(m,n)] = f^{-1}\big(\beta(m_0,n_0), \beta(m_0 + m_1, n_0 + n_1), \beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2)\big) = \int_0^\infty \frac{(x_0+x_1p_1+x_2p_2)^{n-1}}{(1+x_0+x_1p_1+x_2p_2)^{m+n}}. d(x_0 + x_1p_1 + x_2p_2) = \int_0^\infty \frac{(x_0+x_1p_1+x_2p_2)^{m-1}}{(1+x_0+x_1p_1+x_2p_2)^{m+n}}. d(x_0 + x_1p_1 + x_2p_2)$.

2] According to the Properties of classical beta function, we have:

$\beta(m_0, n_0) = 2\int_0^{\frac{\pi}{2}} (\sin\theta_0)^{2m-1}\,(\cos\theta_0)^{2n-1}. d\theta_0$,

$\beta(m_0 + m_1, n_0 + n_1) = 2\int_0^{\frac{\pi}{2}} [\sin(\theta_0 + \theta_1)]^{2(m_0+m_1)-1}\,[\cos(\theta_0 + \theta_1)]^{2(n_0+n_1)-1}. d(\theta_0 + \theta_1)$,

$\beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2) = 2\int_0^{\frac{\pi}{2}} [\sin(\theta_0 + \theta_1 + \theta_2)]^{2(m_0+m_1+m_2)-1}\,[\cos(\theta_0 + \theta_1 + \theta_2)]^{2(n_0+n_1+n_2)-1}. d(\theta_0 + \theta_1 + \theta_2)$,

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$\Longrightarrow \beta_{2-SP}(m,n) = f^{-1}\circ[f\circ\beta_{2-SP}(m,n)] = 2\int_0^{\frac{\pi}{2}}[\sin(\theta_0+\theta_1p_1+\theta_2p_2)]^{2m-1}\,[\cos(\theta_0+\theta_1p_1+\theta_2p_2)]^{2n-1}.\,d(\theta_0+\theta_1p_1+\theta_2p_2)$.

Theorem:

1] $\beta_{2-SP}(m,n) = \beta_{2-SP}(m,n+1) + \beta_{2-SP}(m+1,n)$.

2] $\frac{\beta_{2-SP}(m+1,n)}{m} = \frac{\beta_{2-SP}(m,n+1)}{n} = \frac{\beta_{2-SP}(m,n)}{m+n}$.

Proof:

According to classical beta function, we have:

$\beta(m_0,n_0) = \beta(m_0,n_0+1) + \beta(m_0+1,n_0)$,

$\beta(m_0+m_1,n_0+n_1) = \beta(m_0+m_1,n_0+n_1+1) + \beta(m_0+m_1+1,n_0+n_1+1)$,

$\beta(m_0+m_1+m_2,n_0+n_1+n_2) = \beta(m_0+m_1+m_2,n_0+n_1+n_2+1) + \beta(m_0+m_1+m_2+1,n_0+n_1+n_2)$,

$\frac{\beta(m_0+1,n_0)}{m_0} = \frac{\beta(m_0,n_0+1)}{n_0} = \frac{\beta(m_0,n_0)}{m_0+n_0}$,

$\frac{\beta(m_0+m_1+1,n_0+n_1)}{m_0+m_1} = \frac{\beta(m_0+m_1,n_0+n_1+1)}{n_0+n_1} = \frac{\beta(m_0+m_1,n_0+n_1)}{m_0+m_1+n_0+n_1}$,

$\frac{\beta(m_0+m_1+m_2+1,n_0+n_1+n_2)}{m_0+m_1+m_2} = \frac{\beta(m_0+m_1+m_2,n_0+n_1+n_2+1)}{n_0+n_1+n_2} = \frac{\beta(m_0+m_1+m_2,n_0+n_1+n_2)}{m_0+m_1+m_2+n_0+n_1+n_2}$,

So that:

$\beta(m,n) = f^{-1}\big(\beta(m_0,n_0),\beta(m_0+m_1,n_0+n_1),\beta(m_0+m_1+m_2,n_0+n_1+n_2)\big) = f^{-1}(K_1,K_2,K_3)$; where:

$$\begin{cases} K_1 = \beta(m_0,n_0+1) + \beta(m_0+1,n_0) \\ K_2 = \beta(m_0+m_1,n_0+n_1+1) + \beta(m_0+m_1+1,n_0+n_1) \\ K_3 = \beta(m_0+m_1+m_2,n_0+n_1+n_2+1) + \beta(m_0+m_1+m_2+1,n_0+n_1+n_2) \end{cases}$$

$\Longrightarrow \beta_{2-SP}(m,n) = \beta_{2-SP}(m_0+m_1p_1+m_2p_2,n_0+n_1p_1+n_2p_2) = \beta_{2-SP}(m_0+m_1p_1+m_2p_2,n_0+n_1p_1+n_2p_2+1) + \beta_{2-SP}(m_0+m_1p_1+m_2p_2+1,n_0+n_1p_1+n_2p_2) = \beta_{2-SP}(m+1,n) + \beta_{2-SP}(m,n+1)$.

On the other hand:

$f^{-1}(L_1,L_2,L_3) = f^{-1}(M_1,M_2,M_3) = f^{-1}(N_1,N_2,N_3)$; where:

$$\begin{cases} L_1 = \dfrac{\beta(m_0+1,n_0)}{m_0} \\ L_2 = \dfrac{\beta(m_0+m_1+1,n_0+n_1)}{m_0+m_1} \\ L_3 = \dfrac{\beta(m_0+m_1+m_2+1,n_0+n_1+n_2)}{m_0+m_1+m_2} \end{cases}$$

$$\begin{cases} M_1 = \dfrac{\beta(m_0,n_0+1)}{n_0} \\ M_2 = \dfrac{\beta(m_0+m_1,n_0+n_1+1)}{n_0+n_1} \\ M_3 = \dfrac{\beta(m_0+m_1+m_2,n_0+n_1+n_2+1)}{n_0+n_1+n_2} \end{cases}$$

$$\begin{cases} N_1 = \dfrac{\beta(m_0,n_0)}{m_0+n_0} \\ N_2 = \dfrac{\beta(m_0+m_1,n_0+n_1)}{m_0+m_1+n_0+n_1} \\ N_3 = \dfrac{\beta(m_0+m_1+m_2,n_0+n_1+n_2)}{m_0+m_1+m_2+n_0+n_1+n_2} \end{cases}$$

Hence:

$$\frac{\beta_{2-SP}(m+1,n)}{m} = \frac{\beta_{2-SP}(m_0+m_1p_1+m_2p_2+1,n_0+n_1p_1+n_2p_2)}{m_0+m_1p_1+m_2p_2} =$$

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$$\frac{\beta_{2-SP}(m,n+1)}{n}=\frac{\beta_{2-SP}(m_0+m_1p_1+m_2p_2,n_0+n_1p_1+n_2p_2+1)}{n_0+n_1p_1+n_2p_2}=$$

$\frac{\beta_{2-SP}(m,n)}{m+n}=\frac{\beta_{2-SP}(m_0+m_1p_1+m_2p_2,n_0+n_1p_1+n_2p_2+1)}{(m_0+n_0)+(m_1+n_1)p_1+(m_2+n_2)p_2}$.

Definition:

The ring of Symbolic 3-Plithogenic number is:

$$3-SP_{\mathbb{R}}=\{x+yp_1+zp_2+kp_3\ ,x,y,z,k\in\mathbb{R},p_1^2=p_1,p_2^2=p_2,p_3^2=p_3,p_1p_2=p_2p_1=p_2,p_1p_3=p_3p_1=p_3,p_3p_2=p_2p_3=p_3\}$$

Remark:

$3-SP_{\mathbb{R}}\cong R\times R\times R\times R$, with:

$f:3-SP_{\mathbb{R}}\to R\times R\times R\times R$:

$$f(x+yp_1+zp_2+kp_3)=(x,x+y,x+y+z,x+y+z+k)$$

Remark:

If $g:3-SP_R\to 3-SP_R$ is a function in one variable $X=x_0+x_1p_1+x_2p_2+x_3p_3$, then (g) can be represented by four classical functions $g_1,g_2,g_3,g_4:R\to R$ with: $f\circ g(x):3-SP_R\to R\times R\times R\times R$

$f\circ g(x_0+x_1p_1+x_2p_2+x_3p_3)=(g_1(x_0),g_2(x_0+x_1),g_3(x_0+x_1+x_2),g_4(x_0+x_1+x_2+x_3))$.

Theorem:

1] $\beta_{3-SP}(m,n)=\int_0^{\infty}\frac{(x_0+x_1p_1+x_2p_2+x_3p_3)^{n-1}}{(1+x_0+x_1p_1+x_2p_2+x_3p_3)^{m+n}}.d(x_0+x_1p_1+x_2p_2+x_3p_3)=\int_0^{\infty}\frac{(x_0+x_1p_1+x_2p_2+x_3p_3)^{m-1}}{(1+x_0+x_1p_1+x_2p_2+x_3p_3)^{m+n}}.d(x_0+x_1p_1+x_2p_2+x_3p_3)$.

2] $\beta_{3-SP}(m,n)=2\int_0^{\frac{\pi}{2}}(\sin\theta)^{2m-1}(\cos\theta)^{2n-1}.d\theta\ ;\ \theta=\theta_0+\theta_1p_1+\theta_2p_2+\theta_3p_3$.

Proof:

1] According to the Properties of classical beta function, we have:

$\beta(m_0,n_0)=\int_0^{\infty}\frac{{x_0}^{n-1}}{(1+x_0)^{m+n}}.dx_0=\int_0^{\infty}\frac{{x_0}^{m-1}}{(1+x_0)^{m+n}}.dx_0$,

$\beta(m_0+m_1,n_0+n_1)=\int_0^{\infty}\frac{(x_0+x_1)^{n-1}}{(1+x_0+x_1)^{n+m}}.d(x_0+x_1)=\int_0^{\infty}\frac{(x_0+x_1)^{m-1}}{(1+x_0+x_1)^{m+n}}.d(x_0+x_1)$,

$\beta(m_0+m_1+m_2,n_0+n_1+n_2)=\int_0^{\infty}\frac{(x_0+x_1+x_2)^{n-1}}{(1+x_0+x_1+x_2)^{n+m}}.d(x_0+x_1+x_2)=\int_0^{\infty}\frac{(x_0+x_1+x_2)^{m-1}}{(1+x_0+x_1+x_2)^{n+m}}.d(x_0+x_1+x_2)$, $\beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3)=\int_0^{\infty}\frac{(x_0+x_1+x_2+x_3)^{n-1}}{(1+x_0+x_1+x_2+x_3)^{n+m}}.d(x_0+x_1+x_2+x_3)=\int_0^{\infty}\frac{(x_0+x_1+x_2+x_3)^{m-1}}{(1+x_0+x_1+x_2+x_3)^{n+m}}.d(x_0+x_1+x_2+x_3)$,

$\beta_{3-SP}(m,n)=f^{-1}\circ[f\circ\beta_{3-SP}(m,n)]=f^{-1}\big(\beta(m_0,n_0),\beta(m_0+m_1,n_0+n_1),\beta(m_0+m_1+m_2,n_0+n_1+n_2),\beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3)\big)=\int_0^{\infty}\frac{(x_0+x_1p_1+x_2p_2+x_3p_3)^{n-1}}{(1+x_0+x_1p_1+x_2p_2+x_3p_3)^{m+n}}.d(x_0+x_1p_1+x_2p_2+x_3p_3)=\int_0^{\infty}\frac{(x_0+x_1p_1+x_2p_2+x_3p_3)^{m-1}}{(1+x_0+x_1p_1+x_2p_2+x_3p_3)^{m+n}}.d(x_0+x_1p_1+x_2p_2+x_3p_3)$.

2] According to the Properties of classical beta function, we have:

$\beta(m_0,n_0)=2\int_0^{\frac{\pi}{2}}(\sin\theta_0)^{2m-1}(\cos\theta_0)^{2n-1}.d\theta_0$,

$\beta(m_0+m_1,n_0+n_1)=2\int_0^{\frac{\pi}{2}}[\sin(\theta_0+\theta_1)]^{2(m_0+m_1)-1}[\cos(\theta_0+\theta_1)]^{2(n_0+n_1)-1}.d(\theta_0+\theta_1)$,

$\beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3)=2\int_0^{\frac{\pi}{2}}[\sin(\theta_0+\theta_1+\theta_2+\theta_3)]^{2(m_0+m_1+m_2+m_3)-1}[\cos(\theta_0+\theta_1+\theta_2+\theta_3)]^{2(n_0+n_1+n_2+n_3)-1}.d(\theta_0+\theta_1+\theta_2+\theta_3)$,

$\Longrightarrow\beta_{3-SP}(m,n)=f^{-1}\circ[f\circ\beta_{3-SP}(m,n)]=2\int_0^{\frac{\pi}{2}}[\sin(\theta_0+\theta_1p_1+\theta_2p_2+\theta_3p_3)]^{2m-1}[\cos(\theta_0+\theta_1p_1+\theta_2p_2+\theta_3p_3)]^{2n-1}.d(\theta_0+\theta_1p_1+\theta_2p_2+\theta_3p_3)$.

Theorem:

1] $\beta_{3-SP}(m,n) = \beta_{3-SP}(m,n+1) + \beta_{3-SP}(m+1,n)$.

2] $\frac{\beta_{3-SP}(m+1,n)}{m} = \frac{\beta_{3-SP}(m,n+1)}{n} = \frac{\beta_{3-SP}(m,n)}{m+n}$.

Proof:

According to classical beta function, we have:

$\beta(m_0,n_0) = \beta(m_0,n_0+1) + \beta(m_0+1,n_0)$,

$\beta(m_0+m_1,n_0+n_1) = \beta(m_0+m_1,n_0+n_1+1) + \beta(m_0+m_1+1,n_0+n_1+1)$,

$\beta(m_0+m_1+m_2,n_0+n_1+n_2) = \beta(m_0+m_1+m_2,n_0+n_1+n_2+1) + \beta(m_0+m_1+m_2+1,n_0+n_1+n_2)$,

$$\begin{aligned}\beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3) &= \beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3+1)\\ &+ \beta(m_0+m_1+m_2+m_3+1,n_0+n_1+n_2+n_3)\end{aligned}$$

$\frac{\beta(m_0+1,n_0)}{m_0} = \frac{\beta(m_0,n_0+1)}{n_0} = \frac{\beta(m_0,n_0)}{m_0+n_0}$,

$\frac{\beta(m_0+m_1+1,n_0+n_1)}{m_0+m_1} = \frac{\beta(m_0+m_1,n_0+n_1+1)}{n_0+n_1} = \frac{\beta(m_0+m_1,n_0+n_1)}{m_0+m_1+n_0+n_1}$,

$\frac{\beta(m_0+m_1+m_2+1,n_0+n_1+n_2)}{m_0+m_1+m_2} = \frac{\beta(m_0+m_1+m_2,n_0+n_1+n_2+1)}{n_0+n_1+n_2} = \frac{\beta(m_0+m_1+m_2,n_0+n_1+n_2)}{m_0+m_1+m_2+n_0+n_1+n_2}$,

$$\begin{aligned}\frac{\beta(m_0+m_1+m_2+m_3+1,n_0+n_1+n_2)}{m_0+m_1+m_2+m_3} &= \frac{\beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3+1)}{n_0+n_1+n_2+n_3}\\ &= \frac{\beta(m_0+m_1+m_2,n_0+n_1+n_2+n_3)}{m_0+m_1+m_2+m_3+n_0+n_1+n_2+n_3}\end{aligned}$$

So that:

$\beta(m,n) = f^{-1}\big(\beta(m_0,n_0),\beta(m_0+m_1,n_0+n_1),\beta(m_0+m_1+m_2,n_0+n_1+n_2),\beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3)\big) = f^{-1}(K_1,K_2,K_3)$; where:

$$\begin{cases} K_1 = \beta(m_0,n_0+1) + \beta(m_0+1,n_0) \\ K_2 = \beta(m_0+m_1,n_0+n_1+1) + \beta(m_0+m_1+1,n_0+n_1) \\ K_3 = \beta(m_0+m_1+m_2,n_0+n_1+n_2+1) + \beta(m_0+m_1+m_2+1,n_0+n_1+n_2) \\ K_3 = \beta(m_0+m_1+m_2+m_3,n_0+n_1+n_2+n_3+1) + \beta(m_0+m_1+m_2+m_3+1,n_0+n_1+n_2+n_3) \end{cases}$$

$\Longrightarrow \beta_{3-SP}(m,n) = \beta_{3-SP}(m_0+m_1p_1+m_2p_2+m_3p_3,n_0+n_1p_1+n_2p_2+n_3p_3) = \beta_{2-SP}(m_0+m_1p_1+m_2p_2+m_3p_3,n_0+n_1p_1+n_2p_2+n_3p_3+1) + \beta_{3-SP}(m_0+m_1p_1+m_2p_2+m_3p_3+1,n_0+n_1p_1+n_2p_2+n_3p_3) = \beta_{3-SP}(m+1,n) + \beta_{3-SP}(m,n+1)$.

On the other hand:

$f^{-1}(L_1,L_2,L_3,L_4) = f^{-1}(M_1,M_2,M_3,M_4) = f^{-1}(N_1,N_2,N_3,N_4)$; where:

$$\begin{cases} L_1 = \dfrac{\beta(m_0+1,n_0)}{m_0} \\ L_2 = \dfrac{\beta(m_0+m_1+1,n_0+n_1)}{m_0+m_1} \\ L_3 = \dfrac{\beta(m_0+m_1+m_2+1,n_0+n_1+n_2)}{m_0+m_1+m_2} \\ L_4 = \dfrac{\beta(m_0+m_1+m_2+m_3+1,n_0+n_1+n_2+n_3)}{m_0+m_1+m_2+m_3} \end{cases}$$

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$$\begin{cases} M_1 = \dfrac{\beta(m_0, n_0 + 1)}{n_0} \\ M_2 = \dfrac{\beta(m_0 + m_1, n_0 + n_1 + 1)}{n_0 + n_1} \\ M_3 = \dfrac{\beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2 + 1)}{n_0 + n_1 + n_2} \\ M_4 = \dfrac{\beta(m_0 + m_1 + m_2 + m_3, n_0 + n_1 + n_2 + n_3 + 1)}{n_0 + n_1 + n_2 + n_3} \end{cases}$$

$$\begin{cases} N_1 = \dfrac{\beta(m_0, n_0)}{m_0 + n_0} \\ N_2 = \dfrac{\beta(m_0 + m_1, n_0 + n_1)}{m_0 + m_1 + n_0 + n_1} \\ N_3 = \dfrac{\beta(m_0 + m_1 + m_2, n_0 + n_1 + n_2)}{m_0 + m_1 + m_2 + n_0 + n_1 + n_2} \\ N_4 = \dfrac{\beta(m_0 + m_1 + m_2 + m_3, n_0 + n_1 + n_2 + n_3)}{m_0 + m_1 + m_2 + m_3 + n_0 + n_1 + n_2 + n_3} \end{cases}$$

Hence:

$$\frac{\beta_{3-SP}(m+1, n)}{m} = \frac{\beta_{3-SP}(m_0 + m_1p_1 + m_2p_2 + m_3p_3 + 1, n_0 + n_1p_1 + n_2p_2 + n_3p_3)}{m_0 + m_1p_1 + m_2p_2 + m_3p_3} =$$

$$\frac{\beta_{3-SP}(m, n+1)}{n} = \frac{\beta_{3-SP}(m_0 + m_1p_1 + m_2p_2 + m_3p_3, n_0 + n_1p_1 + n_2p_2 + n_3p_3 + 1)}{n_0 + n_1p_1 + n_2p_2 + n_3p_3} =$$

$$\frac{\beta_{3-SP}(m,n)}{m+n} = \frac{\beta_{3-SP}(m_0+m_1p_1+m_2p_2+m_3p_3, n_0+n_1p_1+n_2p_2+n_3p_3+1)}{(m_0+n_0)+(m_1+n_1)p_1+(m_2+n_2)p_2+(m_3+n_3)p_3}.$$

3. Conclusion

In this paper, we have defined and studied for the first time the concept of beta special function defined over the ring of symbolic 2-plithogenic numbers and symbolic 3-plithogenic numbers, where we proved some of the elementary properties of these two versions of beta function by using the isomorphism that connect plithogenic numbers with the classical real numbers. In addition, we found the relationships between plithogenic beta functions and classical beta functions using the same technique.

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