ƝÇƬ- Confused Neutrosophic Crisp Sets

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Abstract

The importance of Neutrosophic crisp triple sets and their important effects on our daily lives was and still is aturing point in the history of science, especially mathematical sciences. From here, we began a ƝÇƬ–confused, concept that is based on both ƝÇƬ–interior, and ƝÇƬ– exterior, and important characteristic emerged because of mixing the characteristic of ƝÇƬ– interior and ƝÇƬ– exterior sets. We supported this with various examples.

Keywords: Neutrosophic crisp triple set; ƝÇƬ– sets; ƝÇƬ– interior; ƝÇƬ– exterior; ƝÇƬ– confused

1. Introduction

The mathematical basis of our research is neutrosophic crisp triple sets those who laid its first principles and its mathematical foundations were both of Florentin and Salama in [1,2]. It was a path and starting point that resonated with various sciences, especially applied sciences, which made many researchers in different mathematical disciplines passionate about integrating these groups with different mathematical concepts and studying them. Hadi et al. [3] defined a new type of these groups with certain conditions and called it Neutrosophic Axial sets. Abdulsada et al. [4] extend the latter efforts by given new related issues and facts.

2. Neutrosophic Crisp Triple Sets

The following important concepts and facts are presented in this section depending on work in [4]:

Definition 2.1 [4]: Let X be a non-empty set A neutrosophic crisp tripl set $ƝÇƬ_M$ is an object having the style $ƝÇƬ_M =< M_1, M_2, M_3 >$ where $M_1, M_2, M_3 \subseteq X$ satisfying $M_1 \subseteq M_2$ and $M_2 \cap M_3 = \phi$

Definition 2.2 [4]: Let $ƝÇƬ_M =< M_1, M_2, M_3 >$ and $ƝÇƬ_H =< H_1, H_2, H_3 >$ are ƝÇƬ–nonempty sets X through set. Therefore:

1) $ƝÇƬ_M$ is a ƝÇƬ– subset of $ƝÇƬ_H$ if $M_1 \subseteq H_1$, $M_2 \subseteq H_2$ and $H_3 \subseteq M_3$ and write $ƝÇƬ_M \sqsubseteq ƝÇƬ_H$.
2) $ƝÇƬ_M = ƝÇƬ_H$ Iff $ƝÇƬ_M \sqsubseteq ƝÇƬ_H$ and $ƝÇƬ_H \sqsubseteq ƝÇƬ_M$.
3) The ƝÇƬ– complement of ƝÇƬ– set $ƝÇƬ_M$ is $CƝÇƬ_M =< M_3, M_2^c, M_1 >$.
4) The (ƝÇƬ– intersection sets) is the intersection a crisp triple set and defined as $ƝÇƬ_M \sqcap ƝÇƬ_H = < M_1 \cap H_1 , M_2 \cap H_2, M_3 \cup H_3 >$.
5) The (ƝÇƬ– union sets) is the union a crisp triple set and defined as: $ƝÇƬ_M \sqcup ƝÇƬ_H =< M_1 \cup H_1 , M_2 \cup H_2, M_3 \cap H_3 >$.
6) $ƝÇƬ_M - ƝÇƬ_H = ƝÇƬ_M \sqcap CƝÇƬ_H$.
7) Let $\{ƝÇƬ_{Mi} : i \in I\}$ be ƝÇƬ– sets in X and $ƝÇƬ_{Mi} =< M_{i1}, M_{i2}, M_{i3} >$. Then $\sqcup_{i\in I} ƝÇƬ_{Mi} =< \bigcup_{i\in I} M_{i1}, \bigcup_{i\in I} M_{i2}, \bigcap_{i\in I} M_{i3} >$ and $\sqcap_{i\in I} ƝÇƬ_{Mi} =< \bigcap_{i\in I} M_{i1}, \bigcap_{i\in I} M_{i2}, \bigcup_{i\in I} M_{i3} >$.

Definition 2.3 [4]: Let X be a non-empty set then:

1) $ƝÇƬ_X =< X, X, \phi >$ is ƝÇƬ– universal set
2) $ƝÇƬ_\Phi =< \phi, \phi, X >$ is ƝÇƬ– void set. Clearly $CƝÇƬ_\Phi = ƝÇƬ_X$ and $CƝÇ_X = ƝÇ_\Phi$

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The following preposition will clarify most significant relationships ƝÇƬ– union, ƝÇƬ–intersection, ƝÇƬ–complement and de-Morgan's law.

Proposition 2.4 [4]: Let $X \neq \phi$ and let $ƝÇƬ_{Ą} =< Ą_1, Ą_2, Ą_3 >$ and $ƝÇƬ_{ß} =< ß_1, ß_2, ß_3 >$ be ƝÇƬ– sets then:

1) $ƝÇƬ_{Ą} \sqcup ƝÇƬ_{Ą} = ƝÇƬ_{Ą}$
2) $ƝÇƬ_{Ą} \sqcap ƝÇƬ_{Ą} = ƝÇƬ_{Ą}$
3) $ƝÇƬ_{Ą} \sqcap ƝÇƬ_{\Phi} = ƝÇƬ_{\Phi}$
4) $ƝÇƬ_{Ą} \sqcap ƝÇƬ_{X} = ƝÇƬ_{Ą}$
5) $ƝÇƬ_{Ą} \sqcup ƝÇƬ_{\Phi} = ƝÇƬ_{Ą}$
6) $ƝÇƬ_{Ą} \sqcup ƝÇƬ_{X} = ƝÇƬ_{X}$
7) $C(ƝÇƬ_{Ą} \sqcap ƝÇƬ_{ß}) = CƝÇƬ_{Ą} \sqcup CƝÇƬ_{ß}$
8) $C(ƝÇƬ_{Ą} \sqcup ƝÇƬ_{ß}) = CƝÇƬ_{Ą} \sqcap CƝÇƬ_{ß}$

Note 2.5: Let $ƝÇƬ_{M̧} =< M̧_1, M̧_2, M̧_3 >$ and $ƝÇƬ_{Ƕ} =< Ƕ_1, Ƕ_2, Ƕ_3 >$ are ƝÇƬ–a non- empty set X over sets. Through the definition (2.1, 2.2, 2.3) we can conclude the following properties:

1) $ƝÇƬ_{\Phi} \sqsubseteq ƝÇƬ_{M̧}$
2) $ƝÇƬ_{M̧} \sqsubseteq ƝÇƬ_{X}$
3) $ƝÇƬ_{M̧} \sqcap ƝÇƬ_{Ƕ} \sqsubseteq ƝÇƬ_{M̧}$ and $ƝÇƬ_{M̧} \sqcap ƝÇƬ_{Ƕ} \sqsubseteq ƝÇƬ_{Ƕ}$
4) $ƝÇƬ_{M̧} \sqsubseteq ƝÇƬ_{M̧} \sqcup ƝÇƬ_{Ƕ}$ and $ƝÇƬ_{Ƕ} \sqsubseteq ƝÇƬ_{M̧} \sqcup ƝÇƬ_{Ƕ}$
5) $ƝÇƬ_{M̧} \sqsubseteq ƝÇƬ_{Ƕ}$ iff $ƝÇƬ_{M̧} \sqcup ƝÇƬ_{Ƕ} = ƝÇƬ_{Ƕ}$
6) $ƝÇƬ_{M̧} \sqsubseteq ƝÇƬ_{Ƕ}$ iff $ƝÇƬ_{M̧} \sqcap ƝÇƬ_{Ƕ} = ƝÇƬ_{M̧}$
7) $ƝÇƬ_{M̧} \sqsubseteq ƝÇƬ_{Ƕ}$ iff $CƝÇƬ_{Ƕ} \sqsubseteq CƝÇƬ_{M̧}$
8) If $ƝÇƬ_{M̧} \sqcap ƝÇƬ_{Ƕ} = ƝÇƬ_{\Phi}$ then $ƝÇƬ_{M̧} \sqsubseteq CƝÇƬ_{Ƕ}$

Proof 1:

Let $ƝÇƬ_{\Phi} =< \phi, \phi, X >$

Since, $\phi \subseteq M̧ \ \forall \ M̧ \subseteq X$ and for any $ƝÇƬ_{M̧}$, $\phi \subseteq M̧_1$, $\phi \subseteq M̧_2$ and $M̧_3 \subseteq X$

Thus $ƝÇƬ_{\Phi} \sqsubseteq ƝÇƬ_{M̧}$ and 2,3,4.5.6,7, proving directly by applying Definition 2.1, 2.2 and 2.3

We note the opposite part (8) is not necessarily true

Let $X = \{ ã, ƀ, c \}$ and let $ƝÇƬ_{M̧} =< \{ã\}, \{ã\}, \{c\} >$ and $ƝÇƬ_{Ƕ} =< \phi, \{c\}, \{ã\} >$

And $ƝÇƬ_{M̧} \sqsubseteq CƝÇƬ_{Ƕ} =< \{ã\}, \{ã, ƀ\}, \phi >$

But $ƝÇƬ_{M̧} \sqcap ƝÇƬ_{Ƕ} =< \phi, \phi, \{ã, c\} > \neq ƝÇƬ_{\Phi}$

For any $ƝÇƬ_{Ą} =< Ą_1, Ą_2, Ą_3 >$, we get $ƝÇƬ_{\Phi} \sqsubseteq ƝÇƬ_{Ą} \sqcap CƝÇƬ_{Ą}$ and $ƝÇƬ_{Ą} \sqcup CƝÇƬ_{Ą} \sqsubseteq ƝÇƬ_{X}$

Note 2.6: Let $ƝÇƬ_{M}, ƝÇƬ_{K}$ and $ƝÇƬ_{D}$ are ƝÇƬ– a non-empty set X over sets. Then the following in sets theory true but in ƝÇƬ–sets not necessarily

1) $(M - K) \cap K = \phi$ But $ƝÇ_{\Phi} \neq (ƝÇ_{M} - ƝÇ_{K}) \sqcap ƝÇ_{K}$
2) $(M - K) - D = (M - D) - (K - D)$
but $(ƝÇƬ_{M} - ƝÇƬ_{K}) - ƝÇƬ_{D} \sqsubseteq (ƝÇƬ_{M} - ƝÇƬ_{D}) - (ƝÇƬ_{K} - ƝÇƬ_{D})$
3) $M \cap (K - D) = (M \cap K) - (M \cap D)$
but $ƝÇƬ_{M} \sqcap (ƝÇƬ_{K} - ƝÇƬ_{D}) \sqsubseteq (ƝÇƬ_{M} \sqcap ƝÇƬ_{K}) - (ƝÇƬ_{M} \sqcap ƝÇƬ_{D})$
4) $M - (M - K) = M \cap K$ but $ƝÇƬ_{M} - (ƝÇƬ_{M} - ƝÇƬ_{K}) \sqsupseteq ƝÇƬ_{M} \sqcap ƝÇƬ_{K}$

We will explain them. Let $X = \{e, g, h\}$ and $ƝÇƬ_{M} =< \{e\}, \{e, g\}\{h\} >$,

$ƝÇƬ_{K} =< \{h\}, \{g, h\}, \phi >$ then $ƝÇƬ_{\Phi} \neq (ƝÇƬ_{M} - ƝÇƬ_{K}) \sqcap ƝÇƬ_{K} =< \phi, \phi, \{h\} >$

And 4) let $ƝÇƬ_{K} =< \{h\}, \{e, h\}, \{g\} >$

then $ƝÇƬ_{M} - (ƝÇƬ_{M} - ƝÇƬ_{K}) \sqsupseteq ƝÇƬ_{M} \sqcap ƝÇƬ_{K} =< \phi, \{e\}, \{g, h\} >$

Definition 2.7 [4]: Let $X \neq \phi$ and let $P \in X$. Then ƝÇƬ–points ($ƝÇƬ_{p}$) are structure

$ƝÇƬ_{ṗ} =< \{P\}, \{P\}, \{P\}^c >$, $ƝÇƬ_{p̈} =< \phi, \{P\}, \{P\}^c >$ and $ƝÇƬ_{p̈} =< \phi, \phi, \{P\}^c >$

and ƝÇƬ–belong as follows. Let $ƝÇƬ_{M} =< M_1, M_2, M_3 >$ then

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1) $\text{ƝÇT}_{\dot{p}} \in \text{ƝÇ}_{\text{M}}$ *iff* $\text{P} \in \text{M}_1$
2) $\text{ƝÇT}_{\ddot{p}} \in \text{ƝÇ}_{\text{M}}$ *iff* $\text{P} \in \text{M}_2$
3) $\text{ƝÇT}_{\dddot{p}} \in \text{ƝÇ}_{\text{M}}$ *iff* $\text{P} \notin \text{M}_3$

It is soft to see that the ƝÇT–Points are ƝÇT–sets and basic number of these sets are 3n where n is the basic number of universal sets

Proposition 2.8 [4]: Let ƝÇT_M and ƝÇT_K are ƝÇT–sets in non-empty set X .Then:

1) $\text{ƝÇT}_M \sqsubseteq \text{ƝÇT}_H$ *iff* $\forall\, \text{ƝÇT}_{\dot{p}}$ with $\text{ƝÇT}_{\dot{p}} \in \text{ƝÇT}_M \Rightarrow \text{ƝÇT}_{\dot{p}} \in \text{ƝÇT}_\text{K}$, $\forall\, \text{ƝÇT}_{\ddot{p}}$ with $\text{ƝÇT}_{\ddot{p}} \in \text{ƝÇT}_\text{M} \Rightarrow \text{ƝÇT}_{\ddot{p}} \in \text{ƝÇT}_\text{K}$ and $\forall\, \text{ƝÇT}_{\dddot{p}}$ with $\text{ƝÇT}_{\dddot{p}} \in \text{ƝÇT}_\text{M} \Rightarrow \text{ƝÇT}_{\dddot{p}} \in \text{ƝÇT}_\text{K}$
2) Let $\text{ƝÇT}_\text{M} =< \text{M}_1, \text{M}_2, \text{M}_3 >$ be ƝÇT–set on X .
Then: $\text{ƝÇT}_\text{M} = (\{\text{ƝÇT}_{\dot{p}} : \text{ƝÇT}_{\dot{p}} \in \text{ƝÇT}_\text{M}\}) \sqcup (\{\text{ƝÇT}_{\ddot{p}} : \text{ƝÇT}_{\ddot{p}} \in \text{ƝÇT}_\text{M}\}) \sqcup (\{\text{ƝÇT}_{\dddot{p}} : \text{ƝÇT}_{\dddot{p}} \in \text{ƝÇT}_\text{M}\})$

In this section, we explore some of properties generated by ƝÇT–sets, such as interior, exterior and confused.

Definition 2.9 [4]: Let X be a fixed set that is non-empty and let the pair (ƝÇ , $T^X_{\text{ƝÇ}}$) is called neutrosophic crisp topological space (ƝÇT) on ƝÇTX and satisfying the three following condition:

1) $\text{ƝÇT}_\Phi , \text{ƝÇT}_X \in T^X_{\text{ƝÇT}}$
2) $T^X_{\text{ƝÇT}}$ is closed under the finite of ƝÇT– intersection.
3) $T^X_{\text{ƝÇT}}$ is closed under the ƝÇT– union of every subfamily of $T_{\text{ƝÇT}}$.

And element of $T^X_{\text{ƝÇT}}$ can be consider ƝÇT–open and the complement is ƝÇT– closed

i. The ƝÇT– interior of ƝÇT_M is of the mold $\text{ƝÇT}-int(\text{ƝÇT}_\text{M}) = \sqcup \{\text{ƝÇT}_\text{p};\ \exists\ \text{ƝÇT}_\text{H} \in T_{\text{ƝÇT}} \ni \text{ƝÇT}_\text{p} \in \text{ƝÇT}_\text{H} \sqsubseteq \text{ƝÇT}_\text{M}\}$ for any ƝÇT–set ƝÇT_M
ii. The ƝÇT– exterior of ƝÇT_M is of the shape $\text{ƝÇT}-ext(\text{ƝÇT}_\text{M}) = \text{ƝÇT}-int(C\text{ƝÇT}_\text{M})$ for any ƝÇT– set by these definition ƝÇT– interior and ƝÇT– exterior can be considered an ƝÇT– open sets .Also we can consider $\text{ƝÇT}-ext(C\text{ƝÇT}_\text{M}) = \text{ƝÇT}-int(\text{ƝÇT}_\text{M})$

We will explain through the propositions following the most prominent characteristics of the ƝÇT–interior and ƝÇT– exterior.

Proposition 2.10: Let ($\text{ƝÇT}_X, T^X_{\text{ƝÇT}}$) be neutrosophic crisp topological space, let ƝÇT_M and ƝÇT_K are ƝÇT– sets. Then:

1) $\text{ƝÇT}-int(\text{ƝÇT}_X) = \text{ƝÇT}_X$ and $\text{ƝÇT}-int(\text{ƝÇT}_\Phi) = \text{ƝÇT}_\Phi$
2) $\text{ƝÇT}-int(\text{ƝÇT}_\text{M}) \sqsubseteq \text{ƝÇT}_M$
3) $\text{ƝÇT}_\text{M} \sqsubseteq \text{ƝÇT}_\text{K} \Rightarrow \text{ƝÇT}-int(\text{ƝÇT}_\text{M}) \sqsubseteq \text{ƝÇT}-int(\text{ƝÇT}_\text{K})$
4) $\text{ƝÇT}-int(\text{ƝÇT}_\text{M} \sqcap \text{ƝÇT}_\text{K}) = \text{ƝÇT}-int(\text{ƝÇT}_\text{M}) \sqcap \text{ƝÇT}-int(\text{ƝÇT}_\text{K})$
5) $\text{ƝÇT}-int(\text{ƝÇT}_\text{M}) \sqcup \text{ƝÇT}-int(\text{ƝÇT}_\text{K}) \sqsubseteq \text{ƝÇT}-int(\text{ƝÇT}_\text{M} \sqcup \text{ƝÇT}_\text{K})$
6) $\text{ƝÇT}-int(\text{ƝÇT}-int(\text{ƝÇT}_\text{M})) = \text{ƝÇT}-int(\text{ƝÇT}_\text{M})$
7) ƝÇT_M is ƝÇT–open set iff $\text{ƝÇT}-int(\text{ƝÇT}_\text{M}) = \text{ƝÇT}_M$

Proof :(1)
Since it is clear the only ƝÇT–set containing ƝÇT_Φ in $T^X_{\text{ƝÇT}}$ is ƝÇT_Φ and ƝÇT_X.
Thus, it is true. **(2)** let $\text{ƝÇT}_M =< M_1, M_2, M_3 >$ be any ƝÇT– set
And let ƝÇT_p be any type of ƝÇT–point,
Let $\text{ƝÇT}_\text{p} \in \text{ƝÇT}-int(\text{ƝÇT}_\text{M})$
So by definition (2-9 ii) there exist a ƝÇT–set $\text{ƝÇT}_\text{H} =< \text{H}_1, \text{H}_2, \text{H}_3 >$
Such that $\text{ƝÇT}_\text{p} \in \text{ƝÇT}_\text{H} \sqsubseteq \text{ƝÇT}_\text{M} \Rightarrow \text{ƝÇT}_\text{p} \in \text{ƝÇT}_\text{M}$
Thus $\text{ƝÇT}-int(\text{ƝÇT}_\text{M}) \sqsubseteq \text{ƝÇT}_M$

Proposition 2.11: ($\text{ƝÇT}_X, T^X_{\text{ƝÇT}}$) be neutrosophic crisp topological space let ƝÇT_M and ƝÇT_H are ƝÇT– sets. Then:

1) $\text{ƝÇT}-ext(\text{ƝÇT}_\Phi) = \text{ƝÇT}_X$ and $\text{ƝÇT}-ext(\text{ƝÇT}_X) = \text{ƝÇT}_\Phi$
2) $\text{ƝÇT}-ext(\text{ƝÇT}_\text{M}) \sqsubseteq C\text{ƝÇT}_\text{M}$
3) $\text{ƝÇT}_\text{M} \sqsubseteq \text{ƝÇT}_\text{H} \Rightarrow \text{ƝÇT}-ext(\text{ƝÇT}_\text{H}) \sqsubseteq \text{ƝÇT}-ext(\text{ƝÇT}_\text{M})$
4) $\text{ƝÇT}-ext(\text{ƝÇT}_\text{M} \sqcup \text{ƝÇT}_\text{H}) = \text{ƝÇT}-ext(\text{ƝÇT}_\text{M}) \sqcap \text{ƝÇT}-ext(\text{ƝÇT}_\text{H})$
5) $\text{ƝÇT}-int(\text{ƝÇT}_\text{M}) \sqsubseteq \text{ƝÇT}-ext(\text{ƝÇT}-ext(\text{ƝÇT}_\text{M}))$

Note 2.12: for any member ƝÇT_M of $T^X_{\text{ƝÇT}}$ is ƝÇT– closed iff $\text{ƝÇT}-ext(\text{ƝÇT}_\text{M}) = C\text{ƝÇT}_M$.

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We will explain by definition 2-9 that for any ҤÇƬ–set $ҤÇƬ_M =< M_1, M_2, M_3 >$ where M_1, M_2 and M_3 represented first , second and third coordinates on respectively and each element of $T^X_{ҤÇƬ}$ will come out of him three of topologicals space T_1, T_2 *and* T_3 in general,
when X is ending set we will get three topological spaces but when that X is not ending set then T_1 and T_2 are topological spaces while T_3 is base because according to definition of $ҤÇƬ_X$ and $ҤÇƬ_\Phi$ that is not closed at the infinite unions according for arrangement of the union at ҤÇƬ–set .

3. ҤÇƬ– Confused Crisp Set

In this section, we will explain a new concept for these ҤÇƬ–sets, which is the ҤÇƬ– confused concept, which results from the two concepts (ҤÇƬ–interior and ҤÇƬ–exterior) as well as identifying it is most prominent characteristics

Definition 3.1: Let $ҤÇƬ_M$ be ҤÇƬ–set and let $(ҤÇƬ_X, T^X_{ҤÇƬ})$ be neutrosophic crisp topological space, then the ҤÇƬ–confused crisp set of $ҤÇƬ_M$ denoted by $Ƶ(ҤÇƬ_M)$ and defined $Ƶ(ҤÇƬ_M) = ҤÇƬ-int(ҤÇƬ_M) \sqcup ҤÇƬ-ext(ҤÇƬ_M)$, since this concept results from the two previous concepts, $(ҤÇƬ-int(ҤÇƬ_M), ҤÇƬ-ext(ҤÇƬ_M))$,they have an ҤÇƬ–open concept so this concept is also.

We will show that most prominent properties of the ҤÇƬ–confused of $ҤÇƬ_M$

Preposition 3.2: Let $(ҤÇ_X, T^X_{ҤÇ})$ be neutrosophic crisp topological space, for any ҤÇ–sets $ҤÇƬ_M$ and $ҤÇƬ_K$ .Then:

1) $Ƶ(ҤÇƬ_X) = Ƶ(ҤÇƬ_\Phi) = ҤÇƬ_X$
2) $Ƶ(ҤÇƬ_M) = Ƶ(CҤÇƬ_M)$
3) $Ƶ(ҤÇƬ_M) = C[C(ҤÇƬ-int(CҤÇƬ_M)) \sqcap C(ҤÇƬ-int(ҤÇƬ_M))]$
4) $Ƶ(ҤÇƬ_M) \sqsubseteq Ƶ(ҤÇƬ-int(ҤÇƬ_M))$
5) $Ƶ(ҤÇƬ_M) \sqsubseteq Ƶ(ҤÇƬ-ext(ҤÇƬ_M))$
6) $Ƶ(ҤÇƬ_M) \sqcap Ƶ(ҤÇƬ_K) \sqsubseteq Ƶ(ҤÇƬ_M \sqcup ҤÇƬ_K)$
7) $Ƶ(ҤÇƬ_M) \sqcap Ƶ(ҤÇƬ_K) \sqsubseteq Ƶ(ҤÇƬ_M \sqcap ҤÇƬ_K)$

Proof 1:

We take the part

$Ƶ(ҤÇƬ_X) = ҤÇƬ-int(ҤÇƬ_X) \sqcup ҤÇƬ-ext(ҤÇƬ_X)$

$= ҤÇƬ_X \sqcup ҤÇƬ_\Phi = ҤÇ_X$ by (proposition 2-4, 2-10, 2-11) also relative $Ƶ(ҤÇ_\Phi)$

2) if we take the left-hand side

$Ƶ(CҤÇƬ_M) = ҤÇƬ-int(CҤÇƬ_M) \sqcup ҤÇƬ-ext(CҤÇƬ_M)$

$= ҤÇƬ-ext(ҤÇƬ_M) \sqcup ҤÇƬ-int(ҤÇƬ_M)$ by definition (2-9) $= Ƶ(ҤÇƬ_M)$.

We note the convers part 4,5,6 not necessarily true and the following example clarifies that.

Example 3.3: Let $X = \{\alpha, b, \varsigma, d\}, T^X_{ҤÇƬ} = \{ҤÇƬ_X, ҤÇƬ_\Phi, ҤÇƬ_A, ҤÇƬ_B, ҤÇƬ_C, ҤÇƬ_D, ҤÇ_E\}$ such that $ҤÇƬ_A =< \{a\}, x, \phi >$

$ҤÇƬ_C =< \phi, \{b\}, \{d\} >$

$ҤÇƬ_E =< \phi, \{b, \varsigma\}, \phi >$

$ҤÇƬ_B =< \phi, \{c\}, \phi >$

$ҤÇƬ_D =< \phi, \phi, \{d\} >$

4) let $ҤÇƬ_M =< \phi, \{b, d\}, \{\varsigma\} >$ then $Ƶ(ҤÇƬ-int(ҤÇƬ_M)) = ҤÇƬ_X \not\sqsubseteq Ƶ(ҤÇƬ_M) =< \phi, \{\varsigma\}, \phi >$

5) $Ƶ(ҤÇƬ-ext(ҤÇƬ_M)) =< \phi, \{b, \varsigma\}, \phi > \not\sqsubseteq Ƶ(ҤÇƬ_M) =< \phi, \{\varsigma\}, \phi >$

6) let $ҤÇƬ_K =< \{a\}, \{a\}, \phi >$

Then $Ƶ(ҤÇƬ_M \sqcup (ҤÇƬ_K)) =< \phi, \{b\}, \{d\} > \not\sqsubseteq Ƶ(ҤÇƬ_M) \sqcap Ƶ(ҤÇƬ_K) =< \phi, \phi, \{d\} >$

Proposition 3.4: If $ҤÇƬ_M$ be ҤÇƬ–open set then: $Ƶ(ҤÇƬ_M) = C(ҤÇƬ-ext(ҤÇƬ_M)) \sqcap CҤÇƬ_M$

Proof:

Let $C(ƵҤÇƬ_M) = C[ҤÇƬ-int(ҤÇƬ_M) \sqcup ҤÇƬ-ext(ҤÇƬ_M)]$

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$= C(\text{ŊÇT}-ext(\text{ŊÇT}_{\text{M}})) \sqcap C\text{ŊÇT}_M$ (by prop 2.10 (7) and De- Morgan law).

Now through definition (3.1) as well as their ŊÇT–interior and ŊÇT–exterior properties, it can be concluded the following properties:

Proposition 3.5: Let $T^X_{\text{ŊÇT}}$ be neutrosophic crisp topological spase over ŊÇT_X and let ŊÇT_M be ŊÇT–set. Then:

1) $\text{Ƌ}(\text{Ƌ}\text{ŊÇT}_{\text{M}}) \sqsubseteq \text{Ƌ}(\text{ŊÇT}_{\text{M}}) \sqcup C\text{Ƌ}(\text{ŊÇT}_{\text{M}})$
2) $\text{Ƌ}(\text{ŊÇT}_{\text{M}}) \sqcap \text{Ƌ}(\text{ŊÇT}_{\Phi}) = \text{Ƌ}(\text{ŊÇT}_{\text{M}}) \sqcap \text{Ƌ}(\text{ŊÇT}_X) = \text{Ƌ}(\text{ŊÇT}_{\text{M}})$
3) $\text{Ƌ}(\text{ŊÇT}_{\text{M}}) \sqcup \text{Ƌ}(\text{ŊÇT}_{\Phi}) = \text{Ƌ}(\text{ŊÇT}_{\text{M}}) \sqcup \text{Ƌ}(\text{ŊÇT}_X) = \text{ŊÇT}_X$

Proof:

Directly by applying definition (3.1) and proposition 3.2(1)

Note 3.6: The following properties are fulfilled in ordinary topological space but in neutrosophic crisp topological space not necessary.

1) $int(\text{M}) = \text{M} \cap \text{Ƌ}(\text{M})$
2) $ext(\text{M}) = \text{M}^c \cap \text{Ƌ}(\text{M})$.

We will explain. In example (3.3)

Let $\text{ŊÇT}_{\text{M}} = < \phi, \{a, b\}, \{\varsigma\} >$

Then $\text{ŊÇT}-int(\text{ŊÇT}_{\text{M}}) = \text{ŊÇT}_{\Phi} \neq \text{ŊÇT}_M \sqcap \text{Ƌ}(\text{ŊÇT}_{\text{M}}) = < \phi, \phi, \{\varsigma\} >$

Let $\text{ŊÇT}_{\text{M}} = < \{\varsigma\}, \{a, \varsigma\}, \phi >$

Then $\text{ŊÇT}-ext(\text{ŊÇT}_{\text{M}}) = \text{ŊÇT}_{\Phi} \neq C\text{ŊÇT}_M \sqcap \text{Ƌ}(\text{ŊÇT}_{\text{M}}) = < \phi, \phi, \{\varsigma\} >$

Proposition 3.7: Let$(\text{ŊÇT}_X, T^X_{\text{ŊÇT}})$ be neutrosophic crisp topological space and let ŊÇT_β be ŊÇT–set then: If $\text{ŊÇT}-ext(\text{ŊÇT}_\beta) = \text{ŊÇT}_{\Phi}$ then $C\left(\text{Ƌ}(\text{ŊÇT}_\beta)\right) = C(\text{ŊÇT}-int(\text{ŊÇT}_\beta)$

Proof:

Let $\text{ŊÇT}-ext(\text{ŊÇT}_\beta) = \text{ŊÇT}_{\Phi} \rightarrow \text{ŊÇT}-int(C\text{ŊÇT}_\beta) = \text{ŊÇT}_{\Phi}$

Take the complement of two side $\rightarrow C\left(\text{ŊÇT}-int(C\text{ŊÇT}_\beta)\right) = \text{ŊÇT}_X$

But $C\left(\text{Ƌ}(\text{ŊÇT}_\beta)\right) = C\left(\text{ŊÇT}-int(\text{ŊÇT}_\beta)\right) \sqcap C\left(\text{ŊÇT}-ext(\text{ŊÇT}_\beta)\right)$

$= C(\text{ŊÇT}-int(\text{ŊÇT}_\beta) \sqcap \text{ŊÇT}_X = C(\text{ŊÇT}-int(\text{ŊÇT}_\beta))$

Thus $C(\text{Ƌ}(\text{ŊÇT}_\beta)) = C(\text{ŊÇT}-int(\text{ŊÇT}_\beta))$

Proposition 3.8: Let ŊÇT_Z be any ŊÇT–set of neutrosophic crisp topological space then :

1) If $C(\text{Ƌ}(\text{ŊÇT}_Z)) = \text{ŊÇT}_{\Phi}$ then ŊÇT_Z be ŊÇT–closed set
2) If $\text{ŊÇT}_Z \sqcap C(\text{Ƌ}(\text{ŊÇT}_Z)) = \text{ŊÇT}_{\Phi}$ then ŊÇT_Z be ŊÇT–open set

Proof 1:

Let $C\left(\text{Ƌ}(\text{ŊÇT}_Z)\right) = \text{ŊÇT}_{\Phi} \rightarrow C[\text{ŊÇT}-int(\text{ŊÇT}_Z) \sqcup \text{ŊÇT}-ext(\text{ŊÇT}_Z)]$

$= \text{ŊÇT}_{\Phi} \rightarrow C\left(\text{ŊÇT}-int(\text{ŊÇT}_Z)\right) \sqcap C\left(\text{ŊÇT}-int(C\text{ŊÇT}_Z)\right)$

$= \text{ŊÇT}_{\Phi}$ by note 2.5(8)

$C(\text{ŊÇT}-int(C\text{ŊÇ}_Z)) \sqsubseteq \text{ŊÇT}-int(\text{ŊÇT}_Z) \sqsubseteq \text{ŊÇT}_Z$

Take the complement of both side

$C\text{ŊÇT}_Z \sqsubseteq \text{ŊÇT}-int(C\text{ŊÇT}_Z) = \text{ŊÇT}-ext(\text{ŊÇT}_Z)$

This mean $C\text{ŊÇT}_Z \sqsubseteq \text{ŊÇT}-ext(\text{ŊÇT}_Z)$

But $\text{ŊÇT}-ext(\text{ŊÇT}_Z) \sqsubseteq C\text{ŊÇT}_Z \rightarrow \text{ŊÇT}-ext(\text{ŊÇT}_Z) = C\text{ŊÇT}_Z$

Thus by note 2.12 ŊÇT_Z beŊÇT–closed.

2) Let $\text{ŊÇT}_Z \sqcap C\left(\text{Ƌ}(\text{ŊÇT}_Z)\right) = \text{ŊÇT}_{\Phi}$

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$\rightarrow$ $\text{ɈÇƬ}_{Z} \sqcap C[\text{ɈÇƬ}-int(\text{ɈÇƬ}_{Z}) \sqcup \text{ɈÇƬ}-ext(\text{ɈÇƬ}_{Z})] = \text{ɈÇƬ}_{\Phi}$

$\rightarrow$ by De-Morgan law and $(\text{ɈÇƬ}_{Z} \sqsubseteq C(\text{ɈÇƬ}-ext(\text{ɈÇƬ}_{Z}))$

$\rightarrow$ $\text{ɈÇƬ}_{Z} \sqcap C(\text{ɈÇƬ}-int(\text{ɈÇƬ}_{Z})) = \text{ɈÇ}_{\Phi}$

$\rightarrow$ by note 2.5(6) $\rightarrow$ $\text{ɈÇƬ}_{Z} \sqsubseteq \text{ɈÇƬ}-int(\text{ɈÇƬ}_{Z})$

But $\text{ɈÇƬ}-int(\text{ɈÇƬ}_{Z}) \sqsubseteq \text{ɈÇƬ}_{Z}$

This mean that $\text{ɈÇ}_{Z} = \text{ɈÇƬ}-int(\text{ɈÇƬ}_{Z})$

So by preposition 2.10 (7)) ɈÇƬ_{Z} is ɈÇƬ–open set

6. Conclusion

In this research, three important topological concepts (ɈÇƬ– interior, ɈÇƬ– exterior, and ɈÇƬ– confused) were studied which are considered the basis for building all other topological concepts. And the impact of these concepts on it directly, and as we noticed that the ɈÇƬ– interior $< \text{M}_1, \text{M}_2, \text{M}_3 > \neq < T_1 - int\text{M}_1, T_2 - int\text{M}_2, T_3 - int\text{M}_3 >$as example (3-3) and as a result of building the union and intersection as well as defining the complement of ɈÇƬ– sets, and from here we noticed and will notice in the case of studying other topological properties that will be affected by it. On the other hand, we can study W–open in conjunction with the concepts ɈÇ–interior and ɈÇƬ– exterior because the basics of their properties have been prepared, in addition to modifying all concepts studied in [1, 2] and extending them to ɈÇƬ– sets and we can also transform mathematical concepts in [5,6] onto ɈÇƬ-sets.

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