Logarithmic neutrosophic logical communicated to basic interaction aggregating operators using various finite weighted with extension

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Abstract

In this paper, we present novel techniques for the logarithmic neutrosophic interaction (LogNI) aggregating operator. The new averaging and geometric operations of LogNI numbers are studied using the universal aggregation function. The LogNI are satisfied some algebraic properties. Four novel aggregating operators are presented: LogNI weighted averaging, LogNI weighted geometric, generalized LogNI weighted averaging, and generalized LogNI weighted geometric.

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1 Introduction

The uncertainties have led to the development of fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS),[3,4] neutrosophic set (NSS)[5] and Fermatean FS (FFS).[6] For those who make decisions, Zadeh's FS[1] suggests a membership degree (MG). For $\gamma, \zeta \in [0,1]$, each object has MG γ and non-membership degree (NMG) ζ, and fulfills $0 \leq \gamma + \zeta \leq 1$. For this reason, Atanassov[2] proposed an IFS notion. The condition that $\gamma^2 + \zeta^2 \leq 1$ states that PFSs are characterized by their MGs and NMGs, which were created by Yager.[3] IFSs and PFSs have been widely utilized and investigated in a wide range of fields. Palanikumar et al.[7] deals that the concept of aggregation operators via Pythagorean neutrosophic soft set and square root neutrosophic normal interval-valued sets. Cuong and associates[8] by developing the concept of picture FSs (PicFSs). PicFSs has been found to accommodate certain more ambiguity because it is an enhanced version of IFSs. PicFSs states that the MG γ, neutral ζ, and NMG $\breve{m}$ all have $0 \leq \gamma + \zeta + \breve{m} \leq 1$ for $\gamma, \zeta, \breve{m} \in [0,1]$. Using the PicFS, it will ensure that expert opinion messages such as "yes," "abstain," "no," and "refusal" are transmitted. It will also ensure consistency between the assessment data and the actual decision environment and avoid evaluation information from being left out. Despite a large number of PicFSs implementations and studies, the concept has not been fully explored. Shahzaib et al.[9] was established the spherical FS (SFS) for certain

AOs with MADM. $0 \leq \gamma^2 + \zeta^2 + \breve{m}^2 \leq 1$ is required by the SFS, as opposed to $0 \leq \gamma + \zeta + \breve{m} \leq 1$. In MADM problems, the linguistic SFS AOs were discussed by Jin et al.[10] Rafiq and colleagues introduced SFSs and their applications in DM.[11] The concept of an FFS was first proposed by Senapati and associates[6] in 2019. This condition applies to both the MG and NMG: $0 \leq \gamma^3 + \zeta^3 \leq 1$.

2 Basic concepts

Definition 2.1. [3] Let $\Bbbk$ be the universe set. The PFS $\top = \Big\{x, \big\langle \gamma(\imath), \delta(\imath) \big\rangle \big| x \in \Bbbk \Big\}$, where $\gamma, \delta : \Bbbk \to [0,1]$ refers the MG and NMG of $x \in \Bbbk$ to $\top$, respectively and $0 \leq (\gamma(\imath))^2 + (\delta(\imath))^2 \leq 1$. For, $\top = \big\langle \gamma, \delta \big\rangle$ is called the Pythagorean fuzzy number (PFN).

Definition 2.2. [6] A Fermatean fuzzy set $\top = \Big\{x, \big\langle \gamma(\imath), \delta(\imath) \big\rangle \big| x \in \Bbbk \Big\}$, where $\gamma(\imath)$ and $\delta(\imath)$ denote MG and NMG of u respectively, where $\gamma, \delta : \Bbbk \to [0,1]$ and $0 \leq (\gamma(\imath))^3 + (\delta(\imath))^3 \leq 1$. Here, $\top = \big\langle \gamma, \delta \big\rangle$ is represent a Fermatean fuzzy number (FFN).

Definition 2.3. For any PFNs, $\top = \langle \gamma, \delta \rangle$, $\top_1 = \langle \gamma_1, \delta_1 \rangle$ and $\top_2 = \langle \gamma_2, \delta_2 \rangle$, γ, δ denote MG and NMG of u respectively. Then

1. $\top_1 \bigtriangledown \top_2 = \left[\sqrt{(\gamma_1)^2 + (\gamma_2)^2 - (\gamma_1)^2 \cdot (\gamma_2)^2}, (\delta_1 \cdot \delta_2)\right]$
2. $\top_1 \bigtriangleup \top_2 = \left[(\gamma_1 \cdot \gamma_2), \sqrt{(\delta_1)^2 + (\delta_2)^2 - (\delta_1)^2 \cdot (\delta_2)^2}\right]$
3. $\pitchfork \cdot \top = \left[\sqrt{\shortmid - \big(\shortmid - (\gamma)^2\big)^{\pitchfork}}, (\delta)^{\pitchfork}\right]$
4. $\top^{\pitchfork} = \left[(\gamma)^{\pitchfork}, \sqrt{\shortmid - \big(\shortmid - (\delta)^2\big)^{\pitchfork}}\right]$

Definition 2.4. If $\top_1 = \big\langle \gamma_1, \delta_1 \big\rangle$ and $\top_2 = \big\langle \gamma_2, \delta_2 \big\rangle$ are any two PFNs. Then the interaction AO is defined as

1. $\top_1 \bigtriangledown \top_2 = \left[\begin{array}{c} \sqrt{(\gamma_1)^2 + (\gamma_2)^2 - (\gamma_1)^2 \cdot (\gamma_2)^2}, \\ \sqrt{(\delta_1)^2 + (\delta_2)^2 - (\delta_1)^2 \cdot (\delta_2)^2 - (\delta_1)^2 \cdot (\gamma_2)^2 - (\gamma_1)^2 \cdot (\delta_2)^2} \end{array}\right]$
2. $\top_1 \bigtriangleup \top_2 = \left[\begin{array}{c} \sqrt{(\gamma_1)^2 + (\gamma_2)^2 - (\gamma_1)^2 \cdot (\gamma_2)^2 - (\gamma_1)^2 \cdot (\delta_2)^2 - (\delta_1)^2 \cdot (\gamma_2)^2}, \\ \sqrt{(\delta_1)^2 + (\delta_2)^2 - (\delta_1)^2 \cdot (\delta_2)^2} \end{array}\right]$
3. $\pitchfork \cdot \top_1 = \left[\sqrt{\shortmid - \big(\shortmid - (\gamma_1)^2\big)^{\pitchfork}}, \sqrt{\big(\shortmid - (\gamma_1)^2\big)^{\pitchfork} - \big(\shortmid - (\gamma_1 + \delta_1)^2\big)^{\pitchfork}}\right]$
4. $\top_1^{\pitchfork} = \left[\sqrt{\big(\shortmid - (\delta_1)^2\big)^{\pitchfork} - \big(\shortmid - (\gamma_1 + \delta_1)^2\big)^{\pitchfork}}, \sqrt{\shortmid - \big(\shortmid - (\delta_1)^2\big)^{\pitchfork}}\right]$

where $\pitchfork$ be a positive integers.

3 Different AOs for LogNN

Throughout this papar $\log_{\dashv_\imath} = \amalg_{\imath \multimap 1}^{\jmath} \{\gamma_\imath, \varpi_\imath, \delta_\imath\}$.

Definition 3.1. Suppose that $\top_1 = \big\langle \gamma_1, \varpi_1, \delta_1 \big\rangle$ and $\top_2 = \big\langle \gamma_2, \varpi_2, \delta_2 \big\rangle$ be the any two LogNNs. Then

1. $\top_1 \bigtriangledown \top_2 = \left[\begin{array}{c} \sqrt{\langle \log_{\dashv_\imath}^2 \gamma_1 \rangle + \langle \log_{\dashv_\imath}^2 \gamma_2 \rangle - \langle \log_{\dashv_\imath}^2 \gamma_1 \rangle \cdot \langle \log_{\dashv_\imath}^2 \gamma_2 \rangle}, \\ \sqrt{\langle \log_{\dashv_\imath}^2 \varpi_1 \rangle + \langle \log_{\dashv_\imath}^2 \varpi_2 \rangle - \langle \log_{\dashv_\imath}^2 \varpi_1 \rangle \cdot \langle \log_{\dashv_\imath}^2 \varpi_2 \rangle}, \\ \sqrt{\begin{array}{c} \langle \log_{\dashv_\imath}^2 \delta_1 \rangle + \langle \log_{\dashv_\imath}^2 \delta_2 \rangle - \langle \log_{\dashv_\imath}^2 \delta_1 \rangle \cdot \langle \log_{\dashv_\imath}^2 \delta_2 \rangle \\ - \langle \log_{\dashv_\imath}^2 \delta_1 \rangle \cdot \langle \log_{\dashv_\imath}^2 \gamma_2 \rangle - \langle \log_{\dashv_\imath}^2 \gamma_1 \rangle \cdot \langle \log_{\dashv_\imath}^2 \delta_2 \rangle \end{array}} \end{array}\right]$

2. $\top_1 \triangle \top_2 = \begin{bmatrix} \sqrt{\begin{matrix}\langle \log^2_{\exists_\imath} \gamma_1\rangle + \langle \log^2_{\exists_\imath} \gamma_2\rangle - \langle \log^2_{\exists_\imath} \gamma_1\rangle \cdot \langle \log^2_{\exists_\imath} \gamma_2\rangle \\ -\langle \log^2_{\exists_\imath} \gamma_1\rangle \cdot \langle \log^2_{\exists_\imath} \delta_2\rangle - \langle \log^2_{\exists_\imath} \delta_1\rangle \cdot \langle \log^2_{\exists_\imath} \gamma_2\rangle\end{matrix}}, \\ \sqrt{\langle \log^2_{\exists_\imath} \varpi_1\rangle + \langle \log^2_{\exists_\imath} \varpi_2\rangle - \langle \log^2_{\exists_\imath} \varpi_1\rangle \cdot \langle \log^2_{\exists_\imath} \varpi_2\rangle} \\ \sqrt{\langle \log^2_{\exists_\imath} \delta_1\rangle + \langle \log^2_{\exists_\imath} \delta_2\rangle - \langle \log^2_{\exists_\imath} \delta_1\rangle \cdot \langle \log^2_{\exists_\imath} \delta_2\rangle} \end{bmatrix}$

3. $\pitchfork \cdot \top_1 = \begin{bmatrix} \sqrt{\shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1\rangle\rangle^{\pitchfork}}, \sqrt{\shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_1\rangle\rangle^{\pitchfork}}, \\ \sqrt{\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1\rangle\rangle^{\pitchfork} - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\pitchfork}} \end{bmatrix}$

4. $\top_1^{\pitchfork} = \begin{bmatrix} \sqrt{\langle \shortmid - \langle \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\pitchfork} - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\pitchfork}} \\ \sqrt{\shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_1\rangle\rangle^{\pitchfork}}, \sqrt{\shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\pitchfork}} \end{bmatrix}$

3.1 LogNIWA operator

Definition 3.2. Let $\top_\imath = \langle \gamma_\imath, \varpi_\imath, \delta_\imath\rangle$ be the LogNNs, $\imath \multimap 1, 2, ..., \jmath$, $\zeta_\imath$ be the weight of $\top_\imath$ and $\zeta_\imath \geq 0$, $\boxplus^{\jmath}_{\imath \multimap 1} \zeta_\imath = 1$. Then the LogNIWA operator $\langle \top_1, \top_2, ..., \top_\jmath\rangle = \boxplus^{\jmath}_{\imath \multimap 1} \zeta_\imath \top_\imath$.

Theorem 3.3. *Let $\top_\imath = \langle \gamma_\imath, \varpi_\imath, \delta_\imath\rangle$ be the LogNNs, $\imath \multimap 1, 2, ..., \jmath$.*

Then, LogNIWA $\langle \top_1, \top_2, ..., \top_\jmath\rangle = \begin{bmatrix} \sqrt{\shortmid - \divideontimes^{\jmath}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_\imath\rangle\right\rangle^{\zeta_\imath}}, \sqrt{\shortmid - \divideontimes^{\jmath}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_\imath\rangle\right\rangle^{\zeta_\imath}}, \\ \sqrt{\divideontimes^{\jmath}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_\imath\rangle\right\rangle^{\zeta_\imath} - \divideontimes^{\jmath}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_\imath + \log^2_{\exists_\imath} \delta_\imath\rangle\right\rangle^{\zeta_\imath}} \end{bmatrix}.$

Proof. If $\imath \multimap 2$, LogNIWA$\langle \top_1, \top_2\rangle = \zeta_1 \top_1 \triangledown \zeta_2 \top_2$,
where,

$$\zeta_1 \top_1 = \begin{bmatrix} \sqrt{\shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1\rangle\right\rangle^{\zeta_1}}, \sqrt{\shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_1\rangle\right\rangle^{\zeta_1}}, \\ \sqrt{\left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1\rangle\right\rangle^{\zeta_1} - \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1\rangle\right\rangle^{\zeta_1}} \end{bmatrix}$$

and

$$\zeta_2 \top_2 = \begin{bmatrix} \sqrt{\shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_2\rangle\right\rangle^{\zeta_2}}, \sqrt{\shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_2\rangle\right\rangle^{\zeta_2}}, \\ \sqrt{\left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_2\rangle\right\rangle^{\zeta_2} - \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_2 + \log^2_{\exists_\imath} \delta_2\rangle\right\rangle^{\zeta_2}} \end{bmatrix}$$

We get

$$\begin{aligned} \zeta_1 \top_1 \triangledown \zeta_2 \top_2 \quad &= \begin{bmatrix} \sqrt{\begin{matrix} \left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1\rangle\rangle^{\zeta_1}\right\rangle + \left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_2\rangle\rangle^{\zeta_2}\right\rangle \\ -\left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1\rangle\rangle^{\zeta_1}\right\rangle \cdot \left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_2\rangle\rangle^{\zeta_2}\right\rangle \end{matrix}}, \\ \sqrt{\begin{matrix} \left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_1\rangle\rangle^{\zeta_1}\right\rangle + \left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_2\rangle\rangle^{\zeta_2}\right\rangle \\ -\left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_1\rangle\rangle^{\zeta_1}\right\rangle \cdot \left\langle \shortmid - \langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_2\rangle\rangle^{\zeta_2}\right\rangle \end{matrix}}, \\ \sqrt{\begin{matrix} \left\langle \shortmid - \langle \shortmid - \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\zeta_1}\right\rangle + \left\langle \shortmid - \left\langle \shortmid - \log^2_{\exists_\imath} \delta_2\right\rangle\right\rangle^{\zeta_2}\Big\rangle \\ -\left\langle \shortmid - \langle \shortmid - \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\zeta_1}\right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \log^2_{\exists_\imath} \delta_2\right\rangle\right\rangle^{\zeta_2}\Big\rangle \\ -\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1\rangle\rangle^{\zeta_1} \cdot \langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_2 + \log^2_{\exists_\imath} \delta_2\rangle\rangle^{\zeta_2} \end{matrix}} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{\shortmid - \divideontimes^{2}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_\imath\rangle\right\rangle^{\zeta_\imath}}, \sqrt{\shortmid - \divideontimes^{2}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \varpi_\imath\rangle\right\rangle^{\zeta_\imath}}, \\ \sqrt{\divideontimes^{2}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_\imath\rangle\right\rangle^{\zeta_\imath} - \divideontimes^{2}_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\exists_\imath} \gamma_\imath + \log^2_{\exists_\imath} \delta_\imath\rangle\right\rangle^{\zeta_\imath}} \end{bmatrix}. \end{aligned}$$

Ug induction $\imath \geq 3$, LogNIWA$\langle \top_1, \top_2, ..., \top_\jmath \rangle$

$$= \left[\begin{array}{c} \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath}}, \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_\imath \rangle \right\rangle^{\zeta_\imath}}, \\ \sqrt{\circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath} - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath + \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath}}, \end{array} \right].$$

If $\imath \multimap \jmath + 1$, then $LogNIWA\langle \top_1, \top_2, ..., \top_\jmath, \top_{\jmath+1} \rangle$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{c} \boxplus_{\imath \multimap 1}^{\jmath} \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle + \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle \\ - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle \cdot \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle, \end{array}} \\ \sqrt{\begin{array}{c} \boxplus_{\imath \multimap 1}^{\jmath} \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle + \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle \\ - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle \cdot \left\langle \imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle, \end{array}} \\ \sqrt{\begin{array}{c} \boxplus_{\imath \multimap 1}^{\jmath} \left\langle \imath - \left\langle \imath - \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle + \left\langle \imath - \left\langle \imath - \log_{\exists_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle \\ - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \left\langle \imath - \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle \cdot \left\langle \imath - \left\langle \imath - \log_{\exists_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle \\ - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath + \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath} \cdot \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_{\jmath+1} + \log_{\exists_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \end{array}} \end{array} \right]$$

$$= \left[\begin{array}{c} \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath} \cdot \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}}}, \\ \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_\imath \rangle \right\rangle^{\zeta_\imath} \cdot \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}}}, \\ \sqrt{\begin{array}{c} \left\langle \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath} - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath + \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath} \right\rangle \cdot \\ \left\langle \left\langle \langle \log_{\exists_\imath}^2 \gamma_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} - \langle \log_{\exists_\imath}^2 \gamma_{\jmath+1} + \log_{\exists_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^{\zeta_{\jmath+1}} \right\rangle \end{array}} \end{array} \right]$$

$$= \left[\begin{array}{c} \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath+1} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath}}, \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath+1} \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_\imath \rangle \right\rangle^{\zeta_\imath}}, \\ \sqrt{\circledast_{\imath \multimap 1}^{\jmath+1} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath \rangle \right\rangle^{\zeta_\imath} - \circledast_{\imath \multimap 1}^{\jmath+1} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath + \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath}} \end{array} \right]$$

3.2 Interaction weighted geometric(LogNIWG)operator

Definition 3.4. Let $\top_\imath = \langle \gamma_\imath, \varpi_\imath, \delta_\imath \rangle$ be the LogNNs, $\zeta_\imath$ be the weight of $\top_\imath$. Then the LogNIWG operator $\langle \top_1, \top_2, ..., \top_\jmath \rangle = \circledast_{\imath \multimap 1}^{\jmath} \top_\imath^{\zeta_\imath}$.

Theorem 3.5. *If $\top_\imath = \langle \gamma_\imath, \varpi_\imath, \delta_\imath \rangle$ be the LogNNs. Then,*

$$LogNIWG\langle \top_1, \top_2, ..., \top_\jmath \rangle = \left[\begin{array}{c} \sqrt{\circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath} - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_\imath + \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath}}, \\ \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_\imath \rangle \right\rangle^{\zeta_\imath}}, \sqrt{\imath - \circledast_{\imath \multimap 1}^{\jmath} \left\langle \imath - \langle \log_{\exists_\imath}^2 \delta_\imath \rangle \right\rangle^{\zeta_\imath}} \end{array} \right]$$

Proof. If $\imath \multimap 2$, then LogNIWG$\langle \top_1, \top_2 \rangle = \top_1^{\zeta_1} \triangle \top_2^{\zeta_2}$,
where,

$$\top_1^{\zeta_1} = \left[\begin{array}{c} \sqrt{\left\langle \imath - \langle \log_{\exists_\imath}^2 \delta_1 \rangle \right\rangle^{\zeta_1} - \left\langle \imath - \langle \log_{\exists_\imath}^2 \gamma_1 + \log_{\exists_\imath}^2 \delta_1 \rangle \right\rangle^{\zeta_1}}, \\ \sqrt{\imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \varpi_1 \rangle \right\rangle^{\zeta_1}}, \sqrt{\imath - \left\langle \imath - \langle \log_{\exists_\imath}^2 \delta_1 \rangle \right\rangle^{\zeta_1}} \end{array} \right]$$

$$\top_2^{\zeta_2} = \left[\begin{array}{c} \sqrt{\left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2} - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \gamma_2 + \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2}}, \\ \sqrt{\shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_2 \rangle \right\rangle^{\zeta_2}} \ \sqrt{\shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2}} \end{array} \right]$$

We get,

$$\top_1^{\zeta_1} \triangle \top_2^{\zeta_2} = \left[\begin{array}{c} \sqrt{\begin{array}{r} \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_1 \rangle \right\rangle^{\zeta_1} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2} \right\rangle \\ - \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_1 \rangle \right\rangle^{\zeta_1} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2} \right\rangle \\ - \left\langle \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \gamma_1 + \log^2_{\Finv_\imath} \delta_1 \rangle \right\rangle^{\zeta_1} \cdot \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \gamma_2 + \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2} \right\rangle \end{array}} \\ \sqrt{\begin{array}{c} \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_1 \rangle \right\rangle^{\zeta_1} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_2 \rangle \right\rangle^{\zeta_2} \right\rangle \\ - \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_1 \rangle \right\rangle^{\zeta_1} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_2 \rangle \right\rangle^{\zeta_2} \right\rangle \end{array}} \\ \sqrt{\begin{array}{c} \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_1 \rangle \right\rangle^{\zeta_1} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2} \right\rangle \\ - \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_1 \rangle \right\rangle^{\zeta_1} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_2 \rangle \right\rangle^{\zeta_2} \right\rangle \end{array}} \end{array} \right]$$

$$\text{Hence, LogNIWG}\langle \top_1, \top_2 \rangle = \left[\begin{array}{c} \sqrt{\divideontimes^2_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^{\zeta_\imath} - \divideontimes^2_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \gamma_\imath + \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^{\zeta_\imath}}, \\ \sqrt{\shortmid - \divideontimes^2_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_\imath \rangle \right\rangle^{\zeta_\imath}}, \sqrt{\shortmid - \divideontimes^2_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^{\zeta_\imath}} \end{array} \right]$$

$$\text{LogNIWG}\langle \top_1, \top_2, ..., \top_\jmath \rangle = \left[\begin{array}{c} \sqrt{\divideontimes^\jmath_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^{\zeta_\imath} - \divideontimes^\jmath_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \gamma_\imath + \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^{\zeta_\imath}}, \\ \sqrt{\shortmid - \divideontimes^\jmath_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \varpi_\imath \rangle \right\rangle^{\zeta_\imath}}, \sqrt{\shortmid - \divideontimes^\jmath_{\imath \multimap 1} \left\langle \shortmid - \langle \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^{\zeta_\imath}} \end{array} \right]$$

If $\imath \multimap \jmath+1$, then LogNIWG$\langle \top_1, ..., \top_\jmath, \top_{\jmath+1}\rangle$

$$
= \begin{bmatrix}
\sqrt{\begin{array}{c}
\boxplus_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle \\
- \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle \\
- \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\gamma_\imath + \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath} \cdot \left\langle \shortmid - \langle \log^2_{\exists_\imath}\gamma_{\jmath+1} + \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}
\end{array}} \\
\sqrt{\begin{array}{c}
\boxplus_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle \\
- \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle
\end{array}} \\
\sqrt{\begin{array}{c}
\boxplus_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle \\
- \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle
\end{array}}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\sqrt{\begin{array}{c}
\left\langle \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\gamma_\imath + \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}\right\rangle \cdot \\
\left\langle \left\langle \langle \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}} - \langle \log^2_{\exists_\imath}\gamma_{\jmath+1} + \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}\right\rangle
\end{array}}, \\
\sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_\imath\rangle\right\rangle^{\zeta_\imath} \cdot \left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}} \\
\sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath} \cdot \left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_{\jmath+1}\rangle\right\rangle^{\zeta_{\jmath+1}}}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\sqrt{\divideontimes_{\imath\multimap 1}^{\jmath+1}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath+1}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\gamma_\imath + \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}}, \\
\sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath+1}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\varpi_\imath\rangle\right\rangle^{\zeta_\imath}}\sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath+1}\left\langle \shortmid - \langle \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^{\zeta_\imath}}
\end{bmatrix}
$$

3.3 Generalized LogNIWA (GLogNIWA) operator

Definition 3.6. Let $\top_\imath = \langle \gamma_\imath, \varpi_\imath, \delta_\imath\rangle$ be the LogNNs, $\zeta_\imath$ be a weight of $\top_\imath$. Then, the GLogNIWA operator $\langle \top_1, \top_2, ..., \top_\jmath\rangle = \left\langle \boxplus_{\imath\multimap 1}^{\jmath}\zeta_\imath \top_\imath^2\right\rangle^{1/2}$.

Theorem 3.7. *Let* $\top_\imath = \langle \gamma_\imath, \varpi_\imath, \delta_\imath\rangle$ *be the LogNNs. Then GLogNIWA* $\langle \top_1, \top_2, ..., \top_\jmath\rangle =$

$$
\begin{bmatrix}
\left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\right\rangle, \left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\varpi_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\right\rangle, \\
\left\langle \sqrt{\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_\imath + \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\right\rangle
\end{bmatrix}.
$$

Proof. First, we have find that

$$
\boxplus_{\imath\multimap 1}^{\jmath}\zeta_\imath \top_\imath^2 = \begin{bmatrix}
\sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\varpi_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \\
\sqrt{\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_\imath + \log^2_{\exists_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}
\end{bmatrix}.
$$

If $\imath \multimap 2$, then
$$
\zeta_1 \top_1^2 = \begin{bmatrix}
\sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}, \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\varpi_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}, \\
\sqrt{\left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_1\rangle\right\rangle^2\right\rangle^{\zeta_1} - \left\langle \shortmid - \left\langle \langle \log^2_{\exists_\imath}\gamma_1 + \log^2_{\exists_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}
\end{bmatrix}
$$

and

$$\zeta_2 \top_2^2 = \left[\sqrt{\frac{\sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}}, \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}},}{\left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1} - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 + \log^2_{\exists_\imath} \delta_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}}} \right].$$

We get, $\zeta_1 \top_1 \bigtriangledown \zeta_2 \top_2$=

$$\left[\begin{array}{l} \sqrt{\begin{array}{l} \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 + \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2, \\ - \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \cdot \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \end{array}} \\ \sqrt{\begin{array}{l} \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 + \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2, \\ - \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \cdot \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \end{array}} \\ \sqrt{\begin{array}{l} \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 + \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \delta_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2, \\ - \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \cdot \left\langle \sqrt{\imath - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \delta_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \\ - \left[\begin{array}{l} \left\langle \sqrt{\left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1} - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \cdot \\ \left\langle \sqrt{\left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1} - \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_2 + \log^2_{\exists_\imath} \delta_2 \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \end{array} \right] \end{array}} \end{array} \right]$$

$$= \left[\sqrt{\frac{\sqrt{\imath - ⋇^2_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}, \sqrt{\imath - ⋇^2_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}},}{⋇^2_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - ⋇^2_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}} \right]$$

In general,

$$= \left[\sqrt{\frac{\sqrt{\imath - ⋇^{\jmath}_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}, \sqrt{\imath - ⋇^{\jmath}_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \varpi_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}}{⋇^{\jmath}_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - ⋇^{\jmath}_{\imath \multimap 1} \left\langle \imath - \left\langle \langle \log^2_{\exists_\imath} \gamma_1 + \log^2_{\exists_\imath} \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}} \right].$$

If $\imath \multimap \jmath + 1$, then $\boxplus^{\jmath}_{\imath \multimap 1} \zeta_\imath \top^2_\imath + \zeta_{\jmath+1} \top^2_{\jmath+1} = \boxplus^{\jmath+1}_{\imath \multimap 1} \zeta_\imath \top^2_\imath$.
Now, $\boxplus^{\jmath}_{\imath \multimap 1} \zeta_\imath \top^2_\imath + \zeta_{\jmath+1} \top^2_{\jmath+1} = \zeta_1 \top^2_1 \bigtriangledown \zeta_2 \top^2_2 \bigtriangledown \ldots \bigtriangledown \zeta_\jmath \top^2_\jmath \bigtriangledown \zeta_{\jmath+1} \top^2_{\jmath+1}$

$$
= \left[\begin{array}{c}
\sqrt{\begin{array}{c}\left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 + \left\langle \sqrt{\text{ı} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2, \\ - \left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \left\langle \sqrt{\text{ı} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \end{array}} \\
\sqrt{\begin{array}{c}\left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 + \left\langle \sqrt{\text{ı} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2, \\ - \left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \left\langle \sqrt{\text{ı} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \end{array}} \\
\sqrt{\begin{array}{c}\left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 + \left\langle \sqrt{\text{ı} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2, \\ - \left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \left\langle \sqrt{\text{ı} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \\ - \left[\begin{array}{c} \left\langle \sqrt{\divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath + \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \\ \left\langle \sqrt{\left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1} - \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_{\jmath+1} + \log^2_{\Finv_\imath} \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \end{array}\right] \end{array}}
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}, \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}, \\
\sqrt{\divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_1 + \log^2_{\Finv_\imath} \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}
\end{array}\right]
$$

and
$$
\boxplus_{\imath \multimap 1}^{\jmath+1} \left\langle \zeta_\imath \top_\imath^2 \right\rangle = \left[\begin{array}{c}
\left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle, \\
\left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle, \\
\left\langle \sqrt{\divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath \multimap 1}^{\jmath+1} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath + \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle
\end{array}\right].
$$

3.4 Generalized LogNIWG(GLogNIWG) operator

Definition 3.8. Let $\top_\imath = \left\langle \gamma_\imath, \varpi_\imath, \delta_\imath \right\rangle$ be the LogNNs, $\zeta_\imath\rangle$ be the weight of $\top_\imath$, where $\imath \multimap 1, 2, ..., \jmath$. Then, the GLogNIWG$\langle \top_1, \top_2, ..., \top_\jmath \rangle = \frac{1}{\pitchfork} \left\langle \divideontimes_{\imath \multimap 1}^{\jmath} \langle \pitchfork\, \top_\imath \rangle^{\zeta_\imath} \right\rangle$.

Theorem 3.9. *Let $\top_\imath = \left\langle \gamma_\imath, \varpi_\imath, \delta_\imath \right\rangle$ be the collection of LogNNs.*
Then the GLogNIWG operator $\langle \top_1, \top_2, ..., \top_\jmath \rangle =$

$$
\left[\begin{array}{c}
\left\langle \sqrt{\divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \gamma_\imath + \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle, \\
\left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle \left\langle \sqrt{\text{ı} - \divideontimes_{\imath \multimap 1}^{\jmath} \left\langle \text{ı} - \left\langle \langle \log^2_{\Finv_\imath} \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle
\end{array}\right]
$$

Proof. Ug the induction method,

$$\divideontimes_{\imath\multimap 1}^{\jmath}\langle \pitchfork \top_{\imath}\rangle^{\zeta_\imath} = \left[\begin{array}{c}\sqrt{\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_\imath+\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \\ \sqrt{\imath-\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\sqrt{\imath-\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\end{array}\right]$$

If $\imath \multimap 2$, then

$$\langle \pitchfork \top_1\rangle^{\zeta_1} = \left[\begin{array}{c}\sqrt{\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1} - \left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_1+\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}} \\ \sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\ \sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\end{array}\right]$$

and

$$\langle \pitchfork \top_2\rangle^{\zeta_2} = \left[\begin{array}{c}\sqrt{\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1} - \left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_2+\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}} \\ \sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\end{array}\right]$$

We get, $\langle \pitchfork \top_1\rangle^{\zeta_1} \triangle \langle \pitchfork \top_2\rangle^{\zeta_2}$

$$= \left[\begin{array}{c}\sqrt{\begin{array}{c}\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 + \left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 \\ -\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\cdot\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 \\ -\left[\begin{array}{c}\left\langle\sqrt{\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_1+\gamma_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\cdot \\ \left\langle\sqrt{\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_2+\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\end{array}\right]\end{array}}, \\ \sqrt{\begin{array}{c}\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 + \left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 \\ -\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\cdot\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\end{array}} \\ \sqrt{\begin{array}{c}\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 + \left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2 \\ -\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_1\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\cdot\left\langle\sqrt{\imath-\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_2\rangle\right\rangle^2\right\rangle^{\zeta_1}}\right\rangle^2\end{array}}\end{array}\right]$$

$$= \left[\begin{array}{c}\sqrt{\divideontimes_{\imath\multimap 1}^{2}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{2}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_\imath+\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \\ \sqrt{\imath-\divideontimes_{\imath\multimap 1}^{2}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \sqrt{\imath-\divideontimes_{\imath\multimap 1}^{2}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\end{array}\right]$$

If $\imath \multimap \jmath$, then

$$= \left[\begin{array}{c}\sqrt{\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\gamma_\imath+\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \\ \sqrt{\imath-\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\varpi_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}, \sqrt{\imath-\divideontimes_{\imath\multimap 1}^{\jmath}\left\langle \imath-\left\langle\langle\log^2_{\Finv_\imath}\delta_\imath\rangle\right\rangle^2\right\rangle^{\zeta_\imath}}\end{array}\right]$$

If $\imath \multimap \jmath+1$, then $\divideontimes_{\imath\multimap 1}^{\jmath}\langle \pitchfork \top_{\imath}\rangle^{\zeta_\imath} \cdot \langle \pitchfork \top_{\jmath+1}\rangle^{\zeta_{\jmath+1}} = \divideontimes_{\imath\multimap 1}^{\jmath+1}\langle \pitchfork \top_{\imath}\rangle^{\zeta_\imath}$.
Now, $\divideontimes_{\imath\multimap 1}^{\jmath}\langle \pitchfork \top_{\imath}\rangle^{\zeta_\imath} \cdot \langle \pitchfork \top_{\jmath+1}\rangle^{\zeta_{\jmath+1}} = \langle \pitchfork \top_{1}\rangle^{\zeta_1} \triangle \langle \pitchfork \top_{2}\rangle^{\zeta_2} \triangle \ldots \triangle \langle \pitchfork \top_{\jmath}\rangle^{\zeta_\jmath} \triangle \langle \pitchfork \top_{\jmath+1}\rangle^{\zeta_{\jmath+1}}$

$$
= \left[\begin{array}{c}
\sqrt{\begin{array}{c}
\left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 + \left\langle \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \\
- \left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \left\langle \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2
\end{array}}, \\
- \left[\begin{array}{c}
\left\langle \sqrt{\divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \gamma_\imath + \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \\
\left\langle \sqrt{\left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1} - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \gamma_{\jmath+1} + \log_{\daleth_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2
\end{array} \right] \\
\sqrt{\begin{array}{c}
\left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 + \left\langle \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \varpi_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \\
- \left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \left\langle \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \varpi_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2
\end{array}} \\
\sqrt{\begin{array}{c}
\left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 + \left\langle \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2 \\
- \left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle^2 \cdot \left\langle \sqrt{\shortmid - \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_{\jmath+1} \rangle \right\rangle^2 \right\rangle^{\zeta_1}} \right\rangle^2
\end{array}}
\end{array} \right]
$$

$$
= \left[\begin{array}{c}
\sqrt{\divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \gamma_1 + \log_{\daleth_\imath}^2 \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}, \\
\sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \varpi_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}, \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_1 \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}}
\end{array} \right]
$$

Hence

$$
\frac{1}{\pitchfork} \left\langle \divideontimes_{\imath\multimap 1}^{\jmath+1} \langle \pitchfork \top_\imath \rangle^{\zeta_\imath} \right\rangle =
$$

$$
\left[\begin{array}{c}
\left\langle \sqrt{\divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath} - \divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \gamma_\imath + \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle, \\
\left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \varpi_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle \left\langle \sqrt{\shortmid - \divideontimes_{\imath\multimap 1}^{\jmath+1} \left\langle \shortmid - \left\langle \langle \log_{\daleth_\imath}^2 \delta_\imath \rangle \right\rangle^2 \right\rangle^{\zeta_\imath}} \right\rangle
\end{array} \right]
$$

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