A Comprehensive Approach to Solid Waste Management Site Selection Using Simplified Neutrosophic Distance-Based Similarity Measures with N-Valued T-Spherical Fuzzy Neutrosophic Sets

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Abstract

In a neutrosophic environment, a single-valued neutrosophic multi-set, and an intuitionistic fuzzy-valued neutrosophic multi-set are defined by sequences of acceptance, indeterminacy, and rejection grades. The structure of these sets enables the incorporation of multiple layers of information across acceptance, indeterminacy, and rejection grades, making them particularly valuable for multi-criteria decision-making processes. This paper presents the N-valued T-spherical fuzzy neutrosophic set as an advanced extension of neutrosophic sets, aimed at improving uncertainty management and imprecision in complex, real-world scenarios. Building upon previous models such as neutrosophic sets, intuitionistic fuzzy-valued neutrosophic sets, Pythagorean fuzzy neutrosophic sets, and T-spherical fuzzy neutrosophic sets, this new approach introduces greater flexibility in handling indeterminacy. The authors define N-valued T-spherical fuzzy neutrosophic sets and numbers, incorporating new mathematical operations and comparison functions. A significant contribution of the work is the development of simplified neutrosophic-valued distance-based similarity measures for N-valued T-spherical fuzzy neutrosophic sets, along with a score function to rank simplified neutrosophic values. To illustrate the practical utility of this framework, an algorithm is applied to a real-world problem of site selection for solid waste management systems, effectively addressing decision-making scenarios with disjoint criteria. The results and discussions show that the N-valued T-spherical fuzzy neutrosophic set outperforms existing methods by providing more accurate and precise results, specifically in multi-criteria decision-making contexts. The site choice example for solid waste management highlights how this new approach enhances accuracy.

Keywords N-valued T-spherical fuzzy neutrosophic set; Simplified neutrosophic valued distance-based similarity measure; Site selection for solid waste management system; Multi-criteria decision-making technique

1. Introduction

When addressing real-life problems, selecting the best option from multiple alternatives is often essential, and Multi-Criteria Decision Making is a valuable tool in such processes. Decision theory has become a crucial area of study across various scientific fields, especially in addressing the complexities of decision-making under uncertainty. Many decisions are complicated by ambiguity and uncertainty, which must be accounted for when solving real-world issues. One of the key challenges in this process is dealing with uncertain data. To address this, various mathematical theories have been developed, such as Fuzzy Sets (FS) [1], Picture fuzzy sets (PFSs), Pythagorean Fuzzy Sets (PFSs) [2], spherical fuzzy sets (SFS), T-spherical fuzzy sets (T-SFSs), and Intuitionistic Fuzzy Sets (IFSs) [3], where uncertainty is handled through acceptance and rejection functions. In 1998, Smarandache introduced the neutrosophic set (NS) [4] theory as a generalization of these models, incorporating three self-reliant factors: acceptance ($\mathbf{t}$), indeterminacy ($\mathbf{i}$), and rejection ($\boldsymbol{f}$), offering a more flexible approach to managing uncertainty, which provides enhanced tools for decision-making under uncertainty. A unique feature of NSs is that the sum of the supremum of these factors should not invade three, allowing for a more nuanced and accurate representation of real-world data. This innovative approach has attracted widespread attention from

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researchers, leading to numerous studies and further exploration. Ye [5] introduced simplified neutrosophic sets (SNSs) as a specific type within the broader framework of neutrosophic sets (NSs).

Cuong [6] introduced picture fuzzy sets (PFS) to address specific challenges, establishing foundational principles and exploring methods to measure distances between PFSs. Mahmood et al. [7] further advanced fuzzy set theory by developing Spherical Fuzzy Sets (SFS), which are particularly effective for tackling complex multi-attribute, multi-objective problems involving distinct attribute values. This framework has been extended across a variety of uncertain contexts. One of their key innovations is the T-Spherical Fuzzy Set (T-SFS), which includes a unique limitation, $0 \leq Tq + Iq + Fq \leq 1$ for $q \in Z^+$ and $q \geq 1$. This limitation enhances flexibility for decision-makers, minimizing information loss in decision-making processes. T-SFSs provide a broader scope for representing fuzzy information than traditional SFSs, making them highly applicable in decision-making where precision and adaptability are crucial. Additionally, Asharaf al-Quran [8] proposed a novel multi-attribute decision-making method based on T-spherical hesitant fuzzy sets. The frameworks of SFS, T-SFS, and Cubic SNSs find widespread use across multiple decision-making applications in various domains [9-14].

Bhowmik and Pal [15] introduced the intuitionistic fuzzy valued neutrosophic set (IFVNS), defining it with the limitation that the sum of its components should not invade two. Structure on this, Unver et al. [16] extended the concept by developing intuitionistic fuzzy neutrosophic multisets (IFNMSs) and investigating algebraic operations within IFVNSs to create new aggregation operators. Recently, Bozyigit et al. [17] redefined Pythagorean fuzzy-valued neutrosophic sets (PyFVNSs), where each component must satisfy the condition $0 \leq t^2 + f^2 \leq 1$. Rajan and Krishnaswamy [18] subsequently refined clustering models based on similarity measures between Pythagorean FVNSs. However, IFVNSs and Pythagorean FVNSs have limitations in decision-making contexts where intuitionistic and Pythagorean fuzzy values may not fully capture essential decision information. To address this, Asharaf al-Quran et al. [19] expanded the realm of IFVNSs and Pythagorean FVNSs by introducing T-spherical fuzzy values into the neutrosophic set framework, resulting in T-spherical fuzzy-valued neutrosophic sets (T-SFVNSs), where each component follows the condition $0 \leq tq + iq + fq \leq 3$ for $q \in Z^+$ and $q \geq 1$, and the total of the three self-reliant factors should not invade three. This article further broadens the framework of IFVNMSs and T-SFVNSs by incorporating T-spherical fuzzy values into simplified neutrosophic sets (SNS), presenting a novel concept: N-valued T-spherical fuzzy neutrosophic sets (N-valued T-SFVNS).

Distance-based similarity and comparison measures are vital in numerous real-world applications, as they quantify how similar or different two objects are by assigning a numerical value to the comparison. These comparisons are typically based on the attributes or variables of the objects, with dissimilarities often defined using distance metrics, where a smaller distance indicates greater similarity. Similarity and dissimilarity are generally inversely related: as similarity increases, dissimilarity decreases, and vice versa. In some cases, weights are assigned to variables based on their importance to a particular application. While various comparison measures are employed across different domains, they are rooted in similar underlying principles. Classical dissimilarity measures for objects with continuous numerical attributes include Euclidean distance, Manhattan distance, and the broader class of Minkowski distances. These measures are essential for assessing the similarity or deviation between two sets of objects. When applied to neutrosophic sets, such measures have been utilized in fields like medical diagnosis, economics, agriculture, multicriteria decision-making (MCDM), and pattern recognition. Additionally, using cosine, tangent, and cotangent functions, trigonometric similarity measures have applications in several aspects of everyday life.

Over the years, numerous measures for the Single Valued Neutrosophic Sets (SVNS) model have been introduced and developed, encompassing similarity, distance, and entropy measures within the neutrosophic framework. Pioneering contributions from researchers such as Broumi and Smarandache [20], Cui and Ye [21], Ye [22- 25], Ye and Zhang [26], Ye and Fu [27], Majumdar and Samanta [28], and Liu [29] have significantly shaped the field, particularly concerning similarity and distance measures for SVNSs. Pranab Biswas et al. [30], along with Ridvan Şahin and Peide Liu [31], introduced various distance measures for single-valued neutrosophic sets and related these measures to multi-attribute decision-making (MADM) problems within a single-valued neutrosophic hesitant fuzzy set environment to identify the best alternative. They highlighted the importance of applying distance and similarity measures of efficient single-valued hesitant neutrosophic sets in evaluating MADM. Irfan Deli [32] examined the operators on single-valued trapezoidal neutrosophic numbers and SVTN- Group decision-making. Muhammad Naveed Jafar et al. [33] proposed Max-Min operators for distance and similarity measures in neutrosophic hypersoft sets, applying them to site selection for solid waste management systems.

This paper introduces a novel score function, demonstrated through simple examples and applied to a numerical case study. The results and discussions highlight the advantages of this new ranking approach within the neutrosophic environment. An algorithm is presented to determine the ideal solution for multi-criteria decision-making problems and an example for site selection for solid waste management problems. The structure of the paper is as follows: Section 2 covers key concepts associated with neutrosophic sets, T- Spherical fuzzy sets, Pythagorean Fuzzy neutrosophic sets, Intuitionistic fuzzy neutrosophic sets, simplified neutrosophic sets,

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and T-spherical fuzzy neutrosophic sets. Section 3 introduces the N-valued T-spherical fuzzy neutrosophic sets, numbers, and their operations. Section 4 defines a new simplified neutrosophic valued distance-based similarity measures for N-valued T-spherical fuzzy neutrosophic numbers, with a score function for comparison. Also presents a new approach, defining a score function to compare simplified neutrosophic values of N-valued T-spherical fuzzy neutrosophic numbers, with a discussion on why this approach is more realistic and meaningful. Section 5 provides an algorithm to identify the ideal alternative in the decision-making process using the distance-based similarity measure from Section 4, and presents a numerical example to demonstrate the effectiveness of the proposed model and comparison rules in decision-making scenarios. Section 6 presents an in-depth discussion of the results of the proposed technique. Section 7 summarizes the findings and provides the conclusion of the paper.

2. Neutrosophic Preliminaries

Definition 2.1 Neutrosophic Set [4] Let X be a fixed universal set. Neutrosophic set $\breve{A}$ in X is characterized by an acceptance grade $T_{\breve{A}}$, indeterminacy grade $I_{\breve{A}}$, and rejection grade $F_{\breve{A}}$, where $T_{\breve{A}}$,: X → [0, 1], $I_{\breve{A}}$: X → [0, 1], $F_{\breve{A}}$: X → [0, 1]. It can be written as $\breve{A}$ = {(x, $T_{\breve{A}}$(x), $I_{\breve{A}}$(x), $F_{\breve{A}}$(x)) / x ∈ X}, satisfies 0 ≤ $T_{\breve{A}}$(x)+ $I_{\breve{A}}$(x)+ $F_{\breve{A}}$(x) ≤ 3.

Definition 2.2 Single Valued Neutrosophic set [34] Let a fixed set X = $\{x_1, x_2, \ldots. x_n\}$ the single-valued neutrosophic set in X is defined as follows,

$\breve{N}$ = {(x_i, $T_{\breve{N}}$ (x_i), $I_{\breve{N}}$ (x_i), $F_{\breve{N}}$ (x_i)) / x_i ∈ X}, where $T_{\breve{N}}$: X → [0, 1] defines the acceptance grade; $I_{\breve{N}}$: X → [0, 1] defines the indeterminacy grade; $F_{\breve{N}}$: X → [0, 1] defines the rejection grade respectively. Also, satisfies 0 ≤ $T_{\breve{N}}$ (x_i)+ $I_{\breve{N}}$ (x_i)+ $F_{\breve{N}}$ (x_i) ≤ 3 for all x_i ∈ X.

For any two single-valued neutrosophic sets, $\breve{N}_1$ = {(x_i, $T_{\breve{N}_1}$ (x_i), $I_{\breve{N}_1}$ (x_i), $F_{\breve{N}_1}$(x_i)) / x_i ∈ X},

$\breve{N}_2$ = {(x_i, $T_{\breve{N}_2}$ (x_i), $I_{\breve{N}_2}$ (x_i), $F_{\breve{N}_2}$(x_i)) / x_i ∈ X}, the following properties are satisfying,

(i) $\breve{N}_1 \subseteq \breve{N}_2$ iff $T_{\breve{N}_1}$ (x_i) ≤ $T_{\breve{N}_2}$ (x_i); $I_{\breve{N}_1}$ (x_i) ≥ $I_{\breve{N}_2}$ (x_i); $F_{\breve{N}_1}$(x_i) ≥ $F_{\breve{N}_2}$(x_i)

(ii) $\breve{N}_1 = \breve{N}_2$ iff $\breve{N}_1 \subseteq \breve{N}_2$ and $\breve{N}_1 \supseteq \breve{N}_2$

Definition 2.3 Simplified neutrosophic set [5] Let X be a fixed universal set. Neutrosophic set $\breve{A}$ in X is characterized by an acceptance grade $T_{\breve{A}}$, indeterminacy grade $I_{\breve{A}}$, and rejection grade $F_{\breve{A}}$, where $T_{\breve{A}}$: X → [0, 1], $I_{\breve{A}}$: X → [0, 1], $F_{\breve{A}}$: X → [0, 1]. It can be written as A = {(x, $T_{\breve{A}}$(x), $I_{\breve{A}}$(x), $F_{\breve{A}}$: (x)) / x ∈ X}, satisfies 0 ≤ $T_{\breve{A}}$(x)+ $I_{\breve{A}}$(x)+ $F_{\breve{A}}$ (x) ≤ 3. The theory of simplified neutrosophic sets is a subclass of neutrosophic sets.

Take P = {(x; T_P(x), I_P(x), F_P(x)): x ∈ X}, R = {(x; T_R(x), I_R(x), F_R(x)): x ∈ X}, are two simplified neutrosophic sets on the domain X. Then the following conditions are defined,

(i) P⊕R = {x; T_P(x) + T_R(x) - T_P(x) T_R(x), I_P(x) + I_R(x) - I_P(x) I_R(x), F_P(x) + F_R(x) - F_P(x)F_R(x):x ∈ X}

(ii) P⊗R= {x; T_P(x)T_R(x), I_P(x) I_R(x), F_P(x)F_R(x): x ∈ X}.

(iii) γP = {(x; 1 – (1 - T_P(x))$^\gamma$, 1 – (1 - I_P(x))$^\gamma$, 1 – (1 - F_P(x))$^\gamma$): x ∈ X}, γ ≥ 0

(iv) P^γ = {(x; (T_P(x))$^\gamma$, (I_P(x))$^\gamma$, (F_P(x))$^\gamma$: x ∈ X}, γ ≥ 0

Definition 2.4 Spherical fuzzy set [10] Let X be a fixed universal set. A spherical fuzzy set A_s in X is defined by, A_s = {(x, T_{A_s}(x), F_{A_s}(x), π_{A_s}(x)) / x ∈ X}, satisfies 0 ≤ $T^2_{A_s}$(x) + $F^2_{A_s}$(x) + $\pi^2_{A_s}$(x) ≤ 1, where T_{A_s}: X → [0, 1], F_{A_s}: X → [0, 1], π_{A_s}: X → [0, 1]. For each member T_{A_s}(x), F_{A_s}(x), π_{A_s}(x) are the grade of acceptance, rejection, and hesitancy margin of x to A_s respectively.

Definition 2.5 T- Spherical fuzzy set [7] Let X be a fixed universal set. A spherical fuzzy set A_s in X is defined by, E = {(x, T_E(x), F_E(x), π_E(x)) / x ∈ X}, satisfies 0 ≤ T^q_E(x) + F^q_E(x) + π^q_E(x) ≤ 1, where T_E: X → [0, 1], F_E: X → [0, 1], π_E: X → [0, 1]. For each member T_E(x), F_E(x), π_E(x) are the grade of acceptance, rejection, and hesitancy margin of x to E respectively. The hesitancy grade is determined by

π_E(x) = (1 – [(T^q_E(x) + F^q_E(x) + π^q_E(x)])$^{1/q}$

Definition 2.6 Intuitionistic fuzzy valued neutrosophic set [15] Let X = $\{x_1, x_2, \ldots. x_n\}$ be a finite set. An Intuitionistic fuzzy valued neutrosophic set with generic element x in X is categorized as, $\mathscr{J}$ = {(x, $T_\mathcal{J}$(x), $I_\mathcal{J}$(x), $F_\mathcal{J}$(x)) / x ∈ X}, where $T_\mathcal{J}$(x), $I_\mathcal{J}$(x), $F_\mathcal{J}$(x) represent the acceptance grade, indeterminacy grade, and rejection grade of neutrosophic values, each of them is an intuitionistic fuzzy grade, where for every x in X, $T_\mathcal{J}$(x) = ($\mu_{\mathcal{J},t}$ (x), $\vartheta_{\mathcal{J},t}$ (x)) such that $\mu_{\mathcal{J},t}$ (x), $\vartheta_{\mathcal{J},t}$ (x) ∈ [0, 1] also satisfies 0 ≤ $\mu_{\mathcal{J},t}$ (x) + $\vartheta_{\mathcal{J},t}$ (x) ≤ 1; $I_\mathcal{J}$(x) = ($\mu_{\mathcal{J},i}$ (x), $\vartheta_{\mathcal{J},i}$ (x)) such that

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$\mu_{J,i}$ (x), $\vartheta_{J,i}$ (x) ∈ [0, 1] also satisfies $0 \leq \mu_{J,i}(x) + \vartheta_{J,i}(x) \leq 1$; F_J(x) = ($\mu_{J,f}$ (x), $\vartheta_{J,f}$ (x)) such that $\mu_{J,f}$ (x), $\vartheta_{J,f}$ (x) ∈ [0, 1] also satisfies $0 \leq \mu_{J,f}(x) + \vartheta_{J,f}(x) \leq 1$.

Definition 2.7 Pythagorean fuzzy valued neutrosophic set [17] Let X = $\{x_1, x_2, \ldots . x_n\}$ be a finite set. A Pythagorean Fuzzy valued neutrosophic set with generic element x in X is categorized as, Ᵽ = {(x, $T_Ᵽ$(x), $I_Ᵽ$(x), $F_Ᵽ$(x)) / x ∈ X}, where $T_Ᵽ$(x), $I_Ᵽ$(x), $F_Ᵽ$(x) represent the acceptance grade, indeterminacy grade, and rejection grade of neutrosophic values, each of them is a Pythagorean fuzzy grade, where for every x in X, $T_Ᵽ$(x) = ($\mu_{Ᵽ,t}$ (x), $\vartheta_{Ᵽ,t}$ (x)) such that $\mu_{Ᵽ,t}$ (x), $\vartheta_{Ᵽ,t}$ (x) ∈ [0, 1] also satisfies $0 \leq (\mu_{Ᵽ,t}(x))^2 + (\vartheta_{Ᵽ,t}(x))^2 \leq 1$; $I_Ᵽ$(x) = ($\mu_{Ᵽ,i}$ (x), $\vartheta_{Ᵽ,i}$ (x)) such that $\mu_{Ᵽ,i}$ (x), $\vartheta_{Ᵽ,i}$ (x) ∈ [0, 1] also satisfies $0 \leq (\mu_{Ᵽ,i}(x))^2 + (\vartheta_{Ᵽ,i}(x))^2 \leq 1$; $F_Ᵽ$(x) = ($\mu_{Ᵽ,f}$ (x), $\vartheta_{Ᵽ,f}$ (x)) such that $\mu_{Ᵽ,f}$ (x), $\vartheta_{Ᵽ,f}$ (x) ∈ [0, 1] also satisfies $0 \leq (\mu_{Ᵽ,f}(x))^2 + (\vartheta_{Ᵽ,f}(x))^2 \leq 1$.

Definition 2.8 T- Spherical fuzzy valued neutrosophic set [19] Let X = $\{x_1, x_2, \ldots . x_n\}$ be a finite set. A T-Spherical fuzzy-valued neutrosophic set on X is categorized by $\widetilde{N_S}$ = {x, (T_S, I_S, F_S): x ∈ X}, where T_S, I_S, and F_S represent the acceptance grade, indeterminacy grade, and rejection grade of neutrosophic values, each of them is a T-spherical fuzzy grade, where for every x in X, q ∈ Z^+, q ≥ 1, T_S(x) = ($\mu_{s,t}$ (x), $\vartheta_{s,t}$ (x), $\tau_{s,t}$ (x)) such that $\mu_{s,t}$ (x), $\vartheta_{s,t}$ (x),$\tau_{s,t}$ (x) ∈ [0, 1] also satisfies $0 \leq (\mu_{s,t}(x))^q + (\vartheta_{s,t}(x))^q + (\tau_{s,t}(x))^q \leq 1$; I_s(x) = ($\mu_{s,i}$ (x), $\vartheta_{s,i}$ (x), $\tau_{s,i}$ (x)) such that $\mu_{s,i}$ (x), $\vartheta_{s,i}$ (x), $\tau_{s,i}$ (x) ∈ [0, 1] also satisfies $0 \leq (\mu_{s,i}(x))^q + (\vartheta_{s,i}(x))^q + (\tau_{s,i}(x))^q \leq 1$; F_s(x) = ($\mu_{s,f}$ (x), $\vartheta_{s,f}$ (x),$\tau_{s,f}$ (x)) such that $\mu_{s,f}$ (x), $\vartheta_{s,f}$ (x), $\tau_{s,f}$ (x) ∈ [0, 1] also satisfies $0 \leq (\mu_{s,f}(x))^q + (\vartheta_{s,f}(x))^q + (\tau_{s,f}(x))^q \leq 1$. By definition $0 \leq T_S + I_S + F_S \leq 3$. A T- Spherical fuzzy valued neutrosophic set on X will be of the form,

$\widetilde{N_S}$={(x_i, ($\mu_{s,t}$ (x_i), $\vartheta_{s,t}$ (x_i), $\tau_{s,t}$ (x_i)), ($\mu_{s,i}$ (x_i), $\vartheta_{s,i}$ (x_i), $\tau_{s,i}$ (x_i)), ($\mu_{s,f}$ (x_i), $\vartheta_{s,f}$ (x_i),$\tau_{s,f}$ (xi)): x_i ∈ X}

According to the definition, a fixed x_i ,i = 1, 2, … n, a collection of $\mathscr{B}$ = {($\mu_{s,t}$ (x_i), $\vartheta_{s,t}$ (x_i), $\tau_{s,t}$ (x_i)), ($\mu_{s,i}$ (x_i), $\vartheta_{s,i}$ (x_i), $\tau_{s,i}$ (x_i)), ($\mu_{s,f}$ (x_i), $\vartheta_{s,f}$ (x_i),$\tau_{s,f}$ (xi))}is called T- Spherical fuzzy valued neutrosophic number, also satisfies, $0 \leq (\mu_{s,t})^q + (\vartheta_{s,t})^q + (\tau_{s,t})^q \leq 1$;

$0 \leq (\mu_{s,i})^q + (\vartheta_{s,i})^q + (\tau_{s,i})^q \leq 1$; $0 \leq (\mu_{s,f})^q + (\vartheta_{s,f})^q + (\tau_{s,f})^q \leq 1, q \geq 1$.

Example 2.1 Take X = $\{x_1, x_2, x_3\}$. Then $\mathscr{B}$ = {(0.6^3, 0.1^3,0.4^3), (0.9^3,0.2^3,0.5^3), (0.8^3, 0.6^3, 0.5^3); (0.9^3, 0.2^3,0.4^3), (0.4^3,0.2^3,0.6^3), (0.3^3, 0.4^3, 0.5^3); (0.8^3, 0.1^3,0.2^3), (0.5^3,0.2^3,0.3^3), (0.7^3, 0.6^3, 0.5^3)}is a T- Spherical fuzzy valued neutrosophic number with q = 3.

3. In this section, we propose the definition of N-valued T-spherical fuzzy neutrosophic sets and numbers.

Definition 3.1 N-valued T-spherical fuzzy Neutrosophic Set Let X = $\{x_1, x_2, \ldots . x_n\}$ be a finite set. An N-valued T-spherical fuzzy neutrosophic set defined on X is given by,

$\widetilde{N}$ = { (x_i, $(T_N^{i,j})_{j=1}^{k_i}$, $(I_N^{i,j})_{j=1}^{k_i}$, $(F_N^{i,j})_{j=1}^{k_i}$)}; i = 1, 2, … n, j = 1, 2, … k_i, where $(T_N^{i,j})_{j=1}^{k_i}$, $(I_N^{i,j})_{j=1}^{k_i}$, $(F_N^{i,j})_{j=1}^{k_i}$are the acceptance, indeterminacy, and rejection grade sequences of length k respectively.

Here, $T_N^{i,j}$ = (T_N^{j,t_i} , T_N^{j,i_i} , T_N^{j,f_i}) with T_N^{j,t_i} , T_N^{j,i_i} , T_N^{j,f_i} ∈ [0, 1] such that 0 ≤ Sup ($T_N^{j,t_i})^q$ + Sup ($T_N^{j,i_i})^q$ + Sup ($T_N^{j,f_i})^q$ ≤ 1, q ≥ 1;

$I_N^{i,j}$ = (I_N^{j,t_i} , I_N^{j,i_i} , I_N^{j,f_i}) with I_N^{j,t_i} , I_N^{j,i_i} , I_N^{j,f_i} ∈ [0, 1] such that 0 ≤ Sup ($I_N^{j,t_i})^q$ + Sup ($I_N^{j,i_i})^q$ + Sup ($I_N^{j,f_i})^q$ ≤ 1, q ≥ 1;

$F_N^{i,j}$ = (F_N^{j,t_i} , F_N^{j,i_i} , F_N^{j,f_i}) with F_N^{j,t_i} , F_N^{j,i_i} , F_N^{j,f_i} ∈ [0, 1] such that 0 ≤ Sup ($F_N^{j,t_i})^q$ + Sup ($F_N^{j,i_i})^q$ + Sup ($F_N^{j,f_i})^q$ ≤ 1, q ≥ 1. By definition, ≤ $(T_N^{i,j})_{j=1}^{k_i}$, $(I_N^{i,j})_{j=1}^{k_i}$, $(F_N^{i,j})_{j=1}^{k_i}$ ≤ 3.

According to the definition, $\widetilde{N}$ = { (x_i, $(T_N^{i,j})_{j=1}^{k_i}$, $(I_N^{i,j})_{j=1}^{k_i}$, $(F_N^{i,j})_{j=1}^{k_i}$)} , for a fixed i = 1, 2, … n, the sequence $\widetilde{N}_\omega$ = {$(T_\omega^j)_{j=1}^k$, $(I_\omega^j)_{j=1}^k$, $(F_\omega^j)_{j=1}^k$ } be the N-valued T-spherical fuzzy neutrosophic number.

Set operations on N-valued T-spherical fuzzy neutrosophic sets

Here, we introduce and define the core operations of N-valued T-spherical fuzzy neutrosophic sets, building upon the operations of T-spherical fuzzy neutrosophic sets and simplified neutrosophic sets.

Let X = $\{x_1, x_2, \ldots . x_n\}$ be a finite set, and $\widetilde{N_A}, \widetilde{N_B}$ be two n-valued T-spherical fuzzy neutrosophic sets in X. Then some set operations among $\widetilde{N_A}, \widetilde{N_B}$ as follows,

$\widetilde{N_A}$ = {x_i, ($(T_A^{i,j})_{j=1}^k$, $((I_A^{i,j})_{j=1}^k)^{c_q}$, $(F_A^{i,j})_{j=1}^k$); x_i ∈ X}and

$\widetilde{N_B}$ = {x_i, ($(T_A^{i,j})_{j=1}^k$, $((I_A^{i,j})_{j=1}^k)^{c_q}$, $(F_A^{i,j})_{j=1}^k$); x_i ∈ X},

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(I) $\widetilde{N_A} \subseteq_q \widetilde{N_B}$ iff (i) $T_A^{i,j} \subseteq_q T_B^{i,j}$ ie, $T_A^{j,t_i} \le T_B^{j,t_i}$; $T_A^{j,i_i} \ge T_B^{j,i_i}$; $T_A^{j,f_i} \ge T_B^{j,f_i}$ $for\ all\ i,j$

(ii) $I_A^{i,j} \supseteq_q I_B^{i,j}$ ie, $I_A^{j,t_i} \le I_B^{j,t_i}$; $I_A^{j,i_i} \ge I_B^{j,i_i}$; $I_A^{j,f_i} \ge I_B^{j,f_i}$ $for\ all\ i,j$

(iii) $F_A^{i,j} \supseteq_q F_B^{i,j}$ ie, $F_A^{j,t_i} \le F_B^{j,t_i}$; $F_A^{j,i_i} \ge F_B^{j,i_i}$; $F_A^{j,f_i} \ge F_B^{j,f_i}$ $for\ all\ i,j$

For every i = 1, 2, … n; j = 1, 2, … k_i and $\subseteq_q$ represent the n-valued T-spherical neutrosophic subset.

(II) $\widetilde{N_A} = \widetilde{N_B}$ iff (i) $T_A^{i,j} = T_B^{i,j}$ ie, $T_A^{j,t_i} = T_B^{j,t_i}$; $T_A^{j,i_i} = T_B^{j,i_i}$; $T_A^{j,f_i} = T_B^{j,f_i}$ $for\ all\ i,j$

(ii) $I_A^{i,j} = I_B^{i,j}$ ie, $I_A^{j,t_i} = I_B^{j,t_i}$; $I_A^{j,i_i} = I_B^{j,i_i}$; $I_A^{j,f_i} = I_B^{j,f_i}$ $for\ all\ i,j$

(iii) $F_A^{i,j} = F_B^{i,j}$ ie, $F_A^{j,t_i} = F_B^{j,t_i}$; $F_A^{j,i_i} = F_B^{j,i_i}$; $F_A^{j,f_i} = F_B^{j,f_i}$ $for\ all\ i,j$

(III) $\widetilde{N_A^c}$ = {x_i, $((F_A^{i,j})_{j=1}^{k}, ((I_A^{i,j})_{j=1}^{k})^{c_q}, (T_A^{i,j})_{j=1}^{k})$; $x_i \in X$}, $((I_A^{i,j})_{j=1}^{k})^{c_q} =(I_A^{j,f_i}, (1- I_A^{j,f_i}), I_A^{j,t_i}))$,

Where, for every i = 1, 2, … n; j = 1, 2, … k_i and c_q represent the n-valued T-spherical neutrosophic subset.

(IV) $\widetilde{N_A} \cup_q \widetilde{N_B}$ = {x_i, $(T_A^{i,j} \cup_q T_B^{i,j})_{j=1}^{k_i}$, $(I_A^{i,j} \cap_q I_B^{i,j})_{j=1}^{k_i}$, $(F_A^{i,j} \cap F_B^{i,j})_{j=1}^{k_i}; i = 1, 2, \dots n; j = 1, 2, \dots k_i$ }

Where, $(T_A^{i,j} \cup_q T_B^{i,j})_{j=1}^{k_i}$= {$(\max_j (T_A^{j,t_i}, T_B^{j,t_i}), \min_j(T_A^{j,i_i}, T_B^{j,i_i}), \min_j(T_A^{j,f_i}, T_B^{j,f_i}))$}

$(I_A^{i,j} \cap_q I_B^{i,j})_{j=1}^{k_i}$={$(\min_j(I_A^{j,t_i}, I_B^{j,t_i}); \max_j (I_A^{j,i_i}, I_B^{j,i_i}); \max_j (I_A^{j,f_i}, I_B^{j,f_i}))$}

$(F_A^{i,j} \cap_q F_B^{i,j})_{j=1}^{k_i}$={$(\min_j(F_A^{j,t_i}, F_B^{j,t_i}); \max_j (F_A^{j,i_i}, F_B^{j,i_i}) ; \max_j (F_A^{j,f_i}, F_B^{j,f_i})$}, here $\cup_q$, $\cap_q$are the representation of n-valued T-spherical neutrosophic union, intersection respectively.

(V) $\widetilde{N_A} \cap_q \widetilde{N_B}$ = {x_i, $(T_A^{i,j} \cap_q T_B^{i,j})_{j=1}^{k_i}$, $(I_A^{i,j} \cup_q I_B^{i,j})_{j=1}^{k_i}$, $(F_A^{i,j} \cup_q F_B^{i,j})_{j=1}^{k_i}; i = 1, 2, \dots n; j = 1, 2, \dots k_i$}

Where, $(T_A^{i,j} \cap_q T_B^{i,j})_{j=1}^{k_i}$= {$(\min_j(T_A^{j,t_i}, T_B^{j,t_i}), \max_j(T_A^{j,i_i}, T_B^{j,i_i}), \max_j(T_A^{j,f_i}, T_B^{j,f_i}))$}

$(I_A^{i,j} \cup_q I_B^{i,j})_{j=1}^{k_i}$={$(\max_j(I_A^{j,t_i}, I_B^{j,t_i}); \min_j (I_A^{j,i_i}, I_B^{j,i_i}); \min_j (I_A^{j,f_i}, I_B^{j,f_i}))$}

$(F_A^{i,j} \cup_q F_B^{i,j})_{j=1}^{k_i}$={$(\max_j(F_A^{j,t_i}, F_B^{j,t_i}); \min_j (F_A^{j,i_i}, F_B^{j,i_i}) ; \min_j (F_A^{j,f_i}, F_B^{j,f_i})$}, here $\cup_q$, $\cap_q$are the representation of N-valued T-spherical neutrosophic union, intersection respectively.

Example 3.1 If X = {x_1, x_2}such that $\widetilde{N_A}$ = { x_1,([(0.5, 0.4, 0.3),(0.2, 0.1, 0.0),(0.3, 0.4, 0.6)]; [(0.2,0.3,0.1),(0.3,0.4,0.5),(0.4,0.6,0.8)]; [(0.1,0.2,0.4),(0.3,0.2,0.8),(0.1,0.3,0.7)]); x_2, ([(0.3, 0.4, 0.5),(0.5, 0.1, 0.2),(0.4, 0.5, 0.6)];[(0.4, 0.2, 0.3),(0.5, 0.6, 0.8),(0.3, 0.1, 0.0)];[(0.7, 0.8, 0.9),(0.1, 0.2, 0.3),(0.5, 0.4, 0.2)]} and

$\widetilde{N_B}$ = { x_1,([(0.4, 0.5, 0.6),(0.5, 0.8, 0.2),(0.1, 0.7, 0.6)] ; [(0.1,0.2,0.4),(0.6,0.7,0.8),(0.2,0.4,0.6)]; [(0.0,0.1,0.2),(0.1,0.2,0.3),(0.8,0.4,0.5)]); x_2, ([(0.9, 0.7, 0.6),(0.5, 0.3, 0.2),(0.1, 0.2, 0.3)];[(0.8, 0.6, 0.5),(0.4, 0.2, 0.1),(0.2, 0.3, 0.1)];[(0.6, 0.2, 0.3),(0.7, 0.6, 0.5),(0.3, 0.2, 0.4)]} are two n-valued T-spherical fuzzy neutrosophic sets.

Then,

(i) $\widetilde{N_A} \cup_q \widetilde{N_B}$ = {x_1,([(0.5,0.5,0.6),(0.2,0.1,0.0),(0.1,0.4,0.6)]; [(0.1,0.1,0.3),(0.6,0.7,0.8),(0.4,0.6,0.8)], [(0.0,0.1,0.2),(0.3,0.7,0.8),(0.8,0.3,0.7)]); x_2,([(0.9,0.7,0.6),(0.5,0.1,0.2),(0.1,0.2,0.3)]; [(0.4,0.2,0.3),(0.5,0.6,0.8),(0.3,0.3,0.1)], [(0.6,0.2,0.3),(0.7,0.6,0.5),(0.5,0.4,0.4)])}

(ii) $\widetilde{N_A} \cap_q \widetilde{N_B}$ = {x_1,([(0.4, 0.4, 0.3),(0.5, 0.8, 0.2),(0.3,0.7,0.6)],[(0.2,0.2,0.4),(0.3,0.4,0.5),

(0.2,0.4,0.6)],[(0.1,0.2,0.4),(0.1,0.2,0.3),(0.1,0.2,0.3)]); x_2,[((0.3, 0.4, 0.5), (0.5, 0.2, 0.3),(0.4, 0.5,0.6)],[(0.4,0.2,0.3),(0.5,0.6,0.8),(0.3,0.3,0.1)], [(0.7,0.8,0.9),(0.1,0.2,0.3),(0.3,0.2,0.2)])}

(iii) $\widetilde{N_A^c}$ = {x_1,[(0.1,0.2,0.4),(0.3,0.2,0.8),(0.1,0.3,0.7)]; [(0.4,0.6,0.8),(0.5,0.6,0.3),(0.2,0.1,0.3)], [(0.5,0.4,0.3),(0.2,0.1,0.0),(0.3,0.4,0.6)]); x_2,([(0.7,0.8,0.9),(0.1,0.2,0.3),(0.5,0.4,0.2)]; [(0.3,0.1,0.0),(0.8,0.4,0.5),(0.4,0.2,0.3)], [(0.3,0.4,0.5),(0.5,0.1,0.2),(0.4,0.5,0.6)])}

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Proposition Let $\widetilde{N_A}$ = {x_i, $((T_A^{i,j})_{j=1}^{k}, ((I_A^{i,j})_{j=1}^{k})^{c_q}, (F_A^{i,j})_{j=1}^{k})$; $x_i \in X$}, $\widetilde{N_B}$ = {x_i, $((T_B^{i,j})_{j=1}^{k}, ((I_B^{i,j})_{j=1}^{k})^{c_q}, (F_C^{i,j})_{j=1}^{k})$; $x_i \in X$}, and $\widetilde{N_C}$ = {x_i, $((T_C^{i,j})_{j=1}^{k}, ((I_C^{i,j})_{j=1}^{k})^{c_q}, (F_C^{i,j})_{j=1}^{k})$; $x_i \in X$}are the three n-valued T-spherical fuzzy neutrosophic sets. Then the following properties are holds,

(i) $\widetilde{N_A} \cup \widetilde{N_B} = \widetilde{N_B} \cup \widetilde{N_A}$; $\widetilde{N_A} \cap \widetilde{N_B} = \widetilde{N_B} \cap \widetilde{N_A}$

(ii) $(\widetilde{N_A} \cup \widetilde{N_B}) \cup \widetilde{N_C} = \widetilde{N_A} \cup (\widetilde{N_B} \cup \widetilde{N_C})$; $(\widetilde{N_A} \cup \widetilde{N_B}) \cup \widetilde{N_C} = \widetilde{N_A} \cup (\widetilde{N_B} \cup \widetilde{N_C})$.

(iii) $\widetilde{N_A} \cup (\widetilde{N_B} \cap \widetilde{N_C}) = (\widetilde{N_A} \cup \widetilde{N_B}) \cap (\widetilde{N_A} \cup \widetilde{N_C})$; $\widetilde{N_A} \cap (\widetilde{N_B} \cup \widetilde{N_C}) = (\widetilde{N_A} \cap \widetilde{N_B}) \cup (\widetilde{N_A} \cap \widetilde{N_C})$

(iv) $(\widetilde{N_A} \cup \widetilde{N_B})^c = \widetilde{N_A}^c \cap \widetilde{N_B}^c$; $(\widetilde{N_A} \cap \widetilde{N_B})^c = \widetilde{N_A}^c \cup \widetilde{N_B}^c$ (De- Morgan's law)

Proof: For De- Morgan's law,

$\widetilde{N_A} \cup \widetilde{N_B}$ = {x_i, $(T_A^{i,j} \cup_q T_B^{i,j})_{j=1}^{k_i}$, $(I_A^{i,j} \cap_q I_B^{i,j})_{j=1}^{k_i}$, $(F_A^{i,j} \cap_q F_B^{i,j})_{j=1}^{k_i}; i = 1, 2, \ldots n; j = 1, 2, \ldots k_i$ }

$(\widetilde{N_A} \cup \widetilde{N_B})^c$= {$x_i$, $(F_A^{i,j} \cap_q F_B^{i,j})_{j=1}^{k_i}$, $((I_A^{i,j} \cap_q I_B^{i,j})_{j=1}^{k_i})^c$, $(T_A^{i,j} \cup_q T_B^{i,j})_{j=1}^{k_i}$}

= {x_i, $(F_A^{i,j} \cap_q F_B^{i,j})_{j=1}^{k_i}$, $((I_A^{i,j})^c \cup_q (I_B^{i,j})^c)_{j=1}^{k_i}$, $(T_A^{i,j} \cup_q T_B^{i,j})_{j=1}^{k_i}$};

(Where, $(I_A^{i,j})^c)_{j=1}^{k_i} = (I_A^{j,f_i}, 1 - I_A^{j,f_i}, I_A^{j,f_i})$, $(I_B^{i,j})^c)_{j=1}^{k_i} = (I_B^{j,f_i}, 1 - I_B^{j,f_i}, I_B^{j,f_i})$; for all i, j)

$(\widetilde{N_A} \cup \widetilde{N_B})^c = \widetilde{N_A}^c \cap \widetilde{N_B}^c$

$\widetilde{N_A} \cap \widetilde{N_B}$ = {x_i, $(T_A^{i,j} \cap_q T_B^{i,j})_{j=1}^{k_i}$, $(I_A^{i,j} \cup_q I_B^{i,j})_{j=1}^{k_i}$, $(F_A^{i,j} \cup_q F_B^{i,j})_{j=1}^{k_i}; i = 1, 2, \ldots n; j = 1, 2, \ldots k_i$ }

$(\widetilde{N_A} \cap \widetilde{N_B})^c$= {$x_i$, $(F_A^{i,j} \cup_q F_B^{i,j})_{j=1}^{k_i}$, $((I_A^{i,j} \cup_q I_B^{i,j})_{j=1}^{k_i})^c$, $(T_A^{i,j} \cap_q T_B^{i,j})_{j=1}^{k_i}$}

= {x_i, $(F_A^{i,j} \cup_q F_B^{i,j})_{j=1}^{k_i}$, $((I_A^{i,j})^c \cap_q (I_B^{i,j})^c)_{j=1}^{k_i}$, $(T_A^{i,j} \cap_q T_B^{i,j})_{j=1}^{k_i}$}; where

$(I_A^{i,j})^c)_{j=1}^{k_i} = (I_A^{j,t_i}, 1 - I_A^{j,i_i}, I_A^{j,f_i})$, $(I_B^{i,j})^c)_{j=1}^{k_i} = (I_B^{j,t_i}, 1 - I_B^{j,i_i}, I_B^{j,f_i})$; for all i, j

$(\widetilde{N_A} \cap \widetilde{N_B})^c = \widetilde{N_A}^c \cup \widetilde{N_B}^c$ completes the proof.

Algebraic operations for N-valued T-spherical fuzzy neutrosophic numbers

This section introduces a series of algebraic operations for N-valued T-spherical neutrosophic numbers, extending the algebraic principles of T-spherical fuzzy neutrosophic sets and simplified neutrosophic sets.

Let $\widetilde{N_\alpha}$ = {[($T_\alpha^{j,t}$, $T_\alpha^{j,i}$, $T_\alpha^{j,f}$), ($I_\alpha^{j,t}$, $I_\alpha^{j,i}$, $I_\alpha^{j,f}$), ($F_\alpha^{j,t}$, $F_\alpha^{j,i}$, $F_\alpha^{j,f}$)]}; j = 1,2,…k

and $\widetilde{N_\omega}$ = {[($T_\omega^{j,t}$, $T_\omega^{j,i}$, $T_\omega^{j,f}$), ($I_\omega^{j,t}$, $I_\omega^{j,i}$, $I_\omega^{j,f}$), ($F_\omega^{j,t}$, $F_\omega^{j,i}$, $F_\omega^{j,f}$)]}; j = 1,2,…k

(i) $\widetilde{N_\alpha} \oplus \widetilde{N_\omega}$ = {([$(T_\alpha^{j,t})^q + (T_\omega^{j,t})^q - (T_\alpha^{j,t})^q (T_\omega^{j,t})^q]^{1/q}$, $((T_\alpha^{j,i})(T_\omega^{j,i}))$, $((T_\alpha^{j,f})(T_\omega^{j,f}))$); $(((I_\alpha^{j,t})(I_\omega^{j,t}))$, $[(I_\alpha^{j,i})^q + (I_\omega^{j,i})^q - (I_\alpha^{j,i})^q (I_\omega^{j,i})^q]^{1/q}$, $[(I_\alpha^{j,f})^q + (I_\omega^{j,f})^q - (I_\alpha^{j,f})^q (I_\omega^{j,f})^q]^{1/q}$); $(((F_\alpha^{j,t})(F_\omega^{j,t}))$; $([(F_\alpha^{j,i})^q + (F_\omega^{j,i})^q - (F_\alpha^{j,i})^q (F_\omega^{j,i})^q]^{1/q}$, $[(F_\alpha^{j,f})^q + (F_\omega^{j,f})^q - (F_\alpha^{j,f})^q (F_\omega^{j,f})^q]^{1/q}$))}

(ii) $\widetilde{N_\alpha} \otimes \widetilde{N_\omega}$ = {((($(T_\alpha^{j,t})(T_\omega^{j,t})$), $[(T_\alpha^{j,i})^q + (T_\omega^{j,i})^q - (T_\alpha^{j,i})^q (T_\omega^{j,i})^q]^{1/q}$,$[(T_\alpha^{j,f})^q + (T_\omega^{j,f})^q - (T_\alpha^{j,f})^q (T_\omega^{j,f})^q]^{\frac{1}{q}}$); ($[(I_\alpha^{j,t})^q + (I_\omega^{j,t})^q - (I_\alpha^{j,t})^q (I_\omega^{j,t})^q]^{1/q}$), $((I_\alpha^{j,i})(I_\omega^{j,i}))$, $((I_\alpha^{j,f})(I_\omega^{j,f}))$); ($[(F_\alpha^{j,t})^q + (F_\omega^{j,t})^q - (F_\alpha^{j,t})^q (F_\omega^{j,t})^q]^{1/q}$), $((F_\alpha^{j,i})(F_\omega^{j,i}))$, $((F_\alpha^{j,f})(F_\omega^{j,f}))$)}

(iii) $\xi \widetilde{N_\alpha}$ = {([1-(1- $(T_\alpha^{j,f})^q)^\xi]^{1/q}$, $(T_\alpha^{j,i})^\xi$, $(T_\alpha^{j,f})^\xi$);($(I_\alpha^{j,t})^\xi$, [1-(1- $(I_\alpha^{j,i})^q)^\xi]^{1/q}$, $[1 - (1 - (I_\alpha^{j,f})^q)^\xi]^{1/q}$);($(F_\alpha^{j,t})^\xi$, [1-(1- $(F_\alpha^{j,i})^q)^\xi]^{1/q}$, $[1 - (1 - (F_\alpha^{j,f})^q)^\xi]^{1/q}$)}

(iv) $(\widetilde{N_\alpha})^\xi$ = {($(T_\alpha^{j,t})^\xi$, [1-(1- $(T_\alpha^{j,i})^q)^\xi]^{1/q}$, [1-(1- $(T_\alpha^{j,f})^q)^\xi]^{1/q}$);([1-(1- $(I_\alpha^{j,t})^q)^\xi]^{1/q}$,$(I_\alpha^{j,i})^\xi$, $(I_\alpha^{j,f})^\xi$);($[1 - (1 - (F_\alpha^{j,t})^q)^\xi]^{1/q}$, $(F_\alpha^{j,i})^\xi$, $(F_\alpha^{j,f})^\xi$)}

Example 3.2 Let $\widetilde{N_\alpha}$ = {[(T_α^{j,t_i}, T_α^{j,i_i}, T_α^{j,f_i}), (I_α^{j,t_i}, I_α^{j,i_i}, I_α^{j,f_i}), (F_α^{j,t_i}, F_α^{j,i_i}, F_α^{j,f_i})]};i = 1, 2,… n; j = 1,2,…k and $\widetilde{N_\omega}$ = {[(T_ω^{j,t_i}, T_ω^{j,i_i}, T_ω^{j,f_i}), (I_ω^{j,t_i}, I_ω^{j,i_i}, I_ω^{j,f_i}), (F_ω^{j,t_i}, F_ω^{j,i_i}, F_ω^{j,f_i})]};i = 1, 2,… n; j = 1,2,…k, Take i = 1, 2; j = 1, 2, 3; ξ = 4; q = 3

Let $\widetilde{N_\alpha}$ = **{**[⟨(0.5,0.4,0.3), (0.2,0.1,0.0), (0.3,0.4,0.6)⟩**;** ⟨(0.2,0.1,0.3), (0.3,0.4,0.5), (0.4,0.6,0.8)⟩;

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$\langle(0.1,0.2,0.4),(0.3,0.2,0.8),(0.1,0.3,0.7)\rangle]$; $[\langle(0.3,0.4,0.5),(0.5,0.1,0.2),(0.4,0.5,0.6)\rangle$; $\langle(0.4,0.2,0.3),(0.5,0.6,0.8),(0.3,0.1,0.0)\rangle;\langle(0.7,0.8,0.9),(0.1,0.2,0.3),(0.5,0.4,0.2)\rangle]\}$

$\widetilde{N_{\omega}} = \{[\langle(0.5,0.4,0.6),(0.5,0.8,0.2),(0.1,0.7,0.6)\rangle; \langle(0.2,0.1,0.4),(0.6,0.7,0.8),(0.4,0.6,0.2)\rangle;$

$\langle(0.1,0.2,0.0),(0.1,0.2,0.3),(0.8,0.4,0.5)\rangle]$; $[\langle(0.9,0.7,0.6),(0.5,0.3,0.2),(0.1,0.2,0.3)\rangle$; $\langle(0.8,0.6,0.5),(0.4,0.2,0.1),(0.3,0.1,0.2)\rangle;\langle(0.6,0.2,0.3),(0.7,0.6,0.5),(0.3,0.4,0.2)\rangle]\}$

$\widetilde{N_{\alpha}} \oplus \widetilde{N_{\omega}} = \{[\langle(0.57,0.57,0.6),(0.1,0.8,0.0),(0.03,0.28,0.36)\rangle;$
$\langle(0.02,0.02,0.12),(0.62,0.73,0.83),(0.42,0.64,0.85)\rangle;$
$\langle(0.0,0.02,0.08),(0.31,0.25,0.8)1,(0.1,0.45,0.75)\rangle]$; $[\langle(0.9,0.73,0.68),(0.25,0.03,0.04),(0.32,0.2,0.3)\rangle$;
$\langle(0.32,0.12,0.15),(0.57,0.61,0.8),(0.33,0.31,0.1)\rangle;\langle(0.42,0.16,0.27),(0.7,0.11,0.53),(0.53,0.42,0.42)\rangle]\}$

$\widetilde{N_{\alpha}} \otimes \widetilde{N_{\omega}} = \{[\langle(0.2,0.2,0.18),(0.5,0.8,0.2),(0.3,0.73,0.73)\rangle;$
$\langle(0.21,0.21,0.45),(0.18,0.28,0.4),(0.08,0.24,0.48)\rangle;$
$\langle(0.0,0.21,0.42),(0.03,0.04,0.24)1,(0.08,0.12,0.35)\rangle]$; $[\langle(0.27,0.28,0.3),(0.62,0.31,0.35),(0.41,0.51,0.62)\rangle$;
$\langle(0.82,0.61,0.53),(0.2,0.12,0.08),(0.06,0.03,0.0)\rangle;\langle(0.79,0.8,0.9),(0.07,0.12,0.15),(0.15,0.08,0.08)\rangle]\}$

$\xi \, \widetilde{N_{\alpha}} = \{[\langle(0.74,0.61,0.46),(0.1,0.0,0.0),(0.0,0.03,0.13)\rangle;$
$\langle(0.0,0.01,0.01),(0.46,,0.61,0.74),(0.61,0.85,0.98)\rangle;$
$\langle(0.0,0.01,0.02),(0.46,0.031,0.98)1,(0.16,0.46,0.93)\rangle]$; $[\langle(0.46,,0.61,0.74),(0.6,0.0,0.0),(0.03,0.06,0.13)\rangle$;
$\langle(0.03,0.0,0.01),(0.74,0.8,0.98),(0.46,0.16,0.0)\rangle$; $\langle(0.24,0.41,0.66),(0.16,0.31,0.46),(0.74,0.61,0.31)\rangle]\}$

$\widetilde{(N_{\alpha})}^{\xi} = \{[\langle(0.06,0.02,0.01),(0.31,0.16,0.0),(0.46,0.61,0.8)\rangle;$
$\langle(0.31,0.16,0.46),(0.06,0.03,0.01),(0.03,0.13,0.41)\rangle;$
$\langle(0.16,0.31,0.61),(0.01,0.0,0.41)1,(0.0,0.01,0.24)\rangle]$; $[\langle(0.01,0.03,0.06),(0.74,0.16,0.31),(0.61,0.74,0.8)\rangle$;
$\langle(0.16,0.31,0.46),(0.06,0.13,0.41),(0.01,0.03,0.0)\rangle$; $\langle(0.93,0.98,0.99),(0.01,0.02,0.0),(0.06,0.02,0.01\rangle\}$

Distance and Similarity Measures

Finding an optimal solution in decision-making processes often requires evaluating alternatives based on their similarities or differences. Distance and similarity measures are essential tools that help quantify these aspects, enabling decision-makers to make informed choices. This approach leverages N-valued T-spherical neutrosophic numbers and **sets** to effectively capture the inherent uncertainty, indeterminacy, and vagueness in complex decision-making contexts.

Definition 3.2 If $\mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_b})$ be the normalized distance measures for N-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}$ and $\widetilde{N_b}$ then,

(i) $0 \leq \mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_b}) \leq 1$.
(ii) $\mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_b}) = \mathcal{D}_{\rho}(\widetilde{N_b}, \widetilde{N_a})$
(iii) $\mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_a}) = 0$; $\mathcal{D}_{\rho}(\widetilde{N_b}, \widetilde{N_b}) = 0$

Also, the properties are true for $\mathcal{D}_{\rho}^{1}(\widetilde{N_a}, \widetilde{N_b})$; $\mathcal{D}_{\rho}^{2}(\widetilde{N_a}, \widetilde{N_b})$ and $\mathcal{D}_{\rho}^{3}(\widetilde{N_a}, \widetilde{N_b})$ for N-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}, \widetilde{N_b}$.

Definition 3.3 The distance-based similarity measures Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b})$ between two N-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}$ *and* $\widetilde{N_b}$ satisfies the following properties:

(i) $0 \leq$ Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b}) \leq 1$
(ii) Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b})$ = Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b})$
(iii) Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b}) = 1$ iff $\widetilde{N_a} = \widetilde{N_b}$

If $\mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_b})$ be the normalized distance measures for N-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}$ and $\widetilde{N_b}$ then the distance-based similarity measures for N-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}$ and $\widetilde{N_b}$ is Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b}) = 1 - \mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_b})$.

Similarly, if Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b})$ be the distance-based similarity measures for N

-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}$ and $\widetilde{N_b}$ then, $\mathcal{D}_{\rho}(\widetilde{N_a}, \widetilde{N_b}) = 1 -$ Ş$\mathcal{M}(\widetilde{N_a}, \widetilde{N_b})$ be the normalized distance measures for n-valued T-Spherical fuzzy neutrosophic numbers $\widetilde{N_a}$ and $\widetilde{N_b}$.

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4. A Simplified neutrosophic valued normalized distance measures for N-valued T-Spherical fuzzy neutrosophic numbers

Distance-based similarity measures serve as efficient tools in decision-making processes. For N-valued T-spherical fuzzy neutrosophic numbers, a normalized distance measure relies on independent functions, including acceptance, indeterminacy, and rejection grades. In this study, we introduce three simplified neutrosophic valued normalized distance measures specifically designed for N-valued T-spherical fuzzy neutrosophic numbers. To set the foundation, we first revisit the concepts of Simplified Neutrosophic Sets and Simplified Neutrosophic Values.

Definition 4.1 Let X = $\{x_1, x_2, \ldots . x_n\}$, The simplified Neutrosophic set $\breve{N}$ in X is characterized by a membership degree T_A, indeterminacy degree I_A, and non -membership degree F_A, where T_n: X → [0, 1], I_n: X → [0, 1], F_n: X → [0, 1]. It can be written as $\breve{N}$ = {(x_i, $T_n(x_i)$, $I_n(x_i)$, $F_n(xi)$) / x_i ∈ X}, satisfying $0 \leq T_n(x) + I_n(x) + F_n(x) \leq 3$.

Here, $\breve{N}_\vartheta = (T_\vartheta, I_\vartheta, F_\vartheta) = (T_n(x), I_n(x), F_n(x))$ is called a Simplified neutrosophic value.

Definition 4.2 Let X = $\{x_1, x_2, \ldots . x_n\}$ be the universal set for any two N-valued T – SFNSs, $\widetilde{N_a}$ = {xi, $\langle (T_a^{i,j})_{j=1}^k, (I_a^{i,j})_{j=1}^k, (F_a^{i,j})_{j=1}^k \rangle$ / x_i ∈ X} and $\widetilde{N_b}$ = {xi, $\langle (T_b^{i,j})_{j=1}^k, (I_b^{i,j})_{j=1}^k, (F_b^{i,j})_{j=1}^k \rangle$ / x_i ∈ X} then the Normalized Euclidean distance between N-valued T – SFNSs $\widetilde{N_a}$ and $\widetilde{N_b}$ is defined as follows,

$$\text{NED}(\widetilde{N_a}, \widetilde{N_b}) = \{\frac{\{\sum_{i=1}^{n}([({}^+T_a^{j,t})^q - ({}^+T_b^{j,t})^q]^2 + [({}^+T_a^{j,i})^q - ({}^+T_b^{j,i})^q]^2 + [({}^+T_a^{j,f})^q - ({}^+T_b^{j,f})^q]^2)^{1/2}\}}{3n},$$

$$\frac{\{\sum_{i=1}^{n}([({}^+I_a^{j,t})^q - ({}^+I_b^{j,t})^q]^2 + [({}^+I_a^{j,i})^q - ({}^+I_b^{j,i})^q]^2 + [({}^+I_a^{j,f})^q - ({}^+I_b^{j,f})^q]^2)^{1/2}\}}{3n},$$

$$\frac{\{\sum_{i=1}^{n}\left(\left[({}^+F_a^{j,t})^q - ({}^+F_b^{j,t})^q\right]^2 + [({}^+F_a^{j,i})^q - ({}^+F_b^{j,i})^q]^2 + [({}^+F_a^{j,f})^q - ({}^+F_b^{j,f})^q]^2\right)^{\frac{1}{2}}\}}{3n}\}$$

Where, $({}^+T_a^{j,t}) = \max_j(T_a^{j,t})$, $({}^+T_a^{j,i}) = \max_j(T_a^{j,i})$, $({}^+T_a^{j,f}) = \max_j(T_a^{j,f})$; $({}^+I_a^{j,t}) = \max_j(I_a^{j,t})$, $({}^+I_a^{j,i}) = \max_j(I_a^{j,i})$, $({}^+I_a^{j,f}) = \max_j(I_a^{j,f})$; $({}^+F_a^{j,t}) = \max_j(F_a^{j,t})$, $({}^+F_a^{j,i}) = \max_j(F_a^{j,i})$, $({}^+F_a^{j,f}) = \max_j(F_a^{j,f})$; $({}^+T_b^{j,t}) = \max_j(T_b^{j,t})$, $({}^+T_b^{j,i}) = \max_j(T_b^{j,i})$, $({}^+T_b^{j,f}) = \max_j(T_b^{j,f})$; $({}^+I_b^{j,t}) = \max_j(I_b^{j,t})$, $({}^+I_a^{j,i}) = \max_j(I_b^{j,i})$, $({}^+I_b^{j,f}) = \max_j(I_b^{j,f})$; $({}^+F_b^{j,t}) = \max_j(F_b^{j,t})$, $({}^+F_b^{j,i}) = \max_j(F_b^{j,i})$, $({}^+F_b^{j,f}) = \max_j(F_b^{j,f})$.

The above measure is a simplified neutrosophic valued normalized Euclidean distance measure between N-valued T – SFNSs with the same sequence of length k.

Definition 4.3 Let X = $\{x_1, x_2, \ldots . x_n\}$ be the universal set for any two N-valued T – SFNSs, $\widetilde{N_a}$ = {xi, $\langle (T_a^{i,j})_{j=1}^k, (I_a^{i,j})_{j=1}^k, (F_a^{i,j})_{j=1}^k \rangle$ / x_i ∈ X} and $\widetilde{N_b}$ = {xi, $\langle (T_b^{i,j})_{j=1}^k, (I_b^{i,j})_{j=1}^k, (F_b^{i,j})_{j=1}^k \rangle$ / x_i ∈ X} then the Normalized Max-Min distance measure between N-valued T – SFNSs $\widetilde{N_a}$ and $\widetilde{N_b}$ is defined as follows,

Max-Min $(\widetilde{N_a}, \widetilde{N_b})$ =

$$\{\frac{\{\sum_{i=1}^{n}\{min[({}^+T_a^{j,t})^q, ({}^+T_b^{j,t})^q] + \min[({}^+T_a^{j,i})^q, ({}^+T_b^{j,i})^q] + min[({}^+T_a^{j,f})^q, ({}^+T_b^{j,f})^q]\}}{\{\sum_{i=1}^{n}\{max[({}^+T_a^{j,t})^q, ({}^+T_b^{j,t})^q] + \max[({}^+T_a^{j,i})^q, ({}^+T_b^{j,i})^q] + max[({}^+T_a^{j,f})^q, ({}^+T_b^{j,f})^q]\}},$$

$$\frac{\{\sum_{i=1}^{n}\{\min[({}^+I_a^{j,t})^q, ({}^+I_b^{j,t})^q] + \min[({}^+I_a^{j,i})^q, ({}^+I_b^{j,i})^q] + min[({}^+I_a^{j,f})^q, ({}^+I_b^{j,f})^q]\}}{\{\sum_{i=1}^{n}\{max[({}^+I_a^{j,t})^q, ({}^+I_b^{j,t})^q] + \max[({}^+I_a^{j,i})^q, ({}^+I_b^{j,i})^q] + max[({}^+I_a^{j,f})^q, ({}^+I_b^{j,f})^q]\}},$$

$$\frac{\{\sum_{i=1}^{n}\{min[({}^+F_a^{j,t})^q, ({}^+F_b^{j,t})^q] + \min[({}^+F_a^{j,i})^q, ({}^+F_b^{j,i})^q] + min[({}^+F_a^{j,f})^q, ({}^+F_b^{j,f})^q]\}}{\{\sum_{i=1}^{n}\{max[({}^+F_a^{j,t})^q, ({}^+F_b^{j,t})^q] + \max[({}^+F_a^{j,i})^q, ({}^+F_b^{j,i})^q] + max[({}^+F_a^{j,f})^q, ({}^+F_b^{j,f})^q]\}}\}$$

Where, $({}^+T_a^{j,t}) = \max_j(T_a^{j,t})$, $({}^+T_a^{j,i}) = \max_j(T_a^{j,i})$, $({}^+T_a^{j,f}) = \max_j(T_a^{j,f})$; $({}^+I_a^{j,t}) = \max_j(I_a^{j,t})$, $({}^+I_a^{j,i}) = \max_j(I_a^{j,i})$, $({}^+I_a^{j,f}) = \max_j(I_a^{j,f})$; $({}^+F_a^{j,t}) = \max_j(F_a^{j,t})$, $({}^+F_a^{j,i}) = \max_j(F_a^{j,i})$, $({}^+F_a^{j,f}) = \max_j(F_a^{j,f})$; $({}^+T_b^{j,t}) = \max_j(T_b^{j,t})$, $({}^+T_b^{j,i}) = \max_j(T_b^{j,i})$, $({}^+T_b^{j,f}) = \max_j(T_b^{j,f})$; $({}^+I_b^{j,t}) = \max_j(I_b^{j,t})$, $({}^+I_a^{j,i}) = \max_j(I_b^{j,i})$, $({}^+I_b^{j,f}) = \max_j(I_b^{j,f})$; $({}^+F_b^{j,t}) = \max_j(F_b^{j,t})$, $({}^+F_b^{j,i}) = \max_j(F_b^{j,i})$, $({}^+F_b^{j,f}) = \max_j(F_b^{j,f})$.

The above measure is a simplified neutrosophic valued normalized Max-Min distance measure between N-valued T – SFNSs with the same sequence of length k.

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Definition 4.4 Let X = $\{x_1, x_2, \ldots . x_n\}$ be the universal set for any two N-valued T – SFNSs, $\widetilde{N_a}$ = {xi, $\langle(\mathrm{T}_a^{i,j})_{j=1}^k, (\mathrm{I}_a^{i,j})_{j=1}^k, (\mathrm{F}_a^{i,j})_{j=1}^k \rangle$ / x_i ∈ X} and $\widetilde{N_b}$ = {xi, $\langle(\mathrm{T}_b^{i,j})_{j=1}^k, (\mathrm{I}_b^{i,j})_{j=1}^k, (\mathrm{F}_b^{i,j})_{j=1}^k \rangle$ / x_i ∈ X} then the Normalized Hausdroff's distance measure between n-valued T – SFNSs $\widetilde{N_a}$ and $\widetilde{N_b}$ is defined as follows,

HAU $(\widetilde{N_a}, \widetilde{N_b})$ = $\{ \frac{1}{n}\sum_{i=1}^n \max\{[|({}^+\mathrm{T}_a^{j,t})^q - ({}^+\mathrm{T}_b^{j,f})^q|], [|({}^+\mathrm{T}_a^{j,i})^q - ({}^+\mathrm{T}_b^{j,i})^q|], [|({}^+\mathrm{T}_a^{j,f})^q - ({}^+\mathrm{T}_b^{j,f})^q|]\},$

$$\frac{1}{n}\sum_{i=1}^n \max\{[|({}^+\mathrm{I}_a^{j,t})^q - ({}^+\mathrm{I}_b^{j,f})^q|], [|({}^+\mathrm{I}_a^{j,i})^q - ({}^+\mathrm{I}_b^{j,i})^q|], [|({}^+\mathrm{I}_a^{j,f})^q - ({}^+\mathrm{I}_b^{j,f})^q|]\},$$

$$\frac{1}{n}\sum_{i=1}^n \max\{[|({}^+\mathrm{F}_a^{j,t})^q - ({}^+\mathrm{F}_b^{j,f})^q|], [|({}^+\mathrm{F}_a^{j,i})^q - ({}^+\mathrm{F}_b^{j,i})^q|], [|({}^+\mathrm{F}_a^{j,f})^q - ({}^+\mathrm{F}_b^{j,f})^q|]\}\}$$

Where, $({}^+\mathrm{T}_a^{j,t}) = \max_j(\mathrm{T}_a^{j,t})$, $({}^+\mathrm{T}_a^{j,i}) = \max_j(\mathrm{T}_a^{j,i})$, $({}^+\mathrm{T}_a^{j,f}) = \max_j(\mathrm{T}_a^{j,f})$; $({}^+\mathrm{I}_a^{j,t}) = \max_j(\mathrm{I}_a^{j,t})$, $({}^+\mathrm{I}_a^{j,i}) = \max_j(\mathrm{I}_a^{j,i})$, $({}^+\mathrm{I}_a^{j,f}) = \max_j(\mathrm{I}_a^{j,f})$; $({}^+\mathrm{F}_a^{j,t}) = \max_j(\mathrm{F}_a^{j,t})$, $({}^+\mathrm{F}_a^{j,i}) = \max_j(\mathrm{F}_a^{j,i})$, $({}^+\mathrm{F}_a^{j,f}) = \max_j(\mathrm{F}_a^{j,f})$; $({}^+\mathrm{T}_b^{j,t}) = \max_j(\mathrm{T}_b^{j,t})$, $({}^+\mathrm{T}_b^{j,i}) = \max_j(\mathrm{T}_b^{j,i})$, $({}^+\mathrm{T}_b^{j,f}) = \max_j(\mathrm{T}_b^{j,f})$; $({}^+\mathrm{I}_b^{j,t}) = \max_j(\mathrm{I}_b^{j,t})$, $({}^+\mathrm{I}_a^{j,i}) = \max_j(\mathrm{I}_b^{j,i})$, $({}^+\mathrm{I}_b^{j,f}) = \max_j(\mathrm{I}_b^{j,f})$; $({}^+\mathrm{F}_b^{j,t}) = \max_j(\mathrm{F}_b^{j,t})$, $({}^+\mathrm{F}_b^{j,i}) = \max_j(\mathrm{F}_b^{j,i})$, $({}^+\mathrm{F}_b^{j,f}) = \max_j(\mathrm{F}_b^{j,f})$.

The above measure is a simplified neutrosophic valued Normalized Hausdroff's distance measure between N-valued T – SFNSs with the same sequence of length k.

It is clear that $N_\vartheta = (T_\vartheta, I_\vartheta, F_\vartheta)$ is a simplified neutrosophic value, Hence, we need to define a Score Function to Rank the values of normalized distance measures for N-valued T-Spherical fuzzy neutrosophic numbers.

Definition 4.5 Let N = (α, θ, δ) be a simplified neutrosophic value.

A proposed Score Function, $\mathscr{F} = \frac{1 + \alpha\ (1-\theta),(1-\delta)}{3}$. We use the Score Function $\mathscr{F}$ to rank the results of the Normalized distance measure between N-valued T – SFNNs.

Definition 4.6 Let $\widetilde{N}^{s_1}$, $\widetilde{N}^{s_2}$ be two simplified neutrosophic values for the Normalized distance measure between N-valued T – SFNNs. If $\mathscr{F}(\widetilde{N}^{s_1}) < \mathscr{F}(\widetilde{N}^{s_2})$ then $\widetilde{N}^{s_1} < \widetilde{N}^{s_2}$; If $\mathscr{F}(\widetilde{N}^{s_1}) > \mathscr{F}(\widetilde{N}^{s_2})$ then $\widetilde{N}^{s_1} > \widetilde{N}^{s_2}$.

5 Algorithm and Numerical Examples

Multi-criteria decision-making (MCDM) includes a range of methods that help achieve a balanced solution when considering multiple factors, such as determining the most sustainable strategy for solid waste management. This section introduces an algorithm developed from the proposed approach, which is then used to identify appropriate sites for a solid waste management system.

An algorithm based on N-valued T-Spherical Fuzzy Neutrosophic Similarity Measures.

Definition 5.1 Let $\mathscr{L} = \{\mathscr{L}_1, \mathscr{L}_2, \mathscr{L}_3, \ldots \mathscr{L}_m\}$be a set of alternatives, this represents various locations and distinct groups of geographical areas, $\Omega = \{\Omega_1, \Omega_2, \ldots \Omega_n\}$ be a set of criteria's, this is the range of options for solid waste management systems in each geographic field.

Algorithm A decision-making technique enables the evaluation of m alternatives across n criteria under specified parameters ($\wp = \wp_1, \wp_2, \ldots \wp_r$). This approach aids in identifying the most appropriate solid waste management systems tailored to specific geographic regions, ensuring an optimal match for each location.

Next, we'll explain how to apply the proposed distance-based similarity metrics for N-valued T-Spherical Fuzzy Neutrosophic sets.

Step I: The geographical field and suitable solid waste management systems for these areas should be evaluated. Then, regional principles and solid waste management systems should be identified. A decision matrix using N-valued T-Spherical Fuzzy Neutrosophic sets will illustrate the connection between the geographical field.

Step II: Construct a decision matrix using n-valued T-Spherical Fuzzy Neutrosophic sets that should display the relationship between the principles and the alternatives.

Step III: Compute the simplified neutrosophic values by using Normalized Euclidian distance measure, MAX-MIN distance measure, and Normalized Hausdroff's distance measure for each criterion and alternative that is, $\mathcal{D}_\wp^1(\mathscr{L}_m, \Omega_n)$, $\mathcal{D}_\wp^2(\mathscr{L}_m, \Omega_n)$, and $\mathcal{D}_\wp^3(\mathscr{L}_m, \Omega_n)$.

Step IV: Use the different score functions to determine the connection between norms and alternatives and rank them in descending order.

Step V: The ideal choice is identified by selecting the highest value within each geographical category, highlighting the top option for each field.

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Applications Generating solid waste in urban areas has become a significant challenge, requiring careful and effective disposal and management. This complex task involves considering various environmental, economic, ecological, social, and political factors. While the 3R strategy: Reduce, Reuse, and Recycle plays a crucial role in minimizing waste, a residual portion still needs to be managed. Incineration offers a solution by converting waste into energy while composting the organic fraction can produce high-quality fertilizers for agriculture. Ultimately, however, landfilling is the final destination for much of the remaining waste. Selecting suitable sites for incineration, composting, and landfilling is particularly challenging, necessitating a balanced approach to address these factors. This study aims to contribute to solving this global issue by evolving a mathematical technique for optimal site selection, thereby improving waste management practices.

To address this problem, we consider a finite number of distinct geographical regions, each connected by key factors that we will discuss,

Now we will take a set of the available Solid waste management Systems,

$\Omega = \{\Omega_1, \Omega_2, \dots \Omega_n\}$ = {Landfill, Composting, Incinerator, Recycling}

At this point, we will consider the most relevant criteria that influence both fields, carefully selecting those that offer the most profound impact and intersection between them.

Let $\wp = \wp_1, \wp_2, \dots \wp_r$, finite number of parameters. Take Air Quality Index ($\wp_1$); Slope ($\wp_2$); Distance from community ($\wp_3$); and commercial factor ($\wp_4$). Then the mappings from,

β: **($\wp_1, \wp_2, \wp_3, \wp_4$)** $\rightarrow \beta(\mathscr{L})$

Υ: **($\wp_1, \wp_2, \wp_3, \wp_4$)** $\rightarrow \Upsilon(\Omega)$

$\wp_1$: 0 − 50 (Good), 51 – 100 (Moderate), 100 − 150 (Unhealthy for sensitive), 150 − 200 (unhealthy), 200 − 300 (most unhealthy), greater than 300 (Hazardous)

$\wp_2$: 0 % - 2% (ideal), 2% − 09% (good), 10% − 20% (moderate)

$\wp_3$: 0 **-** 1Km, 1Km – 2 Km, greater than 2 km

$\wp_4$: Very costly, costly, moderately costly, and reasonable

Table 1: Multi Criteria Decision Matrix. Here, Alternatives **($\mathscr{L}$),** criteria's **(Ω),** and Parameters **($\wp$)** are in N-valued T-spherical fuzzy neutrosophic numbers

$\mathscr{L}$ \ $\wp$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
$\mathscr{L}_1$	$\mathscr{L}\wp_{11}$	$\mathscr{L}\wp_{12}$	$\mathscr{L}\wp_{13}$	$\mathscr{L}\wp_{14}$
$\mathscr{L}_2$	$\mathscr{L}\wp_{21}$	$\mathscr{L}\wp_{22}$	$\mathscr{L}\wp_{23}$	$\mathscr{L}\wp_{24}$
$\mathscr{L}_3$	$\mathscr{L}\wp_{31}$	$\mathscr{L}\wp_{32}$	$\mathscr{L}\wp_{33}$	$\mathscr{L}\wp_{34}$
$\mathscr{L}_4$	$\mathscr{L}\wp_{41}$	$\mathscr{L}\wp_{42}$	$\mathscr{L}\wp_{43}$	$\mathscr{L}\wp_{44}$
Ω \ $\wp$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$

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Ω_1	$\Omega\wp_{11}$	$\Omega\wp_{12}$	$\Omega\wp_{13}$	$\Omega\wp_{14}$
Ω_2	$\Omega\wp_{21}$	$\Omega\wp_{22}$	$\Omega\wp_{23}$	$\Omega\wp_{24}$
Ω_3	$\Omega\wp_{31}$	$\Omega\wp_{32}$	$\Omega\wp_{33}$	$\Omega\wp_{34}$
Ω_4	$\Omega\wp_{41}$	$\Omega\wp_{42}$	$\Omega\wp_{43}$	$\Omega\wp_{44}$

Table 2: Decision matrix between criteria and geographical fields

Alternatives	$\wp_1$	$\wp_2$
$\mathscr{L}_1$	**{[(0.3,0.2,0.6),(0.2,0.3,0.1),(0.6,0.2,0.4)], [(0.1,0.4,0.2),(0.2,0.5,0.3),(0.3,0.1,0.1)], [(0.5,0.3,0.8),(0.4,0.6,0.3),(0.1,0.9,0.1)]}**	**{[(0.3,0.2,0.1),(0.1,0.2,0.4),(0.3,0.2,0.1)], [(0.1,0.5,0.4),(0.1,0.4,0.2),(0.0,0.3,0.1)], [(0.8,0.9,0.1),(0.6,0.5,0.3),(0.1,0.3,0.1)]}**
$\mathscr{L}_2$	**{[(0.3,0.4,0.5),(0.3,0.6,0.1),(0.8,0.2,0.4)], [(0.5,0.7,0.2),(0.6,0.4,0.5),(0.5,0.6,0.7)], [(0.7,0.9,0.8),(0.8,0.9,0.3),(0.1,0.3,0.1)]}**	**{[(0.5,0.4,0.3),(0.7,0.6,0.5),(0.8,0.2,0.5)], [(0.1,0.1,0.0),(0.8,0.4,0.3),(0.6,0.3,0.1)], [(0.7,0.9,0.5),(0.5,0.4,0.3),(0.4,0.3,0.1)]}**
$\mathscr{L}_3$	**{[(0.3,0.4,0.7),(0.3,0.2,0.1),(0.9,0.2,0.4)], [(0.1,0.3,0.2),(0.5,0.4,0.3),(0.5,0.6,0.7)], [(0.1,0.9,0.4),(0.2,0.5,0.3),(0.8,0.3,0.1)]}**	**{[(0.3,0.4,0.7),(0.3,0.6,0.9),(0.8,0.2,0.4)], [(0.5,0.2,0.4),(0.2,0.1,0.3),(0.5,0.7,0.6)], [(0.1,0.9,0.4),(0.5,0.4,0.3),(0.8,0.3,0.1)]}**
$\mathscr{L}_4$	**{[(0.3,0.4,0.1),(0.3,0.6,0.9),(0.5,0.2,0.4)], [(0.1,0.2,0.6),(0.3,0.4,0.3),(0.2,0.1,0.1)], [(0.7,0.9,0.4),(0.2,0.1,0.3),(0.6,0.3,0.1)]}**	**{[(0.3,0.4,0.9),(0.3,0.2,0.1),(0.1,0.1,0.0)], [(0.1,0.3,0.2),(0.9,0.4,0.3),(0.5,0.9,0.1)], [(0.5,0.3,0.1),(0.2,0.6,0.3),(0.8,0.3,0.1)]}**
Alternatives	$\wp_3$	$\wp_4$

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$\mathscr{L}_1$	**{[(0.8,0.4,0.1),(0.3,0.2,0.5),(0.6,0.5,0.4)], [(0.3,0.6,0.7),(0.8,0.2,0.9),(0.8,0.1,0.6)], [(0.3,0.1,0.2),(0.3,0.1,0.0),(0.5,0.3,0.4)]}**	**{[(0.5,0.4,0.6),(0.2,0.1,0.1),(0.4,0.3,0.5)], [(0.8,0.7,0.6),(0.2,0.1,0.3),(0.4,0.1,0.9)], [(0.8,0.6,0.4),(0.9,0.1,0.8),(0.6,0.3,0.4)]}**
$\mathscr{L}_2$	**{[(0.5,0.4,0.4),(0.3,0.6,0.9),(0.8,0.2,0.4)], [(0.1,0.7,0.9),(0.6,0.4,0.3),(0.5,0.9,0.1)], [(0.7,0.9,0.4),(0.2,0.9,0.3),(0.5,0.3,0.1)]}**	**{[(0.3,0.4,0.7),(0.3,0.6,0.9),(0.8,0.2,0.4)], [(0.1,0.7,0.9),(0.6,0.4,0.3),(0.5,0.9,0.1)], [(0.7,0.9,0.4),(0.2,0.9,0.3),(0.5,0.3,0.1)]}**
$\mathscr{L}_3$	**{[(0.3,0.1,0.2),(0.3,0.6,0.9),(0.8,0.2,0.4)], [(0.1,0.7,0.8),(0.7,0.4,0.3),(0.5,0.9,0.1)], [(0.6,0.0,0.4),(0.8,0.1,0.3),(0.9,0.3,0.1)]}**	**{[(0.3,0.4,0.6),(0.3,0.6,0.9),(0.1,0.2,0.4)], [(0.5,0.7,0.9),(0.8,0.4,0.3),(0.5,0.6,0.1)], [(0.5,0.2,0.4),(0.1,0.0,0.1),(0.8,0.3,0.1)]}**
$\mathscr{L}_4$	**{[(0.3,0.4,0.7),(0.3,0.6,0.9),(0.8,0.2,0.9)], [(0.1,0.2,0.3),(0.6,0.4,0.9),(0.5,0.9,0.1)], [(0.3,0.1,0.4),(0.2,0.8,0.3),(0.9,0.3,0.1)]}**	**{[(0.8,0.4,0.7),(0.3,0.6,0.8),(0.9,0.2,0.4)], [(0.1,0.7,0.8),(0.6,0.4,0.3),(0.5,0.6,0.1)], [(0.1,0.0,0.1),(0.8,0.4,0.3),(0.5,0.3,0.8)]}**

Table 3: Decision matrix between criteria and geographical fields

Criteria's	$\wp_1$	$\wp_2$
Ω_1	**{[(0.3,0.4,0.5),(0.3,0.6,0.7),(0.7,0.2,0.4)], [(0.1,0.7,0.8),(0.7,0.4,0.3),(0.5,0.8,0.1)], [(0.3,0.2,0.1),(0.2,0.9,0.3),(0.6,0.3,0.1)]}**	**{[(0.3,0.2,0.1),(0.3,0.1,0.4),(0.3,0.2,0.1)], [(0.1,0.6,0.7),(0.8,0.4,0.3),(0.7,0.4,0.1)], [(0.8,0.3,0.4),(0.7,0.5,0.3),(0.1,0.3,0.1)]}**
Ω_2	**{[(0.3,0.1,0.2),(0.3,0.2,0.4),(0.5,0.2,0.4)], [(0.1,0.2,0.3),(0.6,0.4,0.8),(0.3,0.2,0.1)], [(0.7,0.9,0.4),(0.2,0.8,0.3),(0.7,0.3,0.1)]}**	**{[(0.3,0.4,0.7),(0.3,0.6,0.3),(0.3,0.2,0.1)], [(0.1,0.7,0.8),(0.9,0.4,0.3),(0.5,0.7,0.1)], [(0.3,0.6,0.4),(0.2,0.5,0.3),(0.4,0.3,0.1)]}**
Ω_3	**{[(0.3,0.4,0.6),(0.3,0.6,0.8),(0.9,0.2,0.4)], [(0.1,0.7,0.8),(0.1,0.2,0.3),(0.2,0.1,0.1)], [(0.7,0.5,0.4),(0.2,0.6,0.3),(0.9,0.3,0.1)]}**	**{[(0.3,0.4,0.9),(0.3,0.4,0.5),(0.9,0.2,0.4)], [(0.1,0.7,0.3),(0.5,0.4,0.3),(0.5,0.6,0.1)], [(0.7,0.8,0.4),(0.2,0.0,0.3),(0.5,0.3,0.9)]}**

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Ω_4	{[(0.3,0.4,0.5),(0.3,0.6,0.7),(0.5,0.2,0.4)], [(0.1,0.7,0.7),(0.9,0.4,0.3),(0.5,0.6,0.1)], [(0.3,0.2,0.4),(0.2,0.4,0.3),(0.5,0.3,0.7)]}	{[(0.3,0.4,0.7),(0.3,0.6,0.3),(0.5,0.2,0.4)], [(0.1,0.5,0.2),(0.2,0.4,0.3),(0.3,0.2,0.1)], [(0.0,0.1,0.0),(0.2,0.1,0.1),(0.7,0.3,0.1)]}
Criteria's	$\wp_3$	$\wp_4$
Ω_1	{[(0.3,0.4,0.8),(0.3,0.6,0.7),(0.3,0.2,0.2)], [(0.1,0.5,0.3),(0.3,0.4,0.3),(0.5,0.7,0.1)], [(0.7,0.9,0.4),(0.2,0.3,0.3),(0.1,0.0,0.1)]}	{[(0.9,0.4,0.7),(0.3,0.2,0.4),(0.8,0.2,0.4)], [(0.1,0.7,0.4),(0.6,0.4,0.3),(0.5,0.3,0.1)], [(0.7,0.8,0.4),(0.2,0.5,0.3),(0.8,0.3,0.1)]}
Ω_2	{[(0.9,0.4,0.7),(0.3,0.4,0.2),(0.5,0.2,0.4)], [(0.1,0.4,0.6),(0.1,0.4,0.3),(0.5,0.9,0.1)], [(0.6,0.6,0.4),(0.2,0.7,0.3),(0.9,0.3,0.1)]}	{[(0.3,0.4,0.9),(0.3,0.6,0.5),(0.7,0.2,0.4)], [(0.1,0.7,0.9),(0.7,0.4,0.3),(0.5,0.4,0.1)], [(0.5,0.0,0.4),(0.3,0.0,0.0),(0.4,0.3,0.1)]}
Ω_3	{[(0.8,0.4,0.7),(0.3,0.4,0.5),(0.3,0.2,0.0)], [(0.1,0.0,0.2),(0.2,0.1,0.0),(0.3,0.1,0.1)], [(0.7,0.6,0.4),(0.2,0.9,0.3),(0.1,0.1,0.1)]}	{[(0.3,0.4,0.9),(0.3,0.2,0.1),(0.1,0.2,0.0)], [(0.1,0.4,0.3),(0.2,0.1,0.0),(0.5,0.7,0.1)], [(0.7,0.8,0.4),(0.8,0.97,0.3),(0.2,0.3,0.1)]}
Ω_4	{[(0.3,0.1,0.2),(0.3,0.6,0.8),(0.5,0.2,0.4)], [(0.1,0.2,0.3),(0.4,0.4,0.3),(0.5,0.9,0.1)], [(0.7,0.8,0.4),(0.2,0.7,0.3),(0.7,0.3,0.1)]}	{[(0.3,0.4,0.9),(0.3,0.1,0.2),(0.0,0.2,0.1)], [(0.1,0.3,0.4),(0.2,0.1,0.3),(0.5,0.7,0.1)], [(0.7,0.5,0.4),(0.2,0.6,0.3),(0.9,0.3,0.7)]}

Table 4: Simplified neutrosophic values.

Distance& Similarity Measures	Ω \ $\mathscr{L}$	Ω_1	Ω_2	Ω_3	Ω_4
	$\mathscr{L}_1$	**(0.2, 0.3, 0.4)**	**(0.5, 0.1, 0.4)**	**(0.7, 0.2, 0.3)**	**(0.1, 0.4, 0.3)**
	$\mathscr{L}_2$	**(0.3, 0.4, 0.5)**	**(0.2, 0.3, 0.5)**	**(0.2, 0.4, 0.6)**	**(0.5, 0.6, 0.7)**

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$\mathcal{D}_P^1$ (β, ϒ)	$\mathcal{L}_3$	**(0.6, 0.3, 0.4)**	**(0.8, 0.2, 0.6)**	**(0.5, 0.4, 0.6)**	**(0.6, 0.2, 0.4)**
	$\mathcal{L}_4$	**(0.4, 0.3, 0.5)**	**(0.1, 0.5, 0.3)**	**(0.2, 0.4, 0.7)**	**(0.6, 0.5, 0.4)**
Distance& Similarity Measures	Ω / $\mathcal{L}$	Ω_1	Ω_2	Ω_3	Ω_4
$\mathcal{D}_P^2$(β, ϒ)	$\mathcal{L}_1$	**(0.5, 0.2, 0.7)**	**(0.3, 0.5, 0.1)**	**(0.1, 0.2, 0.3)**	**(0.3, 0.4, 0.5)**
	$\mathcal{L}_2$	**(0.4, 0.5, 0.6)**	**(0.1, 0.4, 0.2)**	**(0.2, 0.7, 0.5)**	**(0.3, 0.4, 0.7)**
	$\mathcal{L}_3$	**(0.2, 0.4 0.3)**	**(0.7, 0.5, 0.4)**	**(0.1, 0.2, 0.3)**	**(0.3, 0.2, 0.3)**
	$\mathcal{L}_4$	**(0.6, 0.4 0.2)**	**(0.2, 0.1, 0.3)**	**(0.5, 0.4, 0.3)**	**(0.2, 0.1, 0.4)**
Distance& Similarity Measures	Ω / $\mathcal{L}$	Ω_1	Ω_2	Ω_3	Ω_4
$\mathcal{D}_P^3$(β, ϒ)	$\mathcal{L}_1$	**(0.3, 0.4, 02)**	**(0.2, 0.3, 0.5)**	**(0.6, 0.2, 0.5)**	**(0.6, 0.4, 0.7)**
	$\mathcal{L}_2$	**(0.4, 0.5, 0.7)**	**(0.5, 0.7, 0.1)**	**(0.5, 0.7, 0.6)**	**(0.2, 0.3, 0.6)**
	$\mathcal{L}_3$	**(0.5, 0.4, 0.6)**	**(0.7, 0.6, 0.5)**	**(0.4, 0.5, 0.4)**	**(0.4, 0.1, 0.3)**
	$\mathcal{L}_4$	**(0.2, 0.3, 0.1)**	**(0.5, 0.4, 0.3)**	**(0.2, 0.4, 0.5)**	**(0.1, 0.2, 0.3)**

Simplified neutrosophi values in Table IV are converted into scores using a proposed Score Function of Simplified neutrosophi values.

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Table 5: Score values of Simplified neutrosophic values

Distance-based Similarity Measures	Ω \ $\mathscr{L}$	Ω_1	Ω_2	Ω_3	Ω_4
$\mathcal{D}_P^1(\beta, \Upsilon)$	$\mathscr{L}_1$	**0.752**	**0.905**	**0.976**	**0.731**
	$\mathscr{L}_2$	**0.695**	**0.71**	**0.644**	**0.59**
	$\mathscr{L}_3$	**0.836**	**0.788**	**0.68**	**0.884**
	$\mathscr{L}_4$	**0.745**	**0.6925**	**0.608**	**0.74**
Distance-based Similarity Measures	Ω \ $\mathscr{L}$	Ω_1	Ω_2	Ω_3	Ω_4
$\mathcal{D}_P^2(\beta, \Upsilon)$	$\mathscr{L}_1$	**0.68**	**0.7925**	**0.808**	**0.695**
	$\mathscr{L}_2$	**0.64**	**0.764**	**0.59**	**0.617**
	$\mathscr{L}_3$	**0.752**	**0.755**	**0.808**	**0.864**
	$\mathscr{L}_4$	**0.884**	**0.878**	**0.815**	**0.824**
Distance-based Similarity Measures	Ω \ $\mathscr{L}$	Ω_1	Ω_2	Ω_3	Ω_4
$\mathcal{D}_P^3(\beta, \Upsilon)$	$\mathscr{L}_1$	**0.812**	**0.71**	**0.82**	**0.644**
	$\mathscr{L}_2$	**0.605**	**0.7025**	**0.59**	**0.668**

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	$\mathscr{L}_3$	**0.68**	**0.67**	**0.71**	**0.941**
	$\mathscr{L}_4$	**0.878**	**0.815**	**0.68**	**0.808**

The proposed technique, based on $\mathcal{D}_P^1$(β, ϒ), $\mathcal{D}_P^2$(β, ϒ), $\mathcal{D}_P^3$(β, ϒ) Landfill has the highest value corresponding to region $\mathscr{L}_4$; Composting has the highest value corresponding to region $\mathscr{L}_2$; Incinerator has the highest value corresponding to region $\mathscr{L}_1$; Recycling has the highest value corresponding to region $\mathscr{L}_3$ respectively.

Using the proposed distance-based similarity measures, we calculated the proportion of solid waste treatment systems in various regions, as displayed in the tables above. Our findings suggest that this approach is an effective tool for selecting a flexible site for solid waste treatment.

6. Results and Discussions

The introduction of N-valued T-spherical fuzzy neutrosophic models marks a significant advancement in fuzzy set theory, a branch of mathematical logic focused on handling uncertainties and partial truths. This research not only deepens the theoretical underpinnings of fuzzy set theory but also broadens its practical applications by introducing novel distance and similarity measures. The N-valued T-spherical fuzzy neutrosophic model innovatively combines intuitionistic fuzzy-valued neutrosophic multi-sets (IFVNMSs) and T-spherical fuzzy neutrosophic sets (T-SFVNSs) within the Simplified Neutrosophic Sets (SNS) framework, integrating T-spherical fuzzy values to improve the handling of uncertainty. This model allows experts more flexibility in expressing opinions, accommodating varying degrees of certainty without restrictive constraints.

In application, the model's specialized distance-based similarity measures offer valuable tools for selecting sites for Solid Waste Management Systems (SWMS). These measures allow decision-makers to evaluate potential sites with improved accuracy, considering environmental, social, and economic uncertainties. This comprehensive assessment method enables experts to express preferences flexibly, leading to more context-sensitive and reliable site selection for SWMS. Practically, the model's distance-based measures rank alternative sites based on proximity to an ideal solution, while similarity measures assess each site's closeness to a target profile. These metrics refine the decision-making process; ensuring chosen sites support sustainable waste management, address community concerns, and align with regulatory standards.

This study focuses on the mathematical foundation of N-valued T-spherical neutrosophic sets. It suggests that interdisciplinary integration with machine learning, artificial intelligence, and big data could broaden their applicability, especially in complex decision-making scenarios. To address complex decisions under uncertainty, models like neutrosophic cubic hesitant fuzzy sets, neutrosophic cubic spherical fuzzy sets, T-spherical hesitant fuzzy sets, T-spherical fuzzy neutrosophic sets, and intuitionistic fuzzy neutrosophic multi-sets, particularly with aggregation operators, provide significant flexibility. These models manage hesitation, ambiguity, and partial truths, enhancing decision-making by enabling more nuanced solutions. Similarly, the N-valued T-spherical neutrosophic environment offers a comparable framework, empowering decision-makers to capture and manage uncertainty effectively, even under high complexity.

7. Conclusion and scope for future works

In conclusion, the introduction of N-valued T-spherical neutrosophic sets (N-valued T-SFNSs), integrating intuitionistic fuzzy-valued neutrosophic multisets (IFVNMSs) with T-spherical fuzzy neutrosophic sets (T-SFVNSs) through key algebraic operations, provides an innovative solution to Multi-Criteria Decision-Making (MCDM) problems. The proposed model, underpinned by a simplified neutrosophic-valued distance-based similarity measure, proves effective in scenarios involving uncertainty and expert input, as demonstrated in the example of selecting an optimal solid waste disposal site. This model's adaptability, feasibility, and practicality make it an essential tool for real-world decision-making applications. Future research may expand on these findings by developing an N-valued T-spherical rough neutrosophic environment, opening new possibilities for decision-making techniques like TOPSIS, VIKOR, WASPAS, and AHP, further enhancing the model's versatility across diverse fields.

Conflict of interest

The authors affirm no conflicts of interest.

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