New form of weighted interaction aggregating operators communicated with the reciprocal fractional floor function via the neutrosophic set

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Abstract

In this work, we present novel techniques for the reciprocal fractional floor function applied neutrosophic set (RFFFNS) via interaction aggregating operator. The neutrosophic set combined with the reciprocal fractional floor operator. The geometric interaction operations of neutrosophic numbers and their new averaging are studied using the universal aggregation function. The RFFFNS are idempotent, boundedness compatible, commutative, and associative. Four new interaction aggregating operators are introduced: RFFFNS interaction weighted averaging, RFFFNS interaction weighted geometric, generalized RFFFNS interaction weighted averaging, and generalized RFFFNS interaction weighted geometric. The aggregation functions are commonly assumed to be represented by the Euclidean distance, Hamming distance, and score values.

Keywords: Aggregating operator; RFFFNSIWA; RFFFNSIWG; GRFFFNSIWA; GRFFFNSIWG

1 Introduction

The fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS),[3,4] neutrosophic set (NS),[5] and Fermatean FS (FFFS)[6] have all been developed as a consequence of the uncertainties. Zadeh's FS[1] proposes a membership degree (MD) for decision-makers. Since each object possesses MD $\perp$ and non-membership degree (NMD) ς and satisfies $0 \leq \perp + \varsigma \leq 1$, for $\perp, \varsigma \in [0, 1]$, An IFS concept was established by Atanassov.[2] According to the criterion that $\perp^2 + \varsigma^2 \leq 1$, PFSs are defined by their MDs and NMDs, as developed by Yager.[3] In many different disciplines, IFSs and PFSs have been extensively studied and used. Senapati and colleagues[6] introduced the idea of an FFFS in 2019. Both the MD and NMD with the property that $0 \leq \perp^3 + \varsigma^3 \leq 1$. Hatamleh et al.[7] discussed the concept of two-fold fuzzy n-refined neutrosophic rings. Cuong and colleagues[8] by creating the idea of picture FSs (PiFSs). Since PiFSs is an extended form of IFSs, it has been observed that it may support certain extra ambiguities. It is noted in PiFSs that the MD $\perp$, neutral ς, and NMD ι have $0 \leq \perp + \varsigma + \iota \leq 1$; for $\perp, \varsigma, \iota \in [0, 1]$. It will guarantee that expert opinion messages like "yes," "abstain," "no," and "refusal" are sent over the PiFS. Additionally, it will prevent evaluation information from being omitted and guarantee that the evaluation data and the real decision environment are consistent. Although there are many applications and research on PiFSs, the notion has not been thoroughly investigated. For certain AOs with MADM, Shahzaib et al.[9] defined the spherical FS (SFS). The SFS requires that $0 \leq \perp^2 + \varsigma^2 + \iota^2 \leq 1$

instead of $0 \leq \perp + \varsigma + \iota \leq 1$. Jin et al.[10] presented the linguistic SFS AOs, which were talked about in MADM difficulties. In DM, Rafiq et al. presented SFSs and their uses.[11] Decision-making (DM) and the fact that $\perp^2 + \varsigma^2 \geq 1$ are problematic. Palanikumar et al.[12–14] discussed the concept of different operators via trigonometric neutrosophic interval-valued set, complex neutrosophic sets and Diophantine neutrosophic interval valued soft set. Because of this effort, the following contributions are made:

1. Numerous features of algebra, including idempotency, commutativity, and associativity, have been demonstrated.
2. The characteristics of a RFFFNSIWA, WG, GWA and GWG are HD and ED. The ED distance between two RFFFNSs is to be determined using this approach.

2 Different AOs for RFFFNN

If f is a fractional part function, then the fractional part of ι, where ι is a real number, may be written as follows: The formula $\pounds[\iota] = \langle\iota\rangle = \iota - \lfloor\iota\rfloor$. As an alternative, a fractional part function may be used to specify the difference between a real number and its biggest integer value, which is found using the greatest integer function. The fractional component of ι = 0 if ι is an integer. The reciprocal fractional part function $\pounds[\iota] = \frac{1}{\iota}$ is assumed. It is well knowledge that the fractional component of ι equals 0 whenever it is an integer. For $\pounds[\iota] = \frac{1}{\iota}$ to be defined, l must not be an integer. All real numbers, except integers, fall inside the domain of $\pounds[\iota] = \frac{1}{\iota}$.

Definition 2.1. Suppose that $\hbar_1 = \langle \perp_1, \natural_1, \nu_1 \rangle$ and $\hbar_2 = \langle \perp_2, \natural_2, \nu_2 \rangle$ be the any two RFFFNNs. Then

1. $$\hbar_1 \Cup \hbar_2 = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} + \langle\perp_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\perp_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} + \langle\natural_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\natural_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} + \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} \\ -\langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\perp_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} \end{matrix}} \end{pmatrix}$$
2. $$\hbar_1 \Cap \hbar_2 = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} + \langle\perp_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\perp_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} \\ -\langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\perp_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} \end{matrix}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} + \langle\natural_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\natural_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}} \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} + \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}} - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \cdot \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}} \end{pmatrix}$$
3. $$\mathscr{M} \cdot \hbar_1 = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{1 - \left\langle 1 - \langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{1 - \left\langle 1 - \langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle 1 - \langle\perp_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}} - \left\langle 1 - \langle\perp_1 + \nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}}} \end{pmatrix}$$
4. $$\hbar_1^{\mathscr{M}} = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle 1 - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}} - \left\langle 1 - \langle\perp_1 + \nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}}} \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{1 - \left\langle 1 - \langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{1 - \left\langle 1 - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\mathscr{M}}} \end{pmatrix}$$

2.1 RFFFNSIWA operator

Definition 2.2. Let $\hbar_\tau = \langle \perp_\tau, \natural_\tau, \nu_\tau \rangle$ be the RFFFNNs, $\tau \twoheadrightarrow 1, 2, ..., \eta$, ς_τ be the weight of $\hbar_\tau$ and $\varsigma_\tau \geq 0$, $\boxplus_{\tau \twoheadrightarrow 1}^{\eta} \varsigma_\tau = 1$. Then the RFFFNSIWA operator $\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle = \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \varsigma_\tau \hbar_\tau$.

Theorem 2.3. *Let $\hbar_\tau = \langle \perp_\tau, \natural_\tau, \nu_\tau \rangle$ be the RFFFNNs, $\tau \twoheadrightarrow 1, 2, ..., \eta$.*
Then, RFFFNSIWA$\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle$ =

$$\begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{1 - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle 1 - \langle\perp_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{1 - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle 1 - \langle\natural_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle 1 - \langle\perp_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle 1 - \langle\perp_\tau + \nu_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{pmatrix}.$$

Proof. If $\tau \twoheadrightarrow 2$, RFFFNSIWA$\langle \hbar_1, \hbar_2 \rangle = \varsigma_1 \hbar_1 \Cup \varsigma_2 \hbar_2$,
where,

$$\varsigma_1 \hbar_1 = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \left\langle \shortmid - \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \left\langle \shortmid - \langle \natural_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle \shortmid - \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} - \left\langle \shortmid - \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1}} \end{pmatrix}$$

and

$$\varsigma_2 \hbar_2 = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \left\langle \shortmid - \langle \perp_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \left\langle \shortmid - \langle \natural_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle \shortmid - \langle \perp_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} - \left\langle \shortmid - \langle \perp_2 + \nu_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2}} \end{pmatrix}$$

We get

$$\begin{aligned} \varsigma_1 \hbar_1 \Cup \varsigma_2 \hbar_2 &= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \left\langle \shortmid - \left\langle \shortmid - \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \perp_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \\ - \left\langle \shortmid - \left\langle \shortmid - \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \perp_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \end{matrix}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \left\langle \shortmid - \left\langle \shortmid - \langle \natural_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \natural_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \\ - \left\langle \shortmid - \left\langle \shortmid - \langle \natural_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \natural_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \end{matrix}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \left\langle \shortmid - \left\langle \shortmid - \nu_1 \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \Big\rangle + \left\langle \shortmid - \left\langle \shortmid - \nu_2 \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \Big\rangle \\ - \left\langle \shortmid - \left\langle \shortmid - \nu_1 \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \Big\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \nu_2 \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \Big\rangle \\ - \left\langle \shortmid - \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_1} \cdot \left\langle \shortmid - \langle \perp_2 + \nu_2 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_2} \end{matrix}} \end{pmatrix} \\ &= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \shortmid - \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \shortmid - \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \shortmid - \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \shortmid - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{pmatrix}. \end{aligned}$$

Using induction $\tau \geq 3$, RFFFNSIWA$\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\shortmid - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \end{pmatrix}.$$

If $\tau \twoheadrightarrow \eta + 1$, then $RFFFNSIWA\langle \hbar_1, \hbar_2, ..., \hbar_\eta, \hbar_{\eta+1} \rangle$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \shortmid - \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \perp_{\eta+1} \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \shortmid - \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \perp_{\eta+1} \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \end{matrix}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \shortmid - \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle + \left\langle \shortmid - \left\langle \shortmid - \langle \natural_{\eta+1} \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \shortmid - \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \langle \natural_{\eta+1} \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \end{matrix}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix} \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \shortmid - \nu_\tau \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \Big\rangle + \left\langle \shortmid - \left\langle \shortmid - \nu_{\eta+1} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \Big\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \shortmid - \nu_\tau \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \Big\rangle \cdot \left\langle \shortmid - \left\langle \shortmid - \nu_{\eta+1} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \Big\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \cdot \left\langle \shortmid - \langle \perp_{\eta+1} + \nu_{\eta+1} \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \end{matrix}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau} \cdot \left\langle \text{ı} - \langle\perp_{\eta+1}\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_{\eta+1}}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\natural_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau} \cdot \left\langle \text{ı} - \langle\natural_{\eta+1}\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_{\eta+1}}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\begin{matrix}\left\langle \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp_\tau + \nu_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}\right\rangle \cdot \\ \left\langle \left\langle\langle\perp_{\eta+1}\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_{\eta+1}} - \langle\perp_{\eta+1} + \nu_{\eta+1}\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_{\eta+1}}\right\rangle\end{matrix}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle \text{ı} - \langle\perp_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle \text{ı} - \langle\natural_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle \text{ı} - \langle\perp_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle \text{ı} - \langle\perp_\tau + \nu_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

Theorem 2.4. *If $\hbar_\tau = \langle\perp_\tau, \natural_\tau, \nu_\tau\rangle$ be the RFFFNNs and $\hbar_\tau = \hbar$, then the RFFFNSIWA$\langle\hbar_1, \hbar_2, ..., \hbar_\eta\rangle = \hbar$, $\tau \twoheadrightarrow 1, 2, ..., \eta$.*

Proof. Note that, $\langle\perp_\tau, \natural_\tau, \nu_\tau\rangle = \langle\perp, \natural, \nu\rangle$, $\tau \twoheadrightarrow 1, 2, ..., \eta$ and $\boxplus_{\tau\twoheadrightarrow 1}^{\eta}\varsigma_\tau = 1$.
We get, $RFFFNSIWA\langle\hbar_1, \hbar_2, ..., \hbar_\eta\rangle$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\natural\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp + \nu\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\perp\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\boxplus_{\tau\twoheadrightarrow 1}^{\eta}\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\natural\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\boxplus_{\tau\twoheadrightarrow 1}^{\eta}\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle \text{ı} - \langle\perp\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\boxplus_{\tau\twoheadrightarrow 1}^{\eta}\varsigma_\tau} - \left\langle \text{ı} - \langle\perp + \nu\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\boxplus_{\tau\twoheadrightarrow 1}^{\eta}\varsigma_\tau}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\perp\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\natural\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle \text{ı} - \langle\perp\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle - \left\langle \text{ı} - \langle\perp + \nu\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle} \end{pmatrix}$$

$$= \left\langle\perp, \natural, \nu\right\rangle = \hbar$$

2.2 Interaction weighted geometric(RFFFNSIWG) operator

Definition 2.5. Let $\hbar_\tau = \langle\perp_\tau, \natural_\tau, \nu_\tau\rangle$ be the RFFFNNs, ς_τ be the weight of $\hbar_\tau$. Then the RFFFNSIWG operator $\langle\hbar_1, \hbar_2, ..., \hbar_\eta\rangle = \ominus_{\tau\twoheadrightarrow 1}^{\eta}\hbar_\tau^{\varsigma_\tau}$.

Theorem 2.6. *If $\hbar_\tau = \langle\perp_\tau, \natural_\tau, \nu_\tau\rangle$ be the RFFFNNs. Then,*

$$RFFFNSIWG\langle\hbar_1, \hbar_2, ..., \hbar_\eta\rangle = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\nu_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\perp_\tau + \nu_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\natural_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle \text{ı} - \langle\nu_\tau\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

Proof. If $\tau \twoheadrightarrow 2$, then RFFFNSIWG$\langle\hbar_1, \hbar_2\rangle = \hbar_1^{\varsigma_1} \Cap \hbar_2^{\varsigma_2}$,
where,

$$\hbar_1^{\varsigma_1} = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle \text{ı} - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_1} - \left\langle \text{ı} - \langle\perp_1 + \nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_1}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\natural_1\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_1}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\nu_1\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_1}} \end{pmatrix}$$

$$\hbar_2^{\varsigma_2} = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\left\langle \text{ı} - \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_2} - \left\langle \text{ı} - \langle\perp_2 + \nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_2}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\natural_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_2}} \quad \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\text{ı} - \left\langle \text{ı} - \langle\nu_2\rangle^{\frac{1}{\pounds^{\acute{a}}}}\right\rangle^{\varsigma_2}} \end{pmatrix}$$

We get,

$$\hbar_1^{\varsigma_1} \Cap \hbar_2^{\varsigma_2} = \left(\begin{array}{c} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle + \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \\ - \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle \cdot \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \\ - \left\langle \left\langle \text{ı} - \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \cdot \left\langle \text{ı} - \langle \perp_2 + \nu_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \end{array}} \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle + \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \\ - \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle \cdot \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \end{array}} \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle + \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \\ - \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_1 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_1} \right\rangle \cdot \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_2 \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_2} \right\rangle \end{array}} \end{array} \right)$$

Hence, $\text{RFFFNSIWG}\langle \hbar_1, \hbar_2 \rangle = \left(\begin{array}{c} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \text{ı} - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \text{ı} - \langle \natural_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{array} \right)$

$\text{RFFFNSIWG}\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle = \left(\begin{array}{c} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \natural_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{array} \right)$

If $\tau \twoheadrightarrow \eta + 1$, then $\text{RFFFNSIWG}\langle \hbar_1, ..., \hbar_\eta, \hbar_{\eta+1} \rangle$

$$= \left(\begin{array}{c} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle + \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle \cdot \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \cdot \left\langle \text{ı} - \langle \perp_{\eta+1} + \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \end{array}} \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle + \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle \cdot \left\langle \text{ı} - \left\langle \text{ı} - \langle \natural_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \end{array}} \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle + \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \\ - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle \cdot \left\langle \text{ı} - \left\langle \text{ı} - \langle \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \end{array}} \end{array} \right)$$

$$= \left(\begin{array}{c} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\begin{array}{c} \left\langle \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \right\rangle \cdot \\ \left\langle \left\langle \langle \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} - \left\langle \perp_{\eta+1} + \nu_{\eta+1} \right\rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}} \right\rangle \end{array}}, \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \natural_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \cdot \left\langle \text{ı} - \langle \natural_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}}} \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} \cdot \left\langle \text{ı} - \langle \nu_{\eta+1} \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_{\eta+1}}} \end{array} \right)$$

$$= \left(\begin{array}{c} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \text{ı} - \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \text{ı} - \langle \natural_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \sqrt[\frac{1}{\mathcal{L}^{\acute{a}}}]{\text{ı} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \text{ı} - \langle \nu_\tau \rangle^{\frac{1}{\mathcal{L}^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{array} \right)$$

Corollary 2.7. *Let $\hbar_\tau = \langle \perp_\tau, \natural_\tau, \nu_\tau \rangle$ be the RFFFNNs and all are equal and and $\perp \cdot \nu = 0$. Then RFFFNSIWG$\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle = \hbar$.*

2.3 generalized RFFFNSIWA(GRFFFNSIWA) operator

Definition 2.8. Let $\hbar_\tau = \left\langle \perp_\tau, \perp_\tau, \nu_\tau \right\rangle$ be the RFFFNNs, ς_τ be a weight of $\hbar_\tau$. Then, the GRFFFNSIWA operator $\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle = \left\langle \boxplus_{\tau \twoheadrightarrow 1}^{\eta} \varsigma_\tau \hbar_\tau^2 \right\rangle^{1/2}$.

Theorem 2.9. *Let $\hbar_\tau = \left\langle \perp_\tau, \perp_\tau, \nu_\tau \right\rangle$ be the RFFFNNs. Then GRFFFNSIWA $\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle =$*

$$\left(\begin{array}{c} \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \perp_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau}} \right\rangle, \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \natural_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau}} \right\rangle, \\ \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \perp_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau}} \right\rangle \end{array} \right).$$

Proof. First, we have find that

$$\boxplus_{\tau\twoheadrightarrow 1}^{\eta} \varsigma_\tau \hbar_\tau^2 = \left(\begin{array}{c} \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \perp_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \natural_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds\acute{a}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \perp_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta} \left\langle \shortmid - \left\langle \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_\tau}} \end{array} \right).$$

If $\tau \twoheadrightarrow 2$, then $\varsigma_1 \hbar_1^2 = \left(\begin{array}{c} \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}}, \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \natural_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}}, \\ \sqrt[\frac{1}{\pounds\acute{a}}]{\left\langle \shortmid - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1} - \left\langle \shortmid - \left\langle \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \end{array} \right)$

and

$$\varsigma_2 \hbar_2^2 = \left(\begin{array}{c} \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \perp_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}}, \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \natural_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}}, \\ \sqrt[\frac{1}{\pounds\acute{a}}]{\left\langle \shortmid - \left\langle \langle \perp_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1} - \left\langle \shortmid - \left\langle \langle \perp_2 + \nu_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \end{array} \right).$$

We get, $\varsigma_1 \hbar_1 \Cup \varsigma_2 \hbar_2$=

$$\left(\begin{array}{c} \sqrt[\frac{1}{\pounds\acute{a}}]{\begin{array}{c} \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} + \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \perp_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}}, \\ - \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \cdot \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \perp_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \end{array}} \\ \sqrt[\frac{1}{\pounds\acute{a}}]{\begin{array}{c} \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \natural_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} + \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \natural_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}}, \\ - \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \natural_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \cdot \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \natural_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \end{array}} \\ \sqrt[\frac{1}{\pounds\acute{a}}]{\begin{array}{c} \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \nu_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} + \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \nu_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}}, \\ - \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \nu_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \cdot \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\shortmid - \left\langle \shortmid - \left\langle \langle \nu_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \\ - \left(\begin{array}{c} \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\left\langle \shortmid - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1} - \left\langle \shortmid - \left\langle \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \cdot \\ \left\langle \sqrt[\frac{1}{\pounds\acute{a}}]{\left\langle \shortmid - \left\langle \langle \perp_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1} - \left\langle \shortmid - \left\langle \langle \perp_2 + \nu_2 \rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \right\rangle^{\varsigma_1}} \right\rangle^{\frac{1}{\pounds\acute{a}}} \end{array} \right) \end{array}} \end{array} \right)$$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \mathsf{I} - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \mathsf{I} - \left\langle \langle \natural_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \mathsf{I} - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{2} \left\langle \mathsf{I} - \left\langle \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

In general,

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \natural_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{pmatrix}.$$

If $\tau \twoheadrightarrow \eta + 1$, then $\boxplus_{\tau \twoheadrightarrow 1}^{\eta} \varsigma_\tau \hbar_\tau^2 + \varsigma_{\eta+1} \hbar_{\eta+1}^2 = \boxplus_{\tau \twoheadrightarrow 1}^{\eta+1} \varsigma_\tau \hbar_\tau^2$.
Now, $\boxplus_{\tau \twoheadrightarrow 1}^{\eta} \varsigma_\tau \hbar_\tau^2 + \varsigma_{\eta+1} \hbar_{\eta+1}^2 = \varsigma_1 \hbar_1^2 \Cup \varsigma_2 \hbar_2^2 \Cup ... \Cup \varsigma_\eta \hbar_\eta^2 \Cup \varsigma_{\eta+1} \hbar_{\eta+1}^2$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \natural_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \perp_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \perp_1 + \nu_1 \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

and $\boxplus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \varsigma_\tau \hbar_\tau^2 \right\rangle = \begin{pmatrix} \left\langle \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \right\rangle, \\ \left\langle \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \right\rangle, \\ \left\langle \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \perp_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta+1} \left\langle \mathsf{I} - \left\langle \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \right\rangle \end{pmatrix}.$

Corollary 2.10. *Let $\hbar_\tau = \langle \perp_\tau, \natural_\tau, \nu_\tau \rangle$ be the RFFFNNs and all are equal.*
Then GRFFFNSIWA $\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle = \hbar$.

2.4 Generalized RFFFNSIWG(GRFFFNSIWG) operator

Definition 2.11. Let $\hbar_\tau = \langle \perp_\tau, \perp_\tau, \nu_\tau \rangle$ be the RFFFNNs, $\varsigma_\tau \rangle$ be the weight of $\hbar_\tau$, where $\tau \twoheadrightarrow 1, 2, ..., \eta$. Then, the GRFFFNSIWG$\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle = \frac{1}{\mathscr{M}} \left\langle \ominus_{\tau \twoheadrightarrow 1}^{\eta} \langle \mathscr{M} \hbar_\tau \rangle^{\varsigma_\tau} \right\rangle$.

Theorem 2.12. *Let $\hbar_\tau = \langle \perp_\tau, \perp_\tau, \nu_\tau \rangle$ be the collection of RFFFNNs.*
Then the GRFFFNSIWG operator $\langle \hbar_1, \hbar_2, ..., \hbar_\eta \rangle =$

$$\begin{pmatrix} \left\langle \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \right\rangle, \\ \left\langle \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \right\rangle \left\langle \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \right\rangle \end{pmatrix}$$

Proof. Using the induction method, $\ominus_{\tau \twoheadrightarrow 1}^{\eta} \langle \mathscr{M} \hbar_\tau \rangle^{\varsigma_\tau} =$

$$\begin{pmatrix} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \perp_\tau + \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \natural_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \sqrt[\frac{1}{\pounds^{\acute{a}}}]{\mathsf{I} - \ominus_{\tau \twoheadrightarrow 1}^{\eta} \left\langle \mathsf{I} - \left\langle \langle \nu_\tau \rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\frac{1}{\pounds^{\acute{a}}}} \right\rangle^{\varsigma_\tau}} \end{pmatrix}.$$

If $\tau \twoheadrightarrow 2$, then

$$\langle \mathscr{M}\hbar_1\rangle^{\varsigma_1} = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{\left\langle 1-\left\langle \langle \nu_1\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1} - \left\langle 1-\left\langle \langle \perp_1+\nu_1\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1}} \\ \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\left\langle 1-\left\langle \langle \natural_1\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1}} \quad \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\left\langle 1-\left\langle \langle \nu_1\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1}} \end{pmatrix}$$

and

$$\langle \mathscr{M}\hbar_2\rangle^{\varsigma_2} = \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{\left\langle 1-\left\langle \langle \nu_2\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1} - \left\langle 1-\left\langle \langle \perp_2+\nu_2\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1}} \\ \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\left\langle 1-\left\langle \langle \natural_2\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1}} \quad \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\left\langle 1-\left\langle \langle \nu_2\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_1}} \end{pmatrix}$$

We get, $\langle \mathscr{M}\hbar_1\rangle^{\varsigma_1} \Cap \langle \mathscr{M}\hbar_2\rangle^{\varsigma_2}$

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{\ominus_{\tau\twoheadrightarrow 1}^{2}\left\langle 1-\left\langle \langle \nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{2}\left\langle 1-\left\langle \langle \perp_\tau+\nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\ominus_{\tau\twoheadrightarrow 1}^{2}\left\langle 1-\left\langle \langle \natural_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}}, \quad \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\ominus_{\tau\twoheadrightarrow 1}^{2}\left\langle 1-\left\langle \langle \nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

If $\tau \twoheadrightarrow k$, then

$$= \begin{pmatrix} \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle 1-\left\langle \langle \nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle 1-\left\langle \langle \perp_\tau+\nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}}, \\ \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle 1-\left\langle \langle \natural_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}}, \quad \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\ominus_{\tau\twoheadrightarrow 1}^{\eta}\left\langle 1-\left\langle \langle \nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}} \end{pmatrix}$$

If $\tau \twoheadrightarrow \eta+1$, then $\ominus_{\tau\twoheadrightarrow 1}^{\eta}\langle \mathscr{M}\hbar_\tau\rangle^{\varsigma_\tau} \cdot \langle \mathscr{M}\hbar_{\eta+1}\rangle^{\varsigma_{\eta+1}} = \ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\langle \mathscr{M}\hbar_\tau\rangle^{\varsigma_\tau}$.
Now, $\ominus_{\tau\twoheadrightarrow 1}^{\eta}\langle \mathscr{M}\hbar_\tau\rangle^{\varsigma_\tau} \cdot \langle \mathscr{M}\hbar_{\eta+1}\rangle^{\varsigma_{\eta+1}} = \langle \mathscr{M}\hbar_1\rangle^{\varsigma_1} \Cap \langle \mathscr{M}\hbar_2\rangle^{\varsigma_2} \Cap \ldots \Cap \langle \mathscr{M}\hbar_\eta\rangle^{\varsigma_\eta} \Cap \langle \mathscr{M}\hbar_{\eta+1}\rangle^{\varsigma_{\eta+1}}$

Hence

$$\frac{1}{\mathscr{M}}\left\langle \ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\langle \mathscr{M}\hbar_\tau\rangle^{\varsigma_\tau}\right\rangle =$$

$$\begin{pmatrix} \left\langle \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{\ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle 1-\left\langle \langle \nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau} - \ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle 1-\left\langle \langle \perp_\tau+\nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}} \right\rangle, \\ \left\langle \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle 1-\left\langle \langle \natural_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}} \right\rangle \left\langle \sqrt[\frac{1}{\pounds^{\tilde{a}}}]{1-\ominus_{\tau\twoheadrightarrow 1}^{\eta+1}\left\langle 1-\left\langle \langle \nu_\tau\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\frac{1}{\pounds^{\tilde{a}}}}\right\rangle^{\varsigma_\tau}} \right\rangle \end{pmatrix}$$

Corollary 2.13. *Let $\hbar_\tau = \left\langle \perp_\tau, \natural_\tau, \nu_\tau\right\rangle$ be the collection of RFFFNNs and all are equal. Then the GRFFFN-SIWG $\langle \hbar_1, \hbar_2, ..., \hbar_\eta\rangle = \hbar$.*

3 Conclusion

The algebraic accessibility of ED and HD for RFFFNNs is a significant advantage. HD of RFFFNNs may significantly improve data analysis. Strong statistics are used to illustrate the advantages of HD. Examples and suggested models for RFFFNSIWA, RFFFNSIWG, GRFFFNSIWA, and GRFFFNSIWG were shown. In the future, the following subjects will be covered in more detail: The following topics will be covered in further detail: (1) The cubic NS and IVPFS are connected by interaction AOs. (2) The issue may be resolved by using complex RFFFNSIWA, complex RFFFNSIWG, complex GRFFFNSIWA, and complex GRFFFNSIWG.

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