



Continuous Maps and Irresolute Maps Via Fuzzy Hypersoft θ Open set

P. Revathi^{1,2}, B. Premamalini^{2,*}, K. Chitirakala^{3,*}, A. Vadivel^{2,4}

¹Government Polytechnic College, Kuduveli, Chidambaram - 608 305, India

²Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India

³Department of Mathematics, M.Kumarasamy College of Engineering, Karur - 639 113, India

⁴PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal - 637 002, India

Emails: revathimathsau@gmail.com; premamalinips@gmail.com; chitrakalalaksana@gmail.com; avmaths@gmail.com

Abstract

The purpose of this paper is to introduce and study fuzzy hypersoft θ continuous maps, fuzzy hypersoft θ semi continuous maps, fuzzy hypersoft θ pre continuous maps and fuzzy hypersoft θ irresolute maps in fuzzy hypersoft topological spaces with examples. Further, we derived some useful results and properties related to them.

Keywords: Fuzzy hypersoft θ continuous maps; Fuzzy hypersoft θ semi continuous maps; Fuzzy hypersoft θ pre continuous maps; fuzzy hypersoft θ irresolute maps

1 Introduction

The real-world decision-making problems in medical diagnosis, engineering, economics, management computer science, artificial intelligence, social sciences, environmental science and sociology contain more uncertain and inadequate data. Traditional mathematical methods cannot deal with these kinds of problems due to imprecise data. To deal with the problems with uncertainty, Zadeh¹⁶ introduced the fuzzy set in 1965 which contains the membership value in $[0,1]$. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called the membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang⁷ as fuzzy topological space. Molodstov⁹ introduced a new mathematical tool, soft set theory in 1999 to deal with uncertainties in which a soft set is a collection of approximate descriptions of an object. A soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory has several applications in different fields such as decision-making, optimization, forecasting, data analysis etc. Shabir and Naz¹³ presented soft topological spaces.

Smarandache¹⁴ extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing a function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in decision-making issues involving a different kinds of attributes. Saeed et al.^{10,11} studied the fundamentals of hypersoft set theory by introducing aggregate operators, relations, functions, matrices and operations on hypersoft matrices. Abbas et al.¹ defined the basic operations on hypersoft sets and hypersoft point in the fuzzy, intuitionistic and neutrosophic environments. Ajay and Charisma³ introduced fuzzy hypersoft topology, intuitionistic hypersoft

topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology. Ajay et al.⁴ defined fuzzy hypersoft semi-open sets and developed an application in multiattribute group decision-making. The concept of contra continuous function in general topology was introduced by Dontchev⁸ in 1996. Vadivel et al.² introduced generalized fuzzy contra e -continuous functions in fuzzy topological spaces. Ahsan et al.² studied a theoretical and analytical approach for fundamental framework of composite mappings on fuzzy hypersoft classes.

Saha¹² defined δ -open sets and continuous maps in fuzzy topological spaces. The δ -open sets, e -open sets in neutrosophic, neutrosophic soft, fuzzy hypersoft, neutrosophic hypersoft topological spaces are introduced and its maps, separation axioms and compact spaces are studied recently.

The class of sets namely, θ open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. In 1968 Velicko¹⁵ defined θ open set in H -closed Topological Spaces. In,^{5,6} Caldas et al. studied various kinds of θ open sets and their properties in topological spaces. The concept of θ -open sets in fuzzy hypersoft topological spaces is introduced recently.

In this paper, we develop the concept of fuzzy hypersoft θ continuity in fuzzy hypersoft topological spaces and some of their properties are analyzed with examples. Added to that, fuzzy hypersoft θ semi continuous maps, fuzzy hypersoft θ pre continuous maps and fuzzy hypersoft θ irresolute maps are developed and the relation between them are discussed.

2 Preliminaries

Definition 2.1.¹⁶ Let $\mathfrak{M}$ be an initial universe. A function λ from $\mathfrak{M}$ into the unit interval I is called a fuzzy set in $\mathfrak{M}$. For every $m \in \mathfrak{M}$, $\lambda(m) \in I$ is called the grade of membership of m in λ . Some authors say that λ is a fuzzy subset of $\mathfrak{M}$ instead of saying that λ is a fuzzy set in $\mathfrak{M}$. The class of all fuzzy sets from $\mathfrak{M}$ into the closed unit interval I will be denoted by $I^{\mathfrak{M}}$.

Definition 2.2.⁹ Let $\mathfrak{M}$ be an initial universe, Q be a set of parameters and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. A pair $(\tilde{H}, Q)$ is called the a soft set over $\mathfrak{M}$ where $\tilde{H}$ is a mapping $\tilde{H} : Q \rightarrow \mathcal{P}(\mathfrak{M})$. In other words, the soft set is a parametrized family of subsets of the set $\mathfrak{M}$.

Definition 2.3.¹⁴ Let $\mathfrak{M}$ be an initial universe and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. Consider $q_1, q_2, q_3, \dots, q_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $Q_1, Q_2, \dots, Q_n$ with $Q_i \cap Q_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, \dots, n\}$. Then the pair $(\tilde{H}, Q_1 \times Q_2 \times \dots \times Q_n)$ where $\tilde{H} : Q_1 \times Q_2 \times \dots \times Q_n \rightarrow \mathcal{P}(\mathfrak{M})$ is called a hypersoft set over $\mathfrak{M}$.

Definition 2.4.¹ Let $\mathfrak{M}$ be an initial universal set and $Q_1, Q_2, \dots, Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{M})$ be the set of all fuzzy sets of $\mathfrak{M}$. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, \dots, n$. A fuzzy hypersoft set (briefly, $FHSS$) over $\mathfrak{M}$ is defined as the pair $(\tilde{H}, E_1 \times E_2 \times \dots \times E_n)$ where $\tilde{H} : E_1 \times E_2 \times \dots \times E_n \rightarrow \mathcal{P}(\mathfrak{M})$ and $\tilde{H}(E_1 \times E_2 \times \dots \times E_n) = \{(q, \langle m, \mu_{\tilde{H}(q)}(m) \rangle) : m \in \mathfrak{M} : q \in E_1 \times E_2 \times \dots \times E_n \subseteq Q_1 \times Q_2 \times \dots \times Q_n\}$ where $\mu_{\tilde{H}(q)}(m)$ is the membership value such that $\mu_{\tilde{H}(q)}(m) \in [0, 1]$.

Definition 2.5.¹ Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be two $FHSS$'s over $\mathfrak{M}$. Then $(\tilde{H}, \wedge_1)$ is the fuzzy hypersoft subset of $(\tilde{G}, \wedge_2)$ if $\mu_{\tilde{H}(q)}(m) \leq \mu_{\tilde{G}(q)}(m)$.

It is denoted by $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2)$.

Definition 2.6.¹ Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be $FHSS$'s over $\mathfrak{M}$. $(\tilde{H}, \wedge_1)$ is equal to $(\tilde{G}, \wedge_2)$ if $\mu_{\tilde{H}(q)}(m) = \mu_{\tilde{G}(q)}(m)$.

Definition 2.7.¹ A $FHSS$ $(\tilde{H}, \wedge)$ over the universe set $\mathfrak{M}$ is said to be null fuzzy hypersoft set if $\mu_{\tilde{H}(q)}(m) = 0, \forall q \in \wedge$ and $m \in \mathfrak{M}$. It is denoted by $\tilde{0}_{(\mathfrak{M}, Q)}$.

A $FHSS$ $(\tilde{G}, \wedge)$ over the universal set $\mathfrak{M}$ is said to be absolute fuzzy hypersoft set if $\mu_{\tilde{H}(q)}(m) = 1 \forall q \in \wedge$ and $m \in \mathfrak{M}$. It is denoted by $\tilde{1}_{(\mathfrak{M}, Q)}$.

Clearly, $\tilde{0}_{(\mathfrak{M}, Q)}^c = \tilde{1}_{(\mathfrak{M}, Q)}$ and $\tilde{1}_{(\mathfrak{M}, Q)}^c = \tilde{0}_{(\mathfrak{M}, Q)}$.

Definition 2.8. ¹ Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge)$ be $FHSS$ s over $\mathfrak{M}$. $(\tilde{H}, \wedge)^c$ is the complement of $(\tilde{H}, \wedge)$ if $\mu_{\tilde{H}(q)}^c(\mathfrak{m}) = \tilde{1}_{(\mathfrak{M}, Q)} - \mu_{\tilde{H}(q)}(\mathfrak{m})$ where $\forall q \in \wedge$ and $\forall \mathfrak{m} \in \mathfrak{M}$. It is clear that $((\tilde{H}, \wedge)^c)^c = (\tilde{H}, \wedge)$.

Definition 2.9. ¹ Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be $FHSS$'s over $\mathfrak{M}$. Extended union $(\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{G}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_2 - \wedge_1 \\ \max\{\mu_{\tilde{H}(q)}(\mathfrak{m}), \mu_{\tilde{G}(q)}(\mathfrak{m})\} & \text{if } q \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.10. ^{1,3} Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be $FHSS$'s over $\mathfrak{M}$. Extended intersection $(\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{G}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_2 - \wedge_1 \\ \min\{\mu_{\tilde{H}(q)}(\mathfrak{m}), \mu_{\tilde{G}(q)}(\mathfrak{m})\} & \text{if } q \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.11. ³ Let $(\mathfrak{M}, Q)$ be the family of all $FHSS$'s over the universe set $\mathfrak{M}$ and $\tau \subseteq FHSS(\mathfrak{M}, Q)$. Then τ is said to be a fuzzy hypersoft topology (briefly, $FHSt$) on $\mathfrak{M}$ if

- (i) $\tilde{0}_{(\mathfrak{M}, Q)}$ and $\tilde{1}_{(\mathfrak{M}, Q)}$ belongs to τ
- (ii) the union of any number of $FHSS$'s in τ belongs to τ
- (iii) the intersection of finite number of $FHSS$'s in τ belongs to τ .

Then $(\mathfrak{M}, Q, \tau)$ is called a fuzzy hypersoft topological space (briefly, $FHSts$) over $\mathfrak{M}$. Each member of τ is said to be fuzzy hypersoft open set (briefly, $FHSOs$). A $FHSS$ $(\tilde{H}, \wedge)$ is called a fuzzy hypersoft closed set (briefly, $FHSCs$) if its complement $(\tilde{H}, \wedge)^c$ is $FHSOs$.

Definition 2.12. ³ Let $(\mathfrak{M}, Q, \tau)$ be a $FHSts$ over $\mathfrak{M}$ and $(\tilde{H}, \wedge)$ be a $FHSS$ in $\mathfrak{M}$. Then,

- (i) the fuzzy hypersoft interior (briefly, $FHSint$) of $(\tilde{H}, \wedge)$ is defined as $FHSint(\tilde{H}, \wedge) = \cup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq (\tilde{H}, \wedge) \text{ where } (\tilde{G}, \wedge) \text{ is } FHSOs\}$.
- (ii) the fuzzy hypersoft closure (briefly, $FHScl$) of $(\tilde{H}, \wedge)$ is defined as $FHScl(\tilde{H}, \wedge) = \cap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq (\tilde{H}, \wedge) \text{ where } (\tilde{G}, \wedge) \text{ is } FHSCs\}$.

Definition 2.13. ⁴ Let $(\mathfrak{M}, Q, \tau)$ be a $FHSts$ over $\mathfrak{M}$ and $(\tilde{H}, \wedge)$ be a $FHSS$ in $\mathfrak{M}$. Then, $(\tilde{H}, \wedge)$ is called the fuzzy hypersoft semiopen set (briefly, $FHSSos$) if $(\tilde{H}, \wedge) \subseteq FHScl(int(\tilde{H}, \wedge))$.

A $FHSS$ $(\tilde{H}, \wedge)$ is called a fuzzy hypersoft semiclosed set (briefly, $FHSScs$) if its complement $(\tilde{H}, \wedge)^c$ is a $FHSSos$.

Definition 2.14. ² Let $(\mathfrak{M}, L)$ and $(\mathfrak{N}, M)$ be classes of $FHSS$'s over $\mathfrak{M}$ and $\mathfrak{N}$ with attributes L and M respectively. Let $\omega : \mathfrak{M} \rightarrow \mathfrak{N}$ and $\nu : L \rightarrow M$ be mappings. Then a FHS mappings $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ is defined as follows, for a $FHSS$ $(\tilde{H}, \wedge)_A$ in $(\mathfrak{M}, L)$, $f(\tilde{H}, \wedge)_A$ is a $FHSS$ in $(\mathfrak{N}, M)$ obtained as follows, for $\beta \in \nu(L) \subseteq M$ and $\mathfrak{n} \in \mathfrak{N}$, $\mathfrak{h}(\tilde{H}, \wedge)_A(\beta)(\mathfrak{n}) = \bigcup_{\alpha \in \nu^{-1}(\beta) \cap A, s \in \omega^{-1}(\mathfrak{n})} (\alpha) \mu_s \mathfrak{h}(\tilde{H}, \wedge)_A$ is called a fuzzy hypersoft image of a $FHSS$ $(\tilde{H}, \wedge)$. Hence $((\tilde{H}, \wedge)_A, \mathfrak{h}(\tilde{H}, \wedge)_A) \in \mathfrak{h}$, where $(\tilde{H}, \wedge)_A \subseteq (\mathfrak{M}, L)$, $\mathfrak{h}(\tilde{H}, \wedge)_A \subseteq (\mathfrak{N}, M)$.

Definition 2.15. ² If $\mathfrak{h} : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a FHS mapping, then FHS class $(\mathfrak{M}, L)$ is called the domain of $\mathfrak{h}$ and the FHS class $(\tilde{G}, \wedge) \in (\mathfrak{N}, M) : (\tilde{G}, \wedge) = \mathfrak{h}(\tilde{H}, \wedge)$ for some $(\tilde{H}, \wedge) \in (\mathfrak{M}, L)$ is called the range of $\mathfrak{h}$. The FHS class $(\mathfrak{N}, M)$ is called co-domain of $\mathfrak{h}$.

Definition 2.16. ² If $\mathfrak{h} : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping and $(\tilde{G}, \wedge)_B$, a *FHSs* in $(\mathfrak{N}, M)$ where $\omega : \mathfrak{M} \rightarrow \mathfrak{N}$, $\nu : L \rightarrow M$ and $B \subseteq M$. Then $\mathfrak{h}^{-1}(\tilde{G}, \wedge)_B$ is a *FHSs* in $(\mathfrak{M}, L)$ defined as follows, for $\alpha \in \nu^{-1}(B) \subseteq L$ and $\mathfrak{m} \in \mathfrak{M}$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)_B(\alpha)(\mathfrak{m}) = (\nu(\alpha))\mu_p(\mathfrak{m})\mathfrak{h}^{-1}(\tilde{G}, \wedge)_B$ is called a *FHS* inverse image of $(\tilde{G}, \wedge)_B$.

Definition 2.17. ² Let $\mathfrak{h} = (\omega, \nu)$ be a *FHS* mapping of a *FHS* class $(\mathfrak{M}, L)$ into a *FHS* class $(\mathfrak{N}, M)$. Then

- (i) $\mathfrak{h}$ is said to be a one-one (or injection) *FHS* mapping if for both $\omega : \mathfrak{M} \rightarrow \mathfrak{N}$ and $\nu : L \rightarrow M$ are one-one.
- (ii) $\mathfrak{h}$ is said to be a onto (or surjection) *FHS* mapping if for both $\omega : \mathfrak{M} \rightarrow \mathfrak{N}$ and $\nu : L \rightarrow M$ are onto.

If $\mathfrak{h}$ is both one-one and onto, then $\mathfrak{h}$ is called a one-one onto (or bijective) correspondence of *FHS* mapping.

Definition 2.18. ² If $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ and $g = (m, n) : (\mathfrak{N}, M) \rightarrow (P, N)$ are two *FHS* mappings, then their composite $\mathfrak{g} \circ \mathfrak{h}$ is a *FHS* mapping of $(\mathfrak{M}, L)$ into (P, N) such that for every $(\tilde{H}, \wedge)_A \in (\mathfrak{M}, L)$, $(\mathfrak{g} \circ \mathfrak{h})(\tilde{H}, \wedge)_A = \mathfrak{g}(\mathfrak{h}(\tilde{H}, \wedge)_A)$. For $\beta \in n(M) \subseteq N$ and $p \in P$, it is defined as $\mathfrak{g}(\mathfrak{h}(\tilde{H}, \wedge)_A(\beta))(p) = \bigcup_{\alpha \in n^{-1}(\beta) \cap \mathfrak{h}(A), s \in m^{-1}(p)} (\alpha)\mu_s$.

Definition 2.19. ² Let $\mathfrak{h} = (\omega, \nu)$ be a *FHS* mapping where $\omega : \mathfrak{M} \rightarrow \mathfrak{M}$ and $\nu : L \rightarrow L$. Then $\mathfrak{h}$ is said to be a *FHS* identity mapping if for both ω and ν are identity mappings.

Definition 2.20. ² A one-one onto *FHS* mapping $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ is called *FHS* invertable mapping. Its *FHS* inverse mapping is denoted by $\mathfrak{h}^{-1} = (\omega^{-1}, \nu^{-1}) : (\mathfrak{N}, M) \rightarrow (\mathfrak{M}, L)$.

3 Fuzzy Hypersoft θ Continuous Maps

In this section, fuzzy hypersoft θ continuous maps are introduced and its related properties are discussed.

Definition 3.1. Consider any two FHSTs $(\mathfrak{M}, L, \tau)$ and $(\mathfrak{N}, M, \sigma)$. A map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ is said to be *FHS* θ (resp. θS , δS , δP , θP) continuous (in short, *FHS* θCts (resp. *FHS* $\theta SCts$, *FHS* $\delta SCts$, *FHS* $\delta PCts$ & *FHS* $\theta PCts$)) if the inverse image of each *FHS*os in $(\mathfrak{N}, M, \sigma)$ is a *FHS* θ os (resp. *FHS* θ Sos, *FHS* δ Sos, *FHS* δ Pos & *FHS* θ Pos) in $(\mathfrak{M}, L, \tau)$.

Example 3.2. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L)$, $(\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHS*s's $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_1)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$, $(\tilde{H}_5, \wedge_3)$, $(\tilde{H}_6, \wedge_3)$, $(\tilde{H}_7, \wedge_3)$, $(\tilde{H}_8, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \langle (a_1, b_1), \left\{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \right\} \rangle \right\} \\ (\tilde{H}_2, \wedge_1) = \left\{ \langle (a_1, b_1), \left\{ \frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.5} \right\} \rangle \right\} \\ (\tilde{H}_3, \wedge_2) = \left\{ \langle (a_1, b_1), \left\{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.9} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{\mathfrak{m}_1}{0.6}, \frac{\mathfrak{m}_2}{0.4} \right\} \rangle \right\} \\ (\tilde{H}_4, \wedge_3) = \left\{ \langle (a_1, b_1), \left\{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.9} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{\mathfrak{m}_1}{0.6}, \frac{\mathfrak{m}_2}{0.4} \right\} \rangle \right\}$$

$$(\tilde{H}_5, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.7}\} \rangle, \\ &\langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \\ &\langle (a_1, b_2), \{\frac{m_1}{0.6}, \frac{m_2}{0.4}\} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_6, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.9}\} \rangle, \\ &\langle (a_2, b_1), \{\frac{m_1}{0.3}, \frac{m_2}{0.5}\} \rangle, \\ &\langle (a_1, b_2), \{\frac{m_1}{0.6}, \frac{m_2}{0.4}\} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_7, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \\ &\langle (a_2, b_1), \{\frac{m_1}{0.3}, \frac{m_2}{0.5}\} \rangle, \\ &\langle (a_1, b_2), \{\frac{m_1}{0.6}, \frac{m_2}{0.4}\} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_8, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.9}\} \rangle, \\ &\langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \\ &\langle (a_1, b_2), \{\frac{m_1}{0.6}, \frac{m_2}{0.4}\} \rangle \end{aligned} \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_1), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3), (\tilde{H}_8, \wedge_3), \}$ is *FHSts*.

Let the *FHSs's* $(\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_1)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge_1) = \left\{ \begin{aligned} &\langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \\ &\langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.7}\} \rangle \end{aligned} \right\}$$

$$(\tilde{G}_2, \wedge_1) = \left\{ \begin{aligned} &\langle (c_2, d_1), \{\frac{n_1}{0.3}, \frac{n_2}{0.2}\} \rangle, \\ &\langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle \end{aligned} \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_1)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2)$$

$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_2, \wedge_1) = (\tilde{H}_2, \wedge_1),$$

$\therefore \mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ is *FHS θ Cts*.

Proposition 3.3. The statements hold but the converse is not.

- (i) Each *FHS θ Cts* is a *FHS θ SCts*.
- (ii) Each *FHS θ Cts* is a *FHSCts*.
- (iii) Each *FHS θ SCts* is a *FHSSCts*.
- (iv) Each *FHSCts* is a *FHS $\mathcal{P}$ Cts*.
- (v) Each *FHSSCts* is a *FHS θ $\mathcal{P}$ Cts*.
- (vi) Each *FHS $\mathcal{P}$ Cts* is a *FHS θ $\mathcal{P}$ Cts*.

Proof.

Consider the map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$

- (i) Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. As $\mathfrak{h}$ is $FHS\theta Cts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta os$ in $\mathfrak{M}$. Since all $FHS\theta os$ are $FHS\theta S os$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta S os$ in $\mathfrak{M}$. Thus $\mathfrak{h}$ is a $FHS\theta SCts$.
- (ii) Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. As $\mathfrak{h}$ is $FHS\theta Cts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta os$ in $\mathfrak{M}$. Since all $FHS\theta os$ are $FHSos$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHSos$ in $\mathfrak{M}$. Thus $\mathfrak{h}$ is a $FHSCts$.
- (iii) Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. As $\mathfrak{h}$ is $FHS\theta SCts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta S os$ in $\mathfrak{M}$. Since all $FHS\theta S os$ are $FHSSos$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHSSos$ in $\mathfrak{M}$. Thus $\mathfrak{h}$ is a $FHSSCts$.
- (iv) Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. As $\mathfrak{h}$ is $FHSCts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHSos$ in $\mathfrak{M}$. Since all $FHSos$ are $FHS\mathcal{P}os$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\mathcal{P}os$ in $\mathfrak{M}$. Thus $\mathfrak{h}$ is a $FHS\mathcal{P}Cts$.
- (v) Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. As $\mathfrak{h}$ is $FHSSCts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHSSos$ in $\mathfrak{M}$. Since all $FHSSos$ are $FHS\theta\mathcal{P}os$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta\mathcal{P}os$ in $\mathfrak{M}$. Thus $\mathfrak{h}$ is a $FHS\theta\mathcal{P}Cts$.
- (vi) Let $(\tilde{G}, \wedge)$ be a $FHSos$ in $\mathfrak{N}$. As $\mathfrak{h}$ is $FHS\mathcal{P}Cts$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\mathcal{P}os$ in $\mathfrak{M}$. Since all $FHS\mathcal{P}os$ are $FHS\theta\mathcal{P}os$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta\mathcal{P}os$ in $\mathfrak{M}$. Thus $\mathfrak{h}$ is a $FHS\theta\mathcal{P}Cts$.

Example 3.4. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$$

$$Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the FHS s's $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.6} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \right\} \rangle \right\}$$

$$(\tilde{H}_2, \wedge_2) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.2}, \frac{m_2}{0.3} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{m_1}{0.5}, \frac{m_2}{0.4} \right\} \rangle \right\}$$

$$(\tilde{H}_3, \wedge_2) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.7} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{m_1}{0.5}, \frac{m_2}{0.6} \right\} \rangle \right\}$$

$$(\tilde{H}_4, \wedge_3) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.6} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{m_1}{0.5}, \frac{m_2}{0.4} \right\} \rangle \right\}$$

$$(\tilde{H}_5, \wedge_3) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.2}, \frac{m_2}{0.3} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{m_1}{0.5}, \frac{m_2}{0.4} \right\} \rangle \right\}$$

$$(\tilde{H}_6, \wedge_3) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.7} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{m_1}{0.5}, \frac{m_2}{0.6} \right\} \rangle \right\}$$

$$(\tilde{H}_7, \wedge_3) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.6} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \right\} \rangle, \langle (a_1, b_2), \left\{ \frac{m_1}{0.5}, \frac{m_2}{0.6} \right\} \rangle \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)\}$ is $FHSts$.

Let the FHS s $(\tilde{G}_1, \wedge_3)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_3) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.7}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \end{array} \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_3)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\begin{aligned} \omega(\mathfrak{m}_1) &= \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) &= (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2) \end{aligned}$$

$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_6, \wedge_3)$$

$\mathfrak{h}$ is *FHS θ SCts* but not *FHS θ Cts* because $(\tilde{G}_1, \wedge_3)$ is *FHSos* in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_6, \wedge_3)$ is a *FHS θ Sos* but not *FHS θ os*.

Example 3.5. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$\begin{aligned} Q_1 &= \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 &= \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}. \end{aligned}$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHSs*'s $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{array} \right\}$$

$$(\tilde{H}_2, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{array} \right\}$$

$$(\tilde{H}_3, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{array} \right\}$$

$$(\tilde{H}_4, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{array} \right\}$$

$$(\tilde{H}_5, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{array} \right\}$$

$$(\tilde{H}_6, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{array} \right\}$$

$$(\tilde{H}_7, \wedge_3) = \left\{ \begin{array}{l} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{array} \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)\}$ is *FHSts*.

Let the *FHSs* $(\tilde{G}_1, \wedge_1)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_1) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.6}, \frac{n_2}{0.8}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.7}\} \rangle \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{G}_1, \wedge_1)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\begin{aligned} \omega(\mathfrak{m}_1) &= \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) &= (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2) \\ \mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) &= (\tilde{H}_1, \wedge_1) \end{aligned}$$

$\mathfrak{h}$ is *FHSCts* but not *FHS θ Cts* because $(\tilde{G}_1, \wedge_1)$ is *FHSos* in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1)$ is a *FHSos* but not *FHS θ os*.

Example 3.6. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$\begin{aligned} Q_1 &= \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 &= \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}. \end{aligned}$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHS*s's $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$\begin{aligned} (\tilde{H}_1, \wedge_1) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \right\} \\ (\tilde{H}_2, \wedge_2) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_3, \wedge_2) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\} \rangle, \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_4, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_5, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_6, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\} \rangle, \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_7, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \right\} \end{aligned}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)\}$ is *FHSts*.

Let the *FHS*s $(\tilde{G}_1, \wedge_3)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_3) = \left\{ \begin{aligned} &\langle (c_2, d_1), \{ \frac{n_1}{0.6}, \frac{n_2}{0.8} \} \rangle, \\ &\langle (c_1, d_2), \{ \frac{n_1}{0.5}, \frac{n_2}{0.7} \} \rangle, \\ &\langle (c_2, d_2), \{ \frac{n_1}{0.4}, \frac{n_2}{0.5} \} \rangle \end{aligned} \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_3)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\begin{aligned} \omega(\mathfrak{m}_1) &= \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) &= (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2) \end{aligned}$$

$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_4, \wedge_3)$$

$\mathfrak{h}$ is *FHSSCts* but not *FHS θ SCts* because $(\tilde{G}_1, \wedge_3)$ is *FHSos* in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_4, \wedge_3)$ is a *FHSSos* but not *FHS θ Sos*.

Example 3.7. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$\begin{aligned} Q_1 &= \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 &= \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}. \end{aligned}$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHSs*'s $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6} \} \rangle, \\ &\langle (a_2, b_1), \{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_2, \wedge_2) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_3, \wedge_2) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_4, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6} \} \rangle, \\ &\langle (a_2, b_1), \{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_5, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3} \} \rangle, \\ &\langle (a_2, b_1), \{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_6, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7} \} \rangle, \\ &\langle (a_2, b_1), \{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_7, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6} \} \rangle, \\ &\langle (a_2, b_1), \{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6} \} \rangle \end{aligned} \right\}$$

$$(\tilde{H}_8, \wedge_3) = \left\{ \begin{aligned} &\langle (a_1, b_1), \{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7} \} \rangle, \\ &\langle (a_2, b_1), \{ \frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.5} \} \rangle, \\ &\langle (a_1, b_2), \{ \frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6} \} \rangle \end{aligned} \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)\}$ is *FHSts*.

Let the *FHSs* $(\tilde{G}_1, \wedge_3)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_3) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \langle (c_2, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_3)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\begin{aligned} \omega(m_1) &= n_2, \omega(m_2) = n_1, \\ \nu(a_1, b_1) &= (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2) \\ \mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) &= (\tilde{H}_8, \wedge_3) \end{aligned}$$

$\mathfrak{h}$ is *FHSPCts* but not *FHSCts* because $(\tilde{G}_1, \wedge_3)$ is *FHSos* in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_8, \wedge_3)$ is a *FHSPos* but not *FHSos*.

Example 3.8. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$\begin{aligned} Q_1 &= \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 &= \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}. \end{aligned}$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHSs*'s $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$\begin{aligned} (\tilde{H}_1, \wedge_1) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle \right\} \\ (\tilde{H}_2, \wedge_2) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_3, \wedge_2) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.7}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_4, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_5, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_6, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.7}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_7, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_8, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.3}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \end{aligned}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)\}$ is *FHSts*.

Let the *FHSs* $(\tilde{G}_1, \wedge_1)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_3) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.3}, \frac{n_2}{0.2}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \langle (c_2, d_2), \{\frac{n_1}{0.4}, \frac{n_2}{0.5}\} \rangle \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_3)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\begin{aligned} \omega(m_1) &= n_2, \omega(m_2) = n_1, \\ \nu(a_1, b_1) &= (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2) \\ \mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) &= (\tilde{H}_8, \wedge_3) \end{aligned}$$

$\mathfrak{h}$ is *FHS θ PcTs* but not *FHSSCts* because $(\tilde{G}_1, \wedge_3)$ is *FHSos* in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_8, \wedge_3)$ is a *FHS θ Pos* but not *FHSSos*.

Example 3.9. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$\begin{aligned} Q_1 &= \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 &= \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}. \end{aligned}$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHSs*'s $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)$ over the universe $\mathfrak{M}$ be

$$\begin{aligned} (\tilde{H}_1, \wedge_1) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle \right\} \\ (\tilde{H}_2, \wedge_2) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_3, \wedge_2) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.7}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_4, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_5, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \\ (\tilde{H}_6, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.7}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_7, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.6}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.7}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\} \\ (\tilde{H}_8, \wedge_3) &= \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.4}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.3}, \frac{m_2}{0.5}\} \rangle, \langle (a_1, b_2), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\} \end{aligned}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_3), (\tilde{H}_5, \wedge_3), (\tilde{H}_6, \wedge_3), (\tilde{H}_7, \wedge_3)\}$ is *FHSts*.

Let the *FHSs* $(\tilde{G}_1, \wedge_3)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_3) = \left\{ \begin{array}{l} \langle (c_2, d_1), \{\frac{n_1}{0.4}, \frac{n_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{n_1}{0.4}, \frac{n_2}{0.5}\} \rangle \end{array} \right\}$$

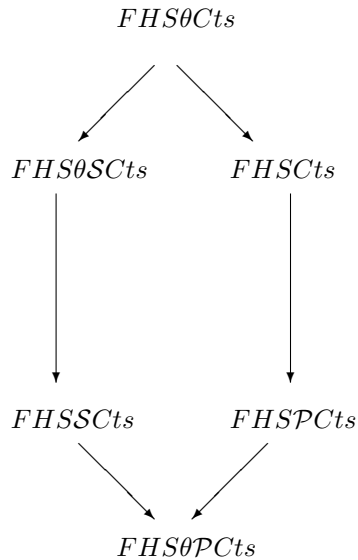
$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_3)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\begin{aligned} \omega(\mathfrak{m}_1) &= \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) &= (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2) \\ \mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) &= (\tilde{H}_8, \wedge_3) \end{aligned}$$

$\mathfrak{h}$ is *FHS θ Pcts* but not *FHS $\mathcal{P}$ Cts* because $(\tilde{G}_1, \wedge_3)$ is *FHSos* in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_3) = (\tilde{H}_8, \wedge_3)$ is a *FHS θ Pos* but not *FHS $\mathcal{P}$ os*.

Remark 3.10. From the results discussed above, the following diagram is obtained.



Theorem 3.11. A map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ is *FHS θ Cts* iff the inverse image of each *FHScs* in $\mathfrak{N}$ is *FHS θ cs* in $\mathfrak{M}$.

Proof. Let $(\tilde{G}, \wedge)$ be a *FHScs* in $\mathfrak{N}$. This implies that $(\tilde{G}, \wedge)^c$ is *FHSos* in $\mathfrak{N}$. Since $\mathfrak{h}$ is *FHS θ Cts*, $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c)$ is *FHS θ os* in $\mathfrak{M}$. Since $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c) = (\mathfrak{h}^{-1}(\tilde{G}, \wedge))^c$, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is a *FHS θ cs* in $\mathfrak{M}$.

Conversely, let $(\tilde{G}, \wedge)$ be a *FHSos* in $\mathfrak{N}$. Then $(\tilde{G}, \wedge)^c$ is a *FHScs* in $\mathfrak{N}$. By hypothesis, $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c)$ is *FHS θ cs* in $\mathfrak{M}$. Since, $\mathfrak{h}^{-1}((\tilde{G}, \wedge)^c) = (\mathfrak{h}^{-1}(\tilde{G}, \wedge))^c$, $(\mathfrak{h}^{-1}(\tilde{G}, \wedge))^c$ is *FHS θ cs* in $\mathfrak{M}$. Therefore, $(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$ is a *FHS θ os* in $\mathfrak{M}$. Hence, $\mathfrak{h}$ is *FHS θ Cts*.

Theorem 3.12. Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ be a *FHS θ Cts* map and $\mathfrak{g} : (\mathfrak{N}, M, \sigma) \rightarrow (P, N, \rho)$ be a *FHS θ Cts*, then $\mathfrak{g} \circ \mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (P, N, \rho)$ is a *FHS θ Cts*.

Proof. Let $(\tilde{K}, \wedge)$ be a *FHSos* in P . Then $\mathfrak{g}^{-1}(\tilde{K}, \wedge)$ is a *FHSos* in $\mathfrak{N}$, by hypothesis. Since $\mathfrak{h}$ is a *FHS θ Cts* map $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{K}, \wedge))$ is a *FHS θ os* in $\mathfrak{M}$. Hence $\mathfrak{g} \circ \mathfrak{h}$ is a *FHS θ Cts* map.

Theorem 3.13. Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ be a *FHS θ Cts* map. Then the following conditions are hold:

- (i) $\mathfrak{h}(FHS\theta cl(\tilde{H}, \wedge)) \leq FHScl(\mathfrak{h}(\tilde{H}, \wedge))$, for all *FHS θ cs* $(\tilde{H}, \wedge)$ in $\mathfrak{M}$.
- (ii) $FHS\theta cl(\mathfrak{h}^{-1}(\tilde{G}, \wedge)) \leq \mathfrak{h}^{-1}(FHScl(\tilde{G}, \wedge))$, for all *FHS θ cs* $(\tilde{G}, \wedge)$ in $\mathfrak{N}$.

Proof. (i) As $FHS\theta cl(\mathfrak{h}(\tilde{H}, \wedge))$ is a *FHS θ cs* in $\mathfrak{N}$ and $\mathfrak{h}$ is *FHS θ Cts*, we have $\mathfrak{h}^{-1}(FHS\theta cl(\mathfrak{h}(\tilde{H}, \wedge)))$ is a *FHS θ cs* in $\mathfrak{M}$. Now, as $(\tilde{H}, \wedge) \leq \mathfrak{h}^{-1}(FHS\theta cl(\mathfrak{h}(\tilde{H}, \wedge)))$, $FHScl(\tilde{H}, \wedge) \leq \mathfrak{h}^{-1}(FHScl(\mathfrak{h}(\tilde{H}, \wedge)))$. Therefore, $\mathfrak{h}(FHS\theta cl(\tilde{H}, \wedge)) \leq FHScl(\mathfrak{h}(\tilde{H}, \wedge))$.

(ii) By replacing $(\tilde{H}, \wedge)$ with $(\tilde{G}, \wedge)$ in (i), we get $\mathfrak{h}(FHS\theta cl(\mathfrak{h}^{-1}(\tilde{G}, \wedge))) \leq FHScl(\mathfrak{h}(\mathfrak{h}^{-1}(\tilde{G}, \wedge))) \leq FHScl(\tilde{G}, \wedge)$. Hence, $FHS\theta cl(\mathfrak{h}^{-1}(\tilde{G}, \wedge)) \leq \mathfrak{h}^{-1}(FHScl(\tilde{G}, \wedge))$.

Remark 3.14. If $\mathfrak{h}$ is *FHS θ Cts* then,

- (i) $\mathfrak{h}(FHS\theta cl(\tilde{H}, \wedge))$ need not be equal to $FHScl(\mathfrak{h}(\tilde{H}, \wedge))$ where $(\tilde{H}, \wedge) \in \mathfrak{M}$.
- (ii) $FHS\theta cl(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$ need not be equal to $\mathfrak{h}^{-1}(FHScl(\tilde{G}, \wedge))$ where $(\tilde{G}, \wedge) \in \mathfrak{N}$.

Example 3.15. Let $\mathfrak{M} = \{m_1, m_2\}$ and $\mathfrak{N} = \{n_1, n_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$$

$$Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHS*s's $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_1), (\tilde{H}_3, \wedge_2)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \langle (a_1, b_1), \{ \frac{m_1}{0.2}, \frac{m_2}{0.3} \} \rangle, \langle (a_1, b_2), \{ \frac{m_1}{0.5}, \frac{m_2}{0.4} \} \rangle \right\}$$

$$(\tilde{H}_2, \wedge_1) = \left\{ \langle (a_1, b_1), \{ \frac{m_1}{0.8}, \frac{m_2}{0.7} \} \rangle, \langle (a_1, b_2), \{ \frac{m_1}{0.5}, \frac{m_2}{0.6} \} \rangle \right\}$$

$$(\tilde{H}_3, \wedge_2) = \left\{ \langle (a_1, b_1), \{ \frac{m_1}{0.8}, \frac{m_2}{0.6} \} \rangle, \langle (a_2, b_1), \{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \} \rangle, \langle (a_1, b_2), \{ \frac{m_1}{0.5}, \frac{m_2}{0.4} \} \rangle \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_1)\}$ is *FHS θ ts*.

Let the *FHS*s's $(\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_1), (\tilde{G}_3, \wedge_2)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge_1) = \left\{ \langle (c_2, d_1), \{ \frac{n_1}{0.3}, \frac{n_2}{0.2} \} \rangle, \langle (c_2, d_2), \{ \frac{n_1}{0.4}, \frac{n_2}{0.5} \} \rangle \right\}$$

$$(\tilde{G}_2, \wedge_1) = \left\{ \langle (c_2, d_1), \{ \frac{n_1}{0.7}, \frac{n_2}{0.8} \} \rangle, \langle (c_2, d_2), \{ \frac{n_1}{0.6}, \frac{n_2}{0.5} \} \rangle \right\}$$

$$(\tilde{G}_3, \wedge_2) = \left\{ \langle (c_2, d_1), \{ \frac{n_1}{0.6}, \frac{n_2}{0.8} \} \rangle, \langle (c_1, d_2), \{ \frac{n_1}{0.5}, \frac{n_2}{0.7} \} \rangle, \langle (c_2, d_2), \{ \frac{n_1}{0.4}, \frac{n_2}{0.5} \} \rangle \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_1)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a FHS mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2)$$

$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_2, \wedge_1) = (\tilde{H}_2, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_3, \wedge_2) = (\tilde{H}_3, \wedge_2)$$

Then $\mathfrak{h}$ is $FHS\theta Cts$.

(i) $\mathfrak{h}(FHS\theta cl(\tilde{H}_3, \wedge_2)) = (\tilde{G}_1, \wedge_1)^c$, but $FHScl(\mathfrak{h}(\tilde{H}_3, \wedge_2)) = (\tilde{G}_1, \wedge_1)^c$.

Hence $\mathfrak{h}(FHS\theta cl(\tilde{H}_3, \wedge_2)) = FHScl(\mathfrak{h}(\tilde{H}_3, \wedge_2))$.

(ii) $FHS\theta cl(\mathfrak{h}^{-1}(\tilde{G}_3, \wedge_2)) = (\tilde{H}_1, \wedge_1)^c$, but $\mathfrak{h}^{-1}(FHScl(\tilde{G}_3, \wedge_2)) = (\tilde{H}_1, \wedge_1)^c$.

Hence $FHScl(\mathfrak{h}^{-1}(\tilde{G}_3, \wedge_2)) = \mathfrak{h}^{-1}(FHScl(\tilde{G}_3, \wedge_2))$.

Theorem 3.16. $\mathfrak{h}$ is $FHS\theta Cts$ iff $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge)) \leq FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$ for all $FHScs$ $(\tilde{G}, \wedge)$ in $\mathfrak{N}$.

Proof. Let $\mathfrak{h}$ be a $FHS\theta Cts$ and $(\tilde{G}, \wedge) \in \mathfrak{N}$. $FHSint(\tilde{G}, \wedge)$ is $FHSos$ in $\mathfrak{N}$ and hence, $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge))$ is a $FHS\theta os$ in $\mathfrak{M}$. Therefore, $FHS\theta int(\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge))) = \mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge))$. Also, $FHSint(\tilde{G}, \wedge) \leq (\tilde{G}, \wedge)$ implies that $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge)) \leq \mathfrak{h}^{-1}(\tilde{G}, \wedge)$. Therefore, $FHS\theta int(\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge))) \leq FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$. That is, $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge)) \leq FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$.

Conversely, let $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge)) \leq FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$ for all subset $(\tilde{G}, \wedge)$ of $\mathfrak{N}$. If $(\tilde{G}, \wedge)$ is $FHSos$ in $\mathfrak{N}$, then $FHSint(\tilde{G}, \wedge) = (\tilde{G}, \wedge)$. By assumption, $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge)) \leq FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$. Thus $(\mathfrak{h}^{-1}(\tilde{G}, \wedge) \leq FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge)))$. But $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge)) = \mathfrak{h}^{-1}(\tilde{G}, \wedge)$. Therefore, $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge)) = \mathfrak{h}^{-1}(\tilde{G}, \wedge)$. That is, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is $FHS\theta os$ in $\mathfrak{M}$, for all $FHSos(\tilde{G}, \wedge)$ in $\mathfrak{N}$. Therefore, $\mathfrak{h}$ is $FHS\theta Cts$ on $\mathfrak{M}$.

Remark 3.17. If $\mathfrak{h}$ is $FHS\theta Cts$, then $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G}, \wedge))$ need not be equal to $\mathfrak{h}^{-1}(FHSint(\tilde{G}, \wedge))$ where $(\tilde{G}, \wedge) \in \mathfrak{N}$.

Example 3.18. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{H}_1, \wedge_1)$ and $(\tilde{H}_2, \wedge_1)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \langle (a_1, b_1), \left\{ \frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7} \right\} \rangle, \right. \\ \left. \langle (a_2, b_1), \left\{ \frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5} \right\} \rangle \right\}$$

$$(\tilde{H}_2, \wedge_1) = \left\{ \langle (a_1, b_1), \left\{ \frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3} \right\} \rangle, \right. \\ \left. \langle (a_2, b_1), \left\{ \frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.5} \right\} \rangle \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_1)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1, \wedge_1)$ and $(\tilde{G}_2, \wedge_1)$ over the universe $\mathfrak{N}$ be defined as

$$(\tilde{G}_1, \wedge_1) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.7}\} \rangle \right\}$$

$$(\tilde{G}_2, \wedge_1) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.3}, \frac{n_2}{0.2}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.5}, \frac{n_2}{0.3}\} \rangle \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{G}_1, \wedge_1)\}$ is *FHSts*.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_2, d_1), \nu(a_1, b_2) = (c_1, d_2), \nu(a_2, b_2) = (c_2, d_2)$$

$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_2, \wedge_1) = (\tilde{H}_2, \wedge_1)$$

Then $\mathfrak{h}$ is *FHS θ Cts*. then *FHS θ int*($\mathfrak{h}^{-1}(\tilde{G}_2, \wedge_1)$) = ($\tilde{H}_2, \wedge_1$)

$\mathfrak{h}^{-1}(\text{FHSint}(\tilde{G}_2, \wedge_1)) = \tilde{0}_{(\mathfrak{M}, Q)}$ where $(\tilde{G}_2, \wedge_1) \in \mathfrak{N}$.

4 Fuzzy Hypersoft θ Irresolute Maps

Fuzzy hypersoft θ irresolute maps are introduced and their relevant properties are discussed in this section.

Definition 4.1. A map $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ is called a *FHS θ irresolute* (in short, *FHS θ Irr*) map if $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is a *FHS θ os* in $(\mathfrak{M}, L, \tau)$ for every *FHS θ os* $(\tilde{G}, \wedge)$ of $(\mathfrak{N}, M, \sigma)$.

Theorem 4.2. Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ be a *FHS θ Irr* map. Then $\mathfrak{h}$ is a *FHS θ Cts* map. But not conversely.

Proof. Let $\mathfrak{h}$ be a *FHS θ Irr* map. Let $(\tilde{G}, \wedge)$ be any *FHSos* on $\mathfrak{N}$. Since every *FHSos* is a *FHS θ os*, $(\tilde{G}, \wedge)$ in $\mathfrak{N}$. By hypothesis, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is a *FHS θ os* in $\mathfrak{M}$. Hence, $\mathfrak{h}$ is a *FHS θ Cts* map.

Example 4.3. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the *FHS* initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q'_1 \times Q'_2$ respectively. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\} \\ Q'_1 = \{c_1, c_2, c_3\}, Q'_2 = \{d_1, d_2\}.$$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of *FHS* sets. Let the *FHSs*'s $(\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_1)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge_1) = \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.2}, \frac{m_2}{0.3}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.5}, \frac{m_2}{0.4}\} \rangle \right\}$$

$$(\tilde{H}_2, \wedge_1) = \left\{ \langle (a_1, b_1), \{\frac{m_1}{0.8}, \frac{m_2}{0.7}\} \rangle, \langle (a_2, b_1), \{\frac{m_1}{0.5}, \frac{m_2}{0.6}\} \rangle \right\}$$

$\tau = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{H}_1, \wedge_1)\}$ is *FHSts*.

Let the *FHSs*'s $(\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_1)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge_1) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.3}, \frac{n_2}{0.2}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.4}, \frac{n_2}{0.5}\} \rangle \right\}$$

$$(\tilde{G}_2, \wedge_1) = \left\{ \langle (c_2, d_1), \{\frac{n_1}{0.7}, \frac{n_2}{0.8}\} \rangle, \langle (c_1, d_2), \{\frac{n_1}{0.6}, \frac{n_2}{0.5}\} \rangle \right\}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N}, Q)}, \tilde{1}_{(\mathfrak{N}, Q)}, (\tilde{G}_1, \wedge_1), (\tilde{G}_2, \wedge_1)\}$ is *FHS*ts.

Let $\mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ be a *FHS* mapping as follows:

$$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1, \\ \nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2), \nu(a_1, b_2) = (c_2, d_2)$$

$$\mathfrak{h}^{-1}(\tilde{G}_1, \wedge_1) = (\tilde{H}_1, \wedge_1), \mathfrak{h}^{-1}(\tilde{G}_2, \wedge_1) = (\tilde{H}_2, \wedge_1),$$

$\therefore \mathfrak{h} = (\omega, \nu) : (\mathfrak{M}, L) \rightarrow (\mathfrak{N}, M)$ is *FHS* θ Cts but not *FHS* θ Irr, because the set $(\tilde{G}_2, \wedge_1)$ is a *FHS* θ os in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_2, \wedge_1) = (\tilde{H}_2, \wedge_1)$ is not *FHS* θ os in $\mathfrak{M}$

Theorem 4.4. Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ and $\mathfrak{g} : (\mathfrak{N}, M, \sigma) \rightarrow (P, N, \rho)$ be *FHS*Irr maps, then $\mathfrak{g} \circ \mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (P, N, \rho)$ is a *FHS*Irr map.

Proof. Let $(\tilde{K}, \wedge)$ be a *FHS* θ os in P . Then $\mathfrak{g}^{-1}(\tilde{K}, \wedge)$ is a *FHS* θ os in $\mathfrak{N}$. Since $\mathfrak{h}$ is a *FHS* θ Irr map, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{K}, \wedge))$ is a *FHS* θ os in $\mathfrak{M}$. Hence $\mathfrak{g} \circ \mathfrak{h}$ *FHS* θ Irr map.

Theorem 4.5. Let $\mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (\mathfrak{N}, M, \sigma)$ be a *FHS* θ Irr map and $\mathfrak{g} : (\mathfrak{N}, M, \sigma) \rightarrow (P, N, \rho)$ is a *FHS* θ Cts map, then $\mathfrak{g} \circ \mathfrak{h} : (\mathfrak{M}, L, \tau) \rightarrow (P, N, \rho)$ is a *FHS* θ Cts map.

Proof. Let $(\tilde{K}, \wedge)$ be a *FHS*os in P . Then $\mathfrak{g}^{-1}(\tilde{K}, \wedge)$ is a *FHS* θ os in $\mathfrak{N}$. Since $\mathfrak{h}$ is a *FHS* θ Irr map, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{K}, \wedge))$ is a *FHS* θ os in $\mathfrak{M}$. Hence, $\mathfrak{g} \circ \mathfrak{h}$ is a *FHS* θ Cts map.

5 Conclusions

In this paper, *FHS* θ Cts, *FHS* θ SCts and *FHS* θ PCts maps are defined using *FHS* θ os and their properties are analysed with examples. Then *FHS*Cts maps are compared with *FHS* θ Cts maps. In addition, these maps are extended to *FHS* θ Irr maps and its relevant properties are discussed. In future, these findings can be extended to *FHS* θ open mapping, *FHS* θ closed mapping and *FHS* homeomorphic functions.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

AMS (2000) subject classification: 03E72, 54A10, 54A40, 54C05, 54C10.

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