The novel algebraic structure trigonometric Pythagorean neutrosophic set provides the basis for interaction weighted aggregating operators

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Abstract

We introduce new methods for the trigonometric Pythagorean neutrosophic set (TPNSS) via interaction aggregating operator in this study. A combination of the trigonometric operator and the Pythagorean neutrosophic set. The universal aggregation function is used to study the novel averaging and geometric interaction operations of Pythagorean neutrosophic numbers. The TPNSS are commutative, associative, idempotent, and boundedness compatible. TPNSS interaction weighted averaging, TPNSS interaction weighted geometric, generalized TPNSS interaction weighted averaging, and generalized TPNSS interaction weighted geometric are the four new interaction aggregating operators that are introduced. The Euclidean distance, Hamming distance, and score values are often assumed to represent the aggregation functions.

Keywords: Aggregating operator; TPNSIWA; TPNSIWG; GTPNSIWA; GTPNSIWG

1 Introduction

The fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS),[3,4] neutrosophic set (NSS),[5] and Fermatean FS (FFS)[6] have all been developed as a consequence of the uncertainties. Zadeh's FS[1] proposes a membership degree (MD) for decision-makers. Since each object possesses MD $\varkappa$ and non-membership degree (NMD) ς and satisfies $0 \leq \varkappa + \varsigma \leq 1$, for $\varkappa, \varsigma \in [0,1]$, An IFS concept was established by Atanassov.[2] According to the criterion that $\varkappa^2 + \varsigma^2 \leq 1$, PFSs are defined by their MDs and NMDs, as developed by Yager.[3] In many different disciplines, IFSs and PFSs have been extensively studied and used. Cuong and colleagues[7] by creating the idea of picture FSs (PiFSs). Since PiFSs is an extended form of IFSs, it has been observed that it may support certain extra ambiguities. It is noted in PiFSs that the MD $\varkappa$, neutral ς, and NMD $\breve{a}$ have $0 \leq \varkappa + \varsigma + \breve{a} \leq 1$; for $\varkappa, \varsigma, \breve{a} \in [0,1]$. It will guarantee that expert opinion messages like "yes," "abstain," "no," and "refusal" are sent over the PiFS. Additionally, it will prevent evaluation information from being omitted and guarantee that the evaluation data and the real decision environment are consistent. Although there are many applications and research on PiFSs, the notion has not been thoroughly investigated. For certain AOs with MADM, Shahzaib et al.[8] defined the spherical FS (SFS). The SFS requires that $0 \leq \varkappa^2 + \varsigma^2 + \breve{a}^2 \leq 1$ instead of $0 \leq \varkappa + \varsigma + \breve{a} \leq 1$. Jin et al.[9] presented the linguistic SFS AOs, which were talked about in MADM difficulties. In DM, Rafiq et al. presented SFSs and their uses.[10] Decision-making (DM) and the fact that $\varkappa^2 + \varsigma^2 \geq 1$ are problematic. Senapati and colleagues[6] introduced the idea of an FFS in 2019. Both the MD and NMD with the property that $0 \leq \varkappa^3 + \varsigma^3 \leq 1$. Because of this effort, the following contributions are made:

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1. Numerous features of algebra, including idempotency, commutativity, and associativity, have been demonstrated.
2. The characteristics of a TPNSIWA, WG, GWA and GWG are HD and ED. The ED distance between two TPNSs is to be determined using this approach.

2 Basic concepts

Definition 2.1. [3] Let $\digamma$ be the universe set. The PFS $\beth = \Big\{x, \big\langle \varkappa^{\daleth}[x], \upsilon^{\Finv}[x]\big\rangle \big| x \in \digamma\Big\}$, where $\varkappa^{\daleth}, \upsilon^{\Finv} : \digamma \to [0,1]$ refers the MD and NMD of $x \in \digamma$ to $\beth$, respectively and $0 \leq [\varkappa^{\daleth}[x]]^2 + [\upsilon^{\Finv}[x]]^2 \leq 1$. For, $\beth = \big\langle \varkappa^{\daleth}, \varkappa^{\Finv}\big\rangle$ is called the Pythagorean fuzzy number [PFN].

Definition 2.2. [6] A Fermatean fuzzy set $\beth = \Big\{x, \big\langle \varkappa^{\daleth}[x], \upsilon^{\Finv}[x]\big\rangle \big| x \in \digamma\Big\}$, where $\varkappa^{\daleth}[x]$ and $\upsilon^{\Finv}[x]$ denote MD and NMD of u respectively, where $\varkappa^{\daleth}, \upsilon^{\Finv} : \digamma \to [0,1]$ and $0 \leq [\varkappa^{\daleth}[x]]^3 + [\upsilon^{\Finv}[x]]^3 \leq 1$. Here, $\beth = \big\langle \varkappa^{\daleth}, \varkappa^{\Finv}\big\rangle$ is represent a Fermatean fuzzy number [FFN].

Definition 2.3. For any PFNs, $\beth = \langle \varkappa^{\daleth}, \upsilon^{\Finv}\rangle$, $\beth_1 = \langle \varkappa_1^{\daleth}, \upsilon_1^{\Finv}\rangle$ and $\beth_2 = \langle \varkappa_2^{\daleth}, \upsilon_2^{\Finv}\rangle$, $\varkappa^{\daleth}, \upsilon^{\Finv}$ denote MD and NMD of u respectively. Then

1. $\beth_1 \curlyvee \beth_2 = \left[\sqrt{[\varkappa_1^{\daleth}]^2 + [\varkappa_2^{\daleth}]^2 - [\varkappa_1^{\daleth}]^2 \cdot [\varkappa_2^{\daleth}]^2}, [\upsilon_1^{\Finv} \cdot \upsilon_2^{\Finv}]\right]$
2. $\beth_1 \curlywedge \beth_2 = \left[[\varkappa_1^{\daleth} \cdot \varkappa_2^{\daleth}], \sqrt{[\upsilon_1^{\Finv}]^2 + [\upsilon_2^{\Finv}]^2 - [\upsilon_1^{\Finv}]^2 \cdot [\upsilon_2^{\Finv}]^2}\right]$
3. $\mho \cdot \beth = \left[\sqrt{1 - \left[1 - [\varkappa^{\daleth}]^2\right]^{\mho}}, [\upsilon^{\Finv}]^{\mho}\right]$
4. $\beth^{\mho} = \left[[\varkappa^{\daleth}]^{\mho}, \sqrt{1 - \left[1 - [\upsilon^{\Finv}]^2\right]^{\mho}}\right]$

Definition 2.4. If $\beth_1 = \big\langle \varkappa_1^{\daleth}, \upsilon_1^{\Finv}\big\rangle$ and $\beth_2 = \big\langle \varkappa_2^{\daleth}, \upsilon_2^{\Finv}\big\rangle$ are any two PFNs. Then the interaction AO is defined as

1. $\beth_1 \curlyvee \beth_2 = \begin{bmatrix} \sqrt{[\varkappa_1^{\daleth}]^2 + [\varkappa_2^{\daleth}]^2 - [\varkappa_1^{\daleth}]^2 \cdot [\varkappa_2^{\daleth}]^2}, \\ \sqrt{[\upsilon_1^{\Finv}]^2 + [\upsilon_2^{\Finv}]^2 - [\upsilon_1^{\Finv}]^2 \cdot [\upsilon_2^{\Finv}]^2 - [\upsilon_1^{\Finv}]^2 \cdot [\varkappa_2^{\daleth}]^2 - [\varkappa_1^{\daleth}]^2 \cdot [\upsilon_2^{\Finv}]^2} \end{bmatrix}$
2. $\beth_1 \curlywedge \beth_2 = \begin{bmatrix} \sqrt{[\varkappa_1^{\daleth}]^2 + [\varkappa_2^{\daleth}]^2 - [\varkappa_1^{\daleth}]^2 \cdot [\varkappa_2^{\daleth}]^2 - [\varkappa_1^{\daleth}]^2 \cdot [\upsilon_2^{\Finv}]^2 - [\upsilon_1^{\Finv}]^2 \cdot [\varkappa_2^{\daleth}]^2}, \\ \sqrt{[\upsilon_1^{\Finv}]^2 + [\upsilon_2^{\Finv}]^2 - [\upsilon_1^{\Finv}]^2 \cdot [\upsilon_2^{\Finv}]^2} \end{bmatrix}$
3. $\mho \cdot \beth_1 = \left[\sqrt{1 - \left[1 - [\varkappa_1^{\daleth}]^2\right]^{\mho}}, \sqrt{\left[1 - [\varkappa_1^{\daleth}]^2\right]^{\mho} - \left[1 - [\varkappa_1^{\daleth} + \upsilon_1^{\Finv}]^2\right]^{\mho}}\right]$
4. $\beth_1^{\mho} = \left[\sqrt{\left[1 - [\upsilon_1^{\Finv}]^2\right]^{\mho} - \left[1 - [\varkappa_1^{\daleth} + \upsilon_1^{\Finv}]^2\right]^{\mho}}, \sqrt{1 - \left[1 - [\upsilon_1^{\Finv}]^2\right]^{\mho}}\right]$

where $\mho$ be a positive integers.

3 Different AOs for TNSN

Throughout this paper, $\sin \pi/2(\cos \pi/2, \tan \pi/2) = \measuredangle$.

Definition 3.1. Suppose that $\beth_1 = \big\langle \varkappa_1^{\daleth}, \backepsilon_1^{\gimel}, \upsilon_1^{\Finv}\big\rangle$ and $\beth_2 = \big\langle \varkappa_2^{\daleth}, \backepsilon_2^{\gimel}, \upsilon_2^{\Finv}\big\rangle$ be the any two TPNSNs. Then

1. $\beth_1 \curlyvee \beth_2 = \begin{bmatrix} \sqrt{[\measuredangle\varkappa_1^{\daleth}]^2 + [\measuredangle\varkappa_2^{\daleth}]^2 - [\measuredangle\varkappa_1^{\daleth}]^2 \cdot [\measuredangle\varkappa_2^{\daleth}]^2}, \\ \sqrt{[\measuredangle\ \backepsilon_1^{\gimel}]^2 + [\measuredangle\ \backepsilon_2^{\gimel}]^2 - [\measuredangle\ \backepsilon_1^{\gimel}]^2 \cdot [\measuredangle\ \backepsilon_2^{\gimel}]^2}, \\ \sqrt{\begin{matrix} [\measuredangle\upsilon_1^{\Finv}]^2 + [\measuredangle\upsilon_2^{\Finv}]^2 - [\measuredangle\upsilon_1^{\Finv}]^2 \cdot [\measuredangle\upsilon_2^{\Finv}]^2 \\ -[\measuredangle\upsilon_1^{\Finv}]^2 \cdot [\measuredangle\varkappa_2^{\daleth}]^2 - [\measuredangle\varkappa_1^{\daleth}]^2 \cdot [\measuredangle\upsilon_2^{\Finv}]^2 \end{matrix}} \end{bmatrix}$

2. $$\beth_1 \curlywedge \beth_2 = \begin{bmatrix} \sqrt{\begin{array}{c}[\measuredangle\varkappa_1^{\daleth}]^2+[\measuredangle\varkappa_2^{\daleth}]^2-[\measuredangle\varkappa_1^{\daleth}]^2\cdot[\measuredangle\varkappa_2^{\daleth}]^2\\ -[\measuredangle\varkappa_1^{\daleth}]^2\cdot[\measuredangle\upsilon_2^{\dashv}]^2-[\measuredangle\upsilon_1^{\dashv}]^2\cdot[\measuredangle\varkappa_2^{\daleth}]^2\end{array}}, \\ \sqrt{[\measuredangle\ \backepsilon_1^{\gimel}]^2+[\measuredangle\ \backepsilon_2^{\gimel}]^2-[\measuredangle\ \backepsilon_1^{\gimel}]^2\cdot[\measuredangle\ \backepsilon_2^{\gimel}]^2} \\ \sqrt{[\measuredangle\upsilon_1^{\dashv}]^2+[\measuredangle\upsilon_2^{\dashv}]^2-[\measuredangle\upsilon_1^{\dashv}]^2\cdot[\measuredangle\upsilon_2^{\dashv}]^2} \end{bmatrix}$$

3. $$\mho\cdot\beth_1 = \begin{bmatrix} \sqrt{1-\left[1-[\measuredangle\varkappa_1^{\daleth}]^2\right]^{\mho}}, \sqrt{1-\left[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2\right]^{\mho}}, \\ \sqrt{\left[1-[\measuredangle\varkappa_1^{\daleth}]^2\right]^{\mho}-\left[1-[\measuredangle\varkappa_1^{\daleth}+\measuredangle\upsilon_1^{\dashv}]^2\right]^{\mho}} \end{bmatrix}$$

4. $$\beth_1^{\mho} = \begin{bmatrix} \sqrt{\left[1-[\measuredangle\upsilon_1^{\dashv}]^2\right]^{\mho}-\left[1-[\measuredangle\varkappa_1^{\daleth}+\measuredangle\upsilon_1^{\dashv}]^2\right]^{\mho}} \\ \sqrt{1-\left[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2\right]^{\mho}}, \sqrt{1-\left[1-[\measuredangle\upsilon_1^{\dashv}]^2\right]^{\mho}} \end{bmatrix}$$

3.1 TPNSIWA operator

Definition 3.2. Let $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\gimel}, \upsilon_{\propto}^{\dashv} \right\rangle$ be the TPNSNs, $\propto = 1, 2, ..., k$, $\varsigma_{\propto}$ be the weight of $\beth_{\propto}$ and $\varsigma_{\propto} \geq 0$, $\biguplus_{\propto=1}^{k} \varsigma_{\propto} = 1$. Then the TPNSIWA operator $[\beth_1, \beth_2, ..., \beth_k] = \biguplus_{\propto=1}^{k} \varsigma_{\propto}\beth_{\propto}$.

Theorem 3.3. *Let* $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\gimel}, \upsilon_{\propto}^{\dashv} \right\rangle$ *be the TPNSNs,* $\propto = 1, 2, ..., k$.

Then, TPNSIWA$[\beth_1, \beth_2, ..., \beth_k] =$
$$\begin{bmatrix} \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}} \end{bmatrix}.$$

Proof. If $\propto = 2$, TPNSIWA$[\beth_1, \beth_2] = \varsigma_1\beth_1 \curlyvee \varsigma_2\beth_2$,
where,

$$\varsigma_1\beth_1 = \begin{bmatrix} \sqrt{1-\left[1-[\measuredangle\varkappa_1^{\daleth}]^2\right]^{\varsigma_1}}, \sqrt{1-\left[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2\right]^{\varsigma_1}}, \\ \sqrt{\left[1-[\measuredangle\varkappa_1^{\daleth}]^2\right]^{\varsigma_1}-\left[1-[\measuredangle\varkappa_1^{\daleth}+\measuredangle\upsilon_1^{\dashv}]^2\right]^{\varsigma_1}} \end{bmatrix}$$

and

$$\varsigma_2\beth_2 = \begin{bmatrix} \sqrt{1-\left[1-[\measuredangle\varkappa_2^{\daleth}]^2\right]^{\varsigma_2}}, \sqrt{1-\left[1-[\measuredangle\ \backepsilon_2^{\gimel}]^2\right]^{\varsigma_2}}, \\ \sqrt{\left[1-[\measuredangle\varkappa_2^{\daleth}]^2\right]^{\varsigma_2}-\left[1-[\measuredangle\varkappa_2^{\daleth}+\measuredangle\upsilon_2^{\dashv}]^2\right]^{\varsigma_2}} \end{bmatrix}$$

We get

$$\begin{aligned} \varsigma_1\beth_1 \curlyvee \varsigma_2\beth_2 &= \begin{bmatrix} \sqrt{\begin{array}{c} \left[1-[1-[\measuredangle\varkappa_1^{\daleth}]^2]^{\varsigma_1}\right]+\left[1-[1-[\measuredangle\varkappa_2^{\daleth}]^2]^{\varsigma_2}\right] \\ -\left[1-[1-[\measuredangle\varkappa_1^{\daleth}]^2]^{\varsigma_1}\right]\cdot\left[1-[1-[\measuredangle\varkappa_2^{\daleth}]^2]^{\varsigma_2}\right] \end{array}}, \\ \sqrt{\begin{array}{c} \left[1-[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2]^{\varsigma_1}\right]+\left[1-[1-[\measuredangle\ \backepsilon_2^{\gimel}]^2]^{\varsigma_2}\right] \\ -\left[1-[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2]^{\varsigma_1}\right]\cdot\left[1-[1-[\measuredangle\ \backepsilon_2^{\gimel}]^2]^{\varsigma_2}\right] \end{array}}, \\ \sqrt{\begin{array}{c} \left[1-[1-\measuredangle\upsilon_1^{\dashv}]^2]^{\varsigma_1}\right]+\left[1-[1-\measuredangle\upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\Big] \\ -\left[1-[1-\measuredangle\upsilon_1^{\dashv}]^2]^{\varsigma_1}\right]\cdot\left[1-[1-\measuredangle\upsilon_2^{\dashv}]^2\right]^{\varsigma_2}\Big] \\ -[1-[\measuredangle\varkappa_1^{\daleth}+\measuredangle\upsilon_1^{\dashv}]^2]^{\varsigma_1}\cdot[1-[\measuredangle\varkappa_2^{\daleth}+\measuredangle\upsilon_2^{\dashv}]^2]^{\varsigma_2} \end{array}} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{1-\Diamond_{\propto=1}^{2}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{2}\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{2}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{2}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}} \end{bmatrix}. \end{aligned}$$

Using induction $\propto \geq 3$, TPNSIWA$[\beth_1, \beth_2, ..., \beth_{\ell}]$

$$= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}}, \end{array}\right].$$

If $\propto = \ell+1$, then $TPNSIWA[\beth_1, \beth_2, ..., \beth_{\ell}, \beth_{\ell+1}]$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{l} \biguplus_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}\right] + \left[1-\left[1-[\measuredangle\varkappa_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}\right]\cdot\left[1-\left[1-[\measuredangle\varkappa_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}}\right], \end{array}} \\ \sqrt{\begin{array}{l} \biguplus_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}\right] + \left[1-\left[1-[\measuredangle\ \backepsilon_{\ell+1}^{\gimel}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}\right]\cdot\left[1-\left[1-[\measuredangle\ \backepsilon_{\ell+1}^{\gimel}]^2\right]^{\varsigma_{\ell+1}}\right], \end{array}} \\ \sqrt{\begin{array}{c} \biguplus_{\propto=1}^{\ell}\left[1-\left[1-\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}\right] + \left[1-\left[1-\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-\left[1-\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}\right]\cdot\left[1-\left[1-\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}\cdot\left[1-[\measuredangle\varkappa_{\ell+1}^{\daleth}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}} \end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}\cdot\left[1-[\measuredangle\varkappa_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}}}, \\ \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}\cdot\left[1-[\measuredangle\ \backepsilon_{\ell+1}^{\gimel}]^2\right]^{\varsigma_{\ell+1}}}, \\ \sqrt{\begin{array}{r} \left[\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}\right]\cdot \\ \left[\left[[\measuredangle\varkappa_{\ell+1}^{\daleth}]^2\right]^{\varsigma_{\ell+1}} - [\measuredangle\varkappa_{\ell+1}^{\daleth}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^2\right]^{\varsigma_{\ell+1}}\right] \end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^2\right]^{\varsigma_{\propto}}} \end{array}\right]$$

Theorem 3.4. *If* $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\gimel}, \upsilon_{\propto}^{\dashv} \right\rangle$ *be the TPNSNs and* $\beth_{\propto} = \beth$ *and* $\varkappa^{\daleth}\cdot\upsilon^{\dashv} = 0$, *then the TPNSIWA*$[\beth_1, \beth_2, ..., \beth_k] = \beth$, $\propto = 1, 2, ..., k$.

Proof. Note that, $[\measuredangle\varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\gimel}, \measuredangle\upsilon_{\propto}^{\dashv}] = [\measuredangle\varkappa^{\daleth}, \backepsilon^{\gimel}, \measuredangle\upsilon^{\dashv}]$, $\propto = 1, 2, ..., k$ and $\biguplus_{\propto=1}^{k}\varsigma_{\propto} = 1$. We get, $TPNSIWA[\beth_1, \beth_2, ..., \beth_k]$

$$\begin{aligned} &= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\varkappa^{\daleth}]^2\right]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\ \backepsilon^{\gimel}]^2\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{k}\left[1-[\measuredangle\varkappa^{\daleth}]^2\right]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{k}\left[1-[\measuredangle\varkappa^{\daleth}+\measuredangle\upsilon^{\dashv}]^2\right]^{\varsigma_{\propto}}} \end{array}\right] \\ &= \left[\begin{array}{c} \sqrt{1-\left[1-[\measuredangle\varkappa^{\daleth}]^2\right]^{\biguplus_{\propto=1}^{k}\varsigma_{\propto}}}, \sqrt{1-\left[1-[\measuredangle\ \backepsilon^{\gimel}]^2\right]^{\biguplus_{\propto=1}^{k}\varsigma_{\propto}}}, \\ \sqrt{\left[1-[\measuredangle\varkappa^{\daleth}]^2\right]^{\biguplus_{\propto=1}^{k}\varsigma_{\propto}} - \left[1-[\measuredangle\varkappa^{\daleth}+\measuredangle\upsilon^{\dashv}]^2\right]^{\biguplus_{\propto=1}^{k}\varsigma_{\propto}}} \end{array}\right] \\ &= \left[\begin{array}{c} \sqrt{1-\left[1-[\measuredangle\varkappa^{\daleth}]^2\right]}, \sqrt{1-\left[1-[\measuredangle\ \backepsilon^{\gimel}]^2\right]}, \\ \sqrt{\left[1-[\measuredangle\varkappa^{\daleth}]^2\right] - \left[1-[\measuredangle\varkappa^{\daleth}+\measuredangle\upsilon^{\dashv}]^2\right]} \end{array}\right] \\ &= \left[\measuredangle\varkappa^{\daleth}, \backepsilon^{\gimel}, \measuredangle\upsilon^{\dashv}\right] = \beth \end{aligned}$$

3.2 Interaction weighted geometric[TPNSIWG] operator

Definition 3.5. Let $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\gimel}, \upsilon_{\propto}^{\Finv} \right\rangle$ be the TPNSNs, $\varsigma_{\propto}$ be the weight of $\beth_{\propto}$. Then the TPNSIWG operator $[\beth_1, \beth_2, ..., \beth_k] = \Diamond_{\propto=1}^{k} \beth_{\propto}^{\varsigma_{\propto}}$.

Theorem 3.6. *If* $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\gimel}, \upsilon_{\propto}^{\Finv} \right\rangle$ *be the TPNSNs. Then,*

$$TPNSIWG[\beth_1, \beth_2, ..., \beth_k] = \begin{bmatrix} \sqrt{\Diamond_{\propto=1}^{k}\Big[1-[\measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{k}\Big[1-[\measuredangle \varkappa_{\propto}^{\daleth} + \measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}}}, \\ \sqrt{1-\Diamond_{\propto=1}^{k}\Big[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\Big]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{k}\Big[1-[\measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}}} \end{bmatrix}$$

Proof. If $\propto= 2$, then TPNSIWG$[\beth_1, \beth_2] = \beth_1^{\varsigma_1} \curlywedge \beth_2^{\varsigma_2}$,
where,

$$\beth_1^{\varsigma_1} = \begin{bmatrix} \sqrt{\Big[1-[\measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1} - \Big[1-[\measuredangle \varkappa_1^{\daleth} + \measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1}}, \\ \sqrt{1-\Big[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2\Big]^{\varsigma_1}}, \sqrt{1-\Big[1-[\measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1}} \end{bmatrix}$$

$$\beth_2^{\varsigma_2} = \begin{bmatrix} \sqrt{\Big[1-[\measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2} - \Big[1-[\measuredangle \varkappa_2^{\daleth} + \measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}}, \\ \sqrt{1-\Big[1-[\measuredangle\ \backepsilon_2^{\gimel}]^2\Big]^{\varsigma_2}} \sqrt{1-\Big[1-[\measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}} \end{bmatrix}$$

We get,

$$\beth_1^{\varsigma_1} \curlywedge \beth_2^{\varsigma_2} = \begin{bmatrix} \sqrt{\begin{array}{r} \Big[1-\Big[1-[\measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1}\Big] + \Big[1-\Big[1-[\measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}\Big] \\ -\Big[1-\Big[1-[\measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1}\Big] \cdot \Big[1-\Big[1-[\measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}\Big] \\ -\Big[\Big[1-[\measuredangle \varkappa_1^{\daleth} + \measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1} \cdot \Big[1-[\measuredangle \varkappa_2^{\daleth} + \measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}\Big] \end{array}} \\ \sqrt{\begin{array}{r} \Big[1-\Big[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2\Big]^{\varsigma_1}\Big] + \Big[1-\Big[1-[\measuredangle\ \backepsilon_2^{\gimel}]^2\Big]^{\varsigma_2}\Big] \\ -\Big[1-\Big[1-[\measuredangle\ \backepsilon_1^{\gimel}]^2\Big]^{\varsigma_1}\Big] \cdot \Big[1-\Big[1-[\measuredangle\ \backepsilon_2^{\gimel}]^2\Big]^{\varsigma_2}\Big] \end{array}} \\ \sqrt{\begin{array}{r} \Big[1-\Big[1-[\measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1}\Big] + \Big[1-\Big[1-[\measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}\Big] \\ -\Big[1-\Big[1-[\measuredangle \upsilon_1^{\Finv}]^2\Big]^{\varsigma_1}\Big] \cdot \Big[1-\Big[1-[\measuredangle \upsilon_2^{\Finv}]^2\Big]^{\varsigma_2}\Big] \end{array}} \end{bmatrix}$$

Hence, TPNSIWG$[\beth_1, \beth_2] = \begin{bmatrix} \sqrt{\Diamond_{\propto=1}^{2}\Big[1-[\measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{2}\Big[1-[\measuredangle \varkappa_{\propto}^{\daleth} + \measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}}}, \\ \sqrt{1-\Diamond_{\propto=1}^{2}\Big[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\Big]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{2}\Big[1-[\measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}}} \end{bmatrix}$

TPNSIWG$[\beth_1, \beth_2, ..., \beth_{\ell}] = \begin{bmatrix} \sqrt{\Diamond_{\propto=1}^{\ell}\Big[1-[\measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}} - \Diamond_{\propto=1}^{\ell}\Big[1-[\measuredangle \varkappa_{\propto}^{\daleth} + \measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}}}, \\ \sqrt{1-\Diamond_{\propto=1}^{\ell}\Big[1-[\measuredangle\ \backepsilon_{\propto}^{\gimel}]^2\Big]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{\ell}\Big[1-[\measuredangle \upsilon_{\propto}^{\Finv}]^2\Big]^{\varsigma_{\propto}}} \end{bmatrix}$

If $\propto = \ell+1$, then TPNSIWG$[\beth_1, ..., \beth_\ell, \beth_{\ell+1}]$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{r} \biguplus_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\right]+\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\right]\cdot\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa^{\daleth}_{\propto}+\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\cdot\left[1-[\measuredangle\varkappa^{\daleth}_{\ell+1}+\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}} \end{array}} \\ \sqrt{\begin{array}{r} \biguplus_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\backepsilon^{\beth}_{\propto}]^2\right]^{\varsigma_\propto}\right]+\left[1-\left[1-[\measuredangle\backepsilon^{\beth}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\backepsilon^{\beth}_{\propto}]^2\right]^{\varsigma_\propto}\right]\cdot\left[1-\left[1-[\measuredangle\backepsilon^{\beth}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \end{array}} \\ \sqrt{\begin{array}{r} \biguplus_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\right]+\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \\ -\Diamond_{\propto=1}^{\ell}\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\right]\cdot\left[1-\left[1-[\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{r} \left[\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\varkappa^{\daleth}_{\propto}+\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\right]\cdot \\ \left[\left[[\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}-[\measuredangle\varkappa^{\daleth}_{\ell+1}+\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}\right] \end{array}}, \\ \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\backepsilon^{\beth}_{\propto}]^2\right]^{\varsigma_\propto}\cdot\left[1-[\measuredangle\backepsilon^{\beth}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}} \\ \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}\cdot\left[1-[\measuredangle\upsilon^{\dashv}_{\ell+1}]^2\right]^{\varsigma_{\ell+1}}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\varkappa^{\daleth}_{\propto}+\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}}, \\ \sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\backepsilon^{\beth}_{\propto}]^2\right]^{\varsigma_\propto}}\sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-[\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^{\varsigma_\propto}} \end{array}\right]$$

Corollary 3.7. *Let $\beth_\propto = \left\langle \varkappa^{\daleth}_{\propto}, \backepsilon^{\beth}_{\propto}, \upsilon^{\dashv}_{\propto}\right\rangle$ be the TPNSNs and all are equal and and $\varkappa^{\daleth}\cdot\upsilon^{\dashv}=0$. Then TPNSIWG$[\beth_1, \beth_2, ..., \beth_k] = \beth$.*

3.3 generalized TPNSIWA [GTPNSIWA] operator

Definition 3.8. Let $\beth_\propto = \left\langle \varkappa^{\daleth}_{\propto}, \varkappa^{\beth}_{\propto}, \upsilon^{\dashv}_{\propto}\right\rangle$ be the TPNSNs, $\varsigma_\propto$ be a weight of $\beth_\propto$. Then, the GTPNSIWA operator $[\beth_1, \beth_2, ..., \beth_k] = \left[\biguplus_{\propto=1}^{k}\varsigma_\propto\beth_\propto^2\right]^{1/2}$.

Theorem 3.9. *Let $\beth_\propto = \left\langle \varkappa^{\daleth}_{\propto}, \varkappa^{\beth}_{\propto}, \upsilon^{\dashv}_{\propto}\right\rangle$ be the TPNSNs. Then GTPNSIWA $[\beth_1, \beth_2, ..., \beth_k] =$*

$$\left[\begin{array}{c} \left[\sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa^{\daleth}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}}\right], \left[\sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\backepsilon^{\beth}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}}\right], \\ \left[\sqrt{\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa^{\daleth}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa^{\daleth}_{\propto}+\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}}\right] \end{array}\right].$$

Proof. First, we have find that

$$\biguplus_{\propto=1}^{k}\varsigma_\propto\beth_\propto^2 = \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa^{\daleth}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}}, \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\backepsilon^{\beth}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa^{\daleth}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa^{\daleth}_{\propto}+\measuredangle\upsilon^{\dashv}_{\propto}]^2\right]^2\right]^{\varsigma_\propto}} \end{array}\right].$$

If $\propto = 2$, then $\varsigma_1 \sqsupset_1^2 = \left[\begin{array}{c} \sqrt{1-\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}, \sqrt{1-\left[1-\left[[\measuredangle\, \backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_1}}, \\ \sqrt{\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1} - \left[1-\left[[\measuredangle\varkappa_1^{\urcorner} + \measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}} \end{array}\right]$

and

$$\varsigma_2 \sqsupset_2^2 = \left[\begin{array}{c} \sqrt{1-\left[1-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}, \sqrt{1-\left[1-\left[[\measuredangle\, \backepsilon_2^{\daleth}]^2\right]^2\right]^{\varsigma_1}}, \\ \sqrt{\left[1-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1} - \left[1-\left[[\measuredangle\varkappa_2^{\urcorner} + \measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}} \end{array}\right].$$

We get, $\varsigma_1 \sqsupset_1 \curlyvee \varsigma_2 \sqsupset_2 =$

$$\left[\begin{array}{c} \sqrt{\begin{array}{c} \left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2 + \left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \cdot \left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \end{array}} \\ \sqrt{\begin{array}{c} \left[\sqrt{1-\left[1-\left[[\measuredangle\, \backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2 + \left[\sqrt{1-\left[1-\left[[\measuredangle\, \backepsilon_2^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{1-\left[1-\left[[\measuredangle\, \backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \cdot \left[\sqrt{1-\left[1-\left[[\measuredangle\, \backepsilon_2^{\daleth}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \end{array}} \\ \sqrt{\begin{array}{c} \left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 + \left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2, \\ -\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \cdot \left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \\ -\left[\begin{array}{c} \left[\sqrt{\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_1} - \left[1-\left[[\measuredangle\varkappa_1^{\urcorner} + \measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \cdot \\ \left[\sqrt{\left[1-\left[[\measuredangle\varkappa_2^{\urcorner}]^2\right]^2\right]^{\varsigma_1} - \left[1-\left[[\measuredangle\varkappa_2^{\urcorner} + \measuredangle\upsilon_2^{\dashv}]^2\right]^2\right]^{\varsigma_1}}\right]^2 \end{array}\right] \end{array}} \end{array}\right]$$

$$= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}, \sqrt{1-\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\, \backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_\propto}}, \\ \sqrt{\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto} - \Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\varkappa_1^{\urcorner} + \measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}} \end{array}\right]$$

In general,

$$= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto}}, \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\, \backepsilon_1^{\daleth}]^2\right]^2\right]^{\varsigma_\propto}} \\ \sqrt{\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_1^{\urcorner}]^2\right]^2\right]^{\varsigma_\propto} - \Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_1^{\urcorner} + \measuredangle\upsilon_1^{\dashv}]^2\right]^2\right]^{\varsigma_\propto}} \end{array}\right].$$

If $\propto = \ell + 1$, then $\biguplus_{\propto=1}^{\ell} \varsigma_\propto \sqsupset_\propto^2 + \varsigma_{\ell+1} \sqsupset_{\ell+1}^2 = \biguplus_{\propto=1}^{\ell+1} \varsigma_\propto \sqsupset_\propto^2$.
Now, $\biguplus_{\propto=1}^{\ell} \varsigma_\propto \sqsupset_\propto^2 + \varsigma_{\ell+1} \sqsupset_{\ell+1}^2 = \varsigma_1 \sqsupset_1^2 \curlyvee \varsigma_2 \sqsupset_2^2 \curlyvee \ldots \curlyvee \varsigma_\ell \sqsupset_\ell^2 \curlyvee \varsigma_{\ell+1} \sqsupset_{\ell+1}^2$

$$= \left[\begin{array}{c} \sqrt{\begin{array}{c} \left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}, \\ -\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2} \end{array}} \\ \sqrt{\begin{array}{c} \left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{\ell+1}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}, \\ -\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{\ell+1}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2} \end{array}} \\ \sqrt{\begin{array}{c} \left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}, \\ -\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2} \\ -\left[\begin{array}{c} \left[\sqrt{\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot \\ \left[\sqrt{\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\daleth}+\measuredangle\upsilon_{\ell+1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2} \end{array}\right] \end{array}} \end{array} \right]$$

$$= \left[\begin{array}{c} \sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{1}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}, \sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\backepsilon_{1}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}, \\ \sqrt{\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{1}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{1}^{\daleth}+\measuredangle\upsilon_{1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}} \end{array} \right]$$

and $\biguplus_{\propto=1}^{\ell+1}\left[\varsigma_{\propto}\beth_{\propto}^{2}\right] = \left[\begin{array}{c} \left[\sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right], \\ \left[\sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right], \\ \left[\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right] \end{array} \right].$

Corollary 3.10. *Let* $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \backepsilon_{\propto}^{\beth}, \upsilon_{\propto}^{\gimel} \right\rangle$ *be the TPNSNs and all are equal. Then GTPNSIWA* $[\beth_1, \beth_2, ..., \beth_k] = \beth$.

3.4 Generalized TPNSIWG [GTPNSIWG] operator

Definition 3.11. Let $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \varkappa_{\propto}^{\beth}, \upsilon_{\propto}^{\gimel} \right\rangle$ be the TPNSNs, $\varsigma_{\propto}]$ be the weight of $\beth_{\propto}$, where $\propto = 1, 2, ..., k$. Then, the GTPNSIWG$[\beth_1, \beth_2, ..., \beth_k] = \frac{1}{\mho}\left[\Diamond_{\propto=1}^{k}[\mho\beth_{\propto}]^{\varsigma_{\propto}}\right]$.

Theorem 3.12. *Let* $\beth_{\propto} = \left\langle \varkappa_{\propto}^{\daleth}, \varkappa_{\propto}^{\beth}, \upsilon_{\propto}^{\gimel} \right\rangle$ *be the collection of TPNSNs.*
Then the GTPNSIWG operator $[\beth_1, \beth_2, ..., \beth_k] =$

$$\left[\begin{array}{c} \left[\sqrt{\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right], \\ \left[\sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\beth}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]\left[\sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right] \end{array} \right]$$

Proof. Using the induction method,

$$\Diamond_{\propto=1}^{k}[\mho\beth_{\propto}]^{\varsigma_{\propto}}=\begin{bmatrix}\sqrt{\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\\ \sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\sqrt{1-\Diamond_{\propto=1}^{k}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\end{bmatrix}$$

If $\propto=2$, then

$$[\mho\beth_{1}]^{\varsigma_{1}}=\begin{bmatrix}\sqrt{\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[1-\left[[\measuredangle\varkappa_{1}^{\daleth}+\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\\ \sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\end{bmatrix}$$

and

$$[\mho\beth_{2}]^{\varsigma_{2}}=\begin{bmatrix}\sqrt{\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[1-\left[[\measuredangle\varkappa_{2}^{\daleth}+\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\\ \sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{2}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\end{bmatrix}$$

We get, $[\mho\beth_{1}]^{\varsigma_{1}}\curlywedge[\mho\beth_{2}]^{\varsigma_{2}}$

$$=\begin{bmatrix}\sqrt{\begin{matrix}\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\begin{bmatrix}\left[\sqrt{\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[1-\left[[\measuredangle\varkappa_{1}^{\daleth}+\varkappa_{1}^{\daleth}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\cdot\\ \left[\sqrt{\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[1-\left[[\measuredangle\varkappa_{2}^{\daleth}+\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{bmatrix}\end{matrix}},\\ \sqrt{\begin{matrix}\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{2}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{2}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{matrix}}\\ \sqrt{\begin{matrix}\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{2}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{matrix}}\end{bmatrix}$$

$$=\begin{bmatrix}\sqrt{\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\\ \sqrt{1-\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\sqrt{1-\Diamond_{\propto=1}^{2}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\end{bmatrix}$$

If $\propto=k$, then

$$=\begin{bmatrix}\sqrt{\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\\ \sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\end{bmatrix}$$

If $\propto=\ell+1$, then $\Diamond_{\propto=1}^{\ell}[\mho\beth_{\propto}]^{\varsigma_{\propto}}\cdot[\mho\beth_{\ell+1}]^{\varsigma_{\ell+1}}=\Diamond_{\propto=1}^{\ell+1}[\mho\beth_{\propto}]^{\varsigma_{\propto}}$.
Now, $\Diamond_{\propto=1}^{\ell}[\mho\beth_{\propto}]^{\varsigma_{\propto}}\cdot[\mho\beth_{\ell+1}]^{\varsigma_{\ell+1}}=[\mho\beth_{1}]^{\varsigma_{1}}\curlywedge[\mho\beth_{2}]^{\varsigma_{2}}\curlywedge\ldots\curlywedge[\mho\beth_{\ell}]^{\varsigma_{\ell}}\curlywedge[\mho\beth_{\ell+1}]^{\varsigma_{\ell+1}}$

$$=\left[\begin{array}{c}\sqrt{\begin{array}{c}\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\begin{array}{c}\left[\sqrt{\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\\ \left[\sqrt{\left[1-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}-\left[1-\left[[\measuredangle\varkappa_{\ell+1}^{\daleth}+\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{array}\right]\end{array}},\\ \sqrt{\begin{array}{c}\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{\ell+1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\backepsilon_{\ell+1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{array}}\\ \sqrt{\begin{array}{c}\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}+\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\\ -\left[\sqrt{1-\Diamond_{\propto=1}^{\ell}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]^{2}\cdot\left[\sqrt{1-\left[1-\left[[\measuredangle\upsilon_{\ell+1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{1}}}\right]^{2}\end{array}}\end{array}\right]$$

$$=\left[\begin{array}{c}\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{1}^{\daleth}+\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\\ \sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\backepsilon_{1}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}},\sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\upsilon_{1}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\end{array}\right]$$

Hence

$$\frac{1}{\mho}\left[\Diamond_{\propto=1}^{\ell+1}[\mho\beth_{\propto}]^{\varsigma_{\propto}}\right]=$$

$$\left[\begin{array}{c}\left[\sqrt{\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\varkappa_{\propto}^{\daleth}+\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right],\\ \left[\sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\backepsilon_{\propto}^{\gimel}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]\left[\sqrt{1-\Diamond_{\propto=1}^{\ell+1}\left[1-\left[[\measuredangle\upsilon_{\propto}^{\dashv}]^{2}\right]^{2}\right]^{\varsigma_{\propto}}}\right]\end{array}\right]$$

Corollary 3.13. *Let* $\beth_{\propto}=\left\langle\varkappa_{\propto}^{\daleth},\backepsilon_{\propto}^{\gimel},\upsilon_{\propto}^{\dashv}\right\rangle$ *be the collection of TPNSNs and all are equal. Then the GTPNSIWG* $[\beth_{1},\beth_{2},\ldots,\beth_{k}]=\beth$.

4 Conclusion

An important benefit of ED and HD for TPNSNs is their algebraic accessibility. Data analysis may be greatly enhanced by HD of TPNSNs. The benefits of HD are demonstrated through the use of powerful statistics. We presented proposed models and examples of TPNSIWA, TPNSIWG, GTPNSIWA, and GTPNSIWG. The following topics will be further discussed in the future: There will be a more thorough discussion of the following subjects: (1) There is a connection between the cubic NS and IVPFS via interaction AOs. (2) Complex TPNSIWA, complex TPNSIWG, complex GTPNSIWA, and complex GTPNSIWG can all be used to fix the problem.

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