Pentapartitioned Neutrosophic Dombi Weighted Aggregation Operator for MADM

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Abstract

This paper introduces the concept of Pentapartitioned Neutrosophic Numbers (PNNs) and proposes score and accuracy functions to effectively rank PNNs based on their score values. Utilizing the adaptability of Dombi operators to accommodate various parameters, the study applies these operators to address complex Multicriteria Attribute Group Decision Making (MAGDM) problems. To achieve precise aggregation under neutrosophic conditions, Dombi T-norm and T-conorm operations for two PNNs are defined. Building upon these Dombi operations, the paper presents two weighted aggregation operators—PNDWAA (Pentapartitioned Neutrosophic Dombi Weighted Arithmetic Average) and PNDWGA (Pentapartitioned Neutrosophic Dombi Weighted Geometric Average)—and investigates their properties within the pentapartitioned neutrosophic environment. Furthermore, the study explores a Multicriteria Attribute Decision Making (MADM) approach that utilizes either the PNDWAA or PNDWGA operators for decision-making.An illustrative example is provided to demonstrate the proposed method, offering a detailed step-by-step process that highlights the effectiveness of the approach in determining the optimal alternative based on ranking order.

Keywords: Pentapartitioned Neutrosophic Set; Score and Accuracy functions; Dombi Weighted Aggregation Operators

1. Introduction

Zadeh introduced fuzzy sets in 1965, allowing elements to have varying degrees of membership within the unit interval [0,1]. In 1983, Atanassov extended this concept by introducing intuitionistic fuzzy sets (IFS), which consider both membership and non-membership degrees. Later, in 1998, Smarandache introduced the neutrosophic set by incorporating an additional component—indeterminacy—alongside truth and falsity membership functions. This framework effectively handles indeterminate and inconsistent information.

In 2010, Wang proposed the Single Valued Neutrosophic Set (SVNS), a generalization of intuitionistic fuzzy sets, interval-valued fuzzy sets, classical sets, and fuzzy sets. Building upon this, Rajashi Chatterjee introduced the Quadripartitioned Single Valued Neutrosophic Set (QSVNS), where the indeterminacy component is divided into two parts: Contradiction (both true and false) and Unknown (neither true nor false). This concept is rooted in Smarandache's *Four Numerical Valued Neutrosophic Logic* and Belnap's *Four-Valued Logic*.

Further advancements came from K. Mohana and M. Mohanasundari, who introduced the Quadripartitioned Single Valued Neutrosophic Rough Set (QSVNRS) by combining rough sets with QSVNS. Subsequently, in 2020, Rama Mallick and Surapathi Pramanik introduced Pentapartitioned Neutrosophic Sets, which extended QSVNS by

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adding an ignorance membership function, based on the concept of Five-Symbol Valued Neutrosophic Logic (FSVNL).

Aggregation operations, such as sum, count, maximum, and minimum, play a critical role in handling numerical tasks within decision-making processes. Multicriteria Attribute Decision Making (MADM) is a widely used approach for selecting the optimal option from a set of alternatives by considering multiple attributes. Researchers have extensively explored correlation coefficients and aggregation operators across various set theories, including fuzzy sets, IFS, SVNS, and QSVNS, to solve MADM problems.

Dombi introduced the Dombi Bonferroni Mean Operator in 1982, known for its excellent flexibility due to the inclusion of a general parameter, making it particularly useful in Multicriteria Attribute Group Decision Making (MAGDM) problems. Building on this, J. Chen and J. Ye investigated Single-Valued Neutrosophic Dombi Weighted Aggregation Operators for solving multiple attribute decision-making problems. This paper is organized as follows:

Section 2 outlines the basic definitions of Pentapartitioned Neutrosophic Sets, score and accuracy functions for Single Valued Neutrosophic Numbers (SVNNs), and the Dombi T-norm and T-conorm operations for two SVNNs, along with their properties. Section 3 and 4 defines the score and accuracy functions for Pentapartitioned Neutrosophic Numbers (PNNs) and introduces the Dombi T-norm and T-conorm operations for two PNNs. Based on these operations, two aggregation operators—PNDWAA (Pentapartitioned Neutrosophic Dombi Weighted Arithmetic Average) and PNDWGA (Pentapartitioned Neutrosophic Dombi Weighted Geometric Average)—are proposed, and their properties are discussed. Section 5 presents a Multicriteria Attribute Decision Making (MADM) method that utilizes the proposed PNDWAA and PNDWGA operators. Section 6 provides an illustrative example to demonstrate the effectiveness and systematic application of the proposed method.

2. Preliminaries

2.1 Pentapartitioned Neutrosophic sets

[1] **Definition 2.1.**

Neutrosophic set is defined over the non-standard unit interval $]^{-}0, 1^{+}[$ whereas single valued neutrosophic set is defined over standard unit interval [0,1]. It means a single valued neutrosophic set A is defined by $A = \{(x, T_A(x), I_A(x), F_A(x)): x \in X\}$ where $T_A(x), I_A(x), F_A(x) : X \to [0,1]$ such that $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

[2] **Definition 2.2.**

Let X be a non-empty set. A Pentapartitioned Neutrosophic set (PNS) A over X characterizes each element in X by a truth-membership function $T_A(x)$, a contradiction membership function $C_A(x)$, an ignorance membership function $G_A(x)$, an unknown membership function $U_A(x)$ and a falsity membership function $F_A(x)$ such that for each $x \in X$, $T_A, C_A, G_A, U_A, F_A \in [0,1]$ and $0 \le T_A(x) + C_A(x) + G_A(x) + U_A(x) + F_A(x) \le 5$ for all $x \in X$.

The set A defined by $A = \{(x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x)): x \in X\}$

Definition 2.3.

The complement of a PNS A is denoted by A^C and is defined as, [3-4]

$A^C = \{(x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x)): x \in X\}$

Definition 2.4.

Consider two PNS A and B, over X. Then A is said to be contained in B, denoted by $A \subseteq B$ iff

$T_A(x) \le T_B(x)$, $C_A(x) \le C_B(x)$, $G_A(x) \ge G_B(x)$, $U_A(x) \ge U_B(x)$, and $F_A(x) \ge F_B(x)$ for all $x \in X$.

Definition 2.5. [5-9]

The union of two PNS A and B is denoted by $A \cup B$ and is defined as,

$A \cup B = \{(x_i, (T_A(x_i) \vee T_B(x_i)), (C_A(x_i) \vee C_B(x_i)), (G_A(x_i) \wedge G_B(x_i)), (U_A(x_i) \wedge U_B(x_i)), (F_A(x_i) \wedge F_B(x_i))) / x_i \in X$ where $i \in I\}$.

Definition 2.6.

The intersection of two PNS A and B is denoted by $A \cap B$ and is defined as, $A \cap B = \{(x_i, (T_A(x_i) \wedge T_B(x_i)), (C_A(x_i) \wedge C_B(x_i)), (G_A(x_i) \vee G_B(x_i)), (U_A(x_i) \vee U_B(x_i)), (F_A(x_i) \vee F_B(x_i))) / x_i \in X$ where $i \in I\}$.

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Definition 2.7.

Let X be a universal set. A *QSVNS N* in X is described by a truth-membership function $t_N(x)$, a contradiction membership function $c_N(x)$, an ignorance membership function $u_N(x)$ and a falsity-membership function $f_N(x)$. Then a *QSVNS N* can be denoted as the following form:

$N = \{(x, t_N(x), c_N(x), u_N(x), f_N(x)):x \in X\}$

where the functions $t_N(x), c_N(x), u_N(x), f_N(x) \in [0,1]$ satisfy the condition $0 \leq t_N(x) + c_N(x)+ u_N(x) + f_N(x) \leq 4$ for

$x \in X$. For convenient expression, a basic element $(x, t_N(x), c_N(x), u_N(x), f_N(x))$ in N is denoted by $s = (t, c, u, f)$ which is called a QSVNN. For any QSVNN $s = (t, c, u, f)$ its score and accuracy functions can be introduced, respectively as follows:

$$E(q) = (3 + t - c - u - f)/4, \qquad E(s) \in [0,1],$$

$$H(q) = t - f, \qquad H(s) \in [-1,1]$$

According to the two functions $E(q)$ and $H(q)$, the comparison and ranking of two QSVNNs are introduced by the following definition.

[10] **Definition 2.8.** Let $q_1 = (t_1, c_1, u_1, f_1)$ and $q_2 = (t_2, c_2, u_2, f_2)$ be two QSVNNs. Then the ranking method for q_1 and q_2 is defined as follows:

(1) If $E(q_1) > E(q_2)$ then $s_1 \succ s_2$

(2) If $E(q_1) = E(q_2)$ and $H(q_1) > H(q_2)$ then $q_1 \succ q_2$,

(3) If $E(q_1) = E(q_2)$ and $H(q_1) = H(q_2)$ then $q_1 = q_2$.

Definition 2.9. Let p and q be any two real numbers. Then, the Dombi T-norm and T-conorm between p and q are defined as follows:

$$O_D(p,q) = \frac{1}{1 + \left\{\left(\frac{1-p}{p}\right)^{\rho} + \left(\frac{1-q}{q}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

$$O_D^C(p,q) = 1 - \frac{1}{1 + \left\{\left(\frac{p}{1-p}\right)^{\rho} + \left(\frac{q}{1-q}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

where $\rho \geq 1$ and $(p,q) \in [0,1] \times [0,1]$.

According to the Dombi T-norm and T-conorm, we define the Dombi operations of QSVNNs.

Definition 2.10. [11-15] Let $q_1 = (t_1, c_1, u_1, f_1)$ and $q_2 = (t_2, c_2, u_2, f_2)$ be two QSVNNs, $\rho \geq 1$, and $\lambda > 0$. Then, the Dombi T-norm and T-conorm operations of QSVNNs are defined below:

(1) $q_1 \oplus q_2 =< 1 - \frac{1}{1+\left\{\left(\frac{t_1}{1-t_1}\right)^{\rho}+\left(\frac{t_2}{1-t_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{c_1}{1-c_1}\right)^{\rho}+\left(\frac{c_2}{1-c_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-u_1}{u_1}\right)^{\rho}+\left(\frac{1-u_2}{u_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-f_1}{f_1}\right)^{\rho}+\left(\frac{1-f_2}{f_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$

(2) $q_1 \otimes q_2 =< \frac{1}{1+\left\{\left(\frac{1-t_1}{t_1}\right)^{\rho}+\left(\frac{1-t_2}{t_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-c_1}{c_1}\right)^{\rho}+\left(\frac{1-c_2}{c_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{u_1}{1-u_1}\right)^{\rho}+\left(\frac{u_2}{1-u_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{f_1}{1-f_1}\right)^{\rho}+\left(\frac{f_2}{1-f_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$

(3) $\lambda q_1 =< 1 - \frac{1}{1+\left\{\lambda\left(\frac{t_1}{1-t_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{c_1}{1-c_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-u_1}{u_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-f_1}{f_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$

(4) $q_1^{\lambda} =< \frac{1}{1+\left\{\lambda\left(\frac{1-t_1}{t_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-c_1}{c_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{u_1}{1-u_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{f_1}{1-f_1}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$

Definition 2.11. Let $q_i = (t_i, c_i, u_i, f_i)$ $(i = 1, 2, ..., n)$ be a collection of QSVNNs and $w = (w_1, w_2, ..., w_n)$ be the weight vector for q_i with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. Then, the QSVNDWAA and QSVNDWGA operators are defined respectively as follows:

QSVNDWAA $(q_1, q_2, \ldots, q_n) = \oplus_{i=1}^{n} w_i q_i$

QSVNDWGA $(q_1, q_2, \ldots, q_n) = \otimes_{i=1}^{n} q_i^{w_i}$

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3. Pentapartitioned Neutrosophhic Dombi Operations

Definition 3.1. For an PNNs $p = (t, c, g, u, f)$ its score and accuracy functions are defined by,

$$E(p) = (3 + t + c - g - u - f)/5, \quad E(p) \in [0,1], \tag{1}$$

$$H(p) = t+c - u - f,\ H(p) \in [-1,1] \tag{2}$$

The following definition defined the comparison and ranking of any two PNNs based on the two functions E(p) and H(p) [14-19].

Definition 3.2. Let $p_1 = (t_1, c_1, g_1, u_1, f_1)$ and $p_2 = (t_2, c_2, g_2, u_2, f_2)$ be two PNNs. Then the ranking method for p_1 and p_2 is defined as follows:

(1) If $E(p_1) > E(p_2)$ then $p_1 \succ p_2$,

(2) If $E(p_1) = E(p_2)$ and $H(p_1) > H(p_2)$ then $p_1 \succ p_2$,

(3) If $E(p_1) = E(p_2)$ and $H(p_1) = H(p_2)$ then $p_1 = p_2$.

[20] **Definition 3.3.** The Dombi T-norm and T-conorm operations of any two PNNs $p_1 = (t_1, c_1, g_1, u_1, f_1)$ and $p_2 = (t_2, c_2, g_2, u_2, f_2)$ are defined as follows:

(1) $$p_1 \oplus p_2 =< 1 - \frac{1}{1+\left\{\left(\frac{t_1}{1-t_1}\right)^\rho + \left(\frac{t_2}{1-t_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{c_1}{1-c_1}\right)^\rho + \left(\frac{c_2}{1-c_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-g_1}{g_1}\right)^\rho + \left(\frac{1-g_2}{g_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-u_1}{u_1}\right)^\rho + \left(\frac{1-u_2}{u_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-f_1}{f_1}\right)^\rho + \left(\frac{1-f_2}{f_2}\right)^\rho\right\}^{\frac{1}{\rho}}} >$$

(2) $$p_1 \otimes p_2 =< \frac{1}{1+\left\{\left(\frac{1-t_1}{t_1}\right)^\rho + \left(\frac{1-t_2}{t_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-c_1}{c_1}\right)^\rho + \left(\frac{1-c_2}{c_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{g_1}{1-g_1}\right)^\rho + \left(\frac{g_2}{1-g_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{u_1}{1-u_1}\right)^\rho + \left(\frac{u_2}{1-u_2}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{f_1}{1-f_1}\right)^\rho + \left(\frac{f_2}{1-f_2}\right)^\rho\right\}^{\frac{1}{\rho}}} >$$

(3) $$\lambda p_1 =< 1 - \frac{1}{1+\left\{\lambda\left(\frac{t_1}{1-t_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{c_1}{1-c_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-g_1}{g_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-u_1}{u_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-f_1}{f_1}\right)^\rho\right\}^{\frac{1}{\rho}}} >$$

(4) $$p_1^\lambda =< \frac{1}{1+\left\{\lambda\left(\frac{1-t_1}{t_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-c_1}{c_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{g_1}{1-g_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{u_1}{1-u_1}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\lambda\left(\frac{f_1}{1-f_1}\right)^\rho\right\}^{\frac{1}{\rho}}} >$$

4. Dombi Weighted Aggregation Operators of PNNs

In this section, we introduce two Dombi weighted aggregation operators PNDWAA and PNDWGA that is based on the Dombi operations of PNNs in Definition 3.3 and studied its properties [21-24].

Definition 4.1. A collection of PNNs is denoted by $p_i = (t_i, c_i, g_i, u_i, f_i)$ $(i = 1, 2, ..., n)$ and $w = (w_1, w_2, ..., w_n)$ be the weight vector for p_i with $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$ Then the PNDWAA and PNDWGA operators are defined as follows.

$$\text{PNDWAA}\ (p_1, p_2, \dots, p_n) = \oplus_{i=1}^n w_i p_i$$

$$\text{PNDWGA}\ (p_1, p_2, \dots, p_n) = \otimes_{i=1}^n p_i^{w_i}$$

Theorem 3.1 A collection of QSVNNs is denoted by $p_i = (t_i, c_i, g_i, u_i, f_i)$ $(i = 1, 2, ..., n)$ and $w = (w_1, w_2, ..., w_n)$ be the weight vector for p_i with $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$ Then the aggregated value of the PNDWAA operator is still a PNN and is calculated by the following formula,

$$\text{PNDWAA}(p_1, p_2, \dots, p_n) = < 1 - \frac{1}{1 + \left\{\sum_{i=1}^n w_i \left(\frac{t_i}{1-t_i}\right)^\rho\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^n w_i \left(\frac{c_i}{1-c_i}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{i=1}^n w_i \left(\frac{1-g_i}{g_i}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{i=1}^n w_i \left(\frac{1-u_i}{u_i}\right)^\rho\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{i=1}^n w_i \left(\frac{1-f_i}{f_i}\right)^\rho\right\}^{\frac{1}{\rho}}} >$$

We can prove this theorem using mathematical induction.

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Proof: When n = 2 by using the Dombi operations of PNNs in Definition (3.3) we can have the following result

$$\mathrm{PNDWAA}(p_1, p_2) = p_1 \oplus p_2$$

$$= < 1 - \frac{1}{1+\left\{w_1\left(\frac{t_1}{1-t_1}\right)^{\rho} + w_2\left(\frac{t_2}{1-t_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{w_1\left(\frac{c_1}{1-c_1}\right)^{\rho} + w_2\left(\frac{c_2}{1-c_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}},$$
$$\frac{1}{1+\left\{w_1\left(\frac{1-g_1}{g_1}\right)^{\rho} + w_2\left(\frac{1-g_2}{g_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{w_1\left(\frac{1-u_1}{u_1}\right)^{\rho} + w_2\left(\frac{1-u_2}{u_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{w_1\left(\frac{1-f_1}{f_1}\right)^{\rho} + w_2\left(\frac{1-f_2}{f_2}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

$$\mathrm{PNDWAA}(p_1, p_2) = p_1 \oplus p_2 = < 1 - \frac{1}{1+\left\{\sum_{i=1}^{2} w_i\left(\frac{t_i}{1-t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{2} w_i\left(\frac{c_i}{1-c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}},$$
$$\frac{1}{1+\left\{\sum_{i=1}^{2} w_i\left(\frac{1-g_i}{g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{2} w_i\left(\frac{1-u_i}{u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{2} w_i\left(\frac{1-f_i}{f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

when $n = k$, Equation (1) becomes,

$$\mathrm{PNDWAA}(p_1, p_2, \dots, p_k) = < 1 - \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{t_i}{1-t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{c_i}{1-c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}},$$
$$\frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-g_i}{g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-u_i}{u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-f_i}{f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

When $n = k + 1$ we have the following result,

$$\mathrm{PNDWAA}(p_1, p_2, \dots, p_k, p_{k+1}) = < 1 - \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{t_i}{1-t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{c_i}{1-c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}},$$
$$\frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-g_i}{g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-u_i}{u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-f_i}{f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$
$$> \oplus\, w_{k+1} p_{k+1}$$

Hence, we proved that Theorem 3.1 is true for $n = k + 1$. Thus, Equation (1) is true for all n.

The operator PNDWAA satisfies the following properties.

(1) Reducibility: If $w = (1/n, 1/n, \dots, 1/n)$, then it is obvious that there exits,

$$\mathrm{PNDWAA}(p_1, p_2, \dots, p_n) = < 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{t_i}{1-t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{c_i}{1-c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}},$$
$$\frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{1-g_i}{g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{1-u_i}{u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{1-f_i}{f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

(2) Idempotency: Let all the PNNs be denoted by $p_i = (t_i, c_i, g_i, u_i, f_i)$ =p (i =1, 2, ..., n).

Then PNDWAA $(p_1, p_2, \dots, p_n) = p$.

(3) Commutativity: Let any PNS $(p'_1, p'_2, \dots, p'_n)$ be any permutation of $(p_1, p_2, \dots, p_n)$. Then there is PNDWAA $(p'_1, p'_2, \dots, p'_n)$ = PNDWAA $(p_1, p_2, \dots, p_n)$.

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(4) Boundedness: Let $p_{min} = min\,(p_1, p_2, \dots, p_n)$ and $p_{max} = max\,(p_1, p_2, \dots, p_n)$. Then $p_{min} \le PNDWAA(p_1, p_2, \dots, p_n) \le p_{max}$

Proof: (1) Given $p_i = (t_i, c_i, g_i, u_i, f_i)$ =p (i =1, 2, ..., n) Property (1) is trivially true based on equation (3)

(2) The following result is derived from the equation (3) and we get,

$$\text{PNDWAA}(p_1, p_2, \dots, p_n) = < 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{t_i}{1-t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{c_i}{1-c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-g_i}{g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-u_i}{u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-f_i}{f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

$$=< 1 - \frac{1}{1+\left\{\left(\frac{t}{1-t}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1+\left\{\left(\frac{c}{1-c}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-g}{g}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-u}{u}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\left(\frac{1-f}{f}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

PNDWAA $(p_1, p_2, \dots, p_n) = p$ holds.

(3) This property is obvious.

(4) Consider $p_{min} = min(p_1, p_2, \dots, p_n) = (t^-, c^-, g^-, u^-, f^-)$ and $p_{max} = max(p_1, p_2, \dots, p_n) = (t^+, c^+, g^+, u^+, f^+)$ Then,

$$t^- = \min_i(t_i), c^- = \min_i(c_i), g^- = \max_i(g_i), u^- = \max_i(u_i), f^- = \max_i(f_i)$$

$$t^+ = \max_i(t_i), c^+ = \max_i(c_i), g^+ = \min_i(g_i), u^+ = \min_i(u_i), f^+ = \min_i(f_i)$$

Therefore, we get the following inequalities.

$$1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{t^-}{1-t^-}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{t_i}{1-t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{t^+}{1-t^+}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

$$1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{c^-}{1-c^-}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{c_i}{1-c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le 1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{c^+}{1-c^+}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

$$\frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-g^-}{g^-}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-g_i}{g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-g^+}{g^+}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

$$\frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-u^-}{u^-}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-u_i}{u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-u^+}{u^+}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

$$\frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-f^-}{f^-}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-f_i}{f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \le \frac{1}{1+\left\{\sum_{i=1}^{n} w_i \left(\frac{1-f^+}{f^+}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

Hence $p_{min} \le PNDWAA(p_1, p_2, \dots, p_n) \le p_{max}$ holds.

[25] **Theorem 3.2** A collection of PNNs is denoted by $p_i = (t_i, c_i, g_i, u_i, f_i)$ (i =1, 2, ..., n) and $w = (w_1, w_2, \dots, w_n)$ be the weight vector for p_i with $w_j \in [0,1]$ and $\sum_{i=1}^{n} w_i$. Then the aggregated value of the PNDWGA operator is still a PNN and is calculated by the following formula

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$$\mathrm{PNDWGA}(p_1, p_2, \dots, p_n) = < \frac{1}{1 + \left\{\sum_{i=1}^{n} w_i \left(\frac{1-t_i}{t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n} w_i \left(\frac{1-c_i}{c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} w_i \left(\frac{g_i}{1-g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} w_i \left(\frac{u_i}{1-u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} w_i \left(\frac{f_i}{1-f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

The proof is similar to the proof of Theorem (3.1).

This PNDWGA operator also satisfies the following properties.

(1) Reducibility: If $w = (1/n, 1/n, \dots, 1/n)$, then it is obvious that there exits,

$$\mathrm{PNDWGA}(p_1, p_2, \dots, p_n) = < \frac{1}{1 + \left\{\sum_{i=1}^{n} \frac{1}{n} \left(\frac{1-t_i}{t_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{i=1}^{n} \frac{1}{n} \left(\frac{1-c_i}{c_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \frac{1}{n} \left(\frac{g_i}{1-g_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \frac{1}{n} \left(\frac{u_i}{1-u_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{i=1}^{n} \frac{1}{n} \left(\frac{f_i}{1-f_i}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

(2) Idempotency: Let all the PNNs be denoted by $p_i = (t_i, c_i, g_i, u_i, f_i) = p$ ($i = 1, 2, \dots, n$).

Then PNDWGA $(p_1, p_2, \dots, p_n) = p$.

(3) Commutativity Let any PNS $(p'_1, p'_2, \dots, p'_n)$ be any permutation of $(p_1, p_2, \dots, p_n)$. Then there is PNDWGA $(p'_1, p'_2, \dots, p'_n)$ = PNDWGA $(p_1, p_2, \dots, p_n)$.

(4) Boundedness: Let $p_{min} = min\ (p_1, p_2, \dots, p_n)$ and $p_{max} = max\ (p_1, p_2, \dots, p_n)$. Then $p_{min} \leq PNDWGA\ (p_1, p_2, \dots, p_n) \leq p_{max}$.

To prove the above properties it is similar to the operator properties of PNDWAA. Hence, it is not repeated here.

5. MADM method using PNDWAA operator or PNDWGA operator

This section deals about the MADM method to handle the MADM problems effectively with PNN information by using the PNDWAA operator or PNDWGA operator [26-30]. Let $A = \{A_1, A_2, \dots, A_m\}$ and $C = \{C_1, C_2, \dots, C_n\}$ be a discrete set of alternatives and attributes respectively. The weight vector of the above attributes is given by $w = \{w_1, w_2, \dots, w_n\}$ such that $w_i \in [0,1]$ and $\sum_{j=1}^{n} w_j$.

To make a better decision to choose the alternative A_i ($i = 1,2,\dots,m$), a decision maker needs to analyse the attributes C_j ($j = 1,2,\dots,n$) by the PNN $p_{ij} = (t_{ij}, c_{ij}, g_{ij}, u_{ij}, f_{ij})$($i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$) then we get a PNN decision matrix $D = (d_{ij})_{m \times n}$

The following decision steps are needed to handle the MADM problems under PNN information by using the operator PNDWAA or PNDWGA.

Step 1: Collect the PNN p_i($i = 1, 2, \dots, m$) for the given alternative A_i($i = 1, 2, \dots, m$) by using the operator PNDWAA

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$$p_i = \text{PNDWAA}(p_{i1}, p_{i2}, \ldots, p_{in}) = < 1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{t_{ij}}{1 - t_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{c_{ij}}{1 - c_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{1 - g_{ij}}{g_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{1 - u_{ij}}{u_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{1 - f_{ij}}{f_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

or by using PNDWGA operator

$$p_i = \text{PNDWGA}(p_{i1}, p_{i2}, \ldots, p_{in}) = < \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{1 - t_{ij}}{t_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{1 - c_{ij}}{c_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{g_{ij}}{1 - g_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{u_{ij}}{1 - u_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, 1 - \frac{1}{1 + \left\{\sum_{j=1}^{n} w_j \left(\frac{f_{ij}}{1 - f_{ij}}\right)^{\rho}\right\}^{\frac{1}{\rho}}} >$$

where $w = (w_1, w_2, \ldots, w_n)$ is the weight vector such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j$

Step 2: Score values $E(p_i)$ can be calculated by using Equation (1) with the collective

PNN p_i $(i = 1, 2, \ldots, m)$

Step 3: Select the best one according to rank given to the alternatives.

6. Illustrative Example

This section presents an illustrative example of a MADM problem focused on evaluating the performance of detergents in cleaning fuel injectors within a Pentapartitioned Neutrosophic Number (PNN) environment. In this study, three detergents, denoted as A_1, A_2, A_3, are considered as alternatives. The evaluation is based on four attributes, C_1, C_2, C_3, C_4, which represent the performance of the detergents on four different engines. The alternatives corresponding to the given attributes are expressed in the form of PNNs. When three alternatives under four attributes are evaluated. we get a Pentapartitioned Neutrosophic decision matrix $D = (p_{ij})_{m\times n}$ where $p_{ij} = (t_{ij}, c_{ij}, g_{ij}, u_{ij}, f_{ij})(i = 1, 2, 3; j = 1, 2, 3, 4)$ which is given below.

$$D = \begin{pmatrix} (0.5, 0.6, 0.8, 0.2, 0.1) & (0.6, 0.5, 0.6, 0.6, 0.1) & (0.3, 0.4, 0.5, 0.6, 0.3) & (0.6, 0.6, 0.5, 0.3, 0.1) \\ (0.5, 0.6, 0.4, 0.3, 0.3) & (0.6, 0.5, 0.8, 0.4, 0.6) & (0.4, 0.5, 0.3, 0.4, 0.4) & (0.3, 0.4, 0.5, 0.4, 0.2) \\ (0.7, 0.6, 0.5, 0.4, 0.4) & (0.7, 0.5, 0.4, 0.6, 0.7) & (0.7, 0.9, 0.4, 0.4, 0.5) & (0.6, 0.4, 0.5, 0.6, 0.6) \end{pmatrix}$$

The weight vector for the above four attributes is given as $w = (0.27, 0.2, 0.3, 0.33)$. Hence the proposed operator of PNDWAA (or) PNDWGA are used here to solve MADM problem under PNN information.

The following steps are needed to solve MADM problem when we use the operator PNDWAA.

Step 1: By using Equation (1) for $\rho = 1$ derive the collective PNNs of p_i for the alternative A_i $(i = 1,2,3)$ which is given below.

$p_1 = (0.62, 0.4256, 0.5268, 0.247, 0.101)$

$p_2 = (0.4003, 0.3125, 0.4166, 0.3017, 0.274)$,

$p_3 = (0.675, 0.854, 0.2529, 0.427, 0.474)$

Step 2: Score values $E(p_i)$ can be calculated by using Equation (1) of the collective PNN $p_i(i = 1,2,3)$ for the alternatives $A_i(i = 1,2,3)$ gives the following results.

$E(p_1) = 0.642$, $E(p_2) = 0.529$, $E(p_3) = 0.7267$

Step 3: The ranking order is given according to the obtained score values

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$p_3 > p_1 > p_2$ and the best one is p_3.

The same MADM problem can also be solved by using another proposed operator that is PNDWGA. The following steps are needed to solve the MADM problem.

Step 1: By using Equation (4) for ρ = 1 derive the collective PNNs of p_i for the alternative $A_i(i = 1,2,3)$ which is given below.

$p_1 = (0.375, 0.408, 0.531, 0.292, 0.13)$

$p_2 = (0.316, 0.382, 0.56, 0.371, 0.431)$,

$p_3 = (0.651, 0.478, 0.338, 0.276, 0.407)$

Step 2: Score values $E(p_i)$ can be calculated by using Equation (1) of the collective PNN $p_i(i = 1,2,3)$ for the alternatives $A_i(i = 1,2,3)$ gives the following results.

$E(p_1) = 0.5659$, $E(p_2) = 0.4672$, $E(p_3) = 0.6215$

Step 3: The ranking order is given according to the obtained score values

$p_3 > p_1 > p_2$ and the best one is p_3.

The following Table 1 and 2 shows the ranking results for the parameters of $\rho \in [1,10]$ of the Pentapartitioned Neutrosophic Dombi weighted arithmetic average (PNDWAA) operator and Pentapartitioned Neutrosophic Dombi weighted geometric average (PNDWGA) operator respectively [31-33].

We can observe the following results from Tables 1 and 2.

1) Different aggregation operators that are PNDWAA and PNDWGA shows different ranking orders. However, the ranking orders due to different operational parameters are same according to the one operator. This results that the operational parameter ρ is not sensitive in this decision-making problem since we get the same ranking orders corresponding to the PNDWAA and PNDWGA operator.

   $E(p_1) = 0.642$, $E(p_2) = 0.529$, $E(p_3) = 0.7267$

Table 1: Ranking results of the operator PNDWAA for different operational parameters.

P	$E(p_1)$	$E(p_2)$	$E(p_3)$	**Ranking Order**
1	0.6420	0.7260	0.5290	$p_2 > p_1 > p_3$
2	0.5598	0.7951	0.5076	$p_2 > p_1 > p_3$
3	0.6238	0.8032	0.5476	$p_2 > p_1 > p_3$
4	0.6731	0.8406	0.5952	$p_2 > p_1 > p_3$
5	0.6221	0.8933	0.6279	$p_2 > p_1 > p_3$
6	0.7736	0.8645	0.6636	$p_2 > p_1 > p_3$
7	0.6054	0.8404	0.6098	$p_2 > p_1 > p_3$
8	0.6088	0.8252	0.61046	$p_2 > p_1 > p_3$
9	0.6270	0.8154	0.60145	$p_2 > p_1 > p_3$
10	0.6442	0.8235	0.6193	$p_2 > p_1 > p_3$

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Table 2: Ranking results of the operator PNDWGA for different operational parameters.

ρ	$E(p_1)$	$E(p_2)$	$E(p_3)$	**Ranking Order**
1	0.5659	0.6215	0.4672	$p_2 > p_1 > p_3$
2	0.5824	0.5709	0.4501	$p_2 > p_1 > p_3$
3	0.5269	0.5495	0.4368	$p_2 > p_1 > p_3$
4	0.5127	0.5833	0.4109	$p_2 > p_1 > p_3$
5	0.5145	0.5168	0.4105	$p_2 > p_1 > p_3$
6	0.4386	0.5037	0.4797	$p_2 > p_1 > p_3$
7	0.4951	0.5013	0.4719	$p_2 > p_1 > p_3$
8	0.4161	0.4915	0.31683	$p_2 > p_1 > p_3$
9	0.4717	0.5664	0.3638	$p_2 > p_1 > p_3$
10	0.4552	0.4928	0.3922	$p_2 > p_1 > p_3$

1. The ranking orders obtained using the PNDWAA and PNDWGA operators are different.
2. The ranking results remain unaffected by variations in the operational parameter ρ∈[0,1]\rho \in [0,1]ρ∈[0,1] for both operators, indicating that ρ\rhoρ is not sensitive in this decision-making problem.
3. The proposed aggregation methods using the PNDWAA and PNDWGA operators offer a novel approach to solving MADM problems within a Pentapartitioned Neutrosophic Number (PNN) environment.

7. Conclusion

In this paper, we introduced and analyzed the properties of two weighted aggregation operators, namely the Pentapartitioned Neutrosophic Dombi Weighted Arithmetic Average (PNDWAA) and the Pentapartitioned Neutrosophic Dombi Weighted Geometric Average (PNDWGA), which are formulated using Dombi T-norm and T-conorm operations. These operators are designed to address problems within the domain of multiple attribute decision making (MADM), a crucial area for selecting the optimal solution from a finite set of alternatives.

To tackle MADM challenges, ranking methods play a central role in determining the best option based on the given attributes. This paper focuses on an MADM approach that leverages the proposed PNDWAA and PNDWGA operators within the Pentapartitioned Neutrosophic Number (PNN) framework. These aggregation operators facilitate the calculation of score functions for various alternatives, enabling the ranking and selection of the most appropriate option. Furthermore, we provide a detailed illustrative example of an MADM problem to display the application and effectiveness of the proposed operators in practice.

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