Advances in Hypothesis Testing with Neutrosophic Sets: A Framework for Uncertainty

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Abstract

In this paper, we extend statistical hypothesis testing to data represented as neutrosophic sets, encompassing membership, indeterminacy, and non-membership components. Two distinct scenarios are considered: first, where all three components are independent, and second, where all three components are dependent.

Keywords: Neutrosophic sets; Statistical hypothesis testing; Means; Decision-making

1 introduction

In 1965, Zadeh[9] initially proposed the concept of a fuzzy set. A fuzzy set is a class of objects with a continuum of grades of membership assigned to each object, where the grade of membership ranges between zero and one. Fuzzy sets are useful for describing ambiguity or vagueness within a given domain.[1] The original concept of fuzzy set theory was later generalized to various other types of fuzzy sets. One popular generalization, known as intuitionistic fuzzy sets, was proposed by Atanassov.[4] An intuitionistic fuzzy set (IFS) specifies both membership and non-membership functions, providing a more comprehensive way to handle uncertainty. A further generalizations of fuzzy sets and intuitionistic fuzzy sets is the neutrosophic set, introduced by Smarandache in 1995.[6] A neutrosophic set primarily consists of three components: membership, indeterminacy, and non-membership.[6,7] This structure enables neutrosophic logic to quantify the proportion of each component within a given context,[8] making neutrosophic sets particularly relevant for decision-making scenarios.
Hypothesis testing is a crucial area of inferential statistics, where researchers make decisions on whether to reject or fail to reject the null hypothesis. This concept extends naturally to fuzzy sets and intuitionistic fuzzy sets, where uncertainty and vagueness are incorporated into the decision-making process. As a result, many studies have explored hypothesis testing for fuzzy sets and intuitionistic fuzzy sets. For instance, Casals, Gil, and Gil,[5] as well as Atalik,[3] investigated the process of testing statistical hypotheses when the outcomes involve fuzzy information. Additionally, hypothesis testing with respect to intuitionistic fuzzy sets has been investigated by many researchers, including Akbari and Hesamian[2] To extend the concepts of generalizations of fuzzy sets and intuitionistic fuzzy sets, we propose a method for hypothesis testing for neutrosophic sets in this paper. We consider two scenarios: first, when all three components (membership, indeterminacy, and non-membership) are independent, and second, when all three components are dependent.

2 Basic Concepts

This section presents key definitions central to the study, providing a foundational understanding of essential concepts relevant to the analysis.

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2.1 Intuitionistic fuzzy sets

Definition 2.1. *Let S be a nonempty set. A* fuzzy set *(FS) $\mathcal{A}$, derived from the set S, is defined as:*

$$\mathcal{A} = \{< x, \mu_A(x) >| \ x \in S\}$$

where $\mu_A : S \to [0,1]$ represents the membership function of the fuzzy set $\mathcal{A}$.

Definition 2.2. *Let S be a nonempty set. An* intuitionistic fuzzy set *(IFS) $\mathcal{A}$ in the set S is defined as:*

$$\mathcal{A} = \{< x, \mu_A(x), \nu_A(x) >| \ x \in S\}$$

where the functions $\mu_A : S \to [0,1]$ and $\nu_A : S \to [0,1]$ represent, respectively, the degree of membership and the degree of non-membership of each element x in the set $\mathcal{A}$, a subset of S. For every element $x \in S$, the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ must hold.

For any FS $\mathcal{A} = \{< x, \mu_A(x) >| \ x \in S\}$, we can define $\nu_A(x) = 1 - \mu_A(x)$, resulting in the set $\{< x, \mu_A(x), \nu_A(x) >| \ x \in S\}$, which forms an IFS. Thus, every FS can be considered as a special case of an IFS.

Definition 2.3. *Let S be a nonempty set. A* neutrosophic set *$\mathcal{A}$, derived from the set S, is defined as follows:*

$$\mathcal{A} = \{< x, \mathcal{T}_A(x), I_A(x), F_A(x) >| \ x \in S\}$$

where $\mathcal{T}_A : S \to [0,1], I_A : S \to [0,1]$ and $F_A : S \to [0,1]$ represent, respectively, the membership, indeterminacy and non-membership functions of the neutrosophic set $\mathcal{A}$.
The functions $\mathcal{T}(x)$, $I(x)$, $F(x)$ are collectively referred to as the neutrosophic components.
For single-valued logic ($\mathcal{T}(x)$, $I(x)$,$F(x)$), the sum of the components satisfies:

$$1 \leq \mathcal{T}(x) + I(x) + F(x) \leq 3$$

assuming that all three components are independent.
If two components are dependent while the third is independent of them, the sum of the components then satisfies:

$$0 \leq \mathcal{T}(x) + I(x) + F(x) \leq 2.$$

Finally, assuming all three components are dependent, the sum satisfies:

$$0 \leq \mathcal{T}(x) + I(x) + F(x) \leq 1.$$

3 Independent T-Test

In this section, we propose tests to examine the difference between the means of two independent populations with small sample sizes, applying concepts related to the neutrosophic set.

Definition 3.1. *Let $\{x_1, x_2, \ldots, x_n\}$ be a random sample of size n from a crisp set X with membership, indeterminacy and non-membership functions $\mathcal{T}_A(x_i)$, $I_A(x_i)$ and $F_A(x_i)$ respectively, associated with the neutrosophic set $\mathcal{A}$. The sample means of the membership, indeterminacy and non-membership of the neutrosophic set $\mathcal{A}$, denoted by $\overline{\mathcal{T}}_A(X)$, $\overline{I}_A(X)$ and $\overline{F}_A(X)$, are defined as follows:*

$$\overline{\mathcal{T}}_A(X) = \frac{\sum_{i=1}^{n} \mathcal{T}_A(x_i)}{n} \overline{I}_A(X) = \frac{\sum_{i=1}^{n} I_A(x_i)}{n} \text{ and } \overline{F}_A(X) = \frac{\sum_{i=1}^{n} F_A(x_i)}{n}.$$

Definition 3.2. *Let $\{x_1, x_2, \ldots, x_n\}$ be a random sample of size n from a crisp set X with membership, indeterminacy and non-membership functions $\mathcal{T}_A(x_i)$, $I_A(x_i)$ and $F_A(x_i)$ respectively, associated with the* *neutrosophic set $\mathcal{A}$. The sample variance of the membership, indeterminacy and non-membership functions of the neutrosophic set $\mathcal{A}$, denoted by $(S_{\mathcal{T}}(X))^2$, $(S_I(X))^2$ and $(S_F(X))^2$respectively defined as follows:*

$$(S_{\mathcal{T}}(X))^2 = \frac{\sum_{i=1}^{n}(\mathcal{T}_A(x_i) - \overline{\mathcal{T}}_A(X))^2}{n-1}$$

$$(S_I(X))^2 = \frac{\sum_{i=1}^{n}(I_A(x_i) - \overline{I}_A(X))^2}{n-1}.$$

and

$$(S_F(X))^2 = \frac{\sum_{i=1}^{n}(F_A(x_i) - \overline{F}_A(X))^2}{n-1}.$$

Next, we present the methods used to address the main problem.

3.1 Significance Testing for the Difference Between Means of Two Independent Populations with Small Sample Sizes Using Neutrosophic Set Concepts

Let X and Y be two crisp populations, and let $\mathcal{A}$ be a neutrosophic set defined on X and Y. Consider a random sample $\{x_1, x_2, \ldots, x_m\}$ of size m drawn from X, with corresponding membership, indeterminacy and non-membership values $\mathcal{T}_A(x_i)$, $I_A(x_i)$, and $F_A(x_i)$, respectively for each $i = 1, 2, \ldots, m$. Similarly, consider a random sample $\{y_1, y_2, \ldots, y_n\}$ of size n drawn from Y, with corresponding membership, indeterminacy and non-membership values $\mathcal{T}_A(y_i)$, $I_A(y_i)$, and $F_A(y_i)$, respectively, for each $i = 1, 2, \ldots, n$. Suppose that $\mathcal{T}_A(x_i)$, $I_A(x_i)$ and $F_A(x_i)$ are normally distributed. Based on these samples, we aim to test that the following hypotheses:

1. The mean membership of population X with respect to $\mathcal{A}$, denoted by $\overline{\mathcal{T}}(A, X)$, is equal to the mean membership of population Y with respect to $\mathcal{A}$, denoted by $\overline{\mathcal{T}}(A, Y)$.
2. The mean indeterminacy of population X with respect to $\mathcal{A}$, denoted by $\overline{I}(A, X)$, is equal to the mean indeterminacy of population Y with respect to $\mathcal{A}$, denoted by $\overline{I}(A, Y)$
3. The mean non-membership of population X with respect to $\mathcal{A}$, denoted by $\overline{F}(A, X)$, is equal to the mean non-membership of population Y with respect to $\mathcal{A}$, denoted by $\overline{F}(A, Y)$

We now define the null hypothesis H_0 as follows:

$$H_0 : \overline{\mathcal{T}}(A, X) = \overline{\mathcal{T}}(A, Y)$$

which states that the mean membership of population X with respect to $\mathcal{A}$ is equal to the mean membership of population Y with respect to $\mathcal{A}$.

We similarly define the null hypothesis H_0 as follows:

$$H_0 : \overline{I}(A, X) = \overline{I}(A, Y)$$

which states that the mean indeterminacy of population X with respect to $\mathcal{A}$ is equal to the mean indeterminacy of population Y with respect to $\mathcal{A}$.
We finally define the null hypothesis H_0 as follows:

$$H_0 : \overline{F}(A, X) = \overline{F}(A, Y)$$

which states that the mean non-membership of population X with respect to $\mathcal{A}$ is equal to the mean non-membership of population Y with respect to $\mathcal{A}$.

If both populations have equal variances with respect to $\mathcal{A}$, we use the following test statistics for testing the null hypotheses:

$$t_{\mathcal{T}} = \frac{\overline{\mathcal{T}}_A(X) - \overline{\mathcal{T}}_A(Y)}{S_{\mathcal{T}}\sqrt{1/n + 1/m}},$$

$$t_I = \frac{\overline{I}_A(X) - \overline{I}_A(Y)}{S_I\sqrt{1/n + 1/m}},$$

$$t_F = \frac{\overline{F}_A(X) - \overline{F}_A(Y)}{S_F\sqrt{1/n + 1/m}},$$

where the pooled standard deviations $S_{\mathcal{T}}$, S_I andS_F of the membership, indeterminacy, and non-membership components, respectively, are given by:

$$S_{\mathcal{T}} = \sqrt{\frac{(m-1)(S_{\mathcal{T}}(X))^2 + (n-1)(S_{\mathcal{T}}(Y))^2}{m+n-2}}$$

,

$$S_I = \sqrt{\frac{(m-1)(S_I(X))^2 + (n-1)(S_I(Y))^2}{m+n-2}}.$$

and

$$S_F = \sqrt{\frac{(m-1)(S_F(X))^2 + (n-1)(S_F(Y))^2}{m+n-2}}.$$

Here, $S_{\mathcal{T}}(X)$, $S_I(X)$, $S_F(X)$, $S_{\mathcal{T}}(Y)$, $S_I(Y)$, and $S_F(Y)$ denote the sample standard deviations for the membership, indeterminacy, and non-membership components, in populations X and Y, respectively. If the variances of the two populations with respect to $\mathcal{A}$ are unequal, we use the following test statistics for testing the null hypotheses:

$$t_{\mathcal{T}} = \frac{\overline{\mathcal{T}}_A(X) - \overline{\mathcal{T}}_A(Y)}{\sqrt{(S_{\mathcal{T}}(X))^2/n + (S_{\mathcal{T}}(Y))^2/m}},$$

$$t_I = \frac{\overline{I}_A(X) - \overline{I}_A(Y)}{\sqrt{(S_I(X))^2/n + (S_I(Y))^2/m}},$$

$$t_F = \frac{\overline{F}_A(X) - \overline{F}_A(Y)}{\sqrt{(S_F(X))^2/n + (S_F(Y))^2/m}},$$

where $S_{\mathcal{T}}(X)$, $S_I(X)$, $S_F(X)$, $S_{\mathcal{T}}(Y)$, $S_I(Y)$, and $S_F(Y)$ denote the sample standard deviations for the membership, indeterminacy, and non-membership components, in populations X and Y, respectively. We define $t = \max\{|t_{\mathcal{T}}|, |t_I|, |t_F|\}$, and let $t_{\alpha,df}$ denote the critical value of t for df degrees of freedom at the significance level α.

If both populations have equal variances, the degree of freedom used in this test are given by $df = m + n - 2$. However, if the variances of the two populations with respect to $\mathcal{A}$ are unequal, the degree of freedom are calculated as:

$$df = \frac{\left(\frac{(S_{\mathcal{T}}(X))^2}{n} + \frac{(S_{\mathcal{T}}(Y))^2}{m}\right)^2}{\frac{\left(\frac{(S_{\mathcal{T}}(X))^2}{n}\right)^2}{n-1} + \frac{\left(\frac{(S_{\mathcal{T}}(Y))^2}{m}\right)^2}{m-1}}$$

For a significance level α, the critical regions for the alternative hypothesis, H_A are as follows:

Alternative Hypothesis	Critical Region
$\overline{\mathcal{T}}(A,X) > \overline{\mathcal{T}}(A,Y)$(upper-tailed test)	$t \geq t_{\alpha,df}$
$\overline{\mathcal{T}}(A,X) < \overline{\mathcal{T}}(A,Y)$(lower-tailed test)	$t \leq -t_{\alpha,df}$
$\overline{\mathcal{T}}(A,X) \neq \overline{\mathcal{T}}(A,Y)$(two-tailed test)	$\lvert t\rvert \geq t_{\alpha/2,df}$

Alternative Hypothesis	Critical Region
$\overline{I}(A,X) > \overline{I}(A,Y)$(upper-tailed test)	$t \geq t_{\alpha,df}$
$\overline{I}(A,X) < \overline{I}(A,Y)$(lower-tailed test)	$t \leq -t_{\alpha,df}$
$\overline{I}(A,X) \neq \overline{I}(A,Y)$(two-tailed test)	$\lvert t\rvert \geq t_{\alpha/2,df}$

Alternative Hypothesis	Critical Region
$\overline{F}(A,X) > \overline{F}(A,Y)$(upper-tailed test)	$t \geq t_{\alpha,df}$
$\overline{F}(A,X) < \overline{F}(A,Y)$(lower-tailed test)	$t \leq -t_{\alpha,df}$
$\overline{F}(A,X) \neq \overline{F}(A,Y)$(two-tailed test)	$\|t\| \geq t_{\alpha/2,df}$

If $|t| \leq |t_{\alpha,df}|$ in a one-tailed test, we cannot reject the null hypothesis. This result indicates insufficient evidence to conclude that $\overline{\mathcal{T}}(A,X)$ is greater than $\overline{\mathcal{T}}(A,Y)$(for an upper-tailed test) or that $\overline{\mathcal{T}}(A,X)$ is less than $\overline{\mathcal{T}}(A,Y)$ (for a lower-tailed test) at the α significance level.
On the other hand, if the test statistic falls outside this range, we reject the null hypothesis, which indicates that $\overline{\mathcal{T}}(A,X) > \overline{\mathcal{T}}(A,Y)$ (for an upper-tailed test) or $\overline{\mathcal{T}}(A,X) < \overline{\mathcal{T}}(A,Y)$ (for a lower- tailed test).

If $|t| \leq |t_{\alpha/2,df}|$ in a two-tailed test, there is insufficient evidence to conclude that the difference between $\overline{\mathcal{T}}(A,X)$ and $\overline{\mathcal{T}}(A,Y)$ is significant at α level. Thus, we do not reject the null hypothesis.
However, if $|t| > |t_{\alpha/2,df}|$ we reject the null hypothesis, indicating that the population means of the membership, indeterminacy, and non-membership components of $\mathcal{A}$ are different.
The conclusions for the other alternative hypotheses, H_A are similar.
We now proceed with the following examples to illustrate the application of these testing methods. The first example demonstrates the case when all three components are independent, while the second example addresses the case when all three components are dependent.

Example 3.3. Let X and Y represent two populations, where $X$is the set of all male students in the final year of high school in Hat Yai, and Y is the set of all female students in the final year of high school in Hat Yai.
We define the neutrosophic set $\mathcal{A}$ as follows: $\mathcal{T}$ represents the membership function indicating students' intent to apply for admission to Prince of Songkla University. Additionally, I represents the indeterminacy function reflecting students' uncertainty about applying specifically to the Faculty of Science at Prince of Songkla University. Finally, F represents the non-membership function, indicating students who do not intend to apply for admission to the Mathematics major within the Faculty of Science at Prince of Songkla University. This neutrosophic set is defined on the populations X and Y.
It is assumed that $\mathcal{T}_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $\mathcal{T}_A(y_i)$, $I_A(y_i)$, and $F_A(y_i)$, are normally distributed for each sample in both populations.
We now proceed to test whether the membership, indeterminacy, and non-membership functions, which represent the goals of the two populations, are similar or different.
Let $S_1 = \{x_1, x_2, x_3, x_4, x_5\}$ be the sample of size five taken from the population of male students in the final year of a high school in Hat Yai (population X) and $S_2 = \{y_1, y_2, y_3, y_4, y_5\}$ be the sample of size five taken from the population of female students in the final year of the same high school in Hat Yai (population Y).

The membership, indeterminacy, and non-membership values for the two given samples, based on their information concerning the neutrosophic set $\mathcal{A}$, are provided below.
Using the numerical example provided below, we illustrate the procedure for the hypothesis testing outlined above.

Table 1: Sample data from male students in the final year of a high school in Hat Yai.

	x_1	x_2	x_3	x_4	x_5
$\mathcal{T}_A(x_i)$	0.7	0.8	0.6	0.5	0.7
$I_A(x_i)$	0.4	0.5	0.6	0.5	0.4
$F_A(x_i)$	0.3	0.4	0.2	0.3	0.5

Table 2: Sample data from female students in the final year of a high school in Hat Yai.

	y_1	y_2	y_3	y_4	y_5
$\mathcal{T}_A(y_i)$	0.6	0.8	0.5	0.5	0.7
$I_A(y_i)$	0.3	0.4	0.6	0.4	0.4
$F_A(y_i)$	0.2	0.4	0.3	0.4	0.5

Since

$$1 \leq \mathcal{T}(x) + I(x) + F(x) \leq 3$$

and

$$1 \leq \mathcal{T}(y) + I(y) + F(y) \leq 3$$

all three components in X and Y are independent.
Our hypotheses are as follows:
H_0: The goals of the two populations, are similar
H_1: The goals of the two populations, are different.
We have the following sample means for the membership, indeterminacy, and non-membership functions:

$$\overline{\mathcal{T}}_A(X) = 0.66,\ \overline{\mathcal{T}}_A(Y) = 0.62,$$
$$\overline{I}_A(X) = 0.48,\ \overline{I}_A(Y) = 0.42,$$
$$\overline{F}_A(X) = 0.34,\ \overline{F}_A(Y) = 0.36.$$

The sample standard deviations for each function are as follows:

$$S_{\mathcal{T}}(X) = 0.11,\ S_{\mathcal{T}}(Y) = 0.13,$$
$$S_I(X) = 0.08, S_I(Y) = 0.11,$$
$$S_F(X) = 0.11, S_F(Y) = 0.11.$$

After the test for equal population variances, the results indicate that $\sigma^2_{\mathcal{T}}(X) = \sigma^2_{\mathcal{T}}(Y)$, $\sigma^2_I(X) = \sigma^2_I(Y)$, $\sigma^2_I(X) = \sigma^2_I(Y)$. we calculate the pooled standard deviations $S_{\mathcal{T}}$, S_I, S_F as follows:

$$S_{\mathcal{T}} = \sqrt{\frac{(m-1)(S_{\mathcal{T}}(X))^2 + (n-1)(S_{\mathcal{T}}(Y))^2}{m+n-2}} = 0.12$$

,

$$S_I = \sqrt{\frac{(m-1)(S_I(X))^2 + (n-1)(S_I(Y))^2}{m+n-2}} = 0.10,$$

and

$$S_F = \sqrt{\frac{(m-1)(S_F)^2 + (n-1)(S_F(Y))^2}{m+n-2}} = 0.11.$$

Therefore, the test statistic are calculated as follows:

$$t_{\mathcal{T}} = \frac{\overline{\mathcal{T}}_A(X) - \overline{\mathcal{T}}_A(Y)}{\sqrt{(S_{\mathcal{T}}(X))^2/n + (S_{\mathcal{T}}(Y))^2/m}} = 0.52$$

and

$$t_I = \frac{\overline{I}_A(X) - \overline{I}_A(Y)}{\sqrt{(S_I(X))^2/n + (S_I(Y))^2/m}} = 0.97$$

$$t_F = \frac{\overline{F}_A(X) - \overline{F}_A(Y)}{\sqrt{(S_F(X))^2/n + (S_F(Y))^2/m}} = -0.28$$

Then, we find $t = \max\{|t_{\mathcal{T}}|, |t_I|, |t_F|\} = \max\{|0.52|, |0.97|, |-0.28|\} = 0.97$. The critical value for $t_{0.05,8}$ are -1.86. and 1.86. Since $-1.86 < 0.97 < 1.83$, we cannot reject the null hypothesis. This result indicates that there is insufficient evidence to conclude that the neutrosophic set $\mathcal{A}$ in population X is different from $\mathcal{A}$ in population Y.

Example 3.4. Let X and Y represent two populations, where X is the set of all male first-year students in the Faculty of Science at Prince of Songkla University, and Y is the set of all female first-year students the Faculty of Science at Prince of Songkla University. We define the neutrosophic set $\mathcal{A}$ as follows: $\mathcal{T}$ represents the membership function, indicating students' intent to choose the Mathematics major in the Faculty of Science at Prince of Songkla University. Additionally, I represents the indeterminacy function, indicating students' uncertainty about choosing the Statistics major. Finally, F represents the non-membership function, indicating students who do not intend to choose the Computer major in the Faculty of Science. This neutrosophic set is defined on the populations X and Y. It is assumed that $\mathcal{T}_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $\mathcal{T}_A(y_i)$, $I_A(y_i)$, and $F_A(y_i)$, are normally distributed for each sample in both populations. We now proceed to test whether the membership, indeterminacy, and non-membership functions, which represent the goals for major selection in the two populations, are similar or different.
Let $S_1 = \{x_1, x_2, x_3, x_4, x_5\}$ represent the sample of size five taken from the population of male students in the Faculty of Science at Prince of Songkla University (population X) and $S_2 = \{y_1, y_2, y_3, y_4, y_5\}$ represent the sample of size five taken from the population of female students in the Faculty of Science at Prince of Songkla University (population Y). The membership, indeterminacy, and non-membership values for the two given samples, based on their information concerning the neutrosophic set $\mathcal{A}$, are provided below. Using the numerical example given, we explain the procedure for the hypothesis testing described above.

Table 3: Sample data from male students in the final year of a high school in Hat Yai.

	x_1	x_2	x_3	x_4	x_5
$\mathcal{T}_A(x_i)$	0.3	0.1	0.2	0.2	0.1
$I_A(x_i)$	0.4	0.5	0.6	0.5	0.4
$F_A(x_i)$	0.3	0.4	0.2	0.3	0.5

Table 4: Sample data from female students in the final year of a high school in Hat Yai.

	y_1	y_2	y_3	y_4	y_5
$\mathcal{T}_A(y_i)$	0.5	0.2	0.1	0.2	0.1
$I_A(y_i)$	0.3	0.4	0.6	0.4	0.4
$F_A(y_i)$	0.2	0.4	0.3	0.4	0.5

Since

$$0 \leq \mathcal{T}(x) + I(x) + F(x) \leq 1$$

and

$$0 \leq \mathcal{T}(y) + I(y) + F(y) \leq 1$$

all three components in X and Y are dependent.
Our hypotheses are as follows:
H_0: The goals for major selection the two populations, are similar
H_1: The goals for major selection the two populations, are different.
We have the following sample means for the membership, indeterminacy, and non-membership functions:

$$\overline{\mathcal{T}}_A(X) = 0.18,\ \overline{\mathcal{T}}_A(Y) = 0.22,$$
$$\overline{I}_A(X) = 0.48,\ \overline{I}_A(Y) = 0.42,$$
$$\overline{F}_A(X) = 0.34,\ \overline{F}_A(Y) = 0.36.$$

The sample standard deviations for each function are as follows:

$$S_{\mathcal{T}}(X) = 0.08,\ S_{\mathcal{T}}(Y) = 0.16,$$
$$S_I(X) = 0.08, S_I(Y) = 0.11,$$
$$S_F(X) = 0.11, S_F(Y) = 0.11.$$

After the test for equal population variances, the results indicate that $\sigma^2_{\mathcal{T}}(X) = \sigma^2_{\mathcal{T}}(Y)$, $\sigma^2_I(X) = \sigma^2_I(Y)$, $\sigma^2_I(X) = \sigma^2_I(Y)$. we calculate the pooled standard deviations $S_{\mathcal{T}}$, S_I, S_F as follows:

$$S_{\mathcal{T}} = \sqrt{\frac{(m-1)(S_{\mathcal{T}}(X))^2 + (n-1)(S_{\mathcal{T}}(Y))^2}{m+n-2}} = 0.13$$

,

$$S_I = \sqrt{\frac{(m-1)(S_I(X))^2 + (n-1)(S_I(Y))^2}{m+n-2}} = 0.10,$$

and

$$S_F = \sqrt{\frac{(m-1)(S_F)^2 + (n-1)(S_F(Y))^2}{m+n-2}} = 0.11.$$

Therefore, the test statistic are calculated as follows:

$$t_{\mathcal{T}} = \frac{\overline{\mathcal{T}}_A(X) - \overline{\mathcal{T}}_A(Y)}{\sqrt{(S_{\mathcal{T}}(X))^2/n + (S_{\mathcal{T}}(Y))^2/m}} = -0.48$$

and

$$t_I = \frac{\overline{I}_A(X) - \overline{I}_A(Y)}{\sqrt{(S_I(X))^2/n + (S_I(Y))^2/m}} = 0.97$$

$$t_F = \frac{\overline{F}_A(X) - \overline{F}_A(Y)}{\sqrt{(S_F(X))^2/n + (S_F(Y))^2/m}} = -0.28$$

Then, we find $t = \max\{|t_T|, |t_I|, |t_F|\} = \max\{|-0.48|, |0.97|, |-0.28|\} = 0.97$. The critical value for $t_{0.05,8}$ are -1.86. and 1.86. Since $-1.86 < 0.97 < 1.83$, we cannot reject the null hypothesis. This result indicates that there is insufficient evidence to conclude that the neutrosophic set $\mathcal{A}$ in population X is different from $\mathcal{A}$ in population Y.

4 Conclusion

In this paper, we have proposed tests of statistical hypotheses for neutrosophic sets under two distinct scenarios. First, we considered the case where all three components (membership, indeterminacy, and non-membership) are independent. Second, we examined the case where all three components are dependent. The proposed hypothesis tests are fundamentally different from conventional statistical hypothesis testing methods, as they are specifically designed for data represented as neutrosophic sets, which generalize fuzzy and intuitionistic fuzzy sets.

We have also outlined the rules for making decisions based on these hypotheses. Our proposed tests provide a valuable tool for decision-makers, offering a systematic approach to obtain appropriate decisions when dealing with uncertainty in neutrosophic data.

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