On fuzzy completely γ-irresolute and weakly completely γ-irresolute functions

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Abstract

In this paper, fuzzy γ-open sets, and fuzzy γ-irresolute functions are used to define and investigate a new class of functions called fuzzy completely γ-irresolute functions, fuzzy completely weakly γ-irresolute between fuzzy topological spaces. We obtain their characterizations and their basic properties.

Keywords: Fuzzy topology; Fuzzy γ-open sets; Fuzzy γ-irresolute functions; Fuzzy γ-open set; Fuzzy completely γ-irresolute; Fuzzy completely weakly γ-irresolute; Fuzzy γ-continuous; Fuzzy γ-connected

1 Introduction and Preliminaries

Generalization topological spaces is a classical subject which is a type of classical topological spaces. The concept of fuzzy has invaded almost all branches of mathematics with the introduction of fuzzy sets by Zadeh[27] of 1965. The fuzzy topological spaces was introduced and developed by.[7] The concept of fuzzy γ-open sets in fuzzy topological spaces was introduced by Benchalli in[4] and generalized by others see.[6,10] Throughout this paper $\mathcal{H}$ and $\mathcal{K}$ are always fuzzy topological spaces. The class of all fuzzy sets on a universe $\mathcal{H}$ will be denoted by $\mathcal{I}^{\mathcal{H}}$. K.K. Azad defined fuzzy semi-open (fuzzy semi-closed) and fuzzy regular closed (fuzzy regular-open) sets.[1] Njastad[18] defined fuzzy α-open (fuzzy α-closed) sets and Mashhour[11] defined fuzzy pre-closed (fuzzy pre-open) sets. The concept of fuzzy semi-pre-closed (fuzzy semi-pre-open) sets was by S.S. Thakur and Surendra Singh.[13] Pu and Liu[12] introduced the concept of quasi-coincident. Weaker forms of fuzzy continuity on fuzzy topological spaces have been considered by many workers using the concepts of fuzzy γ-open sets see.[21,22] In this paper, fuzzy γ-open sets and fuzzy γ-closed sets are used to define and investigate a new class of functions called fuzzy completely weakly γ-irresolute. Relationships between the new class and other classes of functions are established.

$\mathscr{X}$ Throughout this paper $(\mathcal{H}, \mathcal{T})$, $(\mathcal{K}, \sigma)$ and $(\mathcal{Z}, \gamma)$ (or simply $\mathcal{H}$, $\mathcal{K}$ and $\mathcal{Z}$) represent non-empty fuzzy topological spaces. Let S be a fuzzy subset of a space $\mathcal{H}$. The fuzzy closure of S, fuzzy interior of S are denoted by $Cl(S)$, $Int(S)$. A fuzzy subset S of space $\mathcal{H}$ is called fuzzy regular open[1] (resp.fuzzy regular closed) if $S = Int(Cl(S))$ (resp. $S = Cl(Int(S))$. The

Definition 1.1. A fuzzy topology on a nonempty set $\mathcal{H}$ is a family ξ of fuzzy subsets of $\mathcal{H}$ which satisfies the following three conditions:

1. $0, 1 \in \xi$,
2. If $N, M \in \xi$, their $N \cap M \in \xi$,
3. $h_i \in \xi$ for each $i \in I$, then $\bigvee_{i \in I} f_i \in \xi$.

The pair $(\mathcal{H}, \mathcal{T})$ is called a fuzzy topological space.[7]

Definition 1.2. [12] Members of ξ are called fuzzy open sets[7] and complements of fuzzy open sets are called fuzzy closed sets,[7] where the complement of a fuzzy set S, denoted by S^C, is $1 - S$.

Definition 1.3. [12] The fuzzy subset $\kappa_{\mathcal{P}}$ of a non-empty set $\mathcal{H}$, which $x \in \mathcal{H}$ and $0 < \mathcal{P} \leqslant 1$ defined by

$$\kappa_{\mathcal{P}}(P) = \begin{cases} \mathcal{P}, & if \ \ P = x \\ 0, & if \ \ p \neq x \end{cases}$$

is called a fuzzy point in $\mathcal{H}$ with support x and value $\mathcal{P}$. The fuzzy point $\kappa_{\mathcal{P}}$ is called point

Definition 1.4. [12] A fuzzy point $\kappa_{\mathcal{P}}$ belong to a fuzzy set S in $\mathcal{H}$ (denoted by: $\kappa_{\mathcal{P}} \in S$) if and only if $\mathcal{P} \leqslant S(x)$,for some $x \in \mathcal{H}$.

Definition 1.5. [4] A fuzzy set S of a fuzzy topological space $\mathcal{H}$ is said to be fuzzy γ-open if and only if $S \leqslant Int(Cl(S)) \cap Cl(Int(S))$. The complement of fuzzy γ-open set is defined to be fuzzy γ-closed set .

Definition 1.6. [4] Let $\mathcal{H}$ be a fuzzy topological space and S be any fuzzy set in $\mathcal{H}$. 1) The union of all fuzzy open sets contained in S is called the fuzzy interior of S and denoted by $\gamma Int(S)$. i.e; $\gamma Int(S) = \vee\{R : R \leqslant S, \text{R is a fuzzy } \gamma\text{-open set of } \mathcal{H}\}$.
2) The intersection of all fuzzy closed sets containing S is called the fuzzy closure of S, and denoted by $\gamma Cl(S)$ .i.e; $\gamma Cl(S) = \wedge\{C : C \geqslant S, \text{C is a fuzzy } \gamma\text{-closed set of } \mathcal{H}\}$.

Definition 1.7. [15] Let $\mathcal{H}$ and $\mathcal{K}$ are two fuzzy topological spaces, the function $h : \mathcal{H} \to \mathcal{K}$ is called fuzzy γ-irresolute function if $h^{-1}(S)$ is fuzzy γ-open set in $\mathcal{H}$ for every fuzzy γ-open set S in $\mathcal{H}$.

Remark 1.8. [4] In a fuzzy topological space $\mathcal{H}$, S is fuzzy γ-closed(resp. fuzzy γ-open) if and only if $S = f\gamma Cl(S)$ (resp. fuzzy $\gamma Int(S)$).

Definition 1.9. [12] A fuzzy set S in $\mathcal{H}$ is called quasi-coincident with a fuzzy set R in $\mathcal{H}$, denoted by SqR if and only if $S(x) + R(x) \geqslant 1$, for some $x \in \mathcal{H}$. If S is not quasi-coincident with R, then $S(x) + R(x) \leqslant 1$ for every $x \in \mathcal{H}$ and denoted by $S\widetilde{q}R$.

Definition 1.10. [5] Let S be a fuzzy set in a fts $\mathcal{H}$ and $\kappa_{\mathcal{P}}$ be a fuzzy point of $\mathcal{H}$. Then S is called
(1) γ-neighbourhood of $\kappa_{\mathcal{P}}$ if there exists a hb-open set R in $\mathcal{H}$ such that $x_p \in R \leqslant S$.
(2) γ-q-neighbourhood of $\kappa_{\mathcal{P}}$ if there exist a hb-open set R in $\mathcal{H}$ such that $x_p q R \leqslant S$.

Definition 1.11. [5] For a fuzzy set S in $\mathcal{H}$,the image of S under h is the fuzzy set $h(S)$ in $\mathcal{K}$ with membership function $h(S)(y)$, $y \in \mathcal{K}$ defined by:

$$h(S)(y) = \begin{cases} sup_{x \in h^{-1}(y)} S(x), & if \ \ h^{-1}(y) \text{is not empty} \\ 0, & \text{otherewise} \end{cases}$$

where $h^{-1}(y) = \{x : h(x) = y\}$.

Lemma 1.12. [25] *Let $\mathcal{H}$ and $\mathcal{K}$ be two fuzzy topological spaces and let $h : \mathcal{H} \to \mathcal{K}$ be a function ,let $\{S_i\}_{i \in I}$, $\{R_i\}_{i \in I}$ be families of fuzzy sets in $\mathcal{H}$ and $\mathcal{K}$ respectively, then:*
1) $h(\vee_{i \in I} S_i) = \vee_{i \in I} h(S_i)$.
2) $h^{-1}(\wedge_{i \in I} R_i) = \wedge_{i \in I} h^{-1}(R_i)$.

Lemma 1.13. [1] *For functions $h_i : \mathcal{H}_i \to \mathcal{K}_i$, and fuzzy sets λ_i of $\mathcal{K}_i$, $i = 1, 2$, we have $(f_1 \times f_2)^{-1}(\lambda_1 \times \lambda_2) = f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2)$*

Lemma 1.14. [1] *Let $g : \mathcal{H} \longrightarrow \mathcal{H} \times \mathcal{K}$ be the graph of a function $h : \mathcal{H} \to \mathcal{K}$. Then, if λ is a fuzzy set of $\mathcal{H}$ and μ is a fuzzy set of $\mathcal{K}$. $g^{-1}(\lambda \times \mu) = \lambda \cap h^{-1}(\mu)$.*

Definition 1.15. A functions $h : \mathcal{H} \to \mathcal{K}$ is said to be:

1. fuzzy completely continuous[3] if $h^{-1}(M)$ is fuzzy regular open in $\mathcal{H}$ for each fuzzy open set M in $\mathcal{K}$,
2. fuzzy γ-irresolute[15] if $h^{-1}(M)$ is fuzzy γ-open in $\mathcal{H}$ for each fuzzy γ–open set M in $\mathcal{K}$,

3. fuzzy γ-continuous[5] if $h^{-1}(M)$ is fuzzy γ-open in $\mathcal{H}$ for each fuzzy open set M in $\mathcal{K}$,
4. fuzzy totally continuous[17] if $h^{-1}(M)$ is fuzzy clopen in $\mathcal{H}$ for each fuzzy subset M in $\mathcal{K}$,
5. fuzzy open[26] if $h(M)$ is fuzzy open set in $\mathcal{K}$ for each fuzzy open set M in $\mathcal{H}$,
6. fuzzy almost open[20] if $h(M)$ is fuzzy regular open set in $\mathcal{K}$ for each fuzzy regular open set M in $\mathcal{H}$,
7. fuzzy strongly continuous[2] if $h^{-1}(M)$ is fuzzy open fuzzy closed set in $\mathcal{H}$ for every fuzzy set M in $\mathcal{K}$.

Definition 1.16. [15] A family Ω of fuzzy sets is called a cover of a fuzzy set S if and only if $S \leqslant \vee\{C_i, i \in \Omega\}$, and it is called a fuzzy γ-open cover if each member C_i is a fuzzy γ-open set. A sub cover of S is a sub family of Ω which is also a cover of S.

Definition 1.17. [15] Let R be a fuzzy set in a fuzzy topological space $\mathcal{H}$. Then R is said to be a fuzzy γ-compact set if for every γ-open cover of R has a finite sub cover. Let $R = \mathcal{H}$, then $\mathcal{H}$ is called a fuzzy γ-compact space, i.e $\mathcal{H}$ is a fuzzy γ-compact space if for every$i \in \Omega$ and $\vee_{i\in\Omega} R_i = 1_{\mathcal{H}}$, then there are finite many indices $i_1, i_1, ...i_n \in \Omega$ such that $\vee_{j=1}^{n} R_{ij} = 1_{\mathcal{H}}$.

S space $(\mathcal{H}, \mathcal{T})$ is called fuzzy nearly compact[14] if every fuzzy regular open cover of $\mathcal{H}$ has a finite subcover.

Definition 1.18. [24] S space $\mathcal{H}$ is called fuzzy almost normal if for each fuzzy closed set S and each fuzzy regular closed set R such that $S \cap R = \phi$, there exists disjoint fuzzy open sets N and M such that $S \leqslant N$ and $R \leqslant M$.

2 fuzzy completely γ-irresolute function

Definition 2.1. Let $(\mathcal{H}, \mathcal{T})$ and $(\mathcal{K}, \sigma)$ be a fuzzy topological spaces. A function $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$, is said to be a fuzzy completely γ-irresolute function if $h^{-1}(M)$ is fuzzy regular open in $\mathcal{H}$ for every fuzzy γ-open set M of $\mathcal{K}$.

Remark 2.2. Every fuzzy strongly continuous function is fuzzy γ-irresolute, but the converse is not true.

Example 2.3. Let $\mathcal{H} = \mathcal{K} = \{a, b, c\}$. Define fuzzy sets, $\xi_1, \xi_2 : \mathcal{H} \to [0, 1]$ such that $\mathcal{T} = \{0, 1\}$ and $\sigma = \{0_{\mathcal{H}}, 1_{\mathcal{H}}, \xi_1, \xi_2\}$ where $\xi_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, $\xi_2 = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.5}{c}$ Define $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ be the identity function. Then h is fuzzy γ-irresolute but not fuzzy strongly continuous.

Remark 2.4. Every completely γ-irresolute function is fuzzy completely continuous (fuzzy γ-irresolute). That the converse is false.

Example 2.5. Let $\mathcal{H} = [0, 1]$, $\mathcal{T}_1 = \{1_{\mathcal{H}}, 0_{\mathcal{H}}, \xi_1, 1-\xi_1, \xi_1 \wedge (1-\xi_1), \xi_1 \vee (1-\xi_1)\}$ and $\mathcal{T}_2 = \{1_{\mathcal{H}}, 0_{\mathcal{H}}, \xi_1, \xi_2, \xi_1 \vee \xi_2\}$, where

$$\xi_1(P) = \begin{cases} o, & if \ 0 \leqslant t \leqslant \frac{1}{2} \\ 2t-1, & if \ \frac{1}{2} \leqslant t \leqslant 1 \end{cases}$$

$$\xi_2(P) = \begin{cases} 1, & if \ 0 \leqslant t \leqslant \frac{1}{4} \\ -4t+2, & if \ \frac{1}{4} \leqslant t \leqslant \frac{1}{2} \\ 0, & if \ \frac{1}{2} \leqslant t \leqslant 1 \end{cases}$$

Let us consider the mapping $h : (\mathcal{H}, \mathcal{T}_1) \to (\mathcal{K}, \mathcal{T}_2)$ defined by $h(t) = \frac{1}{2}t$ for all $t \in \mathcal{H}$. Then h is clearly fuzzy γ-irresolute. Set ξ_3 in $\mathcal{H}$ given by $\xi_3(t) = t$ for all $t \in \mathcal{H}$. Then $h^{-1}(\xi_3)$ is not fuzzy regularly open in $(\mathcal{H}, \mathcal{T}_1)$. Hence h is not fuzzy completely pre-irresolute.

Example 2.6. Let $\mathcal{H} = \mathcal{K} = \{a, b, c\}$. Define fuzzy sets, $\xi_1, \xi_2 : \mathcal{H} \to [0, 1]$ such that $\mathcal{T} = \{0, 1, \xi_3\}$ and $\sigma = \{0_{\mathcal{H}}, 1_{\mathcal{H}}, \xi_1, \xi_2\}$ where $\xi_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, $\xi_2 = \frac{0.7}{a} + \frac{0.5}{b} + \frac{0.5}{c}$ Define $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ be the identity function. Then h is fuzzy γ-irresolute but not fuzzy completely γ–irresolute.

Theorem 2.7. *$h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is a fuzzy completely γ-irresolute mapping and S is any fuzzy open subset of $\mathcal{H}$, then the restriction $h|_S : S \to \mathcal{K}$ is fuzzy completely γ-irresolute.*

Proof. Let λ be a fuzzy γ-open subset of $\mathcal{K}$. By hypothesis, $h^{-1}(\lambda)$ is fuzzy regular open in $\mathcal{H}$. Since S is fuzzy open in $\mathcal{H}$. Then $(f|_S)^{-1}(\lambda) : h^{-1}(\lambda) \cap S$ is fuzzy regular open in S. Therefore, $h|_S$ is fuzzy completely γ-irresolute □

Corollary 2.8. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ and $g : (\mathcal{K}, \mathcal{T}) \to (\mathcal{Z}, \gamma)$ be a mapping:*
1) If $h : \mathcal{H} \to \mathcal{K}$ is fuzzy completely γ-irresolute and $g : \mathcal{K} \to \mathcal{Z}$ is fuzzy γ-irresolute, then $g \circ h : \mathcal{H} \to \mathcal{Z}$ is fuzzy completely γ-irresolute.
2) If function $h : \mathcal{H} \to \mathcal{K}$ is fuzzy completely continuous and is fuzzy completely γ-irresolute, then $g \circ f : \mathcal{H} \to \mathcal{Z}$ is fuzzy completely γ-irresolute.
3) If $h : \mathcal{H} \to \mathcal{K}$ is fuzzy completely γ-irresolute and $g : \mathcal{K} \to \mathcal{Z}$ is fuzzy γ-continuous, then $g \circ h : \mathcal{H} \to \mathcal{Z}$ is fuzzy completely continuous.

Definition 2.9. A fuzzy topological space $\mathcal{H}$ is said to be fuzzy connected[19] (resp. fuzzy γ-connected[16]) if it cannot be expressed as the union of two fuzzy separated (resp. fuzzy γ-separated) sets. Other wise $(\mathcal{H}, \mathcal{T})$ is fuzzy fuzzy γ-disconnected.

Theorem 2.10. *If a mapping $h : \mathcal{H} \to \mathcal{K}$ is fuzzy completely γ-irresolute surjection and $\mathcal{H}$ is fuzzy almost connected then $\mathcal{K}$ is fuzzy γ-connected.*

Proof. Assume that $\mathcal{H}$ is fuzzy connected and $\mathcal{K}$ is not fuzzy γ-connected. Then $\mathcal{K}$ can be written as $\mathcal{K} = N \cup M$ such that N and M are disjoint nonempty fuzzy γ-open sets. Since h is fuzzy completely γ-irresolute, $h^{-1}(N)$ and $h^{-1}(M)$ are disjoint fuzzy regular open sets and $\mathcal{H} = h^{-1}(N) \cup h^{-1}(M)$ This shows that $\mathcal{H}$ is not fuzzy connected. This is a contradiction. □

Theorem 2.11. *If a mapping $h : \mathcal{H} \to \mathcal{K}$ is fuzzy pre-γ-closed, then for each subset R of $\mathcal{K}$ and a fuzzy γ-open set N of $\mathcal{H}$ containing $h^{-1}(R)$ there exists a fuzzy γ-open set M in $\mathcal{K}$ containing R such that $h^{-1}(M) \leqslant N$.*

Proof. Obvious. □

Theorem 2.12. *If h is fuzzy completely γ-irresolute from an almost regular space $\mathcal{H}$ onto a space $\mathcal{K}$, then $\mathcal{K}$ is fuzzy strongly γ-regular.*

Proof. Let h be fuzzy pre-γ-closed set in $\mathcal{K}$ with $y \notin H$ such that $y = h(k)$. Since h is fuzzy completely γ-irresolute function, $h^{-1}(H)$ is fuzzy regular closed and so fuzzy closed set in $\mathcal{H}$ and hence $x \notin h^{-1}(H)$. By almost regularity of $\mathcal{H}$ there exists disjoint fuzzy open sets N and M such that $x \in N$ and $h^{-1}(H) \leqslant M$. We obtain that $y = h(k) \in h(N)$ and $H \leqslant h(M)$ such that $h(N)$ and $h(M)$ are disjoint fuzzy γ-open sets. Thus $\mathcal{K}$ is fuzzy strongly γ-regular. □

Definition 2.13. A space $\mathcal{H}$ is called fuzzy almost regular[9] (resp. fuzzy strongly γ-regular) if for any fuzzy regular closed (resp. fuzzy γ-closed) set $H \leqslant \mathcal{H}$ and any point $x \in \mathcal{H} - H$, there exists disjoint fuzzy open (resp. fuzzy γ-open) sets N and M such that $k \in N$ and $H \leqslant M$.

Definition 2.14. A function $h : \mathcal{H} \to \mathcal{K}$ is called fuzzy pre-γ-closed if the image of every fuzzy γ-closed subset of $\mathcal{H}$ is fuzzy γ-closed set in $\mathcal{K}$.

Definition 2.15. A space $\mathcal{H}$ is called fuzzy strongly γ-normal if for every pair of disjoint fuzzy γ-closed subsets $\wp$ and β of $\mathcal{H}$ there exists disjoint fuzzy γ-open sets N and M such that $\wp \leqslant N$ and $\beta \leqslant M$.

Theorem 2.16. *If h is fuzzy completely γ-irresolute injective mapping from an fuzzy almost normal spaces $\mathcal{H}$ onto a space $\mathcal{K}$ then $\mathcal{K}$ is fuzzy strongly γ-normal.*

Proof. Let $_{\mathcal{P}}$ and β be disjoint fuzzy γ-closed sets in $\mathcal{K}$. Since h is fuzzy completely γ-irresolute function $h^{-1}(_{\mathcal{P}})$ and $h^{-1}(\beta)$ are disjoint fuzzy regular closed and so fuzzy closed set in $\mathcal{H}$. By fuzzy almost normality of $\mathcal{H}$, there exists disjoint fuzzy open sets N and M such that $h^{-1}(_{\mathcal{P}}) \leqslant N$ and $h^{-1}(\beta) \leqslant M$. We obtain that $_{\mathcal{P}} \leqslant N$ and $\beta \leqslant M$ such that $h(N)$ and $h(M)$ are disjoint fuzzy γ-open. Thus $\mathcal{K}$ is fuzzy strongly γ-normal. □

Definition 2.17. A fuzzy topological space $(\mathcal{H}, \mathcal{T})$ is said to be fuzzy-bT_1[8](resp. fuzzy-rT_1) (fbT_1) if for every pair of fuzzy singletons p_1, p_2 with different supports k_1 and k_2, $k_1 \neq k_2$, there exist hb-open (resp. fuzzy regular open) sets N and M such that, $p_1 \leqslant N \leqslant 1 - p_2$ and $P_2 \leqslant M \leqslant 1 - p_1$.

Theorem 2.18. *If $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy completely γ-irresolute injective mapping and $\mathcal{K}$ is fuzzy bT_1 then $\mathcal{H}$ is fuzzy rT_1.*

Proof. Suppose that $\mathcal{K}$ is fuzzy bT_1. For any two distinct points k and y of $\mathcal{H}$, there exists fuzzy γ-open sets H_1 and H_2 in $\mathcal{K}$ such that $h(k) \in H_1$, $h(y) \in H_2$, $h(k) \notin H_1$ and $h(y) \notin H_2$. Since h injective fuzzy completely γ-irresolute function, we have $\mathcal{H}$ is fuzzy rT_1. □

Definition 2.19. A fuzzy topological space $(\mathcal{H}, \mathcal{T})$ is said to be fuzzy-bT_2[8](resp. fuzzy-rT_2) (fbT_2) if for every pair of fuzzy singletons p_1, p_2 with different supports x_1 and k_2, $k_1 \neq k_2$, there exist hb-open (resp. fuzzy regular open) sets N and M such that, $p_1 \leqslant N \leqslant 1 - p_2$ and $P_2 \leqslant M \leqslant 1 - p_1$ and $N \leqslant 1 - M$.

Theorem 2.20. *If $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy completely γ-irresolute injective mapping and $\mathcal{K}$ is fuzzy-bT_2 then $\mathcal{H}$ is fuzzy-rT_2.*

Proof. Suppose that $\mathcal{K}$ is fuzzy-bT_2. For any two distinct points x and y of $\mathcal{H}$, there exists fuzzy γ-open sets H_1 and H_2 in $\mathcal{K}$ such that $h(x) \in H_1$, $h(y) \in H_2$, $h(x) \notin H_1$ and $h(y) \notin H_2$. Since h injective fuzzy completely γ-irresolute function, we have $\mathcal{H}$ is fuzzy-rT_2. □

3 Fuzzy Weakly Completely γ-Irresolute mapping

A function $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy completely weakly γ-irresolute if and only if the inverse image of each fuzzy γ-open set M in $\mathcal{K}$ is fuzzy open set in $\mathcal{H}$.

Let $h : (I, \mathcal{T}_3) \to (I, \mathcal{T}_2)$ be defined by $g(x) = x$ for each $x \in I$. Then $g^{-1}(1) = (1)$, $g^{-1}(\xi_2) = (\xi_2)$ which is fuzzy open but not regular open in $(I, \mathcal{T}_3)$ Therefore, g is fuzzy completely weakly γ-irresolute but not fuzzy completely γ-irresolute.

Theorem 3.1. *If a mapping $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$, then the following are equivalent:*

1. *h is fuzzy completely weakly γ-irresolute;*
2. *for each fuzzy point $_{\mathcal{P}}$ x in $\mathcal{H}$ and each fuzzy γ-open γ-q-nbd M of $h(\kappa_{\mathcal{P}})$, there exists a fuzzy open q-nbd N of $\kappa_{\mathcal{P}}$ subset that $h(N) \leqslant M$,*
3. *$h(Cl(S)) \leqslant \gamma Cl(h(S))$, for each fuzzy set S in $\mathcal{H}$,*
4. *$Cl(h^{-1}(R)) \leqslant h^{-1}(\gamma Cl(R))$, for each fuzzy set R in $\mathcal{K}$,*
5. *for each fuzzy γ-closed set M in $\mathcal{K}$, $h^{-1}(M)$ is a fuzzy closed set in $\mathcal{H}$,*
6. *$h^{-1}(\gamma Int(R)) \leqslant Int(h^{-1}(R))$, for each fuzzy set R in $\mathcal{K}$.*

Proof. $(1) \to (2)$: Let M be any fuzzy γ-open γ-q-nbd of $h(\kappa_{\mathcal{P}})$ in $\mathcal{K}$. Then $M(h(x)) + _{\mathcal{P}} > 1$. We choose a positive real number δ such that $M(h(x)) > \delta > 1 - _{\mathcal{P}}$ Then M is a fuzzy γ-open set, $h(\kappa_{\mathcal{P}}) \in M$. By hypothesis, there exists fuzzy open set $N_x \in N_{_{\mathcal{P}}}$, such that $h(N) \leqslant M$, $N(\mathcal{H}) > \delta > 1 - _{\mathcal{P}}$. Therefore, N is a fuzzy open q-nbd of $\kappa_{\mathcal{P}}$.
$(2) \to (3)$: Let $\kappa_{\mathcal{P}} \in Cl(S)$ then NqS and $h(N)qh(S)$ implies $Mqh(S)$, $h(\kappa_{\mathcal{P}}) \in \gamma Cl(h(S))$ and $\kappa_{\mathcal{P}} \in$

$h^{-1}(\gamma Cl(h(S))$. Therefore, $Cl(S) \leqslant h^{-1}(\gamma Cl(h(S)))$. Hence, $h(\kappa_{\mathcal{P}}) \in \gamma Cl(h(S))\, h(Cl(S)) \leqslant fh^{-1}(\gamma Cl(h(S)))$.
(3) → (4): Let R be any fuzzy set in $\mathcal{K}$. By (3), we obtain

$$h(h^{-1}(R)) \leqslant \gamma Cl(h(h^{-1}(R)))leq\gamma Cl(R)$$

and hence

$$h^{-1}(R) \leqslant h^{-1}(\gamma Cl(R))$$

(4) → (2): Let $\kappa_{\mathcal{P}}$ be a fuzzy point in $\mathcal{H}$ and M be a fuzzy γ-open, γ-q-nbd of $h(\kappa_{\mathcal{P}})$ and let $h(\kappa_{\mathcal{P}}) \notin \gamma Cl(1-M)$, otherwise since M is a fuzzy γ-open γ-q-nbd of $h(\kappa_{\mathcal{P}}) \notin \gamma Cl(1-M)$, we have $Mq(1-M)$ which is a contradiction. Thus, $\kappa_{\mathcal{P}} \notin h^{-1}(\gamma Cl(1-M))$ and by hypothesis, $\kappa_{\mathcal{P}} \neq Cl(h^{-1}(1-N))$. Then there exists a fuzzy open set N of $\kappa_{\mathcal{P}}$ such that $Nqh^{-1}(1-M)$ which implies that $h(N) \leqslant M$.
(4) → (5): Let H be any γ-closed set in $\mathcal{K}$. By (4), we have $Cl(h^{-1}(H)) \leqslant h^{-1}(\gamma Cl(H)) = h^{-1}(H)$ and so, $h^{-1}(H)$ is fuzzy closed in $\mathcal{H}$.
(5) → (6): For any fuzzy set M in $\mathcal{K}$, γ-$Int(M)$ is fuzzy γ-open set in $\mathcal{K}$ and so $h^{-1}(\gamma Int(M))$ is fuzzy open set in $\mathcal{H}$. Hence $h^{-1}(\gamma Int(M) = Int(h^{-1}(\gamma Int(M))) \leqslant Int(h^{-1}(M))$.
(6) → (4):Let R be a fuzzy set in $\mathcal{K}$. Then $1-\gamma CI(R)$ is fuzzy γ-open in $\mathcal{K}$. By (6), $h^{-1}(1-\gamma CI(R)) = 1-h^{-1}(\gamma Cl(B))$ is fuzzy open in $\mathcal{H}$ and so $h^{-1}(\gamma Cl(R))$ is fuzzy closed in $\mathcal{H}$. Hence $h^{-1}(R) \leqslant [h^{-1}(\gamma Cl(R)) = h^{-1}(\gamma CI(B))$.
(6) → (1): Obvious. □

Theorem 3.2. [23] *Let $(\mathcal{H}, \mathcal{T})$ and $(y, \mathcal{T})$ be any two fuzzy topological spaces such that $\mathcal{H}$ is product related to $\mathcal{K}$ then the product $S_1 \times S_2$ of fuzzy γ-open set S_1 of $\mathcal{H}$ and fuzzy γ-open set S_2 of $\mathcal{K}$ is fuzzy γ-open set of the fuzzy product space $\mathcal{H} \times \mathcal{K}$.*

We can be seen that every fuzzy completely γ-irresolute mapping is fuzzy γ-irresolute. But the converse need not be true as is shown in the following example.

Example 3.3. Let $I = [0, 1]$ and ξ_1 and ξ_2 be fuzzy subsets of I defined as

$$\xi_1(P) = \begin{cases} \frac{1}{3}(5t+1), & if \;\; 0 \leqslant t \leqslant \frac{1}{5} \\ \frac{1}{6}(t+1), & if \;\; \frac{1}{5} \leqslant t \leqslant 1 \end{cases}$$

$$\xi_2(P) = \begin{cases} \frac{1}{9}(5t+1), & if \;\; 0 \leqslant t \leqslant \frac{1}{5} \\ \frac{5}{4}(1-2t), & if \;\; \frac{1}{5} \leqslant t \leqslant 1 \end{cases}$$

Clearly $\mathcal{T}_1 = \{0, 1\}$ and $\mathcal{T}_2 = \{1_{\mathcal{H}}, 0_{\mathcal{H}}, \xi_2\}$ and $\mathcal{T}_3 = \{1_{\mathcal{H}}, 0_{\mathcal{H}}, \xi_1, \xi_2, \xi_1 \vee \xi_2, \xi_1 \wedge \xi_2\}$ are topologies on I. Let $h : (I, \mathcal{T}_1) \to (I, \mathcal{T}_2)$ be defined by $h(t) = t$ for each $t \in I$. Then h is fuzzy γ-irresolute but not fuzzy completely weakly γ-irresolute.

Theorem 3.4. *$p_i : \mathcal{H}_i \to \mathcal{K}_i$ $(i = 1, 2)$ are fuzzy completely weakly γ-irresolute mapping and $\mathcal{K}_1$ is product related to $\mathcal{K}_2$, then $p_i : \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{K}_1 \times \mathcal{K}_2$ is fuzzy completely weakly γ-irresolute.*

Proof. Consider $S = \vee(N_i \times M_i)$ where $N_i's$ and $M_i's$ are fuzzy γ-open sets of $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively. Since $\mathcal{K}_1$ is producted to $\mathcal{K}_2$ then from Theorem 3.2, S is fuzzy γ-open set of $\mathcal{K}_1 \times \mathcal{K}_2$. By Lemma 1.12 and 1.13, $h^{-1}(S) = \vee(h_1^{-1}(N_i) \times h_2^{-1}(M_i))$. Since h_1 and h_2 are completely weakly γ-irresolute, $h^{-1}(S)$ is a fuzzy open in $\mathcal{H}_1 \times \mathcal{H}_2$. □

Theorem 3.5. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ be a maping and $\mathcal{H}$ is product related to $\mathcal{K}$. If the graph $g : \mathcal{H} \to \mathcal{H} \times \mathcal{K}$ of h is fuzzy completely weakly γ-irresolute, then so is h.*

Proof. Let M be a fuzzy γ-open set in $\mathcal{K}$. By Lemma 1.14, we have $h^{-1}(M) = 1 \wedge h^{-1}(M) = g^{-1}(1 \times M)$. Since g is fuzzy completely weakly γ-irresolute and $1 \times M$ is fuzzy γ-open set in $\mathcal{H} \times \mathcal{K}$. $h^{-1}(M)$ is fuzzy open set in $\mathcal{H}$ and so, h is fuzzy completely weakly γ-irresolute. □

Next, the composition and preservation of fuzzy topological structure under the fuzzy completely weakly γ-irresolute which other fuzzy functions are studied.

Corollary 3.6. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ and $g : (\mathcal{K}, \sigma) \to (\mathcal{Z}, \gamma)$ be two functions.*

1. *If h is fuzzy completely weakly γ-irresolute and is g fuzzy completely γ-irresolute, then $g \circ h$ is fuzzy completely γ-irresolute.*
2. *If h is fuzzy completely weakly γ-irresolute and g is fuzzy γ-irresolute, then $g \circ h$ is fuzzy completely weakly $g \circ h$-irresolute*
3. *If h is fuzzy completely continuous and g is fuzzy completely weakly γ-irresolute, then $g \circ h$ is fuzzy completely γ-irresolute.*
4. *If h is fuzzy completely γ-irresolute and g is fuzzy completely weakly γ-irresolute, then $g \circ h$ is fuzzy completely γ-irresolute.*
5. *If h is fuzzy totally continuous and g is fuzzy completely weakly γ-irresolute, then $g \circ h$ is fuzzy completely γ-irresolute.*
6. *If h is fuzzy completely weakly γ-irresolute and g is fuzzy γ-continuous, then $g \circ h$ is fuzzy continuous.*
7. *If h is fuzzy γ-continuous and g is fuzzy completely weakly γ-irresolute, then $g \circ h$ is fuzzy γ-irresolute.*
8. *If h is fuzzy continuous and g is fuzzy completely weakly γ-irresolute, then $g \circ h$ is fuzzy completely weakly γ- irresolute.*

Theorem 3.7. *If $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy almost open surjective and $g : (\mathcal{K}, \sigma) \to (\mathcal{Z}, \gamma)$ is fuzzy mapping such that $g \circ f : (\mathcal{H}, \mathcal{T}) \to (\mathcal{Z}, \gamma)$ is fuzzy completely γ-irresolute, then g is fuzzy completely weakly γ-irresolute.*

Proof. Let M be a fuzzy γ-open set in $\mathcal{Z}$. since $g \circ h$ is fuzzy completely γ-irresolute, $(g \circ h)^{-1}(M) = h^{-1}(g^{-1}(M))$ is fuzzy regular open in $\mathcal{H}$. Since h is fuzzy almost open surjective, $h(h^{-1}(g^{-1}(M)) = g^{-1}(M)$ is fuzzy open in $\mathcal{K}$. Hence g is fuzzy completely weakly γ-irresolute. □

Theorem 3.8. *If $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy open surjective and $g : (\mathcal{K}, \sigma) \to (\mathcal{Z}, \gamma)$ is fuzzy mapping such that $g \circ f : (\mathcal{H}, \mathcal{T}) \to (\mathcal{Z}, \gamma)$ is fuzzy completely weakly γ-irresolute, then g is fuzzy completely weakly γ-irresolute.*

Proof. Similar to the proof of Theorem 3.7. □

Theorem 3.9. *Let P_i be the projection mapping from $\prod \mathcal{H}_i$ onto $\mathcal{H}_i$. If $h : \mathcal{H} \to \prod \mathcal{H}_i$ is fuzzy completely weakly γ-irreolute function, then $(h \circ P_i)$ is also fuzzy completely weakly γ-irresolute.*

Proof. Obvious. □

Theorem 3.10. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is a fuzzy completely weakly γ-irresolute surjective function and $\mathcal{H}$ is fuzzy compact space, then $\mathcal{K}$ is fuzzy γ-compact.*

Proof. Let $\{C_{\mathcal{P}} :_{\mathcal{P}} \in \Lambda\}$ be any fuzzy γ-open cover of $\mathcal{K}$. Then $\{h^{-1}(C_{\mathcal{P}}) :_{\mathcal{P}} \in \Lambda\}$ is a fuzzy open cover of $\mathcal{H}$. Since $\mathcal{H}$ is fuzzy compact there exists a finite subfamily $\{h^{-1}(C_{\mathcal{P}}) : i = 1, 2, ..., n\}$ of $\{h^{-1}(C_{\mathcal{P}}) :_{\mathcal{P}} \in \Lambda\}$ which covers $\mathcal{H}$. Hence, $\mathcal{K}$ is fuzzy γ-compact. □

Corollary 3.11. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy completely weakly γ-irresolute surjective function and $\mathcal{H}$ is fuzzy compact space, then $\mathcal{K}$ is fuzzy γ-compact.*

Definition 3.12. [16] Let $(\mathcal{H}, \mathcal{T})$ be a fuzzy topological space and R, S are fuzzy set in $\mathcal{H}$ then R and S are said to be

1. Fuzzy separated iff $\{Cl(R) \cap S\} = \phi$ and $\{Cl(S) \cap R\} = \phi$, $\forall x \in \mathcal{H}$.

2. Fuzzy γ-separated iff $\gamma Cl(R)\tilde{q}S$ and $\gamma Cl(S)\tilde{q}R$, $\forall x \in \mathcal{H}$.

Definition 3.13. [16] A fuzzy topological space $\mathcal{H}$ is said to be fuzzy connected (resp. fuzzy γ-connected) if it cannot be expressed as the union of two fuzzy separated (resp. fuzzy γ-separated) sets.

Lemma 3.14. *Two non-zero fuzzy sets S and R are fuzzy γ-separated if and only if there exist two fuzzy γ-open sets N and M such that $S \leqslant N$, $R \leqslant M$, $S\tilde{q}M$ and $R\tilde{q}N$.*

Proof. Let S and R be fuzzy γ-separated sets. Putting $N = 1 - \gamma CI(S)$ and $M = 1 - \gamma(R)$, then N and M are fuzzy γ-open such that $S \leqslant N$, $R \leqslant M$, $S\tilde{q}M$ and $R\tilde{q}N$.
Conversely, let N and M be fuzzy γ-open sets such that $S \leqslant N$, $R <\leqslant M$, $S\tilde{q}M$ and $R\tilde{q}N$. Since $1 - M$ and $1 - N$ are fuzzy γ-closed, we have $\gamma CI(S) \leqslant (1 - M) \leqslant (1 - R)$ and $\gamma Cl(R) \leqslant (1 - N) \leqslant (1 - S)$. Thus $\gamma CI(S)\tilde{q}Cl(R)$ and $\gamma CI(R)\tilde{q}CI(S)$. Hence S and R are fuzzy γ-separated. □

Corollary 3.15. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ be a fuzzy completely weakly γ-irresolute surjective function. If N is a fuzzy connected subset in $\mathcal{H}$, then $h(N)$ is also fuzzy γ-connected.*

Theorem 3.16. *If $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy completely weakly γ-irresolute injective function and $\mathcal{K}$ is fuzzy γ-T_1 then $\mathcal{H}$ is fuzzy Hausdorff.*

Proof. Let x, y be any two distinct points of $\mathcal{H}$. Since h is injective, we have $h(x) \neq h(y)$. Since $\mathcal{K}$ is fuzzy γ-T_2, there exists M and W are γ-open sets in $\mathcal{K}$. such that $M \cap W = 0$. Since h is fuzzy completely weakly γ-irresolute, there exists fuzzy open sets G and H in $\mathcal{H}$ such that $h(G) \leqslant M$ and $h(H) \leqslant W$. Hence we obtain $G \cap H = 0$. This shows that $\mathcal{H}$ is fuzzy Hausdorff. □

Theorem 3.17. *A function $h : \mathcal{H} \to \mathcal{K}$ is fuzzy completely weakly γ-irresolute if the graph function $g : \mathcal{H} \to \mathcal{H} \times \mathcal{H}$, defined by $g(x) = (x, g(x))$ for each $x \in \mathcal{H}$ is fuzzy completely weakly γ-irresolute.*

Proof. Let M be any fuzzy γ-open set of $\mathcal{K}$. Then $1 \times M$ is a fuzzy γ-open set of $\mathcal{H} \times \mathcal{K}$. Since g is fuzzy completely γ-irresolute, $h^{-1}(M) = g^{-1}(1 \times M)$ is fuzzy regular open in $\mathcal{H}$. Thus h is fuzzy completely weakly γ-irresolute. □

Theorem 3.18. *If a function $h : \mathcal{H} \to \mathcal{K}$ is a fuzzy completely weakly γ-irresolute surjection and $\mathcal{H}$ is fuzzy connected, then $\mathcal{K}$ is fuzzy γ-connected.*

Proof. Suppose that $\mathcal{K}$ is not fuzzy γ-connected. There exists non empty fuzzy γ-open sets M and W of $\mathcal{K}$ such that $\mathcal{K} = M \cup W$. Since h is fuzzy completely weakly γ-irresolute. $h^{-1}(M)$ and $h^{-1}(W)$ This shows that $\mathcal{H}$ is not fuzzy connected. This is a contradiction. □

Theorem 3.19. *Let $h : (\mathcal{H}, \mathcal{T}) \to (\mathcal{K}, \sigma)$ is fuzzy completely weakly γ-irresolute surjective function. If N is a fuzzy connected subset in $\mathcal{H}$, then $h(N)$ is fuzzy γ-connected in $\mathcal{K}$.*

Proof. Suppose that $h(N)$ is not γ-connected in $\mathcal{K}$. Then there exist fuzzy γ-separated subsets G and H in $\mathcal{K}$. such that $h(N) = G \cap H$. By Lemma 3.14, there exist fuzzy γ-open subsets M and W such that $G \leqslant M$, $H \leqslant W$, GqW and HqM. Since h is fuzzy complete weakly γ-irresolute, $h^{-1}(G)$ and $h^{-1}(H)$ are fuzzy open in $\mathcal{H}$ and $N = h^{-1}(h(N)) = h^{-1}(G \cup H) = h^{-1}(G) \cup h^{-1}(H)$. It is clear that $h^{-1}(G)$ and $h^{-1}(H)$ are fuzzy separated in $\mathcal{H}$. Therefore, N is not fuzzy connected in $\mathcal{H}$. □

4 conclusion

We have defined and proved basic properties of Fuzzy Completely γ-Irresolute Functions and Fuzzy Completely Weakly γ-Irresolute Function. Many results have been established to show how far topological structures are preserved by these γ-Irresolute Functions. We also have provided examples where such properties fail to be preserved.

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5 Author's contributions

This article was written in collaboration by all of the contributors. The final manuscript was read and approved by all writers.

6 Conflicts of interest

There are no competing interests declared by the authors.

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