Lexicographic Approach for Integer Programming Problem under Triangular Neutrosophic Fuzzy Environment and it's Application

Yuvashri P.[1] **, Saraswathi A.**[2,*]**, Broumi Said**[3]
[1]Department of Mathematics, Rajalakshmi Engineering College , Rajalakshmi Nagar, Thandalam, Mevalurkuppam, Tamil Nadu 602105, India
[2]Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chengalpattu- 603203, India
[3]Faculty of Science Ben MSik, University Hassan II, Casablanca, Morocco
Emails: spprakash199@gmail.com; saraswaa@srmist.edu.in; broumisaid78@gmail.com

Abstract

Linear programming is an effective way in mathematical programming for solving optimization problems with linear objectives and linear constraints. There is determinant and indeterminant information in the actual world. As a result, the indeterminate problem is veritable and must be considered in the optimization problem,To handle this situation the neutrosophic theory is formed from extension of fuzzy set theory and is a helpful tool for dealing with inconsistent, indeterminate, and incomplete information.In this paper, we examine the coefficient of single valued triangular neutrosophic numbers to solve the neutrosophic integer programming problem.The neutrosophic integer programming problem are formulated with highest truth membership (T), indeterminancy membership and falsity membership function. The neutrosophic objective function involving a neutrosophic number, and then constructs a neutrosophic integer programming problem technique to handle neutrosophic optimization.In this paper we propose a strategy by using lexicographic approach in fractional dual algorthim to obtaining the basic solution and optimal solution as single valued neutrosophic triangular numbers.To gauge the efficacy of the model we solved few examples.

Keywords: Fractional Dual Algorthim; Lexicographic Technique; Neutrosophic Integer Programming; Neutrosophic Optimal Solution; Single Valued Neutrosophic Triangular fuzzy Numbers

1 Introduction

Neutrosophic Integer programming formulations of situations in which variables are inherently discrete in nature do not pose any problem. there are, many circumstances in which the variables are not uncertainity. With some variation, these problems nevertheless fit within the neutrosophic linear programming environment. Additionally, current methods for uncertain integer programming are not really significant indeterminate programming since they typically aim to convert optimization techniques into crisp objective models in order to find distinct crisp optimal solutions instead of indeterminate solutions in uncertain conditions. Fortunately, there are certain formulation options available for substantially avoiding some of these discrepancies. These imply the establishment of a one or more artificial variables that are restricted to be integers.As progress in the creation of dual fractional algorithms develops, this technique is becoming more viable. Some of the problems discussed by this method are now outlined.

Elhadidi[17] proposed method to solve neutrosophic linear programming problem under trapezoidal neutroshopic number. Abdefattah[1] suggested parametric approach to neutrosophic linear programming problem with coefficients are triangular neutroshopic number. Nafei[29] investegated the interval neutrosophic linear programming problem model with triangular interval neutroshopic number converted into crisp model by ranking function. Eldredge et al.[16] used lexicographic order for primal and dual bounds to find optimal for bounded integer program. Edalpattanah[?] proposed direct method to solve the neutrosophic linear programming, with the triangularneutroshopic number by componentwise. Das and Edalpattanah[?] developed new ranking method for neutrosophic integer programming along with its application with coefficients of triangular neutroshopic number. Khalifa et al.[23] suggested pentagonal neutroshopic integer programming problem with stock portfolio problem. Ferber et.al.[18] suggested a cutting plane solution for iteratively tightens fractional solution to mixed integer programming problem.

Monali[28] solved goal programming problem by modified gomory constraint technique. Barmann[6] approached exact generic method for branch and bound column search. Akay et al.[2] used Artificial bee colony for 0-1 integer and mixed integer programing problem to solve discrete numeric optimization problem. Burachik et al.[10] solved multi-objective mixed integer programming problems by certain algorithm and also solved a problem of rocket injector design with discrete variable . Allaviranloo et al.[3] developed optimum investment on port which is formulated as integer programming model. Brown et al.[9] introduced parametric integer programming was solving through Glimore- Gomory approach through find optimal solution.

Chernov et al.[12] described discrete optimization solving by first algorithm of gomoryconvergence. Cococaine et al.[27] explained lexicographic multi-objective linear programming problem using gorosone method. Huang et al.[19] described petri net models solving through lexicographic multi objective integer programming problem.

Ignizo and Daniel[21] developed a generalized network concept with fuzzy parameters for multi objective zero -one integer programming problem. Ignizo and Thomas[22] described the lexicographic multiobjective problem into an equal single objective programming problem. Kumar et.al[24] defined Acquiring non-linear model approach for integer programming problem with suitable examples. Canedo and Veredegay[11] used a lexicographic method to found optimal fuzzy objective values, also dissusced application in time-cost trade off problem under fuzzy environment. Rasekhipour et.al[31] applied the lexicographic optimization to avoid obstacles via prioritizing the constraints in carsimvechicle model.

Basset et al.[7] discussed the neutrosophic linear programming problem with the parameters are trapezoidal neutrosophic number into crisp linear programming problem. Anderson et al.[4] described mixed integer programming for high dimensional piecewise linear function is used in neural networks. Dash et al.[31] explained the chvatal -gomory inequalities using bound on the variable where 0-1 problems have upper and lower bounds are unbounded. Maiti et al.[26] suggested goal programming to solve multi objective linear programming problem where parameters are neutrosophic numbers. Bera et al.[31] applied post sensitive analysis to solve neutrohopic linear programming along with real life problem. Singh and Mark[33] introduced multi-objective mixed integer programming under intutionistic fuzzy environment into single objective problem. Huda and Essa[20] developed duality in neutrosophic linear programming with application hybrid renewable energy production. Demir[15] investigated lexicographic in multi objective genetic algorithm which transformed into single objective generic algorithm. Das[?] explained new ranking function for triangular neutroshopic number in integer programming problem.

Mai[25] defined ranking for interval neutroshopic number in integer programming problem. Nafei and Nasseri[30] used ranking function to solve triangular neutrosophic number in Integer Praogramming problem. Şahin et al.[32] proposed neutrosophic integer programming problem under triangular neutroshopic number into crisp linear programming problem using ranking function. Jun[34] explained interval neutrosophic linear programming problem obtained neutrosophic optimal solution by using parametric form.
Optimization problems that involve uncertain and imprecise conditions have recently been of great concern to experts. The integrative principles behind neutrosophy, fuzzy logic, and intuitionistic fuzzy sets have developed the theory of Neutrosophic Fuzzy (NF) sets and turned out to be an effective tool for managing these uncertainties. In this research, a new lexicographic method to handle Integer Programming under the Neutrosophic Fuzzy environment is presented. Our novelty is in developing and applying a novel lexicographic algorithm especially tailored for the NF-IP problems. Integer programming is a widely used method for solving optimization problems that involve discrete decision variables. However, when neutrosophic fuzzy elements are introduced into these problems, they bring unique challenges due to the complex nature of neutrosophic fuzzy sets.
Our contribution is primarily in the design and application of a lexicographic method especially designed for NF-IP problems. Integer programming is one of the most well-established techniques to solve optimization

problems with discrete decision variables. However, incorporating neutrosophic fuzzy elements into IP problems presents challenges unique to neutrosophic fuzzy sets.
These are the key contributions of our research:
Formalization of Neutrosophic Fuzzy Integer Programming (NF-IP): We have formally defined the NF-IP problem, which gives a clear mathematical representation that includes neutrosophic fuzzy sets and integer decision variables.This formalization serves as the foundation for our proposed approach.
Lexicographic Objective Function Transformation: To solve NF-IP problems, we propose a lexicographic transformation technique. This approach allows us to maintain the lexicographic order while handling the neutrosophic fuzzy elements.
Algorithm Development: We have developed an efficient algorithm based on the lexicographic transformation for solving NF-IP problems. This algorithm provides a systematic and step-by-step procedure to obtain the lexicographically optimal solution while considering the uncertainties represented by neutrosophic fuzzy sets.
Numerical Experiments: To validate the effectiveness of our approach, we have conducted extensive numerical experiments using real-world data and benchmark Neutrosophic Integer Programming problem instanceproblems compared to existing techniques.
Practical Applications: We show the practical applicability of our approach by presenting it in different real-world scenarios, such as supply chain optimization, project scheduling, and decision-making under uncertainty. Our method is a useful tool for decision-makers to make the right choices in complex uncertain environments.
Theoretical Advancements: Our research contributes to the theoretical foundation of neutrosophic fuzzy optimization, advancing the understanding of how to effectively integrate neutrosophic fuzzy sets into integer programming problems.
In this research paper consist of ten sections , section 2 is discussion about basic definitions and also with established theorem.section 3 presents the formulation of neutroshopic Integer Praogramming problem. Section 4 illustrates the Neutrosophic Optimization Model of integer programming problem . Section 5 demonstrate the proposed stragey for NIPP. By utilization of proposed method section 5 states the suitable numerical example in section 6. Section 7 is demonstrated the application in hospital . section 8 explains the comparability analysis .Section 9 states the advangatages and Section 9 describes the limitations for proposed stragedy. Finally section 10 has reached its conclusion.

2 Preliminaries

Definition 2.1. Let X be a space of points (objects) and $x \in A$. A neutrosophic set A in X is defined by a truth-membership function ($T_A(x)$), an indeterminacy-membership function ($I_A(x)$) and a falsity-membership function $F_A(x).T(x)$, $I(x)$ and $F(x)$ are real standard or real nonstandard subsets of $]0-, 1+[$. That is $T_A(x) : X \to]0-, 1+[, I_A(x) : X \to]0-, 1+[$ and $F_A(x) : X \to]0-, 1+[$. There is no restriction on the sum of (x), x and $F_A(x)$, so $0- \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3+$.

Definition 2.2. [25] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X, A = \{\langle x, T(x), I_A(x), F_A(x)\rangle : x \in X\}$ where $T_A(x) : X \to [0, 1], I_A(x) : X \to [0, 1]$ and $F_A(x) : X \to [0, 1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T(x), I(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. In the following, we write SVN numbers instead of single valued neutrosophic numbers. For convenience, a SVN number is denoted by $A = (a, b, c)$ where $a, b, c \in [0, 1]$ and $a + b + c \leq 3$.

Definition 2.3. Let $\tilde{A}_N = (\tilde{p}_1^N, \tilde{p}_2^N, \tilde{p}_3^N, \mu_a, \alpha_a, \beta_a)$ be a single triangular valued neutrosophic number on the real number set R, is a unique neutrosophic set whose member functions are denoted as truth-membership degree, the indeterminacy-membership degree and the falsity membership degree are defined as

$$u_N = \begin{cases} \dfrac{(x-\tilde{p}_1^N)\mu_a}{\tilde{p}_2^N - \tilde{p}_1^N}, & (\tilde{p}_1^N \leq x \leq \tilde{p}_2^N) \\ \dfrac{(\tilde{p}_3^N - x)\mu_a}{\tilde{p}_3^N - \tilde{p}_2^N}, & (\tilde{p}_2^N \leq x \leq \tilde{p}_3^N) \\ 0, & \text{otherwise} \end{cases}$$

DOI: https://doi.org/10.54216/IJNS.260102

$$w_N = \begin{cases} \dfrac{\tilde{p}_2^N - x + \alpha_a(x - \tilde{p}_1^N)}{\tilde{p}_2^N - \tilde{p}_1^N}, & (\tilde{p}_1^N \leq x \leq \tilde{p}_2^N) \\ \dfrac{(x - \tilde{p}_2^N) + \alpha_a(\tilde{p}_3^N - x)}{\tilde{p}_3^N - \tilde{p}_2^N}, & (\tilde{p}_2^N \leq x \leq \tilde{p}_3^N) \\ 0, & \text{otherwise} \end{cases}$$

$$y_N = \begin{cases} \dfrac{\tilde{p}_2^N - x + \beta_a(x - \tilde{p}_1^N)}{\tilde{p}_2^N - \tilde{p}_1^N}, & (\tilde{p}_1^N \leq x \leq \tilde{p}_2^N) \\ \dfrac{(x - \tilde{p}_2^N) + \beta_a(\tilde{p}_3^N - x)}{\tilde{p}_3^N - \tilde{p}_2^N}, & (\tilde{p}_2^N \leq x \leq \tilde{p}_3^N) \\ 0, & \text{otherwise} \end{cases}$$

2.1 Ordering Function[30]

A ordering function R on $N(\mathcal{R})$ is a map from $N(\mathcal{R})$ to the real numbers line, where the natural ordering there exists. Consider two triangular neutrosophic numbers as: $N = [(a^l, a^m, a^n, \mu_N, \nu_N, \gamma_N)]$ and $M = [(a^l, a^m, a^n, \mu_M, \nu_M, \gamma_M)]$. Then, the ranking

i) If $R(N) > R(M)$ then $N > M$.

ii) If $R(N) < R(M)$ then $N < M$.

iii) If $R(N) = R(M)$ then $N = M$.

2.2 Score Function

Let us consider a single valued triangular neutrosophic number as $\tilde{A}_N = (\tilde{p}_1^N, \tilde{p}_2^N, \tilde{p}_3^N, \mu_a, \alpha_a, \beta_a)$ the score function of Sr -TNN is defined as composition of $\frac{1}{4}(\tilde{p}_1^N + \tilde{p}_2^N + \tilde{p}_3^N)$ and score value of membership function is $(2 + \mu_a - \alpha_a - \beta_a)$.
Thus the score function is $\tilde{S}_N = \frac{1}{4}(\tilde{p}_1^N + \tilde{p}_2^N + \tilde{p}_3^N) \times (2 + \mu_a - \alpha_a - \beta_a)$

2.3 Arithmetic Operations

Let $\tilde{A}_N = (\tilde{p}_1^N, \tilde{p}_2^N, \tilde{p}_3^N, \mu_a, \alpha_a, \beta_a)$ and $\tilde{B}_N = (\tilde{q}_1^N, \tilde{q}_2^N, \tilde{q}_3^N, \mu_b, \alpha_b, \beta_b)$ be two single valued triangular neutrosophic numbers and $\lambda \neq 0$ be any real number. Then,
$\tilde{A}_N + \tilde{B}_N = (\tilde{p}_1^N + \tilde{q}_1^N, \tilde{p}_2^N + \tilde{q}_2^N, \tilde{p}_3^N + \tilde{q}_3^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b$
$\tilde{A}_N - \tilde{B}_N = (\tilde{p}_1^N - \tilde{q}_1^N, \tilde{p}_2^N - \tilde{q}_2^N, \tilde{p}_3^N - \tilde{q}_3^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b$

$$\tilde{A}_N \tilde{B}_N = \begin{cases} \left\langle (\tilde{p}_1^N \tilde{q}_1^N, \tilde{p}_2^N \tilde{q}_2^N, \tilde{p}_3^N \tilde{q}_3^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b \right\rangle (\tilde{p}_3^N > 0, \tilde{q}_3^N > 0) \\ \left\langle (\tilde{p}_1^N \tilde{q}_3^N, \tilde{p}_2^N \tilde{q}_2^N, \tilde{p}_3^N \tilde{q}_1^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b \right\rangle (\tilde{p}_3^N < 0, \tilde{q}_3^N > 0) \\ \left\langle (\tilde{p}_3^N \tilde{q}_3^N, \tilde{p}_2^N \tilde{q}_2^N, \tilde{p}_1^N \tilde{q}_1^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b \right\rangle (\tilde{p}_3^N < 0, \tilde{q}_3^N < 0) \end{cases}$$

$$\tilde{A}_N / \tilde{B}_N = \begin{cases} \left\langle (\tilde{p}_1^N / \tilde{q}_3^N, \tilde{p}_2^N / \tilde{q}_2^N), \tilde{p}_3^N / \tilde{q}_1^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b \right\rangle (\tilde{p}_3^N > 0, \tilde{q}_3^N > 0) \\ \left\langle (\tilde{p}_3^N / \tilde{q}_3^N, \tilde{p}_2^N / \tilde{q}_2^N), \tilde{p}_1^N / \tilde{q}_1^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b \right\rangle (\tilde{p}_3^N < 0, \tilde{q}_3^N > 0) \\ \left\langle (\tilde{q}_1^N / \tilde{p}_3^N, \tilde{p}_2^N / \tilde{q}_2^N), \tilde{p}_1^N / \tilde{q}_3^N), \mu_a \wedge \mu_b, \alpha_a \vee \alpha_b, \beta_a \vee \beta_b \right\rangle (\tilde{p}_3^N < 0, \tilde{q}_3^N < 0) \end{cases}$$

Theorem 2.4. *The fractional dual Algorithm under neutrosophic environment concludes in a finite number of steps.*

Proof:
Let $\tilde{T}_N^S$ represent the optiaml tableau under neutrosophic environment with columns $\tilde{T}_{N_j}^S$ and entries $\tilde{r}_{ij}^{N^S}$ after the rth cut has been added. Since $\tilde{r}_{00}^{N^S}$ is non-increasing because the optimal cost is non-decreasing. Furthermore, the tableau's bottom is always added with cuts. Therefore, $\tilde{T}^1_{\underset{0}{N}} \succ L, \tilde{T}^2_{\underset{0}{N}} \succ L, ...$
Now consider $\tilde{r}_{00}^{N^S}$ to be bound below. Then $\tilde{r}_{00}^{N^S}$ converges to some $\tilde{u}_{00}^N$. Assume that k iterations later, $\tilde{r}_{00}^{N^k} \prec \lfloor \tilde{u}_{00}^N \rfloor + 1$, where $0 \leq \tilde{g}_{00}^{N^k} \prec 1$, write $\tilde{r}_{00}^{N^k} = \lfloor \tilde{u}_{00}^N \rfloor + \tilde{g}_{00}^{N^k}$. The row $i = 0$ will be picked in the following iteration, and the cut is appended.

$$\sum_{j \notin B} (\left\lfloor \tilde{r}_{00}^{N^k} \right\rfloor - \tilde{r}_{00}^{N^k}) x_j + x_{n^k+1} = -\tilde{g}_{00}^{N^k}$$

Let p be the first column of the first pivot while performing dual simplex. As a result of this,

$$\tilde{r}_{00}^{N^{k+1}} = \tilde{r}_{00}^{N^k} + \frac{\tilde{r}_{0p}^{N^k}}{\lfloor \tilde{r}_{0p}^{N^k} \rfloor - \tilde{r}_{0p}^{N^k}} \tilde{g}_{00}^{N^k}$$

Note that $\tilde{r}_{0p}^{N^k} \geq 0$ (since is a reduced cost in the kth optimal tableau), and hence $0 \leq \left\lfloor \tilde{r}_{0p}^{N^k} \right\rfloor \leq \tilde{r}_{0p}^{N^k}$. Thus $\frac{\tilde{r}_{0p}^{N^k}}{\lfloor \tilde{r}_{0p}^{N^k} \rfloor - \tilde{r}_{0p}^{N^k}} \leq -1$, and wherefore

$$\tilde{r}_{00}^{N^{k+1}} \leq \tilde{r}_{00}^{N^k} - \tilde{g}_{00}^{N^k}$$

We know that $\tilde{u}_{00}^N$ is an integer and that $\tilde{r}_{00}^{N^{k+1}} = \tilde{u}_{00}^N$ as of $\tilde{r}_{00}^{N^S}$ is non-increasing and converges to $\tilde{u}_{00}^N$. Additionally, for all iterations of $s \geq k+1$, hence $\tilde{r}_{00}^{N^S} = \tilde{u}_{00}^N$. The same technique is applied to $\tilde{r}_{10}^{N^S}, \tilde{r}_{20}^{N^S}$ and so on. $\tilde{T}_N^S$ is' 0th column is lex-decreasing, therefore we know that $\tilde{r}_{10}^{N^S}$ is convergent to some $\tilde{u}_{10}^N$ (if bounded from below). However, we require that $\tilde{r}_{1p}^{N^k} \geq 0$ in order to apply the technique. Because $\tilde{r}_{00}^{N^k}$ is fixed at $\tilde{u}_{00}^N, \tilde{r}_{0p}^{N^k}$ is equal to 0. However, the dual simplex lexicographic anti-cycling rule makes sure that $\tilde{a}_{ij_p}^{N^k}$ remains lex-positive, signifying that $\tilde{r}_{1p}^{N^k}$ greater then equal to 0.

We repeated this until we get an integral solution for every row that corresponds to a variable in the original neutrosophic integer programming.

3 Neutrosophic Integer Programing Problem[25]

Integer programming problem with neutrosophic coefficients (NIPP) is defined as the following:

$$\begin{aligned} &\text{Maximize } Z = \sum_{j=1}^{n} \tilde{c}_j x_j \\ &\text{Subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij}^n x_j \leq b_i, i = 1, 2, \cdots, m \\ &x_j \geq 0, j = 1, 2, \cdots, n \\ &x_j \text{ are integer } j \in \{0, 1, \cdots, n\} \end{aligned} \tag{1}$$

where $\tilde{c}_j, \tilde{a}_{ij}^n$ are neutrosophic numbres.
The single valued neutrosophic number $(\tilde{a}_{ij}^n)$ is denoted by $A = (a, b, c)$ where $a, b, c \in [0, 1]$ and $a, b, c \leq 3$. The truth- membership function of neutrosophic number $\tilde{a}_{ij}^n$ is defined as:

$$T\tilde{a}_{ij}^n(x) = \begin{cases} \dfrac{(x - a_1)}{a_2 - a_1}, & (a_1 \leq x \leq a_2) \\ \dfrac{(a_2 - x)}{a_3 - a_2}, & (a_2 \leq x \leq a_3) \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

DOI: https://doi.org/10.54216/IJNS.260102

The indeterminacy- membership function of neutrosophic number $\tilde{a}_{ij}^{n}$ is defined as:

$$I\tilde{a}_{ij}^{n}(x)=\begin{cases}\dfrac{(x-b_1)}{b_2-b_1}, & (b_1\le x\le b_2)\\ \dfrac{(b_2-x)}{b_3-b_2}, & (b_2\le x\le b_3)\\ 0, & \text{otherwise}\end{cases}\tag{3}$$

And its falsity membership function of neutrosophic number $\tilde{a}_{ij}^{n}$ is defined as:

$$F\tilde{a}_{ij}^{n}(x)=\begin{cases}\dfrac{(x-c_1)}{c_2-c_1}, & (b_1\le x\le b_2)\\ \dfrac{(c_2-x)}{c_3-c_2}, & (b_2\le x\le b_3)\\ 0, & \text{otherwise}\end{cases}\tag{4}$$

Then we find the maximum and minimum values of the objective function for truth-membership, indeterminacy and falsity membership.
$f^{\max}=\max\{f(x_i^*)\}$ and $f^{\min}=\min\{f(x_i^*)\}$ where $1\le i\le k$.
$f_{\min}^{F}=f_{\min}^{T}$ and $f_{\max}^{F}=f_{\max}^{T}-R(f_{\max}^{T}-f_{\min}^{T})$
$f_{\max}^{I}=f_{\max}^{T}$ and $f_{\min}^{I}=f_{\min}^{T}-S(f_{\max}^{T}-f_{\min}^{T})$
Where R,S are predetermined real number in $(0,1)$. The truth membership, indeterminacy membership, falsity membership of objective function as follows:

$$T^{f}(x)=\begin{cases}1 & \text{if } f\le f^{\min}\\ \dfrac{f^{\max}-f(x)}{f^{\max}-f^{\min}} & \text{if } f^{\min}\prec f(x)\prec f^{\max}\\ 0 & \text{if } f(x)\succ f^{\max}\end{cases}\tag{5}$$

$$I^{f}(x)=\begin{cases}0 & \text{if } f\le f^{\min}\\ \dfrac{f(x)-f^{\max}}{f^{\max}-f^{\min}} & \text{if } f^{\min}\prec f(x)\prec f^{\max}\\ 0 & \text{if } f(x)\succ f^{\max}\end{cases}\tag{6}$$

$$F^{f}(x)=\begin{cases}0 & \text{if } f\le f^{\min}\\ \dfrac{f(x)-f^{\max}}{f^{\max}-f^{\min}} & \text{if } f^{\min}\prec f(x)\prec f^{\max}\\ 1 & \text{if } f(x)\succ f^{\max}\end{cases}\tag{7}$$

The neutrosophic set of the JT decision variable x_J is defined as:

$$T_{x_j}^{(x)}(x)=\begin{cases}1 & \text{if } x_j\le 0\\ \dfrac{d_j-x_j}{d_j} & \text{if } 0\prec x_j\le d_j\\ 0 & \text{if } x_j\succ d_j\end{cases}\tag{8}$$

$$F_{x_j}^{(x)}(x)=\begin{cases}0 & \text{if } x_j\le 0\\ \dfrac{x_j}{d_j+b_j} & \text{if } 0\prec x_j\le d_j\\ 1 & \text{if } x_j\succ d_j\end{cases}\tag{9}$$

$$I_{x_j}^{(x)}(x)=\begin{cases}0 & \text{if } x_j\le 0\\ \dfrac{x_j-d_j}{d_j+b_j} & \text{if } 0\prec x_j\le d_j\\ 0 & \text{if } x_j\succ d_j\end{cases}\tag{10}$$

where d_j,b_j are integer numbers.

4 Neutrosophic Optimization Model of Integer Programming Problem[25]

In our neutrosophic model we want to maximize the degree of acceptance and minimize the degree of rejection and indeterminacy of the neutrosophic objective function and constraints. Neutrosophic optimization model can be defined as:

$$\begin{aligned}&\max T_{(x)}\\&\min F_{(x)}\\&\min I_{(x)}\\&\text{Subject to}\\&T_{(x)} \geq F_{(x)}\\&T_{(x)} \geq I_{(x)}\\&0 \leq T_{(x)} + F_{(x)} + I_{(x)}\\&T_{(x)}, F_{(x)}, I_{(x)} \geq 0\\&x \geq 0 \text{ are integers.}\end{aligned} \tag{11}$$

where $T_{(x)}, F_{(x)}, I_{(x)}$ denotes the degree of acceptance, rejection and indeterminacy of x respectively.
The above problem is equivalent to the following:

$$\begin{aligned}&\max \alpha, \min \beta, \min \theta\\&\text{Subject to}\\&\alpha \leq T_{(x)}\\&\beta \leq F_{(x)}\\&\theta \leq I_{(x)}\\&\alpha \geq \beta\\&\alpha \geq \theta\\&0 \leq \alpha + \beta + \theta \leq 3\\&x \geq 0 \text{ are integers.}\end{aligned} \tag{12}$$

Where α denotes the minimal acceptable degree, β denote the maximal degree of rejection and θ denote maximal degree of indeterminacy. The neutrosophic optimization model can be changed into the following optimization model:

$$\begin{aligned}&\max(\alpha - \beta - \theta)\\&\text{Subject to}\\&\alpha \leq T_{(x)}\\&\beta \leq F_{(x)}\\&\theta \leq I_{(x)}\\&\alpha \geq \beta\\&\alpha \geq \theta\\&0 \leq \alpha + \beta + \theta \leq 3\\&x \geq 0 \text{ are integers.}\end{aligned} \tag{13}$$

The previous model can be written as:

$$\begin{aligned}&\min(1 - \alpha)\beta\theta\\&\text{Subject to}\\&\alpha \leq T_{(x)}\\&\beta \leq F_{(x)}\\&\theta \leq I_{(x)}\\&\alpha \geq \beta\\&\alpha \geq \theta\\&0 \leq \alpha + \beta + \theta \leq 3\\&x \geq 0 \text{ are integers.}\end{aligned} \tag{14}$$

5 Proposed Strategy

Step 1: Formulate the problem as (1).
Step 2: Let $\tilde{a}_{ij}^N$ be an $m \times n$ matrix with SvTNN's are $(\tilde{p}_1^N, \tilde{p}_2^N, \tilde{p}_3^N, \mu_a, \alpha_a, \beta_a)$ and taken into consideration the following neutrosophic integer program is formulated as

$$\begin{aligned} &\text{NIP Max } \tilde{Z}_N = \sum_{j=1}^{n} \tilde{c}_j^N x_j \\ &\text{Subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij}^N x_j \leq b_i^N, \\ &x_j \geq 0, x_j \text{ is integers} \end{aligned} \tag{15}$$

Considering NLP be the neutrosophic linear programming relaxation, where NLP to be optimally solved and yielding an neutrosophic optimal basis as well as an associated with neutrosophic optimal table $\tilde{T}_N$. Let $\tilde{x}^{N^*}$ represent the optimal solution of the NLP relaxation.
Step 3: Let $\tilde{r}_{ij}^N$ represent the entry of the neutrosophic optimal table in the ith row and jth column. We may determine the valid equality from the ith row, where $i = 1, 2, \cdots, m$.

$$\tilde{x}_{B(i)}^N + \sum_{j \notin B} \left\lfloor \tilde{r}_{ij}^N \right\rfloor x_j = \left\lfloor \tilde{r}_{i0}^N \right\rfloor \tag{16}$$

Since all of the x_j of $\tilde{p}_2^N$ are nonnegative, so we have that is an appropriate inequality for NIP.

$$\tilde{x}_{B(i)}^N + \sum_{j \notin B} \left\lfloor \tilde{r}_{ij}^N \right\rfloor x_j \leq \tilde{r}_{i0}^N \tag{17}$$

Therefore, the right side of the inequality is an integer since all of the x_j of $\tilde{p}_2^N$ are integers in NIP. Hence

$$\tilde{x}_{B(i)}^N + \sum_{j \notin B} \left\lfloor \tilde{r}_{ij}^N \right\rfloor x_j \leq \left\lfloor \tilde{r}_{i0}^N \right\rfloor$$

is also a valid inequality for NIP. Subtracting (1) from (2), we obtain the appropriate inequality

$$\sum_{j \notin B} (\left\lfloor \tilde{r}_{ij}^N \right\rfloor - \tilde{r}_{ij}^N) x_j \leq \left\lfloor \tilde{r}_{i0}^N \right\rfloor - \tilde{r}_{i0}^N$$

By adding the slack variable x_{n+1}, In the optimal table we adding the constraints and G_{n+1} to the basis

$$\sum_{j \notin B} (\left\lfloor \tilde{r}_{ij}^N \right\rfloor - \tilde{r}_{ij}^N) x_j + x_{n+1} \leq \left\lfloor \tilde{r}_{i0}^N \right\rfloor - \tilde{r}_{i0}^N \tag{18}$$

Since the table is set to progress. Where $\left\lfloor \tilde{r}_{i0}^N \right\rfloor - \tilde{r}_{i0}^N \prec 0$ and the decreased costs remain non-negative for $\tilde{p}_2^N$, we proceed with the dual simplex algorithm.
Step 4: Notice that $\tilde{x}^{N^*}$ does not satisfy (2), so by introducing the inequality, the feasible region of the neutrosophic linear programming relaxation is reduced. These cuts are added repeatedly until an integral solution is found using the Fractional Dual Algorithm (also known as the The Gomory Cutting Plane Algorithm). Since (2) is satisfied by each and every feasible integral solution to NIP, no integral solutions are lost.
Step 5: We insist that the dual simplex method make use of lexicographic anticycling rules for $\tilde{p}_2^N$ in order to demonstrate that the Fractional Dual Algorithm concludes after a finite number of steps. Thus, we provide an overview of a Fractional Dual Algorithm iteration:

1. Select the highest (i.e. first) row where $\tilde{r}_{i0}^N$ is not integral. (Note that this includes the zeroth row.)
2. Add the cut $\sum_{j \notin B} (\left\lfloor \tilde{r}_{ij}^N \right\rfloor - \tilde{r}_{ij}^N) x_j + x_{n+1} \leq \left\lfloor \tilde{r}_{i0}^N \right\rfloor - \tilde{r}_{i0}^N$ to the bottom of the optimal tableau, and add G_{n+1} to the basis $\tilde{B}_N$.

3. Apply the lexicographic anticycling criteria for $\tilde{p}_2^N$ to the dual simplex algorithm as follows:

   (a) Begin by making all columns $j = 1, 2, \cdots, n+1$ lex-positive.

   (b) Select any row m where $\tilde{r}_{m0}^N \leq 0$ seems to be the pivot row Choose the pivot column j that maximixes the index

$$j : \tilde{r}_{m0}^N \overset{lex-max}{\prec 0} \left[\frac{1}{\tilde{r}_{m0}^N} \tilde{B}_N^{-1} \tilde{a}_{ij}^N \right] \tag{19}$$

observe that the lexicographic anticycling rule for $\tilde{p}_2^N$ ensures that all columns $j = 1, \cdots, n$ continue lexicographically positive.

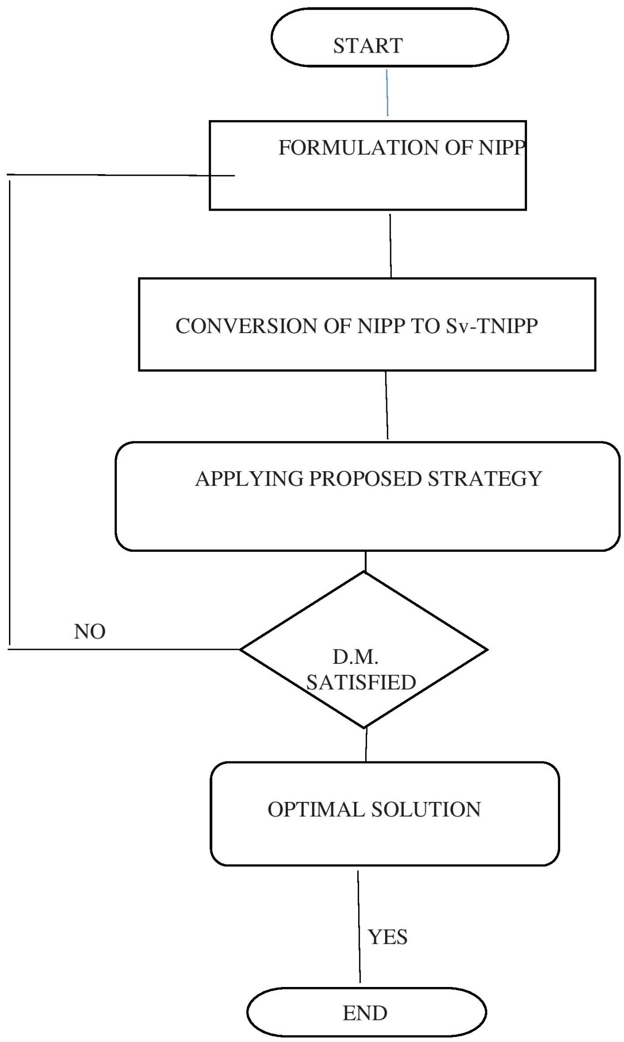


Figure 1: Overview of Proposed Method

6 Numerical Example

The numerical example is illustrate with example and followed with application

6.1 Numerical Example 1[30]

$$\begin{aligned}
&\text{Max } \tilde{Z} \approx \tilde{4}x_1 + \tilde{3}x_2 \\
&\text{subject to} \\
&\tilde{4}x_1 + \tilde{2}x_2 \leq 1\tilde{2} \\
&\tilde{3}x_1 + \tilde{6}x_2 \leq \tilde{5} \\
&x_1, x_2 \geq 0 \\
&\text{where } x_1, x_2 \text{ are integers.}
\end{aligned}$$

Step 1:

$$\begin{aligned}
&\text{Max } \tilde{Z} = [(2,4,6), 0.8, 0.6, 0.4]x_1 + [(1,3,5), 0.75, 0.5, 0.3]x_2 \\
&\text{subject to} \\
&[(0,4,8), 1, 0.0, 0.5]x_1 + [(1,2,3), 1, 0.5, 0.5]x_2 \leq [(5,12,19), 1, 0.25, 0.25] \\
&[(1,3,5), 0.75, 0.0, 0.25]x_1 + [(1,6,11), 1, 0, 0]x_2 \leq [(3,5,7), 0.8, 0.6, 0.4] \\
&x_1, x_2 \geq 0 \\
&\text{where } x_1, x_2 \text{ are integers.}
\end{aligned}$$

Step 2:
The iterations utilising proposed strategy gives in the table 1 and table 2.

Step 3:
By following step 3 of proposed stragegy, formulating the cut

$$\left[\left(\frac{3}{5}, \frac{2}{3}, 7\right), 0.75, 0.6, 0.4\right] = \left[\left(0, \frac{-1}{3}, 0\right), 0, 0, 0.25\right] x_4 + G_1$$

Step 4:
The iterations utilising proposed strategy gives in the table 3 and table 4.

Tables Step 5:
The feasible solution is $x_1 = \left[\left(\frac{3}{5}, 1, 7\right), 0.0, 0.6, 0.4\right]$ and $x_2 = (0,0,0), 0, 0, 0$ and the optimal solution is Max $Z = \left[\left(\frac{6}{5}, 4, 42\right), 0.0, 0.6, 0.4\right]$ It is quite obvious from the following comparison table with existing methods that our technique is always maximizing the desired result for the decision maker.

Table 1: Represents Iteration 1

		C_j	$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(1,3,5);\\0.75,0.5,0.3\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	
$\tilde{C}_{N_B}$	$\tilde{X}_{N_B}$	$\tilde{Y}_{N_B}$	$\tilde{x}_1^N$	$\tilde{x}_2^N$	$\tilde{x}_3^N$	$\tilde{x}_4^N$	$\tilde{\theta}_N$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_3^N$	$\begin{pmatrix}(5,12,19);\\(1,0.25,0.25)\end{pmatrix}$	$\begin{pmatrix}(0,4,8);\\(1,0.0,0.5)\end{pmatrix}$	$\begin{pmatrix}(1,2,3);\\(1,0.5,0.5)\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}\left(\frac{5}{8},3,0\right);\\(1,0.25,0.5)\end{pmatrix}$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_4^N$	$\begin{pmatrix}(3,5,7);\\(0.8,0.6,0.4)\end{pmatrix}$	$\begin{pmatrix}(1,3,5);\\(0.75,0.0,0.25)\end{pmatrix}$	$\begin{pmatrix}(1,6,11);\\(1,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(3,5,7);\\(0.8,0.6,0.4)\end{pmatrix}$
		$\tilde{Z}_{N_j} - \tilde{C}_{N_j}$	$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(1,3,5);\\(0.75,0.0,0.25)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	

Table 2: Represents Iteration 2

$\tilde{C}_{N_B}$	$\tilde{X}_{N_B}$	C_{N_j} $\tilde{Y}_{N_B}$	$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$ $\tilde{x}_1^N$	$\begin{pmatrix}(1,3,5);\\0.75,0.5,0.3\end{pmatrix}$ $\tilde{x}_2^N$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_3^N$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_4^N$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_3^N$	$\begin{pmatrix}\left(-51,\ \frac{16}{3},9\right);\\0.75,0.6,0.5\end{pmatrix}$	$\begin{pmatrix}(-40,0,8);\\0.75,0.0,0.5\end{pmatrix}$	$\begin{pmatrix}(-87-6,3);\\0.75,0.5,0.5\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,-\frac{4}{3}\ ,0);\\0,0,0.5\end{pmatrix}$
$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$	$\tilde{x}_1^N$	$\begin{pmatrix}\left(\frac{3}{5},\frac{5}{3},7\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(\frac{1}{5},1,5);\\0.75,0,0.25\end{pmatrix}$	$\begin{pmatrix}(\frac{1}{5},2,11);\\0.75,0,0.25\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,\frac{1}{3},0);\\(0,0,0)\end{pmatrix}$
$\tilde{Z}_{N_j}-\tilde{C}_{N_j}$		$\begin{pmatrix}\left(\frac{6}{5},\frac{20}{3},42\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}\left(\frac{-28}{5},0,\frac{4}{5}\right);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}\left(\frac{23}{5},5,65\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,\frac{4}{3},0);\\0,0.6,0.4\end{pmatrix}$

Table 3: Represents Iteration 3

$\tilde{C}_{N_B}$	$\tilde{Y}_{N_B}$	C_{N_j} $\tilde{X}_{N_B}$	$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$ $\tilde{x}_1^N$	$\begin{pmatrix}(1,3,5);\\0.75,0.5,0.3\end{pmatrix}$ $\tilde{x}_2^N$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_3^N$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_4^N$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_3^N$	$\begin{pmatrix}\left(-51,\ \frac{16}{3},9\right);\\0.75,0.6,0.5\end{pmatrix}$	$\begin{pmatrix}(-40,0,8);\\0.75,0.0,0.5\end{pmatrix}$	$\begin{pmatrix}(-87-6,3);\\0.75,0.5,0.5\end{pmatrix}$	$\begin{pmatrix}(0,-\frac{4}{3}\ ,0);\\0,0,0.5\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$
$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$	$\tilde{x}_1^N$	$\begin{pmatrix}\left(\frac{3}{5},\frac{5}{3},7\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(\frac{1}{5},1,5);\\0.75,0,0.25\end{pmatrix}$	$\begin{pmatrix}(\frac{1}{5},1,5);\\0.75,0,0.25\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,\frac{1}{3},0);\\(0,0,0)\end{pmatrix}$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	G_1	$\begin{pmatrix}\left(\frac{3}{5},\frac{-2}{3},7\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,\frac{-1}{3},0);\\(0,0,0)\end{pmatrix}$
$\tilde{Z}_{N_j}-\tilde{C}_{N_j}$		$\begin{pmatrix}\left(\frac{6}{5},\frac{20}{3},42\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}\left(\frac{-28}{5},0,\frac{4}{5}\right);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}\left(-\frac{23}{5},5,65\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,\frac{4}{3},0);\\0,0.6,0.4\end{pmatrix}$

Table 4: Represents Iteration 4

$\tilde{C}_{N_B}$	$\tilde{Y}_{N_B}$	C_{N_j} $\tilde{X}_{N_B}$	$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$ $\tilde{x}_1^N$	$\begin{pmatrix}(1,3,5);\\0.75,0.5,0.3\end{pmatrix}$ $\tilde{x}_2^N$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_3^N$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_4^N$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$ $\tilde{G}_{N_1}$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_3^N$	$\begin{pmatrix}(51,\ 8,9);\\0.75,0.6,0.5\end{pmatrix}$	$\begin{pmatrix}(-40,0,8);\\0.75,0.0,0.5\end{pmatrix}$	$\begin{pmatrix}(-87-6,3);\\0.75,0.5,0.5\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0.5)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$
$\begin{pmatrix}(2,4,6);\\0.8,0.6,0.4\end{pmatrix}$	$\tilde{x}_1^N$	$\begin{pmatrix}\left(\frac{3}{5},1,7\right);\\0.75,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(\frac{1}{5},1,5);\\0.75,0,0.25\end{pmatrix}$	$\begin{pmatrix}(\frac{1}{5},2,11);\\0.75,0,0.25\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0.25)\end{pmatrix}$	$\begin{pmatrix}(0,\ 1\ ,0);\\0.0,0.25,0.0\end{pmatrix}$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_4^N$	$\begin{pmatrix}(0,\ 2\ ,0);\\0.0,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,\ 1\ ,0);\\0.0,0.25,0.0\end{pmatrix}$	$\begin{pmatrix}(0,\ 3\ ,0);\\0.0,0.25,0.0\end{pmatrix}$

7 Application in Hospital[25]

In a real-life scenario, consider a hospital's resource problem where the hospital needs to optimize the surgeries for patients while considering uncertain and imprecise factors under a triangular neutrosophic Fuzzy environment. A hospital has a limited number of operating rooms, surgeons,for surgeries. Patients require surgical procedures but the precise duration of each surgery and the patients' medical conditions are uncertain and can be described using Neutrosophic Fuzzy sets. Additionally, the availability of medical personnel varies due to factors like emergencies and sick leaves. NF Parameters: Let's represent the uncertainty in surgical durations like (Organ Transplantation,Craniofacial Reconstruction, Vascular Reconstructions,Neurosurgical Procedures,Orthopedic Reconstructions) and patient conditions using Triangular Neutrosophic Fuzzy sets.
Surgical Duration : The sugergion duration takes place for 5 hours and Patient Condition The pateint condition is stable for 3 hours.
Objective: Optimize the scheduling of surgeries to maximize the utilization of operating rooms and medical personnel while considering the Neutrosophic Fuzzy surgical durations and patient conditions. Constraints: Operating Rooms: The hospital has 12 operating rooms available. Surgeons: There are 6 surgeons with varying availability. Surgery Start Times: Surgeries must start at specified times during the day. Non-negativity: The number of surgeries scheduled must be non-negative.
Triangular Neutrosophic Fuzzy number 5 representing the surgical duration uncertainty for surgery.
Triangular Neutrosophic Fuzzy number 3 representing the patient condition uncertainty for surgery.
Objective Function: Maximize the objective function, which represents the utilization of resources while accommodating uncertainty:
Maximize Z = surgical duration (Organ Transplantation,Craniofacial Reconstruction, Vascular Reconstructions,Neurosurgical Procedures,Orthopedic Reconstructions) + patient condition
Constraints:
Subject to constraints ensuring that the resources are utilized efficiently and that scheduling decisions align with the available resources, including operating room capacity, surgeon availability.
Operating Room Constraints:
the surgery having a particular surgical duration in operating room $\leq$ Operating Room Capacity Surgeon Availability Constraints:
Reperesents the surgery having a particular patient condition in operating room $\leq$ Surgeon Availability
In the above formulation: The objective function maximizes the total number of surgeries scheduled, which represents resource utilization. The constraints ensure that the scheduling decisions comply with the available resources and accommodate the Neutrosophic Fuzzy uncertainty in surgical durations and patient conditions.

Step 1:

$$\begin{aligned}
&\text{Max } \tilde{Z} \approx \tilde{5}x_1 + \tilde{3}x_2\\
&\text{subject to}\\
&\tilde{4}x_1 + \tilde{2}x_2 \leq 1\tilde{2}\\
&\tilde{1}x_1 + \tilde{3}x_2 \leq \tilde{6}\\
&x_1, x_2 \geq 0\\
&\text{where } x_1, x_2 \text{ are integers.}
\end{aligned}$$

Step 2:

$$\begin{aligned}
&\text{Max } \tilde{Z} = [(4,5,6),0.8,0.6,0.4]x_1 + [(2.5,3,3.5),0.75,0.5,0.3]x_2\\
&\text{subject to}\\
&[(3.5,4,4.1),1,0.5,0.0]x_1 + [(2.5,3,3.5),0.75,0.5,0.25]x_2 \leq [(11,12,13),1,0.5,0.0]\\
&[(0,1,2),1,0.5,0.0]x_1 + [(2.8,3,3.2),0.75,0.5,0.0]x_2 \leq [(5.5,6,7.5),0.8,0.6,0.4]\\
&x_1, x_2 \geq 0\\
&\text{where } x_1, x_2 \text{ are integers.}
\end{aligned}$$

Step 3:
The iterations utilising proposed strategy gives in the table 5 and table 6.

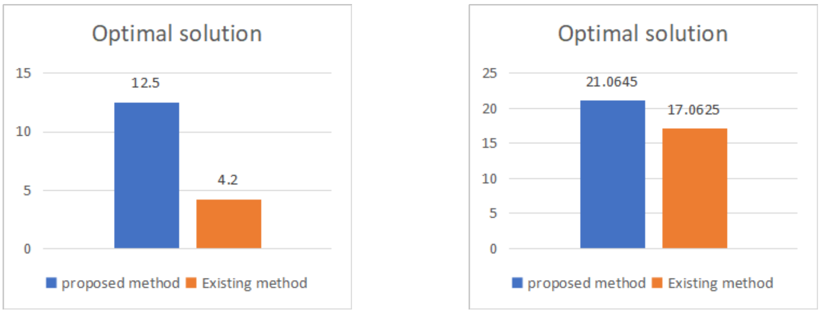
Table 5: Represents Iteration 2

$\tilde{C}_{N_B}$	$\tilde{X}_{N_B}$	C_j / $\tilde{Y}_{N_B}$	$\begin{pmatrix}(4,5,6);\\0.8,0.6,0.4\end{pmatrix}$ $\tilde{x}_1^N$	$\begin{pmatrix}(2.5,3,3.5);\\0.75,0.5,0.25\end{pmatrix}$ $\tilde{x}_2^N$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_3^N$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_4^N$	$\tilde{\theta}_N$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_3^N$	$\begin{pmatrix}(11,12,13);\\1,0.5,0.0\end{pmatrix}$	$\begin{pmatrix}(3.5,4,4.1);\\1,0.5,0.0\end{pmatrix}$	$\begin{pmatrix}(2.5,3,3.5);\\0.75,0.5,0.25\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}\left(\frac{11}{4.1},3,\frac{13}{3.5}\right);\\1,0.5,0.0\end{pmatrix}$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_4^N$	$\begin{pmatrix}(5.5,6,7.5);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(0,1,2);\\1,0.5,0\end{pmatrix}$	$\begin{pmatrix}(2.8,3,3.2)\\0.75,0.5,0.25\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}\left(\frac{5.5}{2},6,0\right);\\0.8,0.6,0.4\end{pmatrix}$
		$\tilde{Z}_{N_j}-\tilde{C}_{N_j}$	$-\begin{pmatrix}(4,5,6);\\0.8,0.6,0.4\end{pmatrix}$	$-\begin{pmatrix}(2.5,3,3.5);\\0.75,0.5,0.25\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	

Table 6: Represents Iteration 1

$\tilde{C}_{N_B}$	$\tilde{X}_{N_B}$	C_{N_j} / $\tilde{Y}_{N_B}$	$\begin{pmatrix}(4,5,6);\\0.8,0.6,0.4\end{pmatrix}$ $\tilde{x}_1^N$	$\begin{pmatrix}(2.5,3,3.5);\\0.75,0.5,0.25\end{pmatrix}$ $\tilde{x}_2^N$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_3^N$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$ $\tilde{x}_4^N$
$\begin{pmatrix}(4,5,6);\\0.8,0.6,0.4\end{pmatrix}$	$\tilde{x}_1^N$	$\begin{pmatrix}\left(\frac{11}{4.1},12,\frac{13}{3.5}\right);\\1,0.5,0.0\end{pmatrix}$	$\begin{pmatrix}(\frac{3.5}{4.1},1,\frac{4.1}{3.5});\\1,0.5,0.0\end{pmatrix}$	$\begin{pmatrix}(\frac{2.5}{4.1},3,\frac{2}{3.5});\\0.75,0.5,0.25\end{pmatrix}$	$\begin{pmatrix}\left(0,\frac{1}{4},0\right);\\0,0,0\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$
$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$	$\tilde{x}_4^N$	$\begin{pmatrix}(6.75,3,7.5);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}(0.55,2,0);\\0.75,0.5,0.25\end{pmatrix}$	$\begin{pmatrix}\left(8.6,\frac{9}{4},0\right);\\0.75,0.5,0.25\end{pmatrix}$	$-\begin{pmatrix}\left(0,\frac{1}{4},0\right);\\0,0,0\end{pmatrix}$	$\begin{pmatrix}(0,1,0);\\(0,0,0)\end{pmatrix}$
		$\tilde{Z}_{N_j}-\tilde{C}_{N_j}$	$\begin{pmatrix}(-8,0,20.6);\\0.8,0.6,0.4\end{pmatrix}$	$\begin{pmatrix}\left(6.5,\frac{3}{4},3.25\right);\\0.75,0.5,0.25\end{pmatrix}$	$\begin{pmatrix}\left(0,\frac{5}{4},0\right);\\0,0.5,0\end{pmatrix}$	$\begin{pmatrix}(0,0,0);\\(0,0,0)\end{pmatrix}$

Step 4:
The above problem had integer optimal solution found after 0-cuts.
Step 5:
Hence the feasible solution is $x_1 = \left(\frac{11}{4.1}, 3, \frac{13}{3.5}\right), 1, 0.5, 0.0$ and $x_2 = (0,0,0), 0, 0, 0$ and the optimal solution is Max $Z = \left[\left(\frac{44}{4.1}, 15, \frac{78}{3.5}\right), 0.8, 0.6, 0.4\right]$.



(a) (b)

Figure 2: (a) Graphical representation of comparision work for example 1; (b) Graphical representation of comparision work for Application in Hospital

8 Comparability Analysis:

In this part, the suggested algorithm and the current approach to single valued neutrosophic integer programming problems are compared. Table 7 exhibit a comparison of the outcomes using current strategies and novel ones. When we compared our suggested technique to the other existing ways in Figs.2 (Graphical comparison with current methods),The comparison for crisp values of produced optimal values and optimal solution by utilising the provided scoring function is shown below. we observed that, when compared to the other approaches, our suggested method had the highest objective value.

Researchers have discussed their efforts to advance theories relating to the neutrosophic domain in the recent years and continuously work to support their sufficient scope applications in many parts of the neutrosophic domain. However, while defending every argument in favour of the SvtNN theory, our major goal is to effectively support the theory with the arguments that follow. Since optamility in a neutroshopic environment led to the conversion of a crisp model with crisp values, in our opinion, no research has demonstrated the outcome of optimality under neutrosophic conditions. Provided that our strategy produces the best result optimality in the neutrosophic domain.

Table 7: Camporision with existing method

Solution	Proposed method	Existing method
Optimal solution for example 1[30]	$Z = 12.5$	$Z = 4.2$
Optimal solution for Application[25]	$Z = 21.0645$	$Z = 17.0625$

9 Advantages

In summary, the research paper's advantages lie in its innovative approach, effective handling of uncertainty, clarity in algorithmic presentation, empirical validation, broad applicability, theoretical contributions, and its potential to enhance decision-making in various domains. These advantages collectively position the paper as a significant contribution to the field of optimization under uncertain conditions.
Introduces a novel lexicographic method for solving Integer Programming in Neutrosophic Fuzzy settings, revolutionizing uncertain problem-solving.
Skillfully incorporates Neutrosophic fuzzy sets, aiding robust decision-making in real-world scenarios plagued by vagueness and uncertainty.
Establishes a clear foundation for understanding complex optimization problems, paving the way for future theoretical advancements.
Demonstrates practical viability through extensive numerical experiments, bolstering confidence in real-world

applicability.
Expands theoretical boundaries by integrating neutrosophic fuzzy sets into integer programming, enriching academic discourse.
Empowers decision-makers in uncertain environments, offering informed choices with far-reaching implications for various industries.
Bridges optimization, fuzzy logic, and neutrosophy, encouraging collaboration and knowledge exchange across diverse disciplines.

10 Limitations

The innovative lexicographic approach may face heightened computational demands for large-scale NF-IP problems, potentially limiting its practicality due to resource and time requirements.
Scalability concerns: While demonstrating effectiveness on small to medium-sized instances, the approach's adaptability to handle more extensive problems remains uncertain and necessitates further exploration.
Defining Neutrosophic Fuzzy sets accurately often demands substantial data input, posing challenges in data gathering and validation, especially concerning real-world data quality limitations.
The success of the approach may hinge on specific parameter choices, potentially introducing subjectivity and requiring meticulous tuning.
Dealing with intricate constraints like non-linearity or dynamics within the NF-IP framework poses unaddressed challenges warranting future exploration for incorporation into the approach.
Primarily designed for single-objective NF-IP problems, extending the approach to handle multi-objective scenarios common in real-world applications remains an important avenue for future research.
Despite advancements, recognizing and addressing these limitations through refined research efforts are crucial to enhance the approach's practicality and impact in solving Neutrosophic Fuzzy Integer Programming problems.

11 Conclusion and Future Scope

Decision makers have wider choice when expressing their preference information by using neutrosophic numbers ,represented by SvTNN.This paper introduces a pioneering lexicographic approach for solving Integer Programming problems in the Neutrosophic Fuzzy environment. Our contributions encompass problem formalization, algorithm development, practical applications, and theoretical advancements, making this research valuable not only to the academic community but also to practitioners seeking robust solutions for decision-making under uncertainty. Our approach opens up new avenues for addressing complex optimization problems with neutrosophic fuzzy elements, facilitating more informed and reliable decision-making processes in various domains. This paper demonstrates the lexicographic anti -cyclic rules in dual fractional algorithm for neutrosophic integer programming . Meanwhile, based on the dual fractional algorithm is defined using operational laws of Neutrosophic numbers . Moreover, neutrosophic integer programming model was constructed to obtain the acceptably no integral solutions are lost. Through analyzing the optimality function of DMs, Some examples were to illustrate the feasibility and effectiveness of the proposed method. The dual fractional algorithm not only keeps the integrity of information, but also enriches the operational laws of NN .Due to uncertainty across every environment and the fact that more researchers are turning the original fuzzy problems into crisp values, our technique creates the answer to the problem. Also the proposed method executed without changing the nature of problem and it shows the best optimality solution along which establishes its superiority over the state of the art through comparison.
Future Research Directions
Enhance the lexicographic approach's scalability for larger Neutrosophic Fuzzy Integer Programming (NF-IP) problems by exploring parallel computing, heuristic algorithms, and hybrid methods.
Extend the approach to handle multi-objective NF-IP scenarios, aiming to find Pareto-optimal solutions amidst conflicting objectives in real-world decision-making.
Investigate integrating and solving NF-IP problems with non-linear constraints, developing optimization techniques within the Neutrosophic Fuzzy framework.

Adapt the approach for swift decision-making in dynamic settings like supply chain management, creating algorithms for near-optimal solutions in time-sensitive situations.
Integrate the approach with other optimization and AI techniques to create hybrid models for more effective handling of NF-IP problems.
Develop precise methodologies to model Neutrosophic Fuzzy parameter uncertainties, enhancing solution reliability.
Validate the approach through diverse industry case studies, identifying strengths and areas needing refinement.
Foster collaboration between optimization, fuzzy logic, and neutrosophy experts to innovate complex decision-making under uncertainty.
By pursuing these paths, the lexicographic approach for Neutrosophic Fuzzy Integer Programming can evolve, offering robust solutions for varied real-world applications.

Funding: Not applicable.

Conflicts of Interest: The authors declare that they do not have any conflict of interest.

References

[1] W. Abdelfattah, . A parametric approach to solve neutrosophic linear programming models. Journal of Information and Optimization Sciences. 1-25, 2020.

[2] B. Akay , D. Karaboga, B. Gorkemli, and E. Kaya, A survey on the Artificial Bee Colony algorithm variants for binary integer and mixed integer programming problems. Applied Soft Computing. 106, 1-35.

[3] M. Allahviranlo, and S. Afandizadeh, Investment optimization on port's development by fuzzy integer programming. European Journal of Operational Research, 186, 423–434, 2008.

[4] R. Anderson, J. Huchette, W. Ma, C. Tjandraatmadja, J.P. Vielma, Strong mixed-integer programming formulations for trained neural networks. Mathematical Programming, 183, 3–39, 2020. https://doi.org/10.1007/s10107-020-01474-5

[5] E. Balas, M. Fischett, and A. Zanette, A hard integer program made easy by lexicography. Math. Program. Ser. A., 135, 509–514 2021

[6] A. Bärmann, A. Müller, and D. Weninger, Lexicographic Branch-and-Bound Column Search, Optimization Online. 1-44, 2022. https://optimization-online.org/?p=19009

[7] M.A. Basset, M. Gunasekaran, M. Mohamed, and F. Smarandache, A novel method for solving the fully neutrosophic linear programming problems. Neural Computing and Applications, 31, 1595–1605,2019 . https://doi.org/10.1007/s00521-018-3404-6

[8] T. Bera, N.K. Mahapatra, Neutrosophic linear programming problem and its application to real life. Afrika Matematika, 31, 709–726, 2020 https://doi.org/10.1007/s13370-019-00754-4

[9] S. Broumi, An Efficient Approach for Solving Time-Dependent Shortest Path Problem under Fermatean Neutrosophic Environment. Neutrosophic Sets and Systems, 63(1), 6, 2024

[10] R.S. Burachik, C.Y. Kaya, M.M. Rizvi, Algorithms for generating Pareto fronts of multi-objective integer and mixed-integer programming problems. Engineering Optimization, 54, 1413–1425, 2022. https://doi.org/10.1080/0305215X.2021.1939695

[11] B.P. Canedo, J.L. Verdegay, . On the Application of a Lexicographic Method to Fuzzy Linear Programming Problems, Journal of Computational and Cognitive Engineering, 00, 1–10, 2022.

[12] S. Chernov, S., L. Chernova, S. Titov, N. unanets, The Analysis of Gomory Algorithm Convergence in Integer Linear Optimization Problem: Projects Management , 1-8,2019.

[13] M. Conforti, M.D. Santis, M.D. Summa, and F. Rinaldi, Scanning integer points with lex-inequalities: A finite cutting plane algorithm for integer programming with linear objective. 1-16, 2020.

[14] S. Dash, O. Günlük, D. Lee, Generalized Chvátal-Gomory closures for integer programs with bounds on variables. Mathematical Programming, 190, 393–425, 2019. https://doi.org/10.1007/s10107-020-01539-5

[15] Y. Demir, An Efficient Lexicographic Approach to Solve Multi-Objective Multi-Port Fabric Dyeing Machine Planning Problem. soft computing, 1-30, 2022.

[16] M. Eldredge, A. Gupte, On Lexicographic Approximations Of Integer Programs. Arxiv, 1-26, 2017. https://doi.org/10.48550/arXiv.1610.06470

[17] M. ElHadidi, O.E. Emam, A.M. Abdelfadel, New Method to Optimize the Fully Neutrosophic Linear Programming Problems. Appl. Math. Inf. Sci., 15, 211-21, 2021.

[18] A. Ferber, B. Wilder, B. Dilkina, M. Tambe, MIPaaL Mixed Integer Program as a Layer. Arxiv, 34443-3452, 2022. https://doi.org/10.48550/arXiv.1907.05912

[19] B. Huang, M.C. Zhou, G. X. Zhang, A.C. Ammari, A. Alabdulwahab, G. Ayman, A.G. Fayoumi, Lexicographic Multiobjective Integer Programming for Optimal and Structurally Minimal Petri Net Supervisors of Automated Manufacturing Systems. IEEE Transactions On Systems, Man, And Cybernetics: Systems, 45, 1459-1470, 2015.

[20] K.E. Huda, A.K. Essa, The Duality Approach of the Neutrosophic Linear Programming. Neutrosophic Sets and Systems, 46, 9-23, 2021. https://digitalrepository.unm.edu/nss_journal/vol46/iss1/29

[21] P. Ignizo, and S.C. Daniels, Fuzzy Multicriteria Integer Programming Via Fuzzy Generalized Networks. fuzzy sets and systems ,10, 261-270, 1983.

[22] P. Ignizo, and L.C. Thomas, An Enchanced conversion scheme for lexicographic , multiobjective integer programs. European Journal of operation research, 18, 57-61, 1984.

[23] H.A.E.W. Khalifa, M. Saeed, A.U. Rahman, and S.E. Morsy, S.E.(2022). An Application of Pentagonal Neutrosophic Linear Programming for Stock Portfolio Optimization. Neutrosophic Sets and Systems, 51, 653-665, 20222.

[24] M. Kumar, S. Teso, L.D. Raedt, Acquiring Integer Programs from Data. Proceedings of the Twenty-Eighth International Joint Conference on Artifificial Intelligence, 1130-1136, 2019 .

[25] M. Mai, M.A. Basset, A.N. Zaied, F. Smarandache, Neutrosophic Integer Programming Problem. Neutrosophic Sets and Systems, **15**, 3-7, 2017 https://digitalrepository.unm.edu/nss_journal/vol15/iss1/2

[26] I. Maiti, T. Mandal, S. Pramanik, S, Neutrosophic goal programming strategy for multi-level multi-objective linear programming problem. Journal of Ambient Intelligence and Humanized Computing, 11, 3175–3186, 2020 https://doi.org/10.1007/s12652-019-01482-0

[27] M. Cococcioni, A.Cudazzo, M. Pappalardo and Y.D. Sergeyev, Solving the Lexicographic Multi-Objective Mixed-Integer Linear Programming Problem using branch-and-bound and grossone methodology. Commun. Nonlinear Sci. Numer. Simulat. 84, 1-20, 2020.

[28] G. Monali, M.G. Dhote, C.P. Ban, Optimum Integer Solution of Goal Programming Problem by Modified Gomory Constraint Technique. Turkish Journal of Computer and Mathematics Education, 12, 3443-3452, 2022.

[29] A. Nafei, W. Yuan, H. Nasseri, H, A New Method for Solving Interval Neutrosophic Linear Programming Problems. GU. J. Sci., 33, 796-808, 2020

[30] A. Nafei and S.H. Nasseri, A new approach for solving neutrosophic integer programming Problem. International Journal of Applied Operational Research, 9,1-9, 2019.

[31] Y. Prakash, S. Appasamy, Optimal solution for fully Spherical Fuzzy Linear Programming Problem. Mathematical Modelling of Engineering Problems, Vol. 10, No. 5, pp. 1611-1618, 2023. https://doi.org/10.18280/mmep.100511

[32] Y. Rasekhipour, I. Fadakar, A. Khajepour, Autonomous driving motion planning with obstacles prioritization using lexicographic optimization. Control Engineering Practice, 77, 235–246, 2018.

[33] S. Karthick, A. Saraswathi, A., & B. Baranidharan, Neutrosophic Linear Fractional Programming Problem using Denominator Objective Restriction Method. Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications and Algorithms, 31(2), 89-101, 2024.

[34] S.K. Singh, and M. Goh, Multi-objective mixed integer programming and an application in a pharmaceutical supply chain. International Journal of Production Research, 1-2, 2018. https://doi.org/10.1080/00207543.2018.1504172

[35] J. ye, Neutrosophic number linear programming method and its application under neutrosophic number environments. Soft Computing, 22, 4639–4646, 2018 .https://doi.org/10.1007/s00500-017-2646-z