The Special Neutrosophic Functions

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Abstract

In this study, we introduce the notion of special neutrosophic functions as new kinds of neutrosophic function defined in a neutrosophic logic. As particular cases, we present the notions of neutrosophic Floor (greatest integer), neutrosophic Absolute Function and neutrosophic Signum Function. Moreover, we draw its neutrosophic graph representation and discuss similarities and differences for these special neutrosophic functions between the classic case and neutrosophic case. We investigate some properties and prove them. However, we often need the definition of absolute value function, especially in the metric space. Therefore, we introduce its initial definition in this study.

Keywords: Neutrosophic relation, Neutrosophic function, Neutrosophic derivative, Neutrosophic integral Neutrosophic representation.

1. Introduction

In our life, there is three main types of logic. The first one is the classical logic which has two values, 'true or false', '0 or 1'. The second one is the fuzzy logic which was first introduced by Dr. Lotfi Zadeh in 1960s. It has more than two values. This means that it has more than 'true or false' because they are considered simple in this type of logic. With fuzzy logic, propositions can be represented with degrees of truth and falseness [1, 2, 3]. The final type of logic is the neutrosophic logic which is an extension of the fuzzy logic in which indeterminacy I is included taking into consideration $(I^n = nI = I, \forall n \in N)$ [4, 5]. This idea has inspired a lot of researchers and opened up a wide range of scientific research in many ways.

Due to the importance of calculus, Florentin Smarandache presented the basic of Neutrosophic Pre-calculus and Neutrosophic Calculus, which studies the neutrosophic functions [6, 7]. A neutrosophic Function $Nf : \mathrm{D} \longrightarrow R$ is a function, which has some indeterminacy, with respect to its domain of definition, to its range, or to the relation that connect between elements in D with elements in R .Especially; he also defined the neutrosophic exponential function and neutrosophic logarithmic function.

The idea of the perception of pentagonal neutrosophic number from different aspects and the score function in pentagonal neutrosophic domain was introduced in [8,9]. Additionally, in [10,11,12,13,14,15,16] the single neutrosophic value and its properties of different kinds have been identified. Moreover, a lot of algebraic neutrosophic structures have been identified, such as neutrosophic R-modules [17,18] and also in the area of neutrosophic topological space [19,20].

DOI: 10.5281/zenodo.3782888

Received: January 27, 2020 Revised: March 06, 2020 Accepted: April 15, 2020

2.Preliminary

In this section, we present the basic definitions that are useful in this research.

2.1 Neutrosophic Subset Relation[6]:

A Neutrosophic Subset Relation β, between two sets A *and* B, is a set of ordered pairs of the form (A', B'), where A' is a subset of A, and B' a subset of B, with some indeterminacy. A neutrosophic relation β, besides sure ordered pairs (A', B') that 100% belong to β, can also contain potential ordered pairs (A'', B''), where A'' is a subset of A, and B'' a subset of B, which may be possible to belong to β, but it is unknown in what degree, or that partially belong to β with the neutrosophic value (T, I, F) where T means degree of appurtenance to β, I means degree of indeterminate appurtenance, and F means degree of non-appurtenance.

2.2 Neutrosophic Functions[6]:

A Neutrosophic Function is a neutrosophic relation in which the vertical line test does not necessarily work. However, in this case, the neutrosophic function coincides with the neutrosophic relation. Generally, a neutrosophic function is a function that has some indeterminacy [with respect to one or more of its formula, domain, or range].

Example [6]**:** Let's we have $f:\{1,2,3\} \rightarrow \{a,b,c,d\}$ is a neutrosophic function defined as: $f(1)=a$, $f(2)=b$, but $f(3)=c \text{ or } d$ [we are not sure], so we can write $f(3)=I$. If we consider a neutrosophic diagram representation of this neutrosophic function, we have:

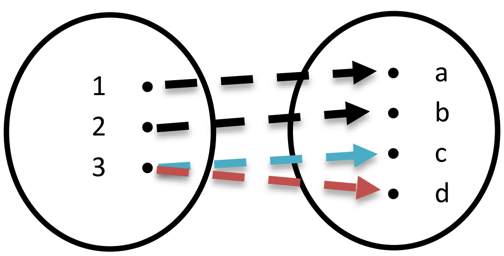
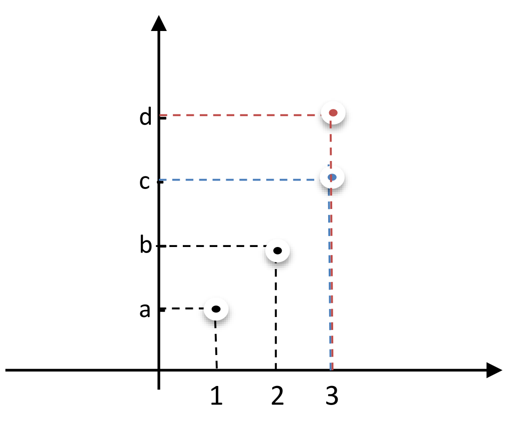


Fig (1)

The color arrows mean that we are not sure if the element 3 is connected to the element c, or if 3 is connected to d. Similarly, for a graph representation:



Fig(2)

DOI: 10.5281/zenodo.3782888

This example can be rephrased in another way, that 3 is connected with d only partially, let's say $(3,d)_{\langle 0.6,\ 0.2,\ 0.5\rangle}$ which means that 60% the 3 is connected with d, 20%. It is not clear whether it is connected or unconnected, and 50% the 3 is not connected with d. The sum of components $0.6+0.2+0.5=1.3$ is more than 1 because the three sources providing information about connection, indeterminacy, non-connection respectively are independent and use different criteria of evaluation.

As we see, this neutrosophic function is neither a function nor a relation in the classical case.

Example: [6] Let's consider $h: R \rightarrow R$ a different type of neutrosophic function defined as: $\forall x \in R, h(x) \in [2,3]$, so we can write $h(x) = I$. Therefore, we just know that this function is bounded by the horizontal lines $y = 2$ and $y = 3$.

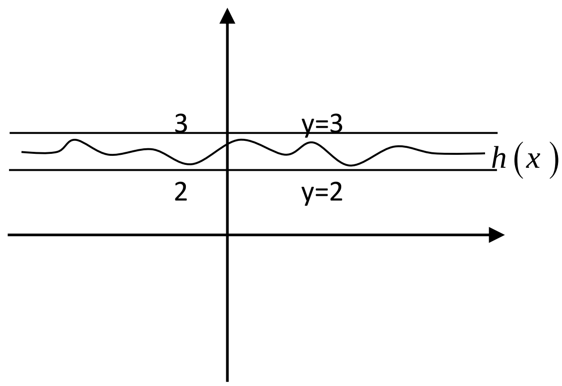


Fig (3)

We can modify $h(x)$ and get a constant neutrosophic function (or thick function): $l: R \rightarrow P(R)$ defined as: $\forall x \in R, l(x) = [2,3]$ Where $P(R)$ is the set of all subsets of R.

For example, is the vertical segment of line [2, 3].

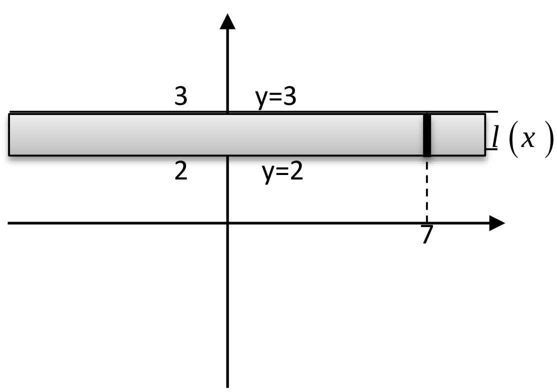


Fig (4)

Example:[6] A non-constant neutrosophic thick function: $k: R \rightarrow P(R)$ defined as: $\forall x \in R, k(x) = [2x, 2x+1]$ whose neutrosophic representation is:

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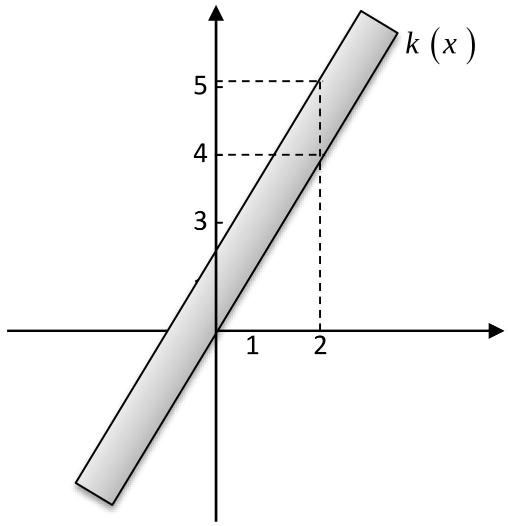

Fig (5)

2.3 Neutrosophic Derivative[6] :

The general definition of the neutrosophic derivative of function $f'_N(x)$ is:

$$f'_N(x)=\lim_{h\to 0}\frac{\left[\inf f(X+h)-\inf f(X),\sup f(X+h)-\sup f(X)\right]}{h}$$

Example: Let's $f:R\to R\cup\{I\}$ a neutrosophic function defined as: $f_N(x)=\left[x^2+2x,x^3\right]$ then :

$$f'_N(X)=\lim_{h\to 0}\frac{\left[(x+h)^2+2(x+h)-x^2-2x,(x+h)^3-x^3\right]}{h}$$

$$=\left[\lim_{h\to 0}\frac{(x+h)^2+2(x+h)-x^2-2x}{h},\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}\right]$$

$$=\left[\frac{d}{dx}\left(x^2+2x\right),\frac{d}{dx}x^3\right]$$

$$=\left[2x+2,3x^2\right]$$

Example: Let $f:R\to R\cup\{I\}$ a neutrosophic function defined as: $f_N(x)=3x-x^2I$ then :

$$f'_N(x)=\lim_{h\to 0}\frac{f_N(x+h)-f_N(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left[3(x+h)-(x+h)^2I\right]-\left[3x-x^2I\right]}{h}$$

$$=\lim_{h\to 0}\frac{h\cdot(3-2xI-hI)}{h}=3-2xI-0\cdot I=3-2xI$$

2.4 Neutrosophic Integral [7]

Using the neutrosophic measure, we will define a neutrosophic integral. The neutrosophic integral of a function Nf is written as: $\int(_X Nf)dv$

DOI: 10.5281/zenodo.3782888

Where X is a neutrosophic measure space, and also the integral is taken with respect to the neutrosophic measure v. Indeterminacy related to integration can occur in various ways: with respect to the value of the integrated function, with respect to the lower or upper limit of integration, or with respect to the space and its measure.

Example: Let $f : R \to R \cup \{I\}$ a neutrosophic function defined as: $f(x) = 2x^3 + (x^2 + 3)I$ then :

$$F(x) = \int [4x^3 + (x^2+3)I]dx$$
$$= \int 4x^3 dx + \int [(x^2+3)I]dx$$
$$= x^4 + \frac{x^3}{3}I + 3xI + c$$

Integration neutrosophic Constant

$c = a + bI : a, b \in R$

3. The Special Neutrosophic Functions:

3.1 Piecewise function:

A Neutrosophic piecewise Function is a piecewise function that has some indeterminacy [with respect to one or more of: its domain, formula, or range].

The Neutrosophic piecewise function is may not be a classical function in general. However, we can say when indeterminacy doesn't exist we will be back the classical case again.

Example: Let's consider a neutrosophic piecewise function which has indeterminacy with respect to its domain:

$$f_1(x) = \begin{cases} x^2 & | x \notin \{-1, 1\} \\ [2,3] & | x = -1 \ \ or \ \ 1 \end{cases}$$

It's clear that is $f_1(-1) \neq f_1(1) \neq 1$ and $f(I) = [2,3]$

As in the classical way we can draw the neutrosophic graph:

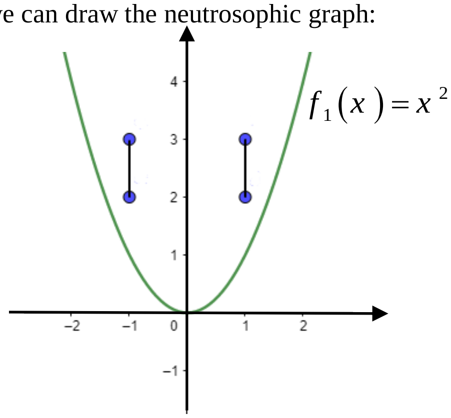

Fig (6)

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Example: Let’s consider a neutrosophic piecewise function, which has indeterminacy with respect to its formula:

$$f_2(x) = \begin{cases} [2x+1, 6x] & | x \neq 0 \\ [1,3) & | x = 0 \end{cases}$$

It's clear that $f_2(x) = I : x \neq 0$ and $f_2(0) = [1,3)$

As in the classical way, we can draw the neutrosophic graph:

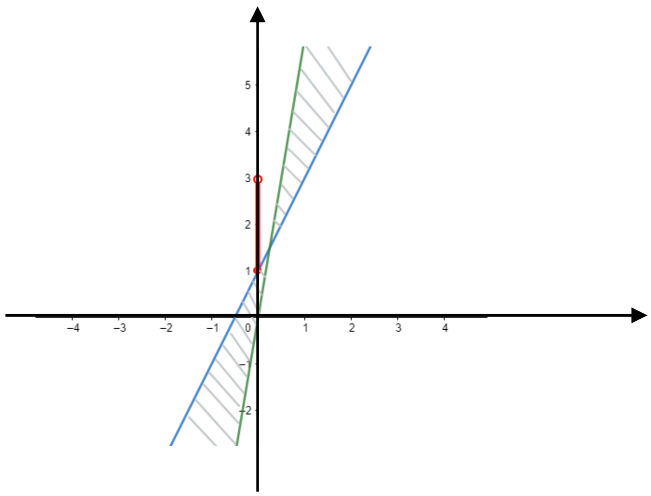


Fig (7)

Example: Let’s consider a neutrosophic piecewise function, which has indeterminacy with respect to its range:

$$f_3(x) = \begin{cases} \dfrac{1}{x-5} & | x \neq 5 \\ 2 \ \ or \ \ 4 & | x = 5 \end{cases}$$

It's clear that $f_3(5) = I$ and $f_3(x) = \dfrac{1}{x-5} : x \neq 5$

As in the classical way, we can draw the neutrosophic graph:

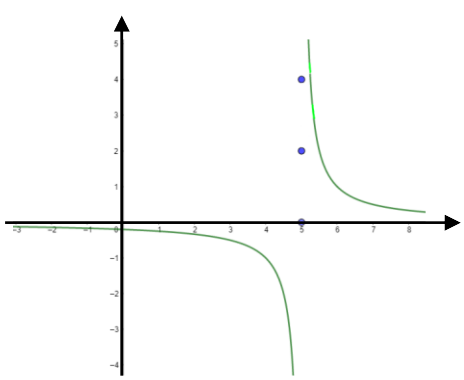


Fig (8)

DOI: 10.5281/zenodo.3782888

3.2 Signum function:

A Neutrosophic signum Function ($N\ \text{sgn}$) is a signum function which has some indeterminacy [with respect to one or more of: its domain, formula, or range] in two ways as follows:

$$1)N\ \text{sgn}\left(x+I\right)=\begin{cases}1: x>0 \quad \textit{and} \quad I=0\\ 0: x=0 \quad \textit{and} \quad I\neq 0\\ -1: x<0 \quad \textit{and} \quad I=0\end{cases}$$

$$2)N\ \text{sgn}\left(x\right)=\begin{cases}1: x>0\\ 0+I: x=0\\ -1: x<0\end{cases}$$

The indeterminacy here is suitable to the problem conditions.

A Neutrosophic signum Function may be continuous at (0) due to the indeterminacy in contrary to the classical case.

Example: Let's consider a neutrosophic signum function, which has indeterminacy with respect to its domain:

$$N\ \text{sgn}(x-3+2I)=\begin{cases}1: x>3 \quad \textit{and} \quad I=0\\ 0: x=3 \quad \textit{and} \quad I\neq 0\\ -1: x<3 \quad \textit{and} \quad I=0\end{cases}$$

As in the classical way, we can draw the neutrosophic graph:

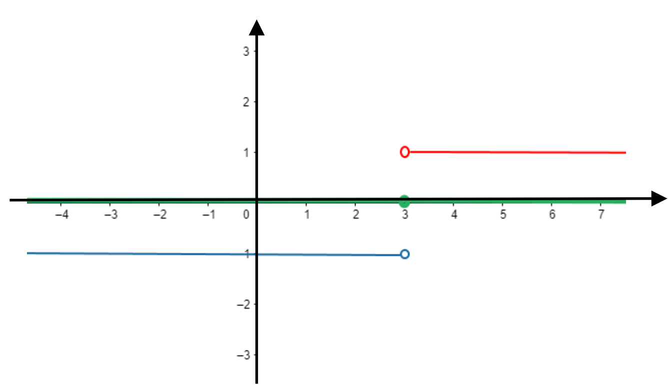


Fig (9)

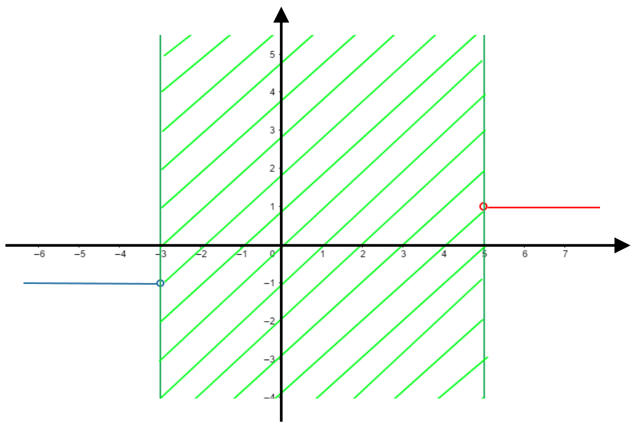
From the graph, we notice that the Neutrosophic signum function is continuous at 3 in the contrary to the classical case, and when there is no indeterminacy the green color will fade. Therefore, we will go back to the classical case.

DOI: 10.5281/zenodo.3782888

Example: Let's consider a neutrosophic signum function which has indeterminacy with respect to its formula and range:

$$N\ \text{sgn}(x+2)=\begin{cases}1 : x>5\\ 0+I : x=-2+I\\ -1 : x<-3\end{cases}$$

As in the classical way we can draw the neutrosophic graph:



Fig(10)

Notice what the indeterminacy has made in the graph. It becomes like a spectrum around zero.

3.3 Neutrosophic Absolute Function

A Neutrosophic Absolute Function ($Nabs$) is an Absolute Function which has some indeterminacy [with respect to one or more of: its domain, formula, or range] in three ways as follows:

$$1) Nabs(x+I)=\begin{cases}x : x>0 \quad and \quad I=0\\ 0+I : x=0 \quad and \quad I\neq 0\\ -x : x<0 \quad and \quad I=0\end{cases}$$

$$2) Nabs(x)=\begin{cases}x : x>0\\ 0+I : x=0\\ -x : x<0\end{cases}$$

$$3) Nabs(x+I)=\begin{cases}x+I : x>0\\ 0+I : x=0\\ -x+I : x<0\end{cases}$$

DOI: 10.5281/zenodo.3782888

Properties:

$$1)Nabs(x+I)=abs(x)+I$$
$$2)Nabs(0+I)=abs(0)+I=I$$
$$3)Nabs(x+I)+I=abs(x)+2I=Nabs(x+I)$$

$$4)Nabs(x+y+I)\le Nabs(x+I)+Nabs(y+I)$$

$proof$:

$$\begin{aligned}Nabs(x+y+I)&=abs(x+y)+I\\&\le abs(x)+abs(y)+I\\&=abs(x)+I+abs(y)+I\\&=Nabs(x+I)+Nabs(y+I)\end{aligned}$$

$$5)Nabs(x+I)=y+I$$
$$\Rightarrow abs(x)+I=y+I$$
$$\Rightarrow abs(x)=y+I\Rightarrow\begin{cases}x=y+I\\x=-y+I\end{cases}$$

Example: Let's consider a neutrosophic Absolute function which has indeterminacy with respect to its domain:

$$Nabs(x+3I)=\begin{cases}x:x>0 \quad and \quad I=0\\0+I:x=0 \quad and \quad I\ne 0\\-x:x<0 \quad and \quad I=0\end{cases}$$

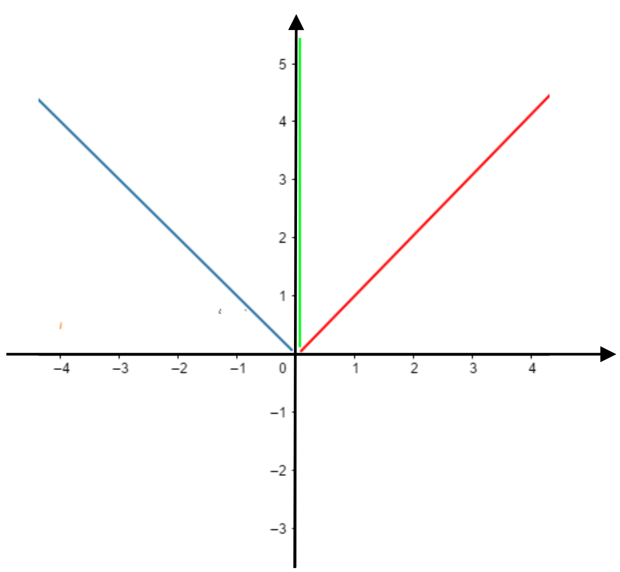


Fig (11)

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Example: Let's consider a neutrosophic Absolute function which has indeterminacy with respect to its formula, and range:

$$Nabs(x-2)=\begin{cases} x-2 : x>4 \\ 0+I : x\in[1,4] \\ 2-x : x<1 \end{cases}$$

As in the classical way, we can draw the neutrosophic graph:

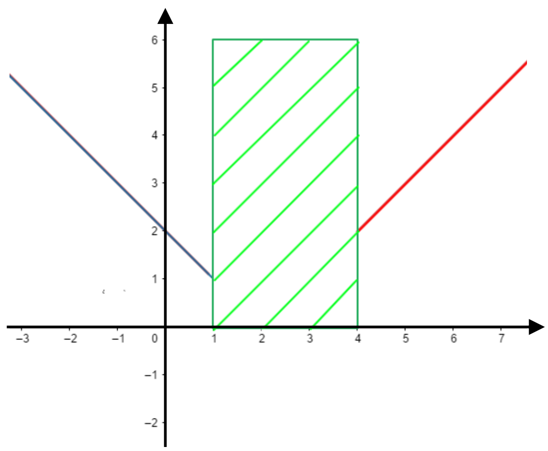

Fig (12)

3.4 Neutrosophic Floor (greatest integer) Function

A Neutrosophic Floor (greatest integer) ($Nfloor[\![\ ]\!]$) is a floor (greatest integer) that has some indeterminacy [with respect to one or more of: its domain, formula, or range] in two ways as follows:

$$1) Nfloor[\![x+I]\!]=\begin{cases} . \\ . \\ -1 \quad : -1\le x+I<0 \\ 0 \quad : 0\le x+I<1 \\ 1 \quad : 1\le x+I<2 \\ . \\ . \end{cases}$$

As in the classical way, we can draw the neutrosophic graph:

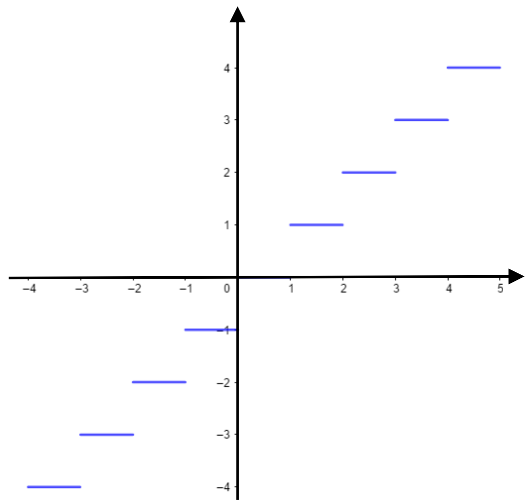

Fig (13)

Here we can say that there is no difference between the neutrosophic case and the classical case.

$$2)Nfloor[\![x]\!]=\begin{cases} . \\ . \\ -1+I & :-1\le x<0 \\ 0+I & :0\le x<1 \\ 1+I & :1\le x<2 \\ . \\ . \end{cases}$$

As in the classical way, we can draw the neutrosophic graph:

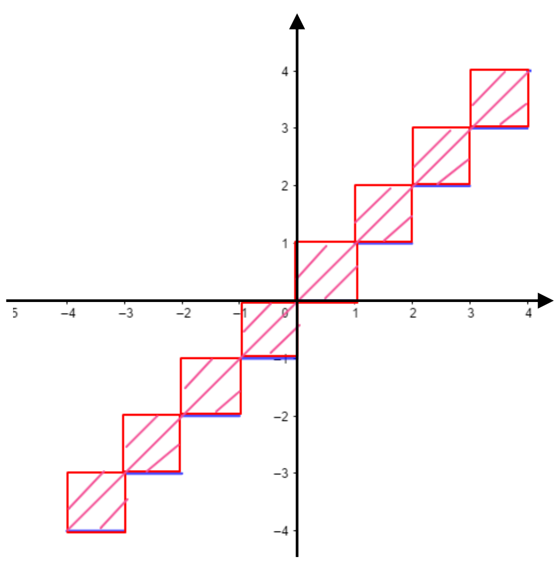

Fig (14)

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Properties:

$$1) Nfloor[\![x+I]\!] = [\![x]\!] + I = x + a + I : 0 \le \text{a} < 1$$

$$2) \forall n \in N\ (I)$$

$$Nfloor[\![n+I]\!] = [\![n]\!] + I$$
$$= n + I$$

$$3) Nfloor[\![x+y+I]\!] \ge Nfloor[\![x+I]\!] + Nfloor[\![y+I]\!]$$

proof :

$$Nfloor[\![x+y+I]\!] = [\![x+y]\!] + I$$
$$\ge [\![x]\!] + [\![y]\!] + I$$
$$= [\![x]\!] + I + [\![y]\!] + I$$
$$= Nfloor[\![x+I]\!] + Nfloor[\![y+I]\!]$$

$$4) \forall n \in N\ (I)$$

$$Nfloor[\![x+n+I]\!] = Nfloor[\![x+I]\!] + n$$

proof :

$$Nfloor[\![x+n+I]\!] = [\![x+n]\!] + I$$
$$= [\![x]\!] + n + I$$
$$= [\![x]\!] + I + n$$
$$= Nfloor[\![x+I]\!] + n$$

4. Conclusions

In this research, we firstly obtained new kinds of neutrosophic functions and focused on the Neutrosophic representation and proved some properties. In addition, we showed that the neutrosophic functions is not a function in the classical case, but in some especial cases there were an coincidence between the neutrosophic case and the classical case.

5. Future Research Directions

As a future work, this article can be extended to include continuity and derivation and integration as well as the definition and applications of the Neutrosophic Cartesian.

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