A STUDY OF NEUTROSOPHIC CUBIC MN SUBALGEBRA

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Abstract

In this paper, we present the new kind of MN-subalgebra for neutrosophic cubic set which is called neutrosophic cubic MN-subalgebra where M represents the initial of author's first name Mohsin and N represents the initial of second author's first name Neha. We investigate this neutrosophic cubic MN-subalgebra on BF-algebra through some significant properties of BF-algebra. We also use R-intersection, p-intersection, p-union upper bound, lower bound and some important characteristics to study the behaviour of neutrosophic cubic MN-subalgebra [NCMNSU] on BF-algebra.

Keywords: BF-algebra, Neutrosophic cubic set, Neutrosophic cubic MN-subalgebra.

1. Introduction

Fuzzy and interval-valued fuzzy sets were prsented by Zadeh [20,21] Jun et al. [3,4] defined the cubic set and proved the axioms for cubic subgroups. Neggers and Kim [7] defined and investigated the B-algebra. Ahn and Ko [9] studied the structure of BF-algebra. Walendziak [19] proved the conditions of B-algebra. Senapati et al. [13] worked on fuzzy dot subalgebra and interval-valued fuzzy subalgebra with respect to t-norm in B-algebra.[14,6] many researches worked on B-algebra and BG-algebra. Khalid et al. [15] studied the neutrosophic soft cubic subalgebra with different characteristics. Khalid et al. [16] studied the effects of magnification of translation for MBJ-neutrosophic set. Khalid et al. [17] investigated the T-ideal under the MBJ-neutrosophic on B-algebra. Khalid et al. [18] presented the multiplication of neutrosophic cubic set. Smarandache [11,12] is the first person who presented the theory of neutrosophy set which invloveed indeterminacy. Jun et al. [5] introduced neutrosophic cubic set. Senapati et al. [22] studied the cubic subalgebras and cubic closed ideals in detailed on B-algebra.

The purpose of this paper is to introduce the idea of neutrosophic cubic MN-subalgebra. We investigate many results to study the neutrosophic cubic MN-subalgebra in detailed way by using different concepts like p-intersection, R-intersection and many others.

2. Preliminaries

In this section, some basic definitions are presented that are necessary for this paper.

Definition 2.1 [19] A nonempty set X with a constant 0 and a binary operation $*$ is called BF–algebra, when it fulfills these conditions for all $t_1, t_2 \in X$.

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1. $t_1 * t_1 = 0$

2. $t_1 * 0 = t_1$

3. $0 * (t_1 * t_2) = t_2 * t_1$ for all $t_1, t_2 \in X$.

A BF-algebra is denoted by $(X,*,0)$.

Definition 2.2 [9] A nonempty subset S of BF-algebra X is called a subalgebra of X, if $t_1 * t_2 \in S\ \forall\ t_1, t_2 \in S$.

Definition 2.4 [9] A mapping $f : X \to Y$ of BF-algebra is called homomorphism if $f(t_1 * t_2) = f(t_1) * f(t_2)\ \forall\ t_1, t_2 \in X$.

Definition 2.4 [21] Let X be the set of elements which are denoted generally by t_1. Then a fuzzy set C in X is defined as $C = \{< t_1, \nu_C(t_1) > |t_1 \in X\}$, where $\nu_C(t_1)$ is called the membership value of t_1 in C and $\nu_C(t_1) \in [0,1]$. For a family $C_i = \{< t_1, \nu_{C_i}(t_1) > |t_1 \in X\}$ of fuzzy sets in X, where $i \in U$ and U is index set, they defined the join ($\vee$) meet ($\wedge$) operations as:

$$\vee_{i\in U} C_i = (\vee_{i\in U} \nu_{C_i})(t_1) = \sup\{\nu_{C_i} | i \in U\}$$

and

$$\wedge_{i\in U} C_i = (\wedge_{i\in U} \nu_{C_i})(t_1) = \inf\{\nu_{C_i} | i \in U\}$$

respectively, $\forall\ t_1 \in X$.

The finding of supremum and infimum between two intervals is not simple. Biswas [2] explained a procedure to find max/sup and min/inf between two intervals or a set of intervals.

Definition 2.5 [2] Let two elements $P_1, P_2 \in P[0,1]$. If $P_1 = [(t_1)_1^-, (t_1)_1^+]$ and $P_2 = [(t_1)_2^-, (t_1)_2^+]$, then $\mathrm{rmax}(P_1, P_2) = [\max((t_1)_1^-, (t_1)_2^-), \max((t_1)_1^+, (t_1)_2^+)]$ which is denoted by $P_1 \vee^r P_2$ and $\mathrm{rmin}(P_1, P_2) = [\min((t_1)_1^-, (t_1)_2^-), \min((t_1)_1^+, (t_1)_2^+)]$ which is denoted by $P_1 \wedge^r P_2$. Thus, if $P_i = [((t_1)_1)_i^-, ((t_1)_2)^+] \in P[0,1]$ for $i = 1,2,3, \ldots$, then we define $\mathrm{rsup}_i(P_i) = [\sup_i(((t_1)_1)_i^-), \sup_i(((t_1)_1)_i^+)]$, i.e., $\vee_i^r P_i = [\vee_i ((t_1)_1)_i^-, \vee_i ((t_1)_1)_i^+]$. In the same way we define $\mathrm{rinf}_i(P_i) = [\inf_i(((t_1)_1)_i^-), \inf_i(((t_1)_1)_i^+)]$, i.e., $\wedge_i^r P_i = [\wedge_i ((t_1)_1)_i^-, \wedge_i ((t_1)_1)_i^+]$. Now we call $P_1 \geq P_2 \Leftarrow (t_1)_1^- \geq (t_1)_2^-$ and $(t_1)_1^+ \geq (t_1)_2^+$. Similarly the relations $P_1 \leq P_2$ and $P_1 = P_2$ are defined.

Definition 2.6 [1] A fuzzy set $C = \{< t_1, \mu_C(t_1) > |t_1 \in X\}$ is called a fuzzy subalgebra of X if $\nu_C(t_1 * t_2) \geq \min\{\nu_C(t_1), \nu_C(t_2)\}\ \forall\ t_1, y \in X$.

Jun et al. [5], defined and investigated neutrosophic cubic set.

Definition 2.7 [5] Let X be a nonempty set. A neutrosophic cubic set in X is pair $\mathcal{C} = (\aleph, S)$ where $\aleph = \{\langle t_1; \aleph_E(t_1), \aleph_I(t_1), \aleph_N(t_1)\rangle\ |t_1 \in X\}$ is an interval neutrosophic set in X and $S = \{\langle t_1; S_E(t_1), S_I(t_1), S_N(t_1)\rangle\ |t_1 \in X\}$ is a neutrosophic set in X.

Definition 2.8 [5] For any $\mathcal{C}_i = (\aleph_i, S_i)$, where $\aleph_i = \{\langle t_1; \aleph_{iE}(t_1), \aleph_{iI}(t_1), \aleph_{iN}(t_1)\rangle | t_1 \in X\}$, $S_i = \{\langle t_1; S_{iE}(t_1), S_{iI}(t_1), S_{iN}(t_1)\rangle\ |t_1 \in X\}$ for $i \in u$, P-union, P-inersection, R-union and R-intersection are defined respectively by

P-union $\cup_{P\, i\in u}\, \mathcal{C}_i = (\cup_{i\in u} \aleph_i, \vee_{i\in u} S_i)$, **P-intersection** $\cap_{P\, i\in u}\, \mathcal{C}_i = (\cap_{i\in u} \aleph_i, \wedge_{i\in u} S_i)$,

R-union $\cup_{R\, i\in u}\, \mathcal{C}_i = (\cup_{i\in u} \aleph_i, \wedge_{i\in u} S_i)$, **R-intersection:** $\cap_{R\, i\in u}\, \mathcal{C}_i = (\cap_{i\in u} \aleph_i, \vee_{i\in u} S_i)$, where

$\cup_{i\in u} \aleph_i = \{\langle t_1; (\cup_{i\in u} \aleph_{iE})(t_1), (\cup_{i\in u} \aleph_{iI})(t_1), (\cup_{i\in u} \aleph_{iN})(t_1)\rangle | t_1 \in X\}$,

$\vee_{i\in u} S_i = \{\langle t_1; (\vee_{i\in u} S_{iE})(t_1), \vee_{i\in u} S_{iI})(t_1), (\vee_{i\in u} S_{iN})(t_1)\rangle | t_1 \in X\}$,

$\cap_{i\in u} \aleph_i = \{\langle t_1; (\cap_{i\in u} \aleph_{iE})(t_1), (\cap_{i\in u} \aleph_{iI})(t_1), (\cap_{i\in u} \aleph_{iN})(t_1)\rangle | t_1 \in X\}$,

$\wedge_{i\in u} S_i = \{\langle t_1; (\wedge_{i\in u} S_{iE})(t_1), (\wedge_{i\in u} S_{iI})(t_1), (\wedge_{i\in u} S_{iN})(t_1)\rangle | t_1 \in X\}$.

Definition 2.9 [22] Let $C = \{\langle t_1, \aleph(t_1), S(t_1)\rangle\}$ be a cubic set, where $\aleph(t_1)$ is an interval-valued fuzzy set in X, $S(t_1)$ is a fuzzy set in X. Then C is cubic subalgebra with following axioms:

C1: $\aleph(t_1 * t_2) \geq \text{rmin}\{\aleph(t_1), \aleph(t_2)\}$,

C2: $S(t_1 * t_2) \leq \max\{S(t_1), S(t_2)\}\ \forall\ t_1, t_2 \in X$.

3. NEUTROSOPHIC CUBIC MN-SUBALGEBRAS

Definition 3.1 Let $\Re = (\aleph, S)$ be a cubic set, where X is subalgebra. Then $\Re$ is NCMNSU under binary operation $*$ if it satisfies the following conditions:

(N1)
$$\aleph_E(t_1 * t_2) \geq rmin\{\aleph_E(t_1), \aleph_E(t_2)\},$$
$$\aleph_I(t_1 * t_2) \leq rmax\{\aleph_I(t_1), \aleph_I(t_2)\},$$
$$\aleph_N(t_1 * t_2) \leq rmax\{\aleph_N(t_1), \aleph_N(t_2)\}.$$

(N2)
$$S_E(t_1 * t_2) \leq max\{S_E(t_1), S_E(t_2)\},$$
$$S_I(t_1 * t_2) \leq min\{S_I(t_1), S_I(t_2)\},$$
$$S_N(t_1 * t_2) \geq min\{S_N(t_1), S_N(t_2)\}.$$

Where E means existenceship/membership value, I means indeterminacy existenceship/membership value and N means non existenceship/membership value.

Example 3.1 Let $X = \{0, t_1, t_2, t_3, t_4, t_5\}$ be a BF-algebra with the following Cayley table.

$*$	0	t_1	t_2	t_3	t_4	t_5
0	0	t_5	t_4	t_3	t_2	t_1
t_1	t_1	0	t_5	t_4	t_3	t_2
t_2	t_2	t_1	0	t_5	t_4	t_3
t_3	t_3	t_2	t_1	0	t_5	t_4
t_4	t_4	t_3	t_2	t_1	0	t_5
t_5	t_5	t_4	t_3	t_2	t_1	0

A neutrosophic cubic set $\Re = (\aleph_\Xi, S_\Xi)$ of X is defined by

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	0	t_1	t_2	t_3	t_4	t_5
$\aleph_E$	[0.2,0.4]	[0.1,0.4]	[0.2,0.4]	[0.1,0.4]	[0.2,0.4]	[0.1,0.4]
$\aleph_I$	[0.7,0.9]	[0.6,0.8]	[0.7,0.9]	[0.6,0.8]	[0.7,0.9]	[0.6,0.8]
$\aleph_N$	[0.3,0.2]	[0.2,0.1]	[0.3,0.2]	[0.2,0.1]	[0.3,0.2]	[0.2,0.1]

	0	t_1	t_2	t_3	t_4	t_5
S_E	0.1	0.3	0.1	0.3	0.1	0.3
S_I	0.3	0.5	0.3	0.5	0.3	0.5
S_N	0.5	0.6	0.5	0.6	0.5	0.6

All the conditions of Definition 3.1 are satisfied by the set $\Re$. Thus $\Re = (\aleph_\Xi, S_\Xi)$ is a NCMNSU of X.

Proposition 3.1 Let $\Re = \{\langle t_1, \aleph_\Xi(t_1), S_\Xi(t_1)\rangle\}$ is a NCMNSU of X, then $\forall\ t_1 \in X$, $\aleph_E(t_1) \geq \aleph_E(0), \aleph_I(t_1) \leq \aleph_I(0), \aleph_N(t_1) \leq \aleph_N(0)$ and$S_E(t_1) \leq S_E(0), S_I(t_1) \geq S_I(0), S_N(t_1) \geq S_N(0)$. Thus, $\aleph_{E,I,N}(0)$ and $S_{E,I,N}(0)$ are the upper bound and lower bound of $\aleph_{E,I,N}(t_1)$ and $S_{E,I,N}(t_1)$ respectively.

Proof. $\forall\ t_1 \in X$, we have $\aleph_E(0) = \aleph_E(t_1 * t_1) \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(t_1)\} = \aleph_E(t_1) \Rightarrow \aleph_E(0) \geq \aleph_E(t_1)$, $\aleph_I(0) = \aleph_I(t_1 * t_1) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_1)\} = \aleph_I(t_1) \Rightarrow \aleph_I(0) \leq \aleph_I(t_1)$, $\aleph_N(0) = \aleph_N(t_1 * t_1) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_1)\} = \aleph_N(t_1) \Rightarrow \aleph_N(0) \leq \aleph_N(t_1)$ and $S_E(0) = S_E(t_1 * t_1) \leq \max\{S_E(t_1), S_E(t_1)\} = S_E(t_1) \Rightarrow S_E(0) \leq S_E(t_1)$, $S_I(0) = S_I(t_1 * t_1) \geq \min\{S_I(t_1), S_I(t_1)\} = S_I(t_1) \Rightarrow S_I(0) \geq S_I(t_1)$, $S_N(0) = S_N(t_1 * t_1) \geq \min\{S_N(t_1), S_N(t_1)\} = S_N(t_1) \Rightarrow S_N(0) \geq S_N(t_1)$.

Theorem 3.1 Let $\Re=\{\langle t_1, \aleph_\Xi(t_1), S_\Xi(t_1)\rangle\}$ be a NCMNSU of X. If there exists a sequence $\{(t_1)_n\}$ of X such that $\lim_{n\to\infty}\aleph_\Xi((t_1)_n) = [1,1]$ and $\lim_{n\to\infty}S_\Xi((t_1)_n) = 0$. Then $\aleph_\Xi(0) = [1,1]$ and $S_\Xi(0) = 0$.

Proof. Using above proposition, $\aleph_E(0) \geq \aleph_E(t_1)\ \forall\ t_1 \in X, \therefore \aleph_E(0) \geq \aleph_E((t_1)_n)$ for $n \in Z^+$. Consider, $[1,1] \geq \aleph_E(0) \geq \lim_{n\to\infty}\aleph_E((t_1)_n) = [1,1]$. So, $\aleph_E(0) = [1,1]$, $\aleph_I(0) \leq \aleph_I(t_1)\ \forall\ t_1 \in X, \therefore \aleph_I(0) \geq \aleph_I((t_1)_n)$ for $n \in Z^+$. Consider, $[1,1] \leq \aleph_I(0) \leq \lim_{n\to\infty}\aleph_I((t_1)_n) = [1,1]$. So, $\aleph_I(0) = [1,1]$, $\aleph_N(0) \leq \aleph_N(t_1)\ \forall\ t_1 \in X, \therefore \aleph_N(0) \leq \aleph_N((t_1)_n)$ for $n \in Z^+$. Consider, $[1,1] \leq \aleph_N(0) \leq \lim_{n\to\infty}\aleph_N((t_1)_n) = [1,1]$. So, $\aleph_N(0) = [1,1]$. Hence, $\aleph_\Xi(0) = [1,1]$. Again, using proposition, $S_E(0) \leq S_E(t_1)\ \forall\ t_1 \in X, \therefore S_E(0) \leq S_E((t_1)_n)$ for $n \in Z^+$. Consider, $0 \leq S_E(0) \leq \lim_{n\to\infty}S_E((t_1)_n) = 0$. So, $S_E(0) = 0$, using proposition, $S_I(0) \geq S_I(t_1)\ \forall\ t_1 \in X, \therefore S_I(0) \geq S_I((t_1)_n)$ for $n \in Z^+$. Consider, $0 \geq S_I(0) \geq \lim_{n\to\infty}S_I((t_1)_n) = 0$. So, $S_I(0) = 0$, using proposition, $S_N(0) \geq S_N(t_1)\ \forall\ t_1 \in X, \therefore S_N(0) \geq S_N((t_1)_n)$ for $n \in Z^+$. Consider, $0 \geq S_N(0) \geq \lim_{n\to\infty}S_N((t_1)_n) = 0$. So, $S_N(0) = 0$. Hence, $S_\Xi(0) = 0$.

Theorem 3.2 The R-intersection of any set of neutrosophic cubic MN-sunalgebra of X is NCMNSU of X.

Proof. Let $\Re_i = \{\langle t_1, (\aleph_i)_\Xi, (S_i)_\Xi\rangle | t_1 \in X\}$ where $i \in k$, is family of sets of NCMNSU of X and $t_1, t_2 \in X$. Then $(\cap (\aleph_i)_E)(t_1 * t_2) = \text{rinf}(\aleph_i)_E(t_1 * t_2) \geq \text{rinf}\,\{\text{rmin}\{(\aleph_i)_E(t_1), (\aleph_i)_E(t_2)\}\} = \text{rmin}\{\text{rinf}(\aleph_i)_E(t_1), \text{rinf}(\aleph_i)_E(t_2)\} = \text{rmin}\{(\cap (\aleph_i)_E)(t_1), (\cap (\aleph_i)_E)(t_2)\} \Rightarrow (\cap (\aleph_i)_E)(t_1 * t_2) \geq \text{rmin}\{(\cap (\aleph_i)_E)(t_1), (\cap (\aleph_i)_E)(t_2)\}$, $(\cap (\aleph_i)_I)(t_1 * t_2) = \text{rinf}(\aleph_i)_I(t_1 * t_2) \leq \text{rinf}\{\text{rmax}\{(\aleph_i)_I(t_1), (\aleph_i)_I(t_2)\}\} = \text{rmax}\{\text{rinf}(\aleph_i)_I(t_1), \text{rinf}(\aleph_i)_I(t_2)\} = \text{rmax}\{(\cap (\aleph_i)_I(t_1), (\cap (\aleph_i)_I)(t_2)\} \Rightarrow (\cap (\aleph_i)_I)(t_1 * t_2) \leq \text{rmax}\{(\cap (\aleph_i)_I)(t_1), (\cap (\aleph_i)_I)(t_2)\}$, $(\cap (\aleph_i)_N)(t_1 * t_2) = \text{rinf}(\aleph_i)_N(t_1 * t_2)$

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$\leq$ rinf{rmax{$(\aleph_i)_N(t_1), (\aleph_i)_N(t_2)$}} = rmax{rinf$(\aleph_i)_N(t_1)$, rinf$(\aleph_i)_N(t_2)$} = rmax{$(\cap (\aleph_i)_N)(t_1), (\cap (\aleph_i)_N)(t_2)$} $\Rightarrow (\cap (\aleph_i)_N)(t_1 * t_2) \leq$ rmax{$(\cap (\aleph_i)_N)(t_1), (\cap (\aleph_i)_N)(t_2)$}, and $(\vee (S_i)_E)(t_1 * t_2) = \sup (S_i)_E(t_1 * t_2) \leq \sup \{ \max\{(S_i)_E(t_1), (S_i)_E(t_2)\}\} = \max \{\sup (S_i)_E(t_1), \sup (S_i)_E(t_2)\} = \max \{(\vee (S_i)_E)(t_1), (\vee (S_i)_E)(t_2)\} \Rightarrow (\vee (S_i)_E) (t_1 * t_2) \leq \max \{(\vee (S_i)_E)(t_1), (\vee (S_i)_E)(t_2)\}$, $(\vee (S_i)_I)(t_1 * t_2) = \sup (S_i)_I(t_1 * t_2) \geq \sup \{\min \{(S_i)_I(t_1), (S_i)_I(t_2)\}\} = \min \{\sup (S_i)_I(t_1), \sup (S_i)_I(t_2)\} = \min \{(\vee (S_i)_I)(t_1), (\vee (S_i)_I)(t_2)\} \Rightarrow (\vee (S_i)_I)(t_1 * t_2) \geq \min \{(\vee (S_i)_I)(t_1), (\vee (S_i)_I)(t_2)\}$, $(\vee (S_i)_N)(t_1 * t_2) = \sup (S_i)_N(t_1 * t_2) \leq \sup \{\min \{(S_i)_N(t_1), (S_i)_N(t_2)\}\} = \min \{\sup(S_i)_N (t_1), \sup(S_i)_N(t_2)\} = \min\{(\vee (S_i)_N)(t_1), (\vee (S_i)_N)(t_2)\} \Rightarrow (\vee (S_i)_N)(t_1 * t_2) \leq \min\{(\vee (S_i)_N)(t_1), (\vee (S_i)_N)(t_2)\}$, which show that R-intersection of $\mathfrak{R}_i$ is NCMNSU of X.

Theorem 3.3 Let $\mathfrak{R}_i = \{\langle t_1, (\aleph_i)_\Xi, (S_i)_\Xi \rangle | t_1 \in X\}$ be a collection of sets of NCMNSU of X, where $i \in k$. If $\inf \{\max \{(S_i)_E(t_1), (S_i)_E(t_1)\}\} = \max \{\inf (S_i)_E(t_1), \inf (S_i)_E(t_1)\}$, $\inf \{\min \{(S_i)_I(t_1), (S_i)_I(t_1)\}\} = \min \{\inf (S_i)_I (t_1), \inf(S_i)_I(t_1)\}$, $\inf\{\min\{(S_i)_N(t_1), (S_i)_N(t_1)\}\} = \min\{\inf(S_i)_N(t_1), \inf(S_i)_N(t_1)\}$ $\forall\, t_1 \in X$, then the P-intersection of $\mathfrak{R}_i$ is also a NCMNSU of X.

Proof. Suppose that $\mathfrak{R}_i = \{\langle t_1, (\aleph_i)_\Xi, (S_i)_\Xi \rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of NCMNSU of X such that $\inf \{\max \{(S_i)_E(t_1), (S_i)_E(t_1)\}\} = \max \{\inf (S_i)_E(t_1), \inf (S_i)_E(t_1)\}$, $\inf \{\min \{(S_i)_I(t_1), (S_i)_I(t_1)\}\} = \min \{\inf (S_i)_I (t_1), \inf(S_i)_I(t_1)\}$, $\inf \{\min \{(S_i)_N(t_1), (S_i)_N(t_1)\}\} = \min \{\inf(S_i)_N(t_1), \inf(S_i)_N(t_1)\}\}\}$ $\forall\, t_1 \in X$. Now for $t_1, t_2 \in X$. Then $(\cap (\aleph_i)_E)(t_1 * t_2)$ = rinf$(\aleph_i)_E(t_1 * t_2) \geq$ rinf{rmin{$(\aleph_i)_E(t_1), (\aleph_i)_E(t_2)$}} = rmin{rinf$(\aleph_i)_E(t_1)$, rinf$(\aleph_i)_E(t_2)$} = rmin{$(\cap (\aleph_i)_E)(t_1), (\cap (\aleph_i)_E)(t_2)$} $\Rightarrow (\cap (\aleph_i)_E)(t_1 * t_2) \geq$ rmin{$(\cap (\aleph_i)_E)(t_1), (\cap (\aleph_i)_E)(t_2)$}, $(\cap (\aleph_i)_I)(t_1 * t_2)$ = rinf$(\aleph_i)_I(t_1 * t_2) \leq$ rinf{rmax{$(\aleph_i)_I(t_1), (\aleph_i)_I(t_2)$}} = rmax{rinf$(\aleph_i)_I(t_1)$, rinf$(\aleph_i)_I(t_2)$} = rmax{$(\cap (\aleph_i)_I)(t_1), (\cap (\aleph_i)_I)(t_2)$} $\Rightarrow (\cap (\aleph_i)_I)(t_1 * t_2) \leq$ rmax{$(\cap (\aleph_i)_I)(t_1), (\cap (\aleph_i)_I)(t_2)$}, $(\cap (\aleph_i)_N)(t_1 * t_2)$=rinf $(\aleph_i)_N (t_1 * t_2) \leq$ rinf{rmax{$(\aleph_i)_N(t_1), (\aleph_i)_N(t_2)$}} = rmax{rinf$(\aleph_i)_N(t_1)$, rinf$(\aleph_i)_N(t_2)$} = rmax{$(\cap (\aleph_i)_N)(t_1), (\cap (\aleph_i)_N)(t_2)$} $\Rightarrow (\cap (\aleph_i)_N)(t_1 * t_2) \leq$ rmax{$(\cap (\aleph_i)_N)(t_1), (\cap (\aleph_i)_N)(t_2)$}, and $(\wedge (S_i)_E)(t_1 * t_2) = \inf (S_i)_E(t_1 * t_2) \leq \inf \{\max \{(S_i)_E(t_1), (S_i)_E(t_2)\}\} = \max \{\inf (S_i)_E(t_1), \inf (S_i)_E(t_2)\} = \max \{(\wedge (S_i)_E)(t_1), (\wedge (S_i)_E)(t_2)\} \Rightarrow (\wedge(S_i)_E(t_1 * t_2) \leq \max \{(\wedge (S_i)_E)(t_1), (\wedge (S_i)_E)(t_2)\}$, $(\wedge (S_i)_I)(t_1 * t_2) = \inf (S_i)_I(t_1 * t_2) \geq \inf \{\min\{(S_i)_I (t_1), (S_i)_I(t_2)\}\} = \min \{\inf (S_i)_I(t_1), \inf (S_i)_I(t_2)\} = \min \{(\wedge (S_i)_I)(t_1), (\wedge (S_i)_I)(t_2)\} \Rightarrow (\wedge (S_i)_I)(t_1 * t_2) \geq \min \{(\wedge (S_i)_I)(t_1), (\wedge (S_i)_I)(t_2)\}$, $(\wedge (S_i)_N)(t_1 * t_2) = \inf (S_i)_N(t_1 * t_2) \geq \inf \{\min\{(S_i)_N(t_1), (S_i)_N(t_2)\}\} = \min \{\inf (S_i)_N(t_1), \inf(S_i)_N(t_2)\} = \min\{(\wedge (S_i)_N)(t_1), (\wedge (S_i)_N)(t_2)\} \Rightarrow (\wedge (S_i)_N)(t_1 * t_2) \geq \min\{(\wedge (S_i)_N)(t_1), (\wedge (S_i)_N)(t_2)\}$. Which show that P-intersection of $\mathfrak{R}_i$ is NCMNSU of X.

Theorem 3.4 Let $\mathfrak{R}_i = \{\langle t_1, (\aleph_i)_\Xi, (S_i)_\Xi \rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of NCMNSU of X. If rsup{rmin{$(\aleph_i)_E(t_1), (\aleph_i)_E(t_2)$}} = rmin{rsup$(\aleph_i)_E(t_1)$, rsup$(\aleph_i)_E(t_2)$}, rsup{rmax {$(\aleph_i)_I(t_1), (\aleph_i)_I(t_2)$}} = rmax {rsup$(\aleph_i)_I(t_1)$, rsup$(\aleph_i)_I(t_2)$}, rsup{rmax {$(\aleph_i)_N(t_1), (\aleph_i)_N(t_2)$}} = rmax {rsup$(\aleph_i)_N(t_1)$, rsup$(\aleph_i)_N(t_2)$}, and $\sup \{\max \{(S_i)_E(t_1), (S_i)_E(t_2)\}\} = \max \{\sup (S_i)_E(t_1), \sup (S_i)_E(t_2)\}$, $\sup \{\min \{(S_i)_I(t_1), (S_i)_I(t_2)\}\} = \min\{\sup(S_i)_I(t_1), \sup(S_i)_I(t_2)\}$, $\sup\{\min\{(S_i)_N(t_1), (S_i)_N(t_2)\}\} = \min\{\sup(S_i)_N(t_1), \sup(S_i)_N(t_2)\}$ $\forall\, t_1, t_2 \in X$. Then P-union of $\mathfrak{R}_i$ is NCMNSU of X.

Proof. Let $\mathfrak{R}_i = \{\langle t_1, (\aleph_i)_\Xi, (S_i)_\Xi \rangle | t_1 \in X\}$ where $i \in k$, be a collection of sets of NCMNSU of X. $\forall\, t_1, t_2 \in X$, we have some conditions mentioned in theorem. Then for $t_1, t_2 \in X$. $(\cup (\aleph_i)_E)(t_1 * t_2)$ = rsup$(\aleph_i)_E(t_1 * t_2) \geq$ rsup{rmin{$(\aleph_i)_E(t_1), (\aleph_i)_E(t_2)$}} = rmin{rsup$(\aleph_i)_E(t_1)$, rsup$(\aleph_i)_E(t_2)$} = rmin{$(\cup (\aleph_i)_E)(t_1), (\cup (\aleph_i)_E)(t_2)$} $\Rightarrow (\cup (\aleph_i)_E)(t_1 * t_2) \geq$ rmin{$(\cup (\aleph_i)_E)(t_1), (\cup (\aleph_i)_E)(t_2)$}, $(\cup (\aleph_i)_I)(t_1 * t_2)$ = rsup$(\aleph_i)_I(t_1 * t_2) \leq$ rsup{rmax{$(\aleph_i)_I(t_1), (\aleph_i)_I(t_2)$}} = rmax{rsup$(\aleph_i)_I(t_1)$, rsup$(\aleph_i)_I(t_2)$} = rmax{$(\cup (\aleph_i)_I)(t_1), (\cup (\aleph_i)_I)(t_2)$} $\Rightarrow (\cup (\aleph_i)_I)(t_1 * t_2) \leq$ rmax{$(\cup (\aleph_i)_I)(t_1), (\cup (\aleph_i)_I)(t_2)$}, $(\cup (\aleph_i)_N)(t_1 * t_2)$ = rsup$(\aleph_i)_N(t_1 * t_2) \leq$ rsup{rmax{$(\aleph_i)_N(t_1), (\aleph_i)_N(t_2)$}} = rmax{rsup$(\aleph_i)_N(t_1)$, rsup$(\aleph_i)_N(t_2)$} = rmax{$(\cup (\aleph_i)_N)(t_1), (\cup (\aleph_i)_N)(t_2)$} $\Rightarrow (\cup (\aleph_i)_N)(t_1 * t_2) \leq$ rmax{$(\cup (\aleph_i)_N)(t_1), (\cup (\aleph_i)_N)(t_2)$}, and $(\vee (S_i)_E)(t_1 * t_2) = \sup (S_i)_E(t_1 * t_2) \leq \sup \{\max \{(S_i)_E(t_1), (S_i)_E(t_2)\}\} = \max \{\sup (S_i)_E(t_1), \sup (S_i)_E(t_2)\} = \max \{(\vee (S_i)_E)(t_1), (\vee (S_i)_E)(t_2)\} \Rightarrow (\vee (S_i)_E)(t_1 * t_2) \leq \max \{(\vee (S_i)_E)(t_1), (\vee (S_i)_E)(t_2)\}$, $(\vee (S_i)_I)(t_1 * t_2) = \sup (S_i)_I(t_1 * t_2) \geq \sup \{\min \{(S_i)_I(t_1), (S_i)_I(t_2)\}\} = \min \{\sup (S_i)_I(t_1), \sup (S_i)_I(t_2)\} = \min \{(\vee (S_i)_I)(t_1), (\vee (S_i)_I)(t_2)\} \Rightarrow (\vee (S_i)_I)(t_1 * t_2) \geq \min \{(\vee (S_i)_I)(t_1), (\vee (S_i)_I)(t_2)\}$, $(\vee (S_i)_N)($

Doi: 10.5281/zenodo.3782870

$t_1 * t_2) = \sup (S_i)_N(t_1 * t_2) \geq \sup\{\min\{(S_i)_N(t_1), (S_i)_N(t_2)\}\} = \min\{\sup (S_i)_N(t_1), \sup (S_i)_N(t_2)\} = \min\{(\vee (S_i)_N)(t_1), (\vee (S_i)_N)(t_2)\} \Rightarrow (\vee (S_i)_N)(t_1 * t_2) \geq \min\{(\vee (S_i)_N)(t_1), (\vee (S_i)_N)(t_2)\}$, which show that P-union of $\mathfrak{R}_i$ is NCMNSU of X.

Theorem 3.5 If neutrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X is subalgebra, then $\forall\ t_1 \in X$, $\aleph_E(0 * t_1) \geq \aleph_E(t_1)$, $\aleph_I(0 * t_1) \leq \aleph_I(t_1)$, $\aleph_N(0 * t_1) \leq \aleph_N(t_1)$ and $S_E(0 * t_1) \leq S_E(t_1)$, $S_I(0 * t_1) \geq S_I(t_1)$, $S_N(0 * t_1) \geq S_N(t_1)$.

Proof. $\forall\ t_1 \in X$, $\aleph_E(0 * t_1) \geq \text{rmin}\{\aleph_E(0), \aleph_E(t_1)\} = \text{rmin}\{\aleph_E(t_1 * t_1), \aleph_E(t_1)\} \geq \text{rmin}\{\text{rmin}\{\aleph_E(t_1), \aleph_E(t_1)\}, \aleph_E(t_1)\} = \aleph_E(t_1)$, $\aleph_I(0 * t_1) \leq \text{rmax}\{\aleph_I(0), \aleph_I(t_1)\} = \text{rmax}\{\aleph_I(t_1 * t_1), \aleph_I(t_1)\} \leq \text{rmax}\{\text{rmax}\{\aleph_I(t_1), \aleph_I(t_1)\}, \aleph_I(t_1)\} = \aleph_I(t_1)$, $\aleph_N(0 * t_1) \leq \text{rmax}\{\aleph_N(0), \aleph_N(t_1)\} = \text{rmax}\{\aleph_N(t_1 * t_1), \aleph_N(t_1)\} \leq \text{rmax}\{\text{rmax}\{\aleph_N(t_1), \aleph_N(t_1)\}, \aleph_N(t_1)\} = \aleph_N(t_1)$ and now $\forall\ t_1 \in X$, $S_E(0 * t_1) \leq \max\{S_E(0), S_E(t_1)\} = \max\{S_E(t_1 * t_1), S_E(t_1)\} \leq \max\{\max\{S_E(t_1), S_E(t_1)\}, S_E(t_1)\} = S_E(t_1)$, $S_I(0 * t_1) \geq \min\{S_I(0), S_I(t_1)\} = \min\{S_I(t_1 * t_1), S_I(t_1)\} \geq \min\{\min\{S_I(t_1), S_I(t_1)\}, S_I(t_1)\} = S_I(t_1)$, $S_N(0 * t_1) \geq \min\{S_N(0), S_N(t_1)\} = \min\{S_N(t_1 * t_1), S_N(t_1)\} \geq \min\{\min\{S_N(t_1), S_N(t_1)\}, S_N(t_1)\} = S_N(t_1)$

Theorem 3.6 If netrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X is subalgebra then $\mathfrak{R}(t_1 * t_2) = \mathfrak{R}(t_1 * (0 * (0 * t_2)))$ $\forall\ t_1, t_2 \in X$.

Proof. Let X be a BF-algebra and $t_1, t_2 \in X$. Then we know by ([13] Proposition 2.5) that $t_2 = 0 * (0 * t_2)$. Hence, $\aleph_\Xi(t_1 * t_2) = \aleph_\Xi(t_1 * (0 * (0 * t_2)))$ and $S_\Xi(t_1 * t_2) = S_\Xi(t_1 * (0 * (0 * t_2)))$. Therefore, $\mathfrak{R}_\Xi(t_1 * t_2) = \mathfrak{R}_\Xi(t_1 * (0 * (0 * t_2)))$.

Theorem 3.7 If netrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X is subalgebra then $\aleph_E(0) \geq \text{rmin}\{\aleph_E(t_2), \aleph_E(t_1\})$, $\aleph_I(0) \leq \text{rmax}\{\aleph_I(t_2), \aleph_I(t_1)\}$, $\aleph_N(0) \leq \text{rmax}\{\aleph_N(t_2), \aleph_N(t_1)\}$, and $S_E(0) \leq \max\{S_E(t_2), S_E(t_1)\}$, $S_I(0) \geq \min\{S_I(t_2), S_I(t_1)\}$, $S_N(0) \geq \min\{S_N(t_2), S_N(t_1)\}$, $\forall\ t_1, t_2 \in X$.

Proof. Here we use the ([13] Proposition 2.5) and above Proposition, now for $t_1, t_2 \in X$ $\aleph_E(0) = \aleph_E(t_1 * t_1) \geq \text{rmax}\{\aleph_E(t_1), \aleph_E(t_1)\} = \text{rmin}\{\aleph_E(0 * (0 * t_1)), \aleph_E(0 * (0 * t_1))\} = \text{rmin}\{\aleph_E(0 * (0 * t_2)), \aleph_E(0 * (0 * t_1))\} = \text{rmin}\aleph_E(t_2), \aleph_E(t_1)\}$, $\aleph_I(0) = \aleph_I(t_1 * t_1) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_1)\} = \text{rmax}\{\aleph_I(0 * (0 * t_1)), \aleph_I(0 * (0 * t_1))\} = \text{rmax}\{\aleph_I(0 * (0 * t_2)), \aleph_I(0 * (0 * t_1))\} = \text{rmax}\{\aleph_I(t_2), \aleph_I(t_1)\}$, $\aleph_N(0) = \aleph_N(t_1 * t_1) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_1)\} = \text{rmax}\{\aleph_N(0 * (0 * t_1)), \aleph_N(0 * (0 * t_1))\} = \text{rmax}\{\aleph_N(0 * (0 * t_2)), \aleph_N(0 * (0 * t_1))\} = \text{rmax}\{\aleph_N(t_2), \aleph_N(t_1)\}$.
Now, $S_E(0) = S_E(t_1 * t_1) \leq \max\{S_E(t_1), S_E(t_1)\} = \max\{S_E(0 * (0 * t_1)), S_E(0 * (0 * t_1))\} = \max\{S_E(0 * (0 * t_2)), S_E(0 * (0 * t_1))\} = \max\{S_E(t_2), S_E(t_1)\}$, $S_I(0) = S_I(t_1 * t_1) \geq \min\{S_I(t_1), S_I(t_1)\} = \min\{S_I(0 * (0 * t_1)), S_I(0 * (0 * t_1))\} = \min\{S_I(0 * (0 * t_2)), S_I(0 * (0 * t_1))\} = \min\{S_I(t_2), S_I(t_1)\}$, $S_N(0) = S_N(t_1 * t_1) \geq \min\{S_N(t_1), S_N(t_1)\} = \min\{S_N(0 * (0 * t_1)), S_N(0 * (0 * t_1))\} = \min\{S_N(0 * (0 * t_2)), S_N(0 * (0 * t_1))\} = \min\{S_N(t_2), S_N(t_1)\}$.

Theorem 3.8 If neutrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X is NCMNSU, then $\forall\ t_1, t_2 \in X$, $\aleph_\Xi(t_1 * (0 * t_2)) \geq \text{rmin}\{\aleph_\Xi(t_1), \aleph_\Xi(t_2)\}$ and $S_\Xi(t_1 * (0 * t_2)) \leq \max\{S_\Xi(t_1), S_\Xi(t_2)\}$.

Proof. Here we use above Proposition for proof. Let $t_1, t_2 \in X$. Then we have $\aleph_E(t_1 * (0 * t_2)) \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(0 * t_2)\} \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\}$, $\aleph_I(t_1 * (0 * t_2)) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(0 * t_2)\} \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\}$, $\aleph_N(t_1 * (0 * t_2)) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(0 * t_2)\} \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\}$ and $S_E(t_1 * (0 * t_2)) \leq \max\{S_E(t_1),$

$S_E(0 * t_2)\} \leq \max\{S_E(t_1), S_E(t_2)\}$, $S_I(t_1 * (0 * t_2)) \geq \min\{S_I(t_1), S_I(0 * t_2)\} \geq \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1 * (0 * t_2)) \geq \min\{S_N(t_1), S_N(0 * t_2)\} \geq \min\{S_N(t_1), S_N(t_2)\}$.

Theorem 3.9 If a neutrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X satisfies the following conditions, then $\mathfrak{R}$ refers to a NCMNSU of X:

Doi: 10.5281/zenodo.3782870

1. $\aleph_E(0*t_1) \geq \aleph_E(t_1)$, $\aleph_I(0*t_1) \leq \aleph_I(t_1)$, $\aleph_N(0*t_1) \leq \aleph_N(t_1)$ and $S_E(0*t_1) \leq S_E(t_1)$, $S_I(0*t_1) \geq S_I(t_1)$, $S_N(0*t_1) \geq S_N(t_1)\ \forall\ t_1 \in X$.

2. $\aleph_E\big(t_1*(0*t_2)\big) \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\}$, $\aleph_I\big(t_1*(0*t_2)\big) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\}$, $\aleph_N\big(t_1*(0*t_2)\big) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\}$ and $S_E\big(t_1*(0*t_2)\big) \leq \max\{S_E(t_1), S_E(t_2)\}$, $S_I\big(t_1*(0*t_2)\big) \geq \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1*(0*t_2)) \geq \min\{S_N(t_1), S_N(t_2)\}$, $\forall\ t_1, t_2 \in X$.

Proof. Assume that the neutrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X satisfies the both axioms above. Then by lemma, we have $\aleph_E(t_1*t_2) = \aleph_E(t_1*(0*(0*t_2))) \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(0*t_2)\} \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\}$, $\aleph_I(t_1*t_2) = \aleph_I(t_1*(0*(0*t_2))) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(0*t_2)\} \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\}$, $\aleph_N(t_1*t_2) = \aleph_N(t_1*(0*(0*t_2))) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(0*t_2)\} \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\}$, and $S_E(t_1*t_2) = S_E(t_1*(0*(0*t_2))) \leq \max\{S_E(t_1), S_E(0*t_2)\} \leq \max\{S_E(t_1), S_E(t_2)\}$, $S_I(t_1*t_2) = S_I(t_1*(0*(0*t_2))) \geq \min\{S_I(t_1), S_I(0*t_2)\} \geq \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1*t_2) = S_N(t_1*(0*(0*t_2))) \geq \min\{S_N(t_1), S_N(0*t_2)\} \geq \min\{S_N(t_1), S_N(t_2)\}\ \forall\ t_1, t_2 \in X$. Hence, $\mathfrak{R}$ is NCMNSU of X.

Theorem 3.10 A neutrosophic cubic set $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ of X is NCMNSU of X $\Leftarrow \aleph_{\Xi^-}, \aleph_{\Xi^+}$ and S_Ξ are fuzzy subalgebra of X.

Proof. Let $\aleph_\Xi^-, \hat{\kappa}^{t+}{}_\Xi$ and S_Ξ are fuzzy subalgebra of X and $t_1, t_2 \in X$. Then $\aleph_E^-(t_1*t_2) \geq \min\{\aleph_E^-(t_1), \aleph_E^-(t_2)\}$, $\aleph_I^-(t_1*t_2) \leq \max\{\aleph_I^-(t_1), \aleph_I^-(t_2)\}$, $\aleph_N^-(t_1*t_2) \leq \max\{\aleph_N^-(t_1), \aleph_N^-(t_2)\}$, $\aleph_E^+(t_1*t_2) \geq \min\{\aleph_E^+(t_1), \aleph_E^+(t_2)\}$, $\aleph_I^+(t_1*t_2) \leq \max\{\aleph_I^+(t_1), \aleph_I^+(t_2)\}$, $\aleph_N^+(t_1*t_2) \leq \max\{\aleph_N^+(t_1), \aleph_N^+(t_2)\}$, and $S_E(t_1*t_2) \leq \max\{S_E(t_1), S_E(t_2)\}$, $S_I(t_1*t_2) \geq \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1*t_2) \geq \min\{S_N(t_1), S_N(t_2)\}$. Now, $\aleph_E(t_1*t_2) = [\aleph_E^-(t_1*t_2), \aleph_E^+(t_1*t_2)] \geq [\min\{\aleph_E^-(t_1), \aleph_E^-(t_2)\}, \min\{\aleph_E^+(t_1), \aleph_E^+(t_2)\}] \geq \text{rmin}\{[\aleph_E^-(t_1), \aleph_E^+(t_2)], [\aleph_E^-(t_1), \aleph_E^+(t_2)]\} = \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\}$, $\aleph_I(t_1*t_2) = [\aleph_I^-(t_1*t_2), \aleph_I^+(t_1*t_2)] \leq [\max\{\aleph_I^-(t_1), \aleph_I^-(t_2)\}, \max\{\aleph_I^+(t_1), \aleph_I^+(t_2)\}] \leq \text{rmax}\{[\aleph_I^-(t_1), \aleph_I^+(t_2)], [\aleph_I^-(t_1), \aleph_I^+(t_2)]\} = \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\}$, $\aleph_N(t_1*t_2) = [\aleph_N^-(t_1*t_2), \aleph_N^+(t_1*t_2)] \leq [\max\{\aleph_N^-(t_1), \aleph_N^-(t_2)\}, \max\{\aleph_N^+(t_1), \aleph_N^+(t_2)\}] \leq \text{rmax}\{[\aleph_N^-(t_1), \aleph_N^+(t_2)], [\aleph_N^-(t_1), \aleph_N^+(t_2)]\} = \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\}$. Therefore, $\mathfrak{R}$ is NCMNSU of X. Conversely, assume that $\mathfrak{R}$ is a NCMNSU of X. For any $t_1, t_2 \in X$, $\{\aleph_E^-(t_1*t_2), \aleph_E^+(t_1*t_2)\} = \aleph_E(t_1*t_2) \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\} = \text{rmin}\{[\aleph_E^-(t_1), \aleph_E^+(t_1)], [\aleph_E^-(t_2), \aleph_E^+(t_2)]\} = [\min\{\aleph_E^-(t_1), \aleph_E^-(t_2)\}, \min\{\aleph_E^+(t_1), \aleph_E^+(t_2)\}]$, $\{\aleph_I^-(t_1*t_2), \aleph_I^+(t_1*t_2)] = \aleph_I(t_1*t_2) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\} = \text{rmax}\{[\aleph_I^-(t_1), \aleph_I^+(t_1)], [\aleph_I^-(t_2), \aleph_I^+(t_2)]\} = [\max\{\aleph_I^-(t_1), \aleph_I^-(t_2)\}, \max\{\aleph_I^+(t_1), \aleph_I^+(t_2)\}]$, $[\aleph_N^-(t_1*t_2), \aleph_N^+(t_1*t_2)] = \aleph_N(t_1*t_2) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\} = \text{rmax}\{[\aleph_N^-(t_1), \aleph_N^+(t_1)], [\aleph_N^-(t_2), \aleph_N^+(t_2)]\} = [\max\{\aleph_N^-(t_1), \aleph_N^-(t_2)\}, \max\{\aleph_N^+(t_1), \aleph_N^+(t_2)\}]$. Thus, $\aleph_E^-(t_1*t_2) \geq \min\{\aleph_\Xi^-(t_1), \aleph_\Xi^-(t_2)\}$, $\aleph_I^-(t_1*t_2) \leq \max\{\aleph_I^-(t_1), \aleph_I^-(t_2)\}$, $\aleph_N^-(t_1*t_2) \leq \max\{\aleph_N^-(t_1), \aleph_N^-(t_2)\}$, $\aleph_E^+(t_1*t_2) \geq \min\{\aleph_\Xi^+(t_1), \aleph_\Xi^+(t_2)\}$, $\aleph_I^+(t_1*t_2) \leq \max\{\aleph_I^+(t_1), \aleph_I^+(t_2)\}$, $\aleph_N^+(t_1*t_2) \leq \max\{\aleph_N^+(t_1), \aleph_N^+(t_2)\}$, and $S_E(t_1*t_2) \leq \max\{S_E(t_1), S_E(t_2)\}$, $S_I(t_1*t_2) \geq \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1*t_2) \geq \min\{S_N(t_1), S_N(t_2)\}$. Hence $\aleph_\Xi^+, \aleph_\Xi^-$ and S_Ξ are fuzzy subalgebra of X.

Theorem 3.11 Let $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ be a NCMNSU of X and $n \in \mathbb{Z}^+$(the set of positive integer). Then **1**. $\aleph_E(\daleth^n t_1 * t_1) \geq \aleph_E(t_1)$ for $n \in \mathbb{O}$. **2**. $\aleph_I(\daleth^n t_1 * t_1) \leq \aleph_I(t_1)$ for $n \in \mathbb{O}$. **3**. $\aleph_N(\daleth^n t_1 * t_1) \leq \aleph_N(t_1)$ for $n \in \mathbb{O}$. **4**. $S_E(\daleth^n t_1 * t_1) \leq \aleph_E(t_1)$ for $n \in \mathbb{O}$. **5**. $S_I(\daleth^n t_1 * t_1) \geq \aleph_I(t_1)$ for $n \in \mathbb{O}$. **6**. $S_N(\daleth^n t_1 * t_1) \geq \aleph_N(t_1)$ for $n \in \mathbb{O}$. **7**. $\aleph_\Xi(\daleth^n t_1 * t_1) = \aleph_\Xi(t_1)$ for $n \in \mathbb{E}$. **8**. $S_\Xi(\daleth^n t_1 * t_1) = \aleph_\Xi(t_1)$ for $n \in \mathbb{E}$.

Proof. Let $t_1 \in X$ and n is odd. Then $n = 2q - 1$ for some positive integer q. We prove the theorem by induction.

Now $\aleph_E(t_1*t_1) = \aleph_E(0) \geq \aleph_E(t_1)$, $\aleph_I(t_1*t_1) = \aleph_I(0) \leq \aleph_I(t_1)$, $\aleph_N(t_1*t_1) = \aleph_N(0) \leq \aleph_N(t_1)$ and $S_E(t_1*t_1) = S_E(0) \leq S_E(t_1)$, $S_I(t_1*t_1) = S_I(0) \geq S_I(t_1)$, $S_N(t_1*t_1) = S_N(0) \geq S_N(t_1)$. Suppose that $\aleph_E(\daleth^{2q-1}t_1 * t_1) \geq \aleph_E(t_1)$, $\aleph_I(\daleth^{2q-1}t_1 * t_1) \leq \aleph_I(t_1)$, $\aleph_N(\daleth^{2q-1}t_1 * t_1) \leq \aleph_N(t_1)$ and $S_E(\daleth^{2q-1}t_1 * t_1) \leq S_E(t_1)$, $S_I(\daleth^{2q-1}t_1 * t_1) \geq S_I(t_1)$, $S_N(\daleth^{2q-1}t_1 * t_1) \geq S_N(t_1)$. Then by assumption, $\aleph_E(\daleth^{2(q+1)-1}t_1 * t_1) = \aleph_E(\daleth^{2q+1}t_1 * t_1) = \aleph_E(\daleth^{2q-1}t_1 * (t_1 * (t_1 * t_1))) = \aleph_E(\daleth^{2q-1}t_1 * t_1) \geq \aleph_E(t_1)$, $\aleph_I(\daleth^{2(q+1)-1}t_1 * t_1) = \aleph_I(\daleth^{2q+1}t_1 * t_1) = \aleph_I(\daleth^{2q-1}t_1 * (t_1 * (t_1 * t_1))) = \aleph_I(\daleth^{2q-1}t_1 * t_1) \leq \aleph_I(t_1)$, $\aleph_N(\daleth^{2(q+1)-1}t_1 * t_1) = \aleph_N(\daleth^{2q+1}t_1 * t_1) = \aleph_N(\daleth^{2q-1}t_1 * (t_1 * (t_1 * t_1))) = \aleph_N(\daleth^{2q-1}t_1 * t_1) \leq \aleph_N(t_1)$ and $S_E(\daleth^{2(q+1)-1}t_1 * t_1) = S_E(\daleth^{2q+1}t_1 * t_1) = S_E(\daleth^{2q-1}t_1 * (t_1 * (t_1 * t_1))) = S_E(\daleth^{2q-1}t_1 * t_1) \leq S_E(t_1)$,

$S_I(\daleth^{2(q+1)-1}t_1 * t_1)$=$S_I(\daleth^{2q+1}t_1 * t_1) = S_I(\daleth^{2q-1}t_1 * (t_1 * (t_1 * t_1))) = S_I(\daleth^{2q-1}t_1 * t_1) \geq S_I(t_1)$, $S_N(\daleth^{2(q+1)-1}t_1 * t_1)$=$S_N(\daleth^{2q+1}t_1 * t_1) = S_N(\daleth^{2q-1}t_1 * (t_1 * (t_1 * t_1))) = S_N(\daleth^{2q-1}t_1 * t_1) \geq S_N(t_1)$, which prove (1),(2),(3),(4),(5) and (6), similarly we can prove the remaining cases (7) and (8).

Note: The sets denoted by $I_{\aleph_\Xi}$ and I_{S_Ξ} are also subalgebras of X, which are defined as: $I_{\aleph_\Xi}$=$\{t_1 \in X | \aleph_\Xi(t_1) = \aleph_\Xi(0)\}$, I_{S_Ξ}=$\{t_1 \in X | S_\Xi(t_1) = S_\Xi(0)\}$.

Theorem 3.12 Let $\Re = (\aleph_\Xi, S_\Xi)$ be a NCMNSU of X. Then the sets $I_{\aleph_\Xi}$ and I_{S_Ξ} are subalgebras of X.

Proof. Let $t_1, t_2 \in I_{\aleph_\Xi}$. Then $\aleph_\Xi(t_1) = \aleph_\Xi(0) = \aleph_\Xi(t_2)$ and $\aleph_\Xi(t_1 * t_2) \geq \text{rmin}\{\aleph_\Xi(t_1), \aleph_\Xi(t_2)\} = \aleph_\Xi(0)$. By using Proposition 2.3, we know that $\aleph_\Xi(t_1 * t_2) = \aleph_\Xi(0)$ or equivalently $t_1 * t_2 \in I_{\aleph_\Xi}$.
Let $t_1, t_2 \in I_{\aleph_\Xi}$. Then $S_\Xi(t_1) = S_\Xi(0) = S_\Xi(t_2)$ and $S_\Xi(t_1 * t_2) \leq \max\{S_\Xi(t_1), S_\Xi(t_2)\}$ =$S_\Xi(0)$. Again by using Proposition 2.3, we know that $S_\Xi(t_1 * t_2) = S_\Xi(0)$ or equivalently $t_1 * t_2 \in I_{\aleph_\Xi}$. Hence the sets $I_{\aleph_\Xi^t}$ and I_{S_Ξ} are subalgebras of X.

Theorem 3.13 Let B be a nonempty subset of X and $\Re = (\aleph_\Xi, S_\Xi)$ be a neutrosophic cubic set of X defined by

$$\aleph_\Xi(t_1) = \begin{cases} [\kappa_{\Xi_1}, \kappa_{\Xi_2}], & \text{if } t_1 \in B \\ [\varphi_{\Xi_1}, \varphi_{\Xi_2}] & \text{otherwise,} \end{cases}, S_T(t_1) = \begin{cases} \omega_\Xi, & \text{if } t_1 \in B \\ \varrho_\Xi, & \text{otherwise} \end{cases}$$

$\forall$ $[\kappa_{\Xi_1}, \kappa_{\Xi_2}]$,$[\varphi_{\Xi_1}, \varphi_{\Xi_2}] \in D[0,1]$ and ω_Ξ, $\varrho_\Xi \in [0,1]$ with $[\kappa_{E_1}, \kappa_{E_2}] \geq [\varphi_{E_1}, \varphi_{E_2}]$, $[\kappa_{I_1}, \kappa_{I_2}] \leq [\varphi_{I_1}, \varphi_{I_2}]$, $[\kappa_{N_1}, \kappa_{N_2}] \leq [\varphi_{N_1}, \varphi_{N_2}]$, and $\omega_E \leq \varrho_E$, $\omega_I \geq \varrho_I$, $\omega_N \geq \varrho_N$. Then $\Re$ is a NCMNSU of X $\Leftarrow$ B is a subalgebra of X. Moreover, $I_{\aleph_\Xi}$=B=I_{S_Ξ}.

Proof. Let $\Re$ be a NCMNSU of X and $t_1, t_2 \in X$ such that $t_1, t_2 \in B$. Then $\aleph_E(t_1 * t_2) \geq \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\} = \text{rmin}\{[\kappa_{E_1}, \kappa_{E_2}], [\kappa_{E_1}, \kappa_{E_2}]\} = [\kappa_{E_1}, \kappa_{E_2}]$, $\aleph_I(t_1 * t_2) \leq \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\} = \text{rmax}\{[\kappa_{I_1}, \kappa_{I_2}], [\kappa_{I_1}, \kappa_{I_2}]\} = [\kappa_{I_1}, \kappa_{I_2}]$, $\aleph_N(t_1 * t_2) \leq \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\} = \text{rmax}\{[\kappa_{N_1}, \kappa_{N_2}], [\kappa_{N_1}, \kappa_{N_2}]\} = [\kappa_{N_1}, \kappa_{N_2}]$ and $S_E(t_1 * t_2) \leq \max\{S_E(t_1), S_E(t_2)\} = \max\{\omega_E, \omega_E\} = \omega_E$, $S_I(t_1 * t_2) \geq \min\{S_I(t_1), S_I(t_2)\} = \min\{\omega_I, \omega_I\} = \omega_I$, $S_N(t_1 * t_2) \geq \min\{S_N(t_1), S_N(t_2)\} = \min\{\omega_N, \omega_N\} = \omega_N$,. Therefore $t_1 * t_2 \in B$. Hence, B is a subalgebra of X. Conversely, suppose that B is a subalgebra of X and $t_1, t_2 \in X$. Consider two cases.

Case 1: If $t_1, t_2 \in B$ then $t_1 * t_2 \in B$, thus $\aleph_E(t_1 * t_2) = [\kappa_{E_1}, \kappa_{E_2}] = \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\}$, $\aleph_I(t_1 * t_2) = [\kappa_{I_1}, \kappa_{I_2}] = \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\}$, $\aleph_N(t_1 * t_2) = [\kappa_{N_1}, \kappa_{N_2}] = \text{rmax}\{\aleph_N(t_1), \aleph_N(t_2)\}$, and $S_E(t_1 * t_2) = \omega_E = \max\{S_E(t_1), S_E(t_2)\}$, $S_I(t_1 * t_2) = \omega_I = \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1 * t_2) = \omega_N = \min\{S_N(t_1), S_N(t_2)\}$.

Case 2: If $t_1 \notin B$ or $t_2 \notin B$, then $\aleph_E(t_1 * t_2) \geq [\varphi_{E_1}, \varphi_{E_2}] = \text{rmin}\{\aleph_E(t_1), \aleph_E(t_2)\}$, $\aleph_I(t_1 * t_2) \leq [\varphi_{I_1}, \varphi_{I_2}] = \text{rmax}\{\aleph_I(t_1), \aleph_I(t_2)\}$, $\aleph_N(t_1 * t_2) \leq [\varphi_{N_1}, \varphi_{N_2}] = \text{rmin}\{\aleph_N(t_1), \aleph_N(t_2)\}$, and $S_E(t_1 * t_2) \leq \varrho_E = \max\{S_E(t_1), S_E(t_2)\}$, $S_I(t_1 * t_2) \geq \varrho_I = \min\{S_I(t_1), S_I(t_2)\}$, $S_N(t_1 * t_2) \geq \varrho_I = \min\{S_N(t_1), S_N(t_2)\}$. Hence $\Re$ is a NCMNSU of X.
Now, $I_{\aleph_\Xi}$=$\{t_1 \in X, \aleph_\Xi(t_1) = \aleph_\Xi(0)\}$=$\{t_1 \in X, \aleph_\Xi(t_1) = [\kappa_{\Xi_1}, \kappa_{\Xi_2}]\}$ = B, and I_{S_Ξ}= $\{t_1 \in X, S_\Xi(t_1) = S_\Xi(0)\}$ = $\{t_1 \in X, S_\Xi(t_1) = \omega_\Xi\}$ = B.

Theorem 3.14 Let $\Re = (\aleph_\Xi, S_\Xi)$ be a neutrosophic cubic set of X. For $[s_{E_1}, s_{E_2}], [s_{I_1}, s_{I_2}], [s_{N_1}, s_{N_2}] \in D[0,1]$ and $t_{E_1}, t_{I_1}, t_{N_1} \in [0,1]$, the set $U(\aleph_\Xi | ([s_{E_1}, s_{E_2}], [s_{I_1}, s_{I_2}], [s_{N_1}, s_{N_2}]))$ =$\{t_1 \in X | \aleph_E(t_1) \geq [s_{E_1}, s_{E_2}], \aleph_I(t_1) \leq [s_{I_1}, s_{I_2}], \aleph_N(t_1) \leq [s_{N_1}, s_{N_2}]\}$ is called upper $([s_{E_1}, s_{E_2}], [s_{I_1}, s_{I_2}], [s_{N_1}, s_{N_2}])$-level of $\Re$ and $L(S_\Xi | (t_{E_1}, t_{I_1}, t_{N_1}))$ =$\{t_1 \in X | S_E(t_1) \leq t_{E_1}, S_I(t_1) \geq t_{I_1}, S_N(t_1) \geq t_{N_1}\}$ is called lower $(t_{E_1}, t_{I_1}, t_{N_1})$-level of $\Re$. If $\Re = (\aleph_\Xi, S_\Xi)$ is NCMNSU of X, then the upper $[s_{\Xi_1}, s_{\Xi_2}]$-level and lower t_{Ξ_1}-level of $\Re$ are subalgebras of X.

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Proof. Let $t_1, t_2 \in U(\aleph_\Xi|[s_{\Xi_1}, s_{\Xi_2}])$. Then $\aleph_E(t_1) \geq [s_{E_1}, s_{E_2}]$ and $\aleph_E(t_2) \geq [s_{E_1}, s_{E_2}]$. It follows that $\aleph_E(t_1 * t_2) \geq rmin\{\aleph_E(t_1), \aleph_E(t_2)\} \geq [s_{\Xi_1}, s_{\Xi_2}] \Rightarrow t_1 * t_2 \in U(\aleph_E|[s_{E_1}, s_{E_2}])$, $\aleph_I(t_1) \leq [s_{I_1}, s_{I_2}]$ and $\aleph_I(t_2) \leq [s_{I_1}, s_{I_2}]$. It follows that $\aleph_I(t_1 * t_2) \leq rmax\{\aleph_I(t_1), \aleph_I(t_2)\} \leq [s_{I_1}, s_{I_2}] \Rightarrow t_1 * t_2 \in U(\aleph_I|[s_{I_1}, s_{I_2}])$, $\aleph_N(t_1) \leq [s_{N_1}, s_{N_2}]$ and $\aleph_N(t_2) \leq [s_{N_1}, s_{N_2}]$. It follows that $\aleph_N(t_1 * t_2) \leq rmax\{\aleph_N(t_1), \aleph_N(t_2)\} \leq [s_{N_1}, s_{N_2}] \Rightarrow t_1 * t_2 \in U(\aleph_N|[s_{N_1}, s_{N_2}])$. Hence, $U(\aleph_\Xi|[s_{\Xi_1}, s_{\Xi_2}]$ is a subalgebra of X. Let $t_1, t_2 \in L(S_\Xi|t_{\Xi_1})$. Then $S_E(t_1) \leq t_{E_1}$ and $S_E(t_2) \leq t_{E_1}$. It follows that $S_E(t_1 * t_2) \leq max\{S_E(t_1), S_E(t_2)\} \leq t_{E_1} \Rightarrow t_1 * t_2 \in L(S_E|t_{E_1})$, $S_I(t_1) \geq t_{I_1}$ and $S_I(t_2) \geq t_{I_1}$. It follows that $S_I(t_1 * t_2) \geq min\{S_I(t_1), S_I(t_2)\} \geq t_{I_1} \Rightarrow t_1 * t_2 \in L(S_I|t_{I_1})$, $S_N(t_1) \geq t_{N_1}$ and $S_N(t_2) \geq t_{N_1}$. It follows that $S_N(t_1 * t_2) \geq min\{S_N(t_1), S_N(t_2)\} \geq t_{N_1} \Rightarrow t_1 * t_2 \in L(S_N|t_{N_1})$, Hence $L(S_\Xi|t_{\Xi_1})$ is a subalgebra of X.

Theorem 3.15 Let $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ is NCMNSU of X. Then $\aleph([s_{\Xi_1}, s_{\Xi_2}]; t_{\Xi_1}) = U(\aleph_\Xi|[s_{\Xi_1}, s_{\Xi_2}]) \cap L(S_\Xi|t_{\Xi_1}) = \{t_1 \in X|\aleph_E(t_1) \geq [s_{E_1}, s_{E_2}], \aleph_I(t_1) \leq [s_{I_1}, s_{I_2}], \aleph_N(t_1) \leq [s_{N_1}, s_{N_2}], S_E(t_1) \leq t_{E_1}, S_I(t_1) \geq t_{I_1}, S_N(t_1) \geq t_{N_1}\}$ is a subalgebra of X.

Proof. This theorem can be proved by using Theorem 3.14. The converse of Theorem 3.15 is not valid, for which we present the example.

Example 3.2 Let $X = \{0, t_1, t_2, t_3, t_4, t_5\}$ be a BF-algebra used in above example and $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ is a neutrosophic cubic set defined by

	0	t_1	t_2	t_3	t_4	t_5
$\aleph_T$	[0.5,0.7]	[0.6,0.7]	[0.6,0.7]	[0.2,0.3]	[0.4,0.5]	[0.4,0.5]
$\aleph_I$	[0.4,0.6]	[0.5,0.6]	[0.5,0.6]	[0.5,0.7]	[0.4,0.4]	[0.4,0.8]
$\aleph_F$	[0.3,0.5]	[0.3,0.6]	[0.3,0.6]	[0.3,0.6]	[0.2,0.3]	[0.2,0.3]

	0	t_1	t_2	t_3	t_4	t_5
S_T	0.2	0.4	0.4	0.6	0.4	0.6
S_I	0.3	0.5	0.5	0.7	0.5	0.7
S_F	0.4	0.6	0.6	0.8	0.6	0.8

Now $\aleph([s_{\Xi_1}, s_{\Xi_2}]; t_{\Xi_1}) = U(\aleph_\Xi|[s_{\Xi_1}, s_{\Xi_2}]) \cap L(S_\Xi|t_{\Xi_1}) = \{t_1 \in X|\aleph_T(t_1) \geq [s_{T_1}, s_{T_1}],\ S_T(t_1) \leq t_T\} = \{0, t_1, t_3\} \cap \{0, t_1, t_2, t_3\} = \{0, t_1, t_3\}$ is a subalgebra of X, similarly we can find this for indterminate and non membership elelments. But $\mathfrak{R} = (\aleph_\Xi, S_\Xi)$ is not a neutrosophic cubic subalgebra, since $\aleph_T(t_1 * t_4) = [0.2,0.3] \ngeq [0.4,0.5] = rmin\{\aleph_T(t_1), \aleph_T(t_4)\}$, $\aleph_I(t_1 * t_2) = [0.4,0.8] \nleq [0.4,0.6] = rmax\{\aleph_I(t_1), \aleph_I(t_2)\}$, similarly we can find this for non membership elelment and $\lambda_T(a_2 * a_4) = 0.6 \nleq 0.4 = max\{\lambda_T(a_2), \lambda_T(a_4)\}$, similarly we can find this for indeterminate and non membership elelments.

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4. Conclusion

In this paper, neutrosophic cubic MN-subalgebra was introduced and its few helpful results and new characteristics were studied. The investigation of this new sort of subalgebra will help analysts to apply this subalgebra on various algebras. We are recommending some ideas like multiplication, and cartesian product to apply this work.

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