



The Analysis of Pentagonal Fuzzy Numbers in a Neutrosophic Fuzzy Inventory Management Modelling with Minimal Insufficient Supply Required and Fuzzy Consumption

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Abstract

The fuzzy stock administration demonstrates displayed in this work employments neutrosophic set hypothesis, pentagonal fuzzy numbers, and the Graded mean Integration Representation (GMIR) strategy for defuzzification. Request rates, arrange amounts, utilization rates, holding costs, setup costs, and deficiency costs are all spoken to as fuzzy parameters within the demonstrate to account for the inborn instability and vacillation. To reduce by and large costs, the whole cost work is calculated, taking setup, holding, and shortage costs into consideration. In arrange to speak to the combined impacts of a few fetched components, the overall taken a toll work is rearranged and the ideal arrange amount is built up beneath fuzzy conditions utilizing pentagonal fuzzy parameters. The demonstrate is assessed beneath different degrees of instability through a case-based investigation, advertising an exhaustive system for making choices on stock administration in equivocal and dubious circumstances. The results appear how versatile and capable the show is for improving fetched advancement and stock control.

Keywords: Fuzzy Inventory Management; Pentagonal Fuzzy Numbers; Neutrosophic Set Theory; Graded Mean Integration Representation (GMIR); Defuzzification; Total Cost Function; Optimization; Uncertainty; Inventory Control; Cost Minimization; Fuzzy Decision-Making; Supply Chain Management

1 Introduction

To ensure continuous operations, effective inventory management is essential for the efficiency and effectiveness of supply chain operations. However, inventory management becomes more complex when faced with unpredictable supply and demand conditions. These changes complicate accurate forecasting and decision-making and are often associated with inconsistent supplier performance, changing consumer preferences, and volatile market conditions.²Traditionally, fuzzy logic models have been used to manage uncertainty in inventory management by collecting fuzzy data and representing ambiguous information. These models, called fuzzy sets, are based on the idea of degrees of membership and allow you to control conditions that reflect demand levels, manage costs, and eliminate the uncertainty associated with missing data. These models offer

a flexible approach to solving supply chain problems that accounts for data variability and inaccuracies. The asymmetrical character of real- world demand distributions, which might be often skewed in the direction of precise stages, is captured by means of those figures, which painting demand as five specific factors: minimum, decrease probably, most probably, better probably, and maximum values.²The framework improves inventory structures' potential to handle uncertainty by means of fusing pentagonal fuzzy numbers with neutrosophic principle, which results in more truthful choice-making.³This technique boosts reactivity to changing deliver chain situations, optimises useful resource allocation, lowers costs, and improves forecasting accuracy. By means of offering a more bendy and sturdy approach of managing uncertainties and opening the door for wider programs in intricate and unpredictable fields, the incorporation of those modern thoughts represents a tremendous development in inventory control.

Managing stock can be extremely hard while call for and supply situations are unpredictable.⁴Correctly simulating the complexity of real- world situations is impossible with conventional deterministic fashions, which rely on set values for traits like call for, supply prices, and keeping fees. Fuzzy logic is utilized in fuzzy stock fashions to account for the imprecision and uncertainty found in those traits, presenting an extra adaptable and dynamic approach.⁵For corporations like prescribed drugs, perishable items, or high- call for purchaser products, where stockouts ought to bring about tremendous sales loss or customer discontent, fuzzy fashions are very useful in stopping stock shortages.

These models seize the variety and volatility that can occur over the years through expressing deliver and demand as fuzzy variables.⁶Deterministic methods, however, typically underestimate the variety of deliver and demand and rely upon fixed numbers. keeping consistent product availability whilst decreasing prices is the essential objective of fuzzy stock models in a situation when there may be no scarcity. for instance, fuzzy numbers may be used to enhance a monetary Order quantity (EOQ) model, where variables such as ordering price, protecting price, and demand fee are handled as fuzzy as opposed to fixed values. This enables the system's uncertainty to be reflected more appropriately.⁷The order quantities and reorder points that these fuzzy models compute assist optimise stock selections through considering.

There are various makes use of for those fuzzy fashions in inventory control. They can be applied in supply chains while inventory selections want to regulate to transferring intake trends because of various call for.⁸In production planning for seasonal products, where call for might fluctuate significantly all through the 12 months, fuzzy fashions are mainly crucial. furthermore, they aid in the green use of storage space, guaranteeing that items are on hand without going overboard. Fuzzy inventory fashions increase client happiness, save emergency procurement costs, and improve ordinary operational performance via putting off shortages.

In assessment to greater conventional fuzzy models which includes trapezoidal or triangular fuzzy numbers, Pentagonal fuzzy numbers (PFNs) are a specific kind of fuzzy numbers that provide a greater flexible and thorough illustration of uncertainty. The minimal, most possibly, lower possibly, better possibly, and maximum values are the 5 parameters that make up PFNs.⁹Due to their multi-parameter shape, PFNs can explicit asymmetric and non-linear uncertainty, which makes them mainly beneficial in situations in which the statistics distribution is not symmetrical or uniform. PFNs are especially beneficial in inventory management for coping with a ramification of intricate conditions in which uncertainty is gift and varies in more than one dimensions. Demand forecasting is one of the main methods that PFNs are utilized in stock manage. for the reason that demand is hardly ever constant, especially in sectors with seasonal or variable patterns, PFNs can be used to account for different degrees of likelihood and reflect the range of capability demand consequences.¹⁰As an instance, demand for some products can also display splendid asymmetry during top seasons, with an more likelihood of surpassing, in place of falling underneath, the anticipated demand. the usage of PFNs to simulate this lets in stock managers to more precisely predict demand and alter stock degrees.

PFNs can play a sizable position in fee estimates by means of modelling the uncertainty in stock prices. The prices associated with stock, including keeping prices, ordering prices, and lack prices, are not regular and might change because of external causes or changes within the marketplace.¹¹Those prices may be flexibly modelled with PFNs, imparting an extra accurate depiction of fee behaviour in unpredictable situations. PFNs can also be utilized in reorder factor computations, permitting dynamic adjustments consistent with call for variations, consumption fees, and fuzzy lead times. PFNs can be used, as an example, by a enterprise to simulate erratic purchaser demand over the vacation season. a pentagonal fuzzy wide variety may be used to symbolize demand, permitting the merchant to determine the quality order portions to in shape consumer demand even as minimising ordinary expenses. by offering a stronger framework for making selections within the face of uncertainty, this approach improves inventory control in dynamic contexts.

Fuzzy and intuitionistic fuzzy sets are prolonged by way of neutrosophic sets, which have been created to cope with indeterminacy in addition to imprecision and vagueness.¹²In contrast to traditional fuzzy sets, which use a single membership cost (fact) to represent uncertainty, neutrosophic sets introduce 3 specific membership functions: indeterminacy (I), which captures the degree of uncertainty or war within the data; falsity (F),

which represents the degree of contradiction or falsehood; and fact (T), which indicates the degree of fact or settlement. given that many selection-making tactics involve partial, contradictory, or puzzling records, neutrosophic sets are mainly useful in these circumstances due to their triadic nature. By using thinking of both the degree of indeterminacy— in which facts or results are ambiguous or contradictory—and what’s true or fake, neutrosophic units offer a extra thorough approach for making decisions.¹³Typically, especially imperative in complex stock control frameworks where choices are frequently made based on fragmented or negating information. For instance, when comparing numerous suppliers, the data that’s open on fetched, quality, and conveyance unwavering quality may be conflicting or conflicting. To assist them choose the pleasant alternative, neutrophilic units allow selection-makers to provide club values to every provider in step with the fact, falsehood, and indeterminacy of the information at hand. Additionally useful in stock control threat assessment are neutrophilic sets. they’re capable of simulate the ambiguity and contradictory facts associated with variables like price swings, supply chain interruptions, and call for variations.¹⁴Neutrosophic sets enable an extra nuanced examination of risks by shooting the indeterminacy in these elements, which aids corporations in making extra educated selections. similarly, neutrosophic evaluation may be utilized in product lifecycle control, mainly in conditions regarding decisions on product advent or discontinuation, in which market facts is often ambiguous and contradictory.

A production business enterprise, as an instance, might also encounter contradictory statistics concerning future demand, supply chain barriers, and cost implications while determining whether or not to increase production capability.¹⁵It’s miles feasible that traditional fuzzy models merely think about the level of demand (reality) truth. The firm can, however, additionally account for the diploma of uncertainty (indeterminacy) and any opposing views (falsity) by means of employing neutrosophic sets, which results in an extra thorough and reliable selection-making technique.¹⁶With this technique, the business enterprise will be capable of handle uncertainty higher and make strategic selections that could face up to ambiguity and contradicting records. Utilizing neutrosophic sets makes selection-making processes extra strong, adaptable, and capable of manipulate a greater variety of uncertainty. this is particularly helpful in regions like inventory management, in which making precise choices is vital to maximising

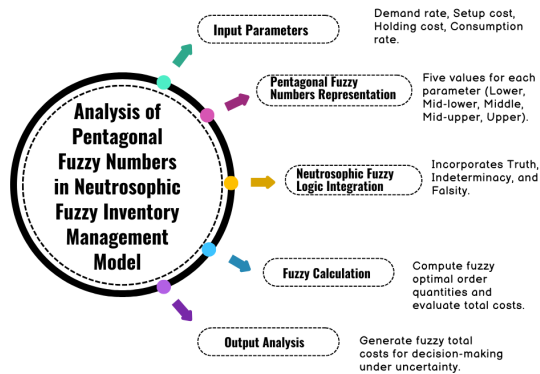
2 Methodology For Inventory Management Using Pentagonal Fuzzy Numbers and Neutrosophic Fuzzy Logic

Making a strong methodology for stock administration within the confront of uncertainty and capriciousness is the most objective of the investigation of pentagonal fuzzy numbers in a neutrosophic fuzzy stock administration demonstrate. Five one-of-a-kind values—lower, mid-lower, center, mid-upper, and upper limits—that capture distinctive degrees of plausibility are utilized to depict Pentagonal fuzzy numbers, which are utilized to speak to dubious characteristics counting request, setup fetched, holding taken a toll, and utilization.¹⁷Comparing this exact representation to customary fuzzy numbers, a more adaptable and nuanced displaying of vulnerability is conceivable. Besides, by including three levels of evaluation—truth, indeterminacy, and falsity—neutrosophic fuzzy rationale permits the demonstrate to bargain with supply and request uncertainty, inconsistencies, and questions.¹⁸Whereas guaranteeing that stock levels fulfill demands with minor deviations, the methodology lays a solid accentuation on decreasing deficiencies in arrange to extend operational productivity and client fulfillment. Moreover, cushy usage is guaranteed, whereby unpredictable and whimsical use plans utilizing soft basis imitate real-world stream. The exhibit gives a perfect stock approach that adjusts to moving supply and request designs through the improvement of holding, asking, and mishap charges. This strategy is particularly valuable in businesses where demands change bundles or supply chain issues are apparent, such as fabricating, retail, and healthcare. The adaptable decision-making made conceivable by the combination of pentagonal feathery numbers and neutrosophic thinking gives an exhaustive comprehension of dangers and potential results.

3 Preliminaries

3.1 Representation and Membership Function of Pentagonal Fuzzy Numbers

A pentagonal fuzzy number is represented by the fuzzy set $\tilde{D} = (a, b, c, d, e)$ where a, b, c, d and e are the five parameters: minimum value, lower likely value, most likely value, upper likely value, and maximum value,



respectively.

The membership function $\mu_{\tilde{D}}(x)$ for a pentagonal fuzzy number is defined as

$$\mu_{\tilde{D}}(x) = \begin{cases} 0, & x < a \text{ or } x > e \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b < x \leq c \\ \frac{e-x}{e-d}, & d < x \leq e \end{cases}$$

This function describes the degree of membership of x in the fuzzy set, capturing the uncertainty and variability in the range of possible value.

Given pentagonal fuzzy number: $\tilde{A} = (1, 3, 5, 7, 9)$

$$\beta(\tilde{A}) = \frac{1 + 2(3) + 2(5) + 2(7) + 9}{8}$$

$$\beta(\tilde{A}) = \frac{1 + 6 + 10 + 14 + 9}{8}$$

$$\beta(\tilde{A}) = \frac{40}{8} = 5.875$$



3.2 Comprehensive Understanding of Neutrosophic Sets and Their Role in Modelling Uncertainty

A Neutrosophic Set is a set where each element xx has three membership values: Truth ($T(x)T(x)$), Falsity ($F(x)F(x)$), and Indeterminacy ($I(x)I(x)$), representing the degree of certainty, falsehood, and uncertainty, respectively. It is defined as:

$$A \sim = \{(x, T(x), F(x), I(x)) \mid x \in X\}$$

Where $0 \leq T(x), F(x), I(x) \leq 1$ and $T(x) + F(x) + I(x) \leq 3$. This allows for modeling uncertainty, ambiguity, and conflicting information in decision-making.

3.3 Defuzzification of Pentagonal Fuzzy Numbers Using the Graded Mean Integration Representation (GMIR) Method

The Graded Mean Integration Representation (GMIR) method for defuzzifying a pentagonal fuzzy number is defined as follows. Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ represent a pentagonal fuzzy number. The crisp value $\beta(\tilde{A})$ is computed using the formula:

$$\beta(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{8}$$

4 Notations

To deal with uncertainty, fuzzy variables are used to express different parameters in fuzzy inventory management.

Symbol	Description
θ	Demand rate
ψ	Fuzzy order quantity (decision variable)
φ	Fuzzy consumption rate
κ	Holding cost per unit per time
λ	Setup cost per order
ξ	Shortage cost per unit
ζ	Minimal insufficient supply (fuzzy constraint)
ω	Total cost function

5 Total Cost Function Simplification in Fuzzy Inventory Management

The overall fetched work could be a comprehensive measurement in fuzzy stock administration that takes into consideration a variety of fetched components beneath uncertain circumstances. Typical operating costs incorporate setup, deficiencies, fuzzy capacity, and other costs. Many of the factors taken under consideration within the add up to fetched work by speaking to these costs as fuzzy factors are request, arrange amount, and utilization rate. In expansion to bringing down costs, this approach creates a sound system for decision-making that takes under consideration the complexities of real capacity frameworks.

Total cost function

$$\omega = \frac{\lambda \cdot \theta}{\psi} + \frac{\kappa \cdot \psi}{2} + \xi \cdot \zeta$$

Differentiating

$$\frac{d\omega}{d\psi} = -\frac{\lambda \cdot \theta}{\psi^2} + \frac{\kappa}{2}$$

Setting $\frac{d\omega}{d\psi} = 0$, solve for ψ :

$$\begin{aligned} -\frac{\lambda \cdot \theta}{\psi^2} + \frac{\kappa}{2} &= 0 \\ \psi^2 &= \frac{2 \cdot \lambda \cdot \theta}{\kappa} \\ \therefore \psi &= \sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}} \end{aligned}$$

The overall cost function demonstrates how different factors affect the total cost and aims to lower the expenses associated with a particular factor. To accomplish this, we examine how costs vary when a specific factor is modified by deriving a cost function pertinent to that factor. The assumption suggests that an increase or decrease in one factor will result in a corresponding increase or decrease in the overall cost; by setting the derivative to zero, we guarantee that the cost remains constant irrespective of how the factors change, which is

essential as it denotes the potential minimum or maximum cost. To address this matter, the factor of interest must be separated, and its value should be evaluated in relation to the other variables within the cost function. The resulting equation produces the optimal factor values that reduce the total cost by balancing the effects of various components of the cost function.

By varying the configuration, the capacity, and request rate expenses, Fuzzy Stock Control determines the optimal arrangement quantity. Set the subsystem of the generally fetched job to zero in order to talk to the smallest taken a toll point. Enhancing the specifications, the square of the organise constitute can be divided by the quantity of inventory taken a toll and proportionate to the item of the setup taken a toll and request rate. The optimal arrangement amount is determined by taking the reciprocal of the square root of this connection and accounting for variations in request, toll, and utilisation rates in order to reduce total costs. This outcome is essential for effective management of inventory in a fuzzy setting.

Total cost in terms of ψ .

Substitute $\psi = \sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}}$ into ω

$$\omega = \frac{\lambda \cdot \theta}{\sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}}} + \frac{\kappa \cdot \sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}}}{2} + \xi \cdot \zeta$$

An addition to accomplished job in stock administration can be designated as the ideal arrangement quantity. Configuration, and grasping, and shortcoming suspicions are computed using the perfect esteem, which is embedded throughout the complete taken-a-toll task using a developed equation. In contrast to keeping costs, which are directly tied to the amount arranged, setup costs are inversely proportional to it. Furthermore, the expected amount of shortages is determined by the least advantage for poor care. Together, these elements form the entire fetched work, that supplies a reliable fetched figure that does, in fact, support prudent stock administration decisions in unexpected emergencies.

Simplify each term:

First term:

$$\frac{\lambda \cdot \theta}{\sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}}} = \sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa}$$

Second term:

$$\frac{\kappa \cdot \sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}}}{2} = \frac{\sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa}}{2}$$

Thus, total cost becomes:

$$\begin{aligned} \omega &= \sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa} + \frac{\sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa}}{2} + \xi \cdot \zeta \\ \omega &= \frac{3}{2} \sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa} + \xi \cdot \zeta \end{aligned}$$

In order to rearrange the typically taken toll work, each phrase is first examined separately. This first phrase is essentially twice the sum of the three variables: setup taken expenses, encourage rate, and the number of records fetched. In short, the momentous term is numerous times the sum of the intersects of the capacity generated, the request for information rate, and the setup fetched. This illustrates the ability to fetch more than the planned sum. Both of these elements have been incorporated to the total taken-a-toll work along with the inadequate levels fetched term, which manages the least amount of deficiency supply, once the two terms have been separated. The final fetched condition that explains the overall effect of setup is below fluffy circumstances.

5.1 Fuzzy Inventory Management with Pentagonal Parameters: Case-Based Analysis and Total Cost Function

To further demonstrate the erratic and variable nature of various expenses and amounts associated with the management of stock, fluffy investments models employ Pentagonal fluffy parameters. Every one of the five possible values for each parameter exhibits a unique level of instability or changeability. For example, pentagons converse to the simplest availability aperture, holdings captured, assembled suffered a toll, the demand rate, and stockpiled taken a toll. There are five different settings for these parameters. While determining the

right numbers can be challenging, these figures provide for a range of potential outcomes, facilitating a more flexible and accurate depiction of the actual stock structure. These pentagonal fluffy requirements can much more effectively account for the characteristics of vulnerabilities in elements like supply, toll, and request limits.

For Fuzzy models, consider the pentagonal fuzzy parameters:

$$\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$$

$$\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$$

$$\kappa = (\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5)$$

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5)$$

$$\zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5)$$

Fuzzy stock fashions use ambiguity with inside the parameters to decide the proper order amount and overall cost. A variety of capacity values primarily based totally at the pentagonal fuzzy parameters—which constitute various ranges of uncertainty for the setup cost, call for rate, and keeping cost—are produced through adjusting the formulation for every fuzzy parameter with a view to get the proper order amount. These values are calculated for 5 exclusive levels, which constitute the variety of those parameters: minimum, midway, and most values.

Fuzzy optimal ψ :

$$\psi = \sqrt{\frac{2 \cdot \lambda \cdot \theta}{\kappa}}$$

$$\psi = \left(\sqrt{\frac{2 \cdot \lambda_1 \cdot \theta_1}{\kappa_1}}, \sqrt{\frac{2 \cdot \lambda_2 \cdot \theta_2}{\kappa_2}}, \dots, \sqrt{\frac{2 \cdot \lambda_5 \cdot \theta_5}{\kappa_5}} \right)$$

Fuzzy total cost:

$$\omega = \frac{3}{2} \cdot \sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa} + \xi \cdot \zeta$$

Break this into its fuzzy components:

$$\omega = \left(\frac{3}{2} \cdot \sqrt{2 \cdot \lambda_1 \cdot \theta_1 \cdot \kappa_1} + \xi_1 \cdot \zeta_1, \dots, \frac{3}{2} \cdot \sqrt{2 \cdot \lambda_5 \cdot \theta_5 \cdot \kappa_5} + \xi_5 \cdot \zeta_5 \right)$$

Case-Based analysis

$$\text{Case 1: } \omega_{\min} = \frac{3}{2} \cdot \sqrt{2 \cdot \lambda_1 \cdot \theta_1 \cdot \kappa_1} + \xi_1 \cdot \zeta_1$$

Case 2: Midpoint values

$$\omega_{\text{mid}} = \frac{3}{2} \cdot \sqrt{2 \cdot \lambda_3 \cdot \theta_3 \cdot \kappa_3} + \xi_3 \cdot \zeta_3$$

Case 3: maximum values

$$\omega_{\max} = \frac{3}{2} \cdot \sqrt{2 \cdot \lambda_5 \cdot \theta_5 \cdot \kappa_5} + \xi_5 \cdot \zeta_5$$

$$\omega = \left(\frac{3}{2} \cdot \sqrt{2 \cdot \lambda_1 \cdot \theta_1 \cdot \kappa_1} + \xi_1 \cdot \zeta_1, \dots, \frac{3}{2} \cdot \sqrt{2 \cdot \lambda_5 \cdot \theta_5 \cdot \kappa_5} + \xi_5 \cdot \zeta_5 \right)$$

The total cost function also takes into account the fuzzy parameters. For each of the three scenarios, the total cost (minimum, average, and maximum) is determined using the appropriate setup cost, demand rate, inventory cost, shortage cost, and minimum shortage. In the case-based approach, three different scenarios are considered: minimum, medium, and maximum. The range of possible outcomes is illustrated by the fact that each of these scenarios produces a different total cost depending on the uncertainty in the fuzzy parameters.

Crisp model:

$$\omega = \frac{3}{2} \cdot \sqrt{2 \cdot \lambda \cdot \theta \cdot \kappa} + \xi \cdot \zeta$$

Fuzzy model:

$$\omega = \left(\frac{3}{2} \cdot \sqrt{2 \cdot \lambda_1 \cdot \theta_1 \cdot \kappa_1} + \xi_1 \cdot \zeta_1, \dots, \frac{3}{2} \cdot \sqrt{2 \cdot \lambda_5 \cdot \theta_5 \cdot \kappa_5} + \xi_5 \cdot \zeta_5 \right)$$

Ultimately, everything that happened are summed up in two approaches: an inventive demonstration that uses the right measures for the parameters, and a fluffy display that includes a range of potential ranges for each fluffy parameter. With a more thoughtful and adaptable approach to stock management, the fluffy clearly demonstrate takes into account the flaws seen in real arrangements.

6 Determining the Ideal Order Quantity and Total Cost in Inventory Management

Streaming rates, structure costs, lack, and unbalanced specifications are used to determine the ideal arrangement amount. With a 500 unit request rate every year, the initial configuration captured is 30 units depending on the course of action. Holding expenses are two units annually, and each deficit results in a fetched of ten units, for a total of fifty units. Reducing establishment, possessing, and request the incidence yields the optimal arrange amount. The resource of the demand and expenses associated with setup are divided by expenses associated with holding and making adjustments for shortcomings to determine the outcome.

- $\lambda=500$ (demand rate),
- $\theta=30$ (setup cost per order),
- $\kappa=2$ (holding cost per unit per year),
- $\xi=10$ (shortage cost per unit),
- $\zeta=50$ (number of shortages).

$$\psi = \frac{2 \cdot \lambda \cdot \theta}{\kappa}$$

Substitute the values

$$\psi = \sqrt{2 \cdot 500 \cdot 30} = \sqrt{15000} = 122.47.$$

Substitute $\psi = 122.47$ into ω :

$$\omega = \frac{500 \cdot 30}{122.47} + \frac{2 \cdot 122.47}{2} + 10 \cdot 50.$$

Simplify each term:

First term

$$\frac{500 \cdot 30}{122.47} = 122.45,$$

Second term

$$\frac{2 \cdot 122.47}{2} = 122.47$$

Making use of the specified request rate, setup taken toll, and keeping taken toll values, the ideal arrangement amount is determined to be approximately 122.47 units to play down add up to taken toll. The total amount taken is then determined by adding the requesting, holding, and deficit fetched. The requesting taken toll is approximately 122.45, which is determined by subtracting the setup taken toll item from the request rate by the correct arrange amount. Duplicating the holding taken toll per unit and dividing the result by half of the ideal arrange amount yields the holding taken toll of about 122.47. Lastly, the shortfall taken a toll, which is calculated by multiplying the number of units of the deficit taken a toll.

Third term

$$10 \cdot 50 = 500$$

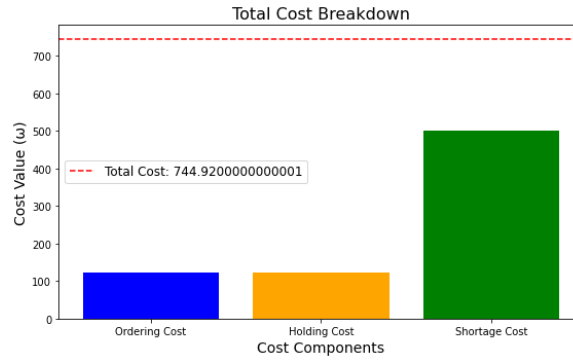
Thus,

$$\omega = 122.45 + 122.47 + 500 = 744.92.$$

Crisp total cost

$$\omega = 744.92$$

When these elements are added together, the precise total cost comes to about 744.92 units of cash. This price is the lowest overall cost that can be obtained by maximising the order quantity under the specified circumstances. The realistic appears how the requesting taken a toll, holding fetched, and shortage fetched are the three primary components of the in general fetched ω . The fetched of setting orders to meet request is spoken to by the blue bar, which is the requesting fetched of 122.45. The holding fetched of 122.47, which speaks to the taken a toll of keeping inventories, is shown within the orange bar. When there's not sufficient stock to meet request, the deficiency taken a toll of 500 is appeared by the green bar. Ultimately, a ruddy dashed line speaking to the whole taken a toll of 744.92 outlines how the total of these components decides the ultimate taken a toll. The noteworthiness of effectively controlling each cost to diminish the by and large taken a toll of stock administration is underscored by this realistic.



7 Fuzzy Optimal Order Quantity and Total Cost Analysis in Inventory Management

The fuzzy demand rate, setup cost, and holding cost are taken into consideration while utilising pentagonal fuzzy numbers to determine the fuzzy optimal order quantity. There are five values for each of these parameters, each of which represents a different degree of uncertainty. The figures for the setup cost, holding cost, and demand rate range from 28 to 32, 480 to 520, and 1.8 to 2.2, respectively. The lowest, middle, and upper limits of the corresponding parameters are represented by these values.

$$\lambda = (480, 490, 500, 510, 520),$$

$$\theta = (28, 29, 30, 31, 32),$$

$$\kappa = (1.8, 1.9, 2.0, 2.1, 2.2),$$

$$\xi = (9, 9.5, 10, 10.5, 11),$$

$$\zeta = (45, 47, 50, 53, 55).$$

Fuzzy the optimal ψ is:

$$\psi = \frac{2 \cdot \lambda \cdot \theta}{\kappa}$$

These pentagonal fuzzy numbers are included within the fuzzy equation for calculating the perfect arrange figure. To decide the fitting arrange amount, the equation is utilized for each combination of the request rate, setup fetched, and holding fetched. Each of the five fuzzy arrange amounts that are delivered as a result speaks to a level of the fuzzy parameters.

In arrange to account for errors and changeability within the parameters, the fuzzy show computes the arrange amount for each level. As a result, the stock demonstrate is stronger and more versatile to real-world instabilities, and the resulting order numbers offer a assortment of conceivable outcomes that will be custom-made to different scenarios.

For each fuzzy component:

$$\psi = \sqrt{\frac{2 \cdot 480 \cdot 28}{1.8}}, \psi = \sqrt{\frac{2 \cdot 490 \cdot 29}{1.9}}, \psi = \sqrt{\frac{2 \cdot 500 \cdot 30}{2.0}}, \psi = \sqrt{\frac{2 \cdot 510 \cdot 31}{2.1}}, \psi = \sqrt{\frac{2 \cdot 520 \cdot 32}{2.2}}$$

Calculate each

$$\psi_1 = \sqrt{\frac{2 \cdot 480 \cdot 28}{1.8}} = \sqrt{14933.33} = \sqrt{122.21},$$

$$\psi_2 = \sqrt{\frac{2 \cdot 490 \cdot 29}{1.9}} = \sqrt{14942.11} = 122.22,$$

$$\psi_3 = \sqrt{\frac{2 \cdot 500 \cdot 30}{2.0}} = \sqrt{15000} = 122.47,$$

$$\psi_4 = \sqrt{\frac{2 \cdot 510 \cdot 31}{2.1}} = \sqrt{15000.95} = 122.50,$$

$$\psi_5 = \sqrt{\frac{2 \cdot 520 \cdot 32}{2.2}} = \sqrt{15127.27} = 122.96$$

Thus

$$\psi = (122.21, 122.22, 122.47, 122.50, 122.96)$$

In order to determine the fuzzy total cost, we first use the pentagonal fuzzy numbers for demand rate, setup cost, and holding cost to evaluate each part of the fuzzy order quantity. Five distinct values for the optimal order amount are obtained by applying the formula; each value corresponds to a different degree of parameter uncertainty. The demand rate, setup cost, and holding cost combinations are represented by these figures, which vary from 122.21 to 122.96.

The fuzzy total cost is

$$\omega = \frac{\lambda \cdot \theta}{\psi} + \frac{\kappa \cdot \psi}{2} + \xi \cdot \zeta$$

For each fuzzy component

$$\begin{aligned} \omega_1 &= \frac{480 \cdot 28}{122.21} + \frac{1.8 \cdot 122.21}{2} + 9 \cdot 45 = 109.96 + 109.99 + 405 = 624.95, \\ \omega_2 &= \frac{490 \cdot 29}{122.22} + \frac{1.9 \cdot 122.22}{2} + 9 \cdot 5 \cdot 47 = 116.25 + 116.11 + 446.5 = 678.86, \\ \omega_3 &= \frac{500 \cdot 30}{122.47} + \frac{2.0 \cdot 122.47}{2} + 10 \cdot 50 = 122.45 + 122.47 + 500 = 744.92, \\ \omega_4 &= \frac{510 \cdot 31}{122.50} + \frac{2.1 \cdot 122.50}{2} + 10 \cdot 5 \cdot 53 = 128.99 + 128.63 + 556.5 = 814.12, \\ \omega_5 &= \frac{520 \cdot 32}{122.96} + \frac{2.2 \cdot 122.96}{2} + 11 \cdot 55 = 135.48 + 135.26 + 605 = 875.74. \end{aligned}$$

Thus

$$\omega = (624.95, 678.86, 744.92, 814.12, 875.74).$$

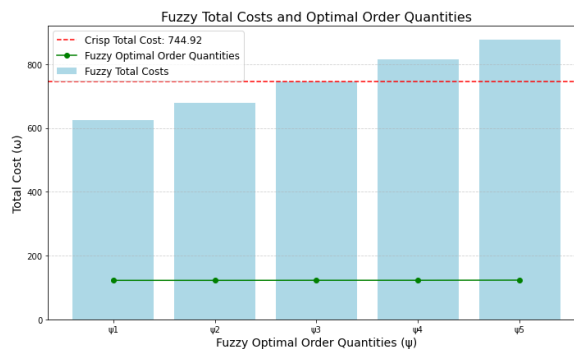
The request rate, holding fetched, and shortage taken a toll are at that point included to these figures to decide the fuzzy add up to fetched. We utilize the comparing fuzzy values to calculate the generally taken a toll for each fuzzy component. Five unmistakable add up to costs are hence created, each of which speaks to the fluctuating costs for each level of the fuzzy arrange amount. These fuzzy add up to costs, which change from 624.95 to 875.74, speak to the run of potential add up to costs based on the parameter values. Crisp total cost

$$\omega = 744.92,$$

Fuzzy total cost

$$\omega = (624.95, 678.86, 744.92, 814.12, 875.74).$$

744.92 is the new sum that is obtained when the fuzzy demonstration is used to create the ideal arrangement amount. The most likely sum to be retrieved is displayed in this figure by averaging the fuzzy values. In the event that there are instabilities within the parameters, the fuzzy add-up to fetched provides a more comprehensive range of possible prices, enabling flexibility in decision-making. In this sense, the fuzzy method provides a more thorough analysis of the generally accepted cost of stock administration while also taking capriciousness into consideration. In conjunction with the fresh add up to taken a toll, the perception contrasts the fuzzy add



up to costs and the related fuzzy ideal arrange amounts. The cloudy add up to costs are appeared by the light blue bars, and they shift from 624.95 to 875.74. Each bar speaks to a particular fuzzy ideal arrange amount, and the setup, holding, and request rate instability are reflected within the fetched values. As a reference point within the setting of the fuzzy demonstrate, the ruddy dashed line appears the exact, non-variable add up to

fetches of 744.92, which is decided beneath indicated suspicions. The fuzzy ideal arrange amounts are too shown by the green line, and they extend from 122.21 to 122.96. The run of potential ideal orders depending on changing degrees of instability is spoken to by these numbers, which are computed for each combination of fuzzy parameters. By taking parameter inconstancy under consideration, the fuzzy approach offers a more exhaustive picture of in general costs and grants decision-making adaptability, as the chart outlines. A settled point for comparison is given by the fresh add up to taken a toll, but the fuzzy show speaks to the energetic character of real stock administration circumstances.

8 Conclusion

Stock administration that consolidates neutrosophic set integration and pentagonal fuzzy numbers offers a solid system for managing with supply chain eccentrics and instability. Indeed, with uncertain or deficient information, this approach permits for more adaptable decision-making and upgraded stock control by including fuzzy parameters for request, costs, and other imperative contemplations. Way better choices are made when defuzzification strategies just like the Graded Mean Integration Representation (GMIR) are utilized to extend the exactness of taken a toll gauges and stock figures. beneath common sense applications, this approach can help companies with taken a toll reduction, resource allotment, stock-out lessening, and common operational productivity, which can eventually boost benefit and competitiveness beneath eccentric advertise circumstances.

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