Neutrosophic Cordial Labeling on Helm and Closed Helm Graph

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Abstract

The Neutrosophic Cordial Labeling Graph integrates both neutrosophic labeling and Cordial Labeling. Building on our previous work, we have extended our study to include Neutrosophic Cordial Labeling for Helm and Closed Helm Graphs. This extension allows us to explore the application of Neutrosophic Cordial Labeling in more complex graph structures, providing insights into their properties and relationships. One of the key aspects of our research is investigating the relationship between Cordial and Neutrosophic Cordial Labeling. By comparing and contrasting these labeling techniques [4], we aim to uncover similarities, differences, and potential synergies between them. This analysis contributes to a deeper understanding of graph labeling methodologies and their implications in various graph-theoretic applications [18]. Our research contributes to the advancement of graph labeling theory, particularly in the context of Neutrosophic Cordial Labeling and its applications in Helm and Closed Helm Graphs. By exploring these concepts and relationships, we aim to enhance the theoretical foundation and practical utility of graph labeling techniques in diverse domains [16,17].

Keywords*:* Neutrosophic; Labeling; Cordial Labeling; Graphs; Helm; Closed Helm

1. Introduction

Graph labeling [10] has become a crucial tool in graph theory, with applications across various domains [14]. The essence of graph labeling lies in assigning labels or values to the vertices, edges, or both, based on certain predefined rules or criteria [8]. This process enables researchers and practitioners to encode meaningful information into graphs, leading to a deeper comprehension of their properties and behaviors. One of the key motivations behind graph labeling is to facilitate the representation and analysis of complex networks in various domains, including computer science, mathematics, biology, and social sciences. By labeling vertices or edges with relevant attributes, such as colors, numbers, or symbols, researchers can study connectivity patterns, identify specific graph features, and solve graph-related problems efficiently [12,13].

DOI: https://doi.org/10.54216/IJNS.250439

Among the many labeling techniques [1,6,7,11], the Neutrosophic Cordial Labeling Graph [9] stands out for its fusion of neutrosophic [3,15] and Cordial Labeling concepts. Our previous research introduced this unique graph labeling approach, setting the stage for further exploration into its properties and practical uses.

This paper extends our investigation into Neutrosophic Cordial Labeling by examining its application to Helm and Closed Helm Graphs. We not only establish Neutrosophic Cordial Labeling for these graph structures but also delve into the intricate relationship between Cordial Labeling [5] and Neutrosophic Cordial Labeling [2]. This exploration contributes significantly to the ongoing development of graph theory, shedding light on how different labeling methodologies interact and influence our understanding of graphs [19,20].

The paper is structured into two main sections. The first section provides a thorough exploration of foundational concepts, laying the groundwork for our subsequent analysis. The second section presents our research findings, focusing specifically on Helm and Closed Helm Graphs and their compatibility with Neutrosophic Cordial Labeling. Additionally, we introduce a lemma that elucidates the connection between Cordial Labeling Graphs and Neutrosophic Cordial Labeling Graphs.

2. Preliminaries

2.1 Graph

A Graph G = (V(G), E(G)) consists of two sets V(G) = $\{v_1, v_2, ...\}$ called the vertex set of G and E(G) = $\{e_1, e_2, ...\}$ called the edge set of G. Sometimes we denote the vertex set of G as V and edge set as E. Elements of V(G) and E(G) are called vertices and edges respectively.

2.2 Simple Graph

A simple graph is a graph with no loop edges or multiple edges. Edges in a simple graph may be specified by a set {vi , vj} of the two vertices that the edge makes adjacent.

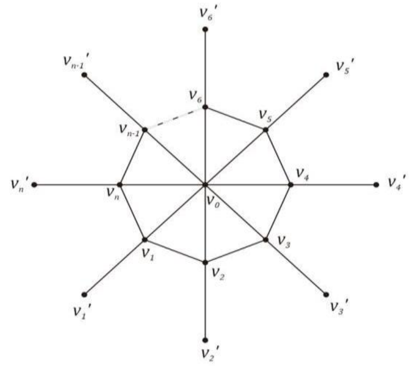
2.3 Multi Graph and Pseudo Graph

A graph with more than one edge between a pair of vertices is called a multigraph while a graph with loop edges is called a pseudograph.

2.4 Helm Graph

The helm graph H_n is the graph obtained from an n - wheel graph by adjoining a pendant edge at each node of the cycle.

Example:



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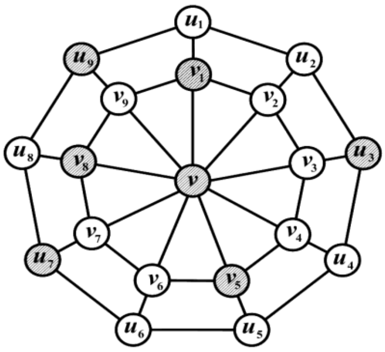
2.5 Closed Helm Graph

A closed helm CH_n is the graph obtained by taking a helm H_n and adding edges between the pendant vertices.

Example: CH_9



2.6 Graph Labeling

A Graph Labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

2.7 Cordial Labeling Graph

The cordial labeling of a graph G is an injection f : V(G) → {0, 1} such that each edge uv in G is assigned the label |f(u)-f(v)| with the property that, |vf (0) - vf (1)| ≤ 1 and |ef (0) - ef (1)| ≤ 1 where vf (i) for i = 0 and 1 denotes the number of vertices with label i and ef (i) for i = 0 and 1 denotes the number of edges with label i. A graph is called cordial if it admits cordial labeling.

2.8 Neutrosophic Graph [2]

A Neutrosphic graph is of the form $G = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1 + v_2 + v_3 + \cdots . +v_n\}$ such that $T_1: V \rightarrow [0,1], I_1: V \rightarrow [0,1] and\ F_1: V \rightarrow [0,1]$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the vertex $v_i \in V$ respectively and

$0 \leq T_1 + I_1 + F_1 \leq 3 \ \forall v_i \in V (i = 1,2,3, \dots n)$

(ii) $T_2: V \rightarrow [0,1], I_2: V \rightarrow [0,1] and\ F_2: V \rightarrow [0,1]$ where $T_2(v_i, v_j), I_2(v_i, v_j), F_2(v_i, v_j)$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the edge (v_i, v_j) respectively such that for every (v_i, v_j)

$T_2(v_i, v_j) \leq \min\{T_1(v_i), T_1(v_j)\}$

$I_2(v_i, v_j) \leq \min\{I_1(v_i), I_1(v_j)\}$

$F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\}$ and

$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$

2.9 Neutrosophic Labeling Graph

A neutrosophic graph $G = (V, \sigma, \mu)$ is said to be an neutrosophic labeling graph if $T_1: V \rightarrow [0,1]$, $I_1: V \rightarrow [0,1] and\ F_1: V \rightarrow [0,1]$ and $T_2: V \times V \rightarrow [0,1], I_2: V \times V \rightarrow [0,1] and\ F_2: V \times V \rightarrow [0,1]$ is bijective such that truth – membership function, indeterminacy – membership function and falsity – membership function of the vertices and edges are distinct and for every edges (v_i, v_j)

DOI: https://doi.org/10.54216/IJNS.250439

$T_2(v_i, v_j) \leq \min\{T_1(v_i), T_1(v_j)\}$
$I_2(v_i, v_j) \leq \max\{I_1(v_i), I_1(v_j)\}$
$F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\}$ and
$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$

In this paper, we introduce Neutrosophic Cordial Labeling graph and prove Cycle C_n , Wheel W_n and Helm H_n admits Neutrosophic Cordial Labeling.

2.10 Neutrosophic Cordial Labeling Graph

A neutrosophic graph $G = (V, \sigma, \mu)$ is said to be an neutrosophic cordial labeling graph if $T_1: V \to \{0,1\}$, $I_1: V \to \{0,1\}$ *and* $F_1: V \to \{0,1\}$ and $T_2: V \times V \to \{0,1\}, I_2: V \times V \to \{0,1\}$ *and* $F_2: V \times V \to \{0,1\}$ is bijective such that $|T_1(0) - T_1(1)| \leq 1, |I_1(0) - I_1(1)| \leq 1$ *and* $|F_1(0) - F_1(1)| \leq 1$ for vertices and $|T_2(0) - T_2(1)| \leq 1, |I_2(0) - I_2(1)| \leq 1$ *and* $|F_2(0) - F_2(1)| \leq 1$ for edges

$$T_2(v_i, v_j) = \begin{Bmatrix} \max \{T_1(v_i), T_1(v_j)\} \ if \ v_i > v_j \ or \ v_j > v_i \\ 0 \ if \ v_i = v_j \end{Bmatrix}$$

$$I_2(v_i, v_j) = \begin{Bmatrix} \max \{I_1(v_i), I_1(v_j)\} \ if \ v_i > v_j \ or \ v_j > v_i \\ 0 \ if \ v_i = v_j \end{Bmatrix}$$

$$F_2(v_i, v_j) = \begin{Bmatrix} \max \{F_1(v_i), F_1(v_j)\} \ if \ v_i > v_j \ or \ v_j > v_i \\ 0 \ if \ v_i = v_j \end{Bmatrix}$$

and

$$0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$$

Theorem 1

Helm Graph H_n admits Neutrsophic Cordial Labeling

Proof:

Helm graph is obtained from a wheel W_n by attaching a pendant edge to each vertex of the wheel's rim. It contains three types of vertex. An Apex of degree n, n vertices of degree 4 and n pendant vertices. The apex vertex is denoted as v, n vertices of degree 4 is denoted as $v_i: 1 \leq i \leq n$ and pendant vertex is denoted as u_n. Edge adjacent to the apex vertex is denoted as $e_i: 1 \leq i \leq n$, the edges of the wheels rim is denoted as $g_i: 1 \leq i \leq n$ and the edges of the pendant vertices incident on the rim of the wheel is denoted as $h_i: 1 \leq i \leq n$
Let G = (V, σ, μ) be a neutrosophic graph where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$, $V = [v, \{v_i: i \geq 1\}, \{u_i: i \geq 1\}]$ such that $T_1: V \to \{0,1\}, I_1: V \to \{0, 1\}$ *and* $F_1: V \to \{0, 1\}$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the vertex $v, v_i, u_i \in V$ and $T_2: V \to \{0,1\}, I_2: V \to \{0,1\}$*and* $F_2: V \to \{0,1\}$ where $T_2(e_i), I_2(e_i), F_2(e_i)$, $T_2(g_i), I_2(g_i), F_2(g_i)$, $T_2(h_i), I_2(h_i), F_2(h_i)$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the edge.

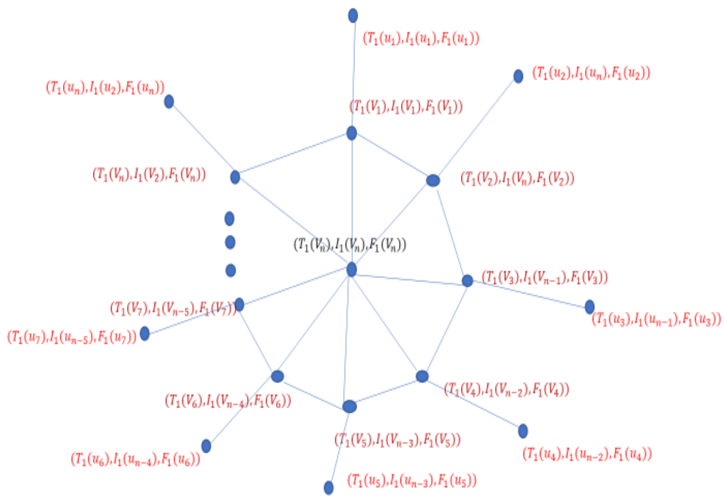
Vertices are denoted as follows

$T_1(v_i)$ denote the truth – membership function and are labeled in the clockwise direction, $I_1(v_i)$ denote an indeterminacy – membership function and are labeled in the anticlockwise direction and $F_1(v_i)$ denote the

DOI: https://doi.org/10.54216/IJNS.250439

falsity – membership function and are labeled in the clockwise direction. Similarly, $T_1(u_i)$ denote the truth – membership function and are labeled in the clockwise direction, $I_1(u_i)$ denote an indeterminacy – membership function and are labeled in the anticlockwise direction and $F_1(u_i)$ denote the falsity – membership function and are labeled in the clockwise direction and

Truth – membership function, indeterminacy – membership function and falsity – membership function of the vertices and edges are distinct.



Then,

Total number of vertices is 2n+1 and for edges 3n.

Case 1:

For n = 2m : m ≥ 1

Define Vertex labeling as follows

$$T_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$I_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$F_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \end{Bmatrix}$$

$$T_1(v) = \begin{Bmatrix} 1 \ \ if \ \ m \ \ is \ \ even \\ 0 \ \ if \ \ m \ \ is \ \ odd \end{Bmatrix}$$

$$I_1(v) = \begin{Bmatrix} 1 \ \ if \ \ m \ \ is \ \ even \\ 0 \ \ if \ \ m \ \ is \ \ odd \end{Bmatrix}$$

$$F_1(v) = \begin{Bmatrix} 1 \ \ if \ \ m \ \ is \ \ odd \\ 0 \ \ if \ \ m \ \ is \ \ even \end{Bmatrix}$$

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$$T_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i \ \ is \ \ even \\ 0 \ \ if \ \ i \ \ is \ \ odd \end{Bmatrix}$$

$$I_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i \ \ is \ \ even \\ 0 \ \ if \ \ i \ \ is \ \ odd \end{Bmatrix}$$

$$F_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i \ \ is \ \ odd \\ 0 \ \ if \ \ i \ \ is \ \ even \end{Bmatrix}$$

Then the edges are labeled as follows

$$T_2(e_i) = \begin{Bmatrix} \max \{T_1(v), T_1(v_i)\} \ \ if \ \ v > v_i \ \ or \ \ v_i > v \\ 0 \ \ if \ \ v_i = v \end{Bmatrix}$$

$$I_2(e_i) = \begin{Bmatrix} \max \{I_1(v), I_1(v_j)\} \ \ if \ \ v > v_i \ \ or \ \ v_i > v \\ 0 \ \ if \ \ v_i = v \end{Bmatrix}$$

$$F_2(e_i) = \begin{Bmatrix} \max \{F_1(v), F_1(v_j)\} \ \ if \ \ v > v_i \ \ or \ \ v_i > v \\ 0 \ \ if \ \ v_i = v \end{Bmatrix}$$

$$T_2(g_i) = \begin{Bmatrix} \max \{T_1(v_i), T_1(v_j)\} \ \ if \ \ v_i > v_j \ \ or \ \ v_j > v_i \\ 0 \ \ if \ \ v_i = v_j \end{Bmatrix}$$

$$I_2(g_i) = \begin{Bmatrix} \max \{I_1(v_i), I_1(v_j)\} \ \ if \ \ v_i > v_j \ \ or \ \ v_j > v_i \\ 0 \ \ if \ \ v_i = v_j \end{Bmatrix}$$

$$F_2(g_i) = \begin{Bmatrix} \max \{F_1(v_i), F_1(v_j)\} \ \ if \ \ v_i > v_j \ \ or \ \ v_j > v_i \\ 0 \ \ if \ \ v_i = v_j \end{Bmatrix}$$

$$T_2(h_i) = \begin{Bmatrix} \max \{T_1(u_i), T_1(v_j)\} \ \ if \ \ u_i > v_j \ \ or \ \ v_j > u_i \\ 0 \ \ if \ \ u_i = v_j \end{Bmatrix}$$

$$I_2(h_i) = \begin{Bmatrix} \max \{I_1(u_i), I_1(v_j)\} \ \ if \ \ u_i > v_j \ \ or \ \ v_j > u_i \\ 0 \ \ if \ \ u_i = v_j \end{Bmatrix}$$

$$F_2(h_i) = \begin{Bmatrix} \max \{F_1(u_i), F_1(v_j)\} \ \ if \ \ u_i > v_j \ \ or \ \ v_j > u_i \\ 0 \ \ if \ \ u_i = v_j \end{Bmatrix}$$

Then for truth membership function

$$T_1 \ \ [1] = \ \ n + 1$$

In other words, for $T_1(v_i)$ the number of vertices labeled one are *n+1*

$$T_1 \ \ [0] = \ \ n$$

In other words, for $T_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |T_1(0) - T_1(1)| \le 1$$

In other words, for $T_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

DOI: https://doi.org/10.54216/IJNS.250439

Then for indeterminacy membership function

$$I_1\ [1] = \ n+1$$

In other words, for $I_1(v_i)$ the number of vertices labeled one are $n+1$

$$I_1\ [0] = \ n$$

In other words, for $I_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |I_1(0) - I_1(1)| \le 1$$

In other words, for $I_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for falsity membership function

$$F_1\ [1] = \ n$$

In other words, for $F_1(v_i)$ the number of vertices labeled one are n

$$F_1\ [0] = \ n+1$$

In other words, for $F_1(v_i)$ the number of vertices labeled Zero are n+1

$$\Rightarrow |F_1(0) - F_1(1)| \le 1$$

In other words, for $F_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

For edges (v_i, v_j)

$$T_2\ [1] = \frac{3n}{2}$$

In other words, for $T_2\ (v_i, v_j)$ the number of edges labeled one are $\frac{3n}{2}$

$$T_2\ [0] = \frac{3n}{2}$$

In other words, for $T_2\ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n}{2}$

$$\Rightarrow |T_2(0) - T_2(1)| \le 1$$

Similarly,

$$I_2\ [1] = \frac{3n}{2}$$

In other words, for $I_2\ (v_i, v_j)$ the number of edges labeled one are $\frac{3n}{2}$

$$I_2\ [0] = \frac{3n}{2}$$

In other words, for $I_2\ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n}{2}$

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$$\Rightarrow |I_2(0) - I_2(1)| \leq 1$$

And

$$F_2 \ [1] = \frac{3n}{2}$$

In other words, for $F_2 \ (v_i, v_j)$ the number of edges labeled one are $\frac{3n}{2}$

$$F_2 \ [0] = \frac{3n}{2}$$

In other words, for $F_2 \ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n}{2}$

$$\Rightarrow |F_2(0) - F_2(1)| \leq 1$$

Therefore, W_n: n = 2m : m ≥ 1 satisfies Neutrosophic Cordial Labeling.

Case 2:

For n = 4m + 1 : m ≥ 1

Define Vertex labeling as follows

$$T_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$I_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$F_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \end{Bmatrix}$$

$$T_1(v) = \{0\}$$

$$I_1(v) = \{0\}$$

$$F_1(v) = \{1\}$$

$$T_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i \ \ is \ \ even \\ 0 \ \ if \ \ i \ \ is \ \ odd \end{Bmatrix}$$

$$I_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i \ \ is \ \ even \\ 0 \ \ if \ \ i \ \ is \ \ odd \end{Bmatrix}$$

$$F_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i \ \ is \ \ odd \\ 0 \ \ if \ \ i \ \ is \ \ even \end{Bmatrix}$$

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Then the edges are labeled as follows

$$T_2(e_i) = \begin{Bmatrix} \max\ \{T_1(v), T_1(v_i)\}\ \ if\ \ v > v_i\ \ or\ \ v_i > v \\ 0\ \ if\ \ v_i = v \end{Bmatrix}$$

$$I_2(e_i) = \begin{Bmatrix} \max\ \{I_1(v), I_1(v_j)\}\ \ if\ \ v > v_i\ \ or\ \ v_i > v \\ 0\ \ if\ \ v_i = v \end{Bmatrix}$$

$$F_2(e_i) = \begin{Bmatrix} \max\ \{F_1(v), F_1(v_j)\}\ \ if\ \ v > v_i\ \ or\ \ v_i > v \\ 0\ \ if\ \ v_i = v \end{Bmatrix}$$

$$T_2(g_i) = \begin{Bmatrix} \max\ \{T_1(v_i), T_1(v_j)\}\ \ if\ \ v_i > v_j\ \ or\ \ v_j > v_i \\ 0\ \ if\ \ v_i = v_j \end{Bmatrix}$$

$$I_2(g_i) = \begin{Bmatrix} \max\ \{I_1(v_i), I_1(v_j)\}\ \ if\ \ v_i > v_j\ \ or\ \ v_j > v_i \\ 0\ \ if\ \ v_i = v_j \end{Bmatrix}$$

$$F_2(g_i) = \begin{Bmatrix} \max\ \{F_1(v_i), F_1(v_j)\}\ \ if\ \ v_i > v_j\ \ or\ \ v_j > v_i \\ 0\ \ if\ \ v_i = v_j \end{Bmatrix}$$

$$T_2(h_i) = \begin{Bmatrix} \max\ \{T_1(u_i), T_1(v_j)\}\ \ if\ \ u_i > v_j\ \ or\ \ v_j > u_i \\ 0\ \ if\ \ u_i = v_j \end{Bmatrix}$$

$$I_2(h_i) = \begin{Bmatrix} \max\ \{I_1(u_i), I_1(v_j)\}\ \ if\ \ u_i > v_j\ \ or\ \ v_j > u_i \\ 0\ \ if\ \ u_i = v_j \end{Bmatrix}$$

$$F_2(h_i) = \begin{Bmatrix} \max\ \{F_1(u_i), F_1(v_j)\}\ \ if\ \ u_i > v_j\ \ or\ \ v_j > u_i \\ 0\ \ if\ \ u_i = v_j \end{Bmatrix}$$

Then for truth membership function

$$T_1\ \ [1] =\ \ n$$

In other words, for $T_1(v_i)$ the number of vertices labeled one are n

$$T_1\ \ [0] =\ \ n + 1$$

In other words, for $T_1(v_i)$ the number of vertices labeled Zero are n+1

$$\Rightarrow |T_1(0) - T_1(1)| \leq 1$$

In other words, for $T_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for indeterminancy membership function

$$I_1\ \ [1] =\ \ n$$

In other words, for $I_1(v_i)$ the number of vertices labeled one are n

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$$I_1 \ [0] = \ n+1$$

In other words, for $I_1(v_i)$ the number of vertices labeled Zero are n +1

$$\Rightarrow |I_1(0) - I_1(1)| \leq 1$$

In other words, for $I_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for falsity membership function

$$F_1 \ [1] = \ n+1$$

In other words, for $F_1(v_i)$ the number of vertices labeled one are n + 1

$$F_1 \ [0] = \ n$$

In other words, for $F_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |F_1(0) - F_1(1)| \leq 1$$

In other words, for $F_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

For edges (v_i, v_j)

$$T_2 \ [1] = \frac{3n+1}{2}$$

In other words, for $T_2 \ (v_i, v_j)$ the number of edges labeled one are $\frac{3n+1}{2}$

$$T_2 \ [0] = \frac{3n-1}{2}$$

In other words, for $T_2 \ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n-1}{2}$

$$\Rightarrow |T_2(0) - T_2(1)| \leq 1$$

Similarly,

$$I_2 \ [1] = \frac{3n+1}{2}$$

In other words, for $I_2 \ (v_i, v_j)$ the number of edges labeled one are $\frac{3n+1}{2}$

$$I_2 \ [0] = \frac{3n-1}{2}$$

In other words, for $I_2 \ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n-1}{2}$

$$\Rightarrow |I_2(0) - I_2(1)| \leq 1$$

And

$$F_2 \ [1] = \frac{3n-1}{2}$$

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In other words, for $F_2 \ (v_i, v_j)$ the number of edges labeled one are $\frac{3n-1}{2}$

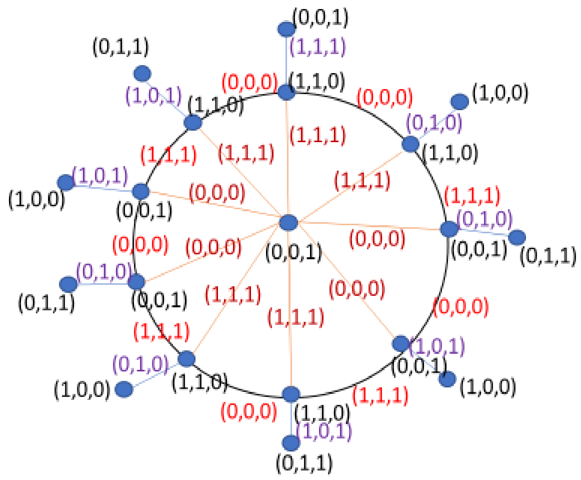
$$F_2 \ [0] = \frac{3n+1}{2}$$

In other words, for $F_2 \ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n+1}{2}$

$$\Rightarrow |F_2(0) - F_2(1)| \leq 1$$

Therefore, W_n: n = 4m + 1 : m ≥ 1 satisfies Neutrosophic Cordial Labeling.

- H_9



Case 3:

For n = 4m - 1 : m ≥ 1

Define Vertex labeling as follows

$$T_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$I_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$F_1(v_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ even \\ 0 \ \ if \ \ i = 2n-1 \ \ and \ \ 2n \ \ iff \ \ n \ \ is \ \ odd \end{Bmatrix}$$

$$T_1(v) = \ \{1\}$$
$$I_1(v) = \ \{1\}$$
$$F_1(v) = \ \{0\}$$

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$$T_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 1 \\ 0 \ \ if \ \ i = 2n \ \ and \ \ 2n+1 \ \ iff \ \ n \ \ is \ \ odd \\ 1 \ \ if \ \ i = 2n \ \ and \ \ 2n+1 \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$I_1(u_i) = \begin{Bmatrix} 1 \ \ if \ \ i = 1 \\ 0 \ \ if \ \ i = 2n \ \ and \ \ 2n+1 \ \ iff \ \ n \ \ is \ \ odd \\ 1 \ \ if \ \ i = 2n \ \ and \ \ 2n+1 \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

$$F_1(u_i) = \begin{Bmatrix} 0 \ \ if \ \ i = 1 \\ 1 \ \ if \ \ i = 2n \ \ and \ \ 2n+1 \ \ iff \ \ n \ \ is \ \ odd \\ 0 \ \ if \ \ i = 2n \ \ and \ \ 2n+1 \ \ iff \ \ n \ \ is \ \ even \end{Bmatrix}$$

Then the edges are labeled as follows

$$T_2(e_i) = \begin{Bmatrix} \max \ \{T_1(v), T_1(v_i)\} \ \ if \ \ v > v_i \ \ or \ \ v_i > v \\ 0 \ \ if \ \ v_i = v \end{Bmatrix}$$

$$I_2(e_i) = \begin{Bmatrix} \max \ \{I_1(v), I_1(v_j)\} \ \ if \ \ v > v_i \ \ or \ \ v_i > v \\ 0 \ \ if \ \ v_i = v \end{Bmatrix}$$

$$F_2(e_i) = \begin{Bmatrix} \max \ \{F_1(v), F_1(v_j)\} \ \ if \ \ v > v_i \ \ or \ \ v_i > v \\ 0 \ \ if \ \ v_i = v \end{Bmatrix}$$

$$T_2(g_i) = \begin{Bmatrix} \max \ \{T_1(v_i), T_1(v_j)\} \ \ if \ \ v_i > v_j \ \ or \ \ v_j > v_i \\ 0 \ \ if \ \ v_i = v_j \end{Bmatrix}$$

$$I_2(g_i) = \begin{Bmatrix} \max \ \{I_1(v_i), I_1(v_j)\} \ \ if \ \ v_i > v_j \ \ or \ \ v_j > v_i \\ 0 \ \ if \ \ v_i = v_j \end{Bmatrix}$$

$$F_2(g_i) = \begin{Bmatrix} \max \ \{F_1(v_i), F_1(v_j)\} \ \ if \ \ v_i > v_j \ \ or \ \ v_j > v_i \\ 0 \ \ if \ \ v_i = v_j \end{Bmatrix}$$

$$T_2(h_i) = \begin{Bmatrix} \max \ \{T_1(u_i), T_1(v_j)\} \ \ if \ \ u_i > v_j \ \ or \ \ v_j > u_i \\ 0 \ \ if \ \ u_i = v_j \end{Bmatrix}$$

$$I_2(h_i) = \begin{Bmatrix} \max \ \{I_1(u_i), I_1(v_j)\} \ \ if \ \ u_i > v_j \ \ or \ \ v_j > u_i \\ 0 \ \ if \ \ u_i = v_j \end{Bmatrix}$$

$$F_2(h_i) = \begin{Bmatrix} \max \ \{F_1(u_i), F_1(v_j)\} \ \ if \ \ u_i > v_j \ \ or \ \ u_j > v_i \\ 0 \ \ if \ \ u_i = v_j \end{Bmatrix}$$

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Then for truth membership function

$T_1 \; [1] = \; n + 1$

In other words, for $T_1(v_i)$ the number of vertices labeled one are n *+1*

$$T_1 \; [0] = \; n$$

In other words, for $T_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |T_1(0) - T_1(1)| \leq 1$$

In other words, for $T_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for indeterminacy membership function

$I_1 \; [1] = \; n$ +1

In other words, for $I_1(v_i)$ the number of vertices labeled one are $n + 1$

$$I_1 \; [0] = \; n$$

In other words, for $I_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |I_1(0) - I_1(1)| \leq 1$$

In other words, for $I_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for falsity membership function

$$F_1 \; [1] = \; n$$

In other words, for $F_1(v_i)$ the number of vertices labeled one are n

$F_1 \; [0] = \; n$ +1

In other words, for $F_1(v_i)$ the number of vertices labeled Zero are n + 1

$$\Rightarrow |F_1(0) - F_1(1)| \leq 1$$

In other words, for $F_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

For edges (v_i, v_j)

$$T_2 \; [1] = \; \frac{3n+1}{2}$$

In other words, for $T_2 \; (v_i, v_j)$ the number of edges labeled one are $\frac{3n+1}{2}$

$$T_2 \; [0] = \; \frac{3n-1}{2}$$

In other words, for $T_2 \; (v_i, v_j)$ the number of edges labeled zero are $\frac{3n-1}{2}$

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$$\Rightarrow |T_2(0) - T_2(1)| \leq 1$$

Similarly,

$$I_2\ \ [1] = \frac{3n+1}{2}$$

In other words, for $I_2\ \ (v_i, v_j)$ the number of edges labeled one are $\frac{3n+1}{2}$

$$I_2\ \ [0] = \frac{3n-1}{2}$$

In other words, for $I_2\ \ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n-1}{2}$

$$\Rightarrow |I_2(0) - I_2(1)| \leq 1$$

And

$$F_2\ \ [1] = \frac{3n-1}{2}$$

In other words, for $F_2\ \ (v_i, v_j)$ the number of edges labeled one are $\frac{3n-1}{2}$

$$F_2\ \ [0] = \frac{3n+1}{2}$$

In other words, for $F_2\ \ (v_i, v_j)$ the number of edges labeled zero are $\frac{3n+1}{2}$

$$\Rightarrow |F_2(0) - F_2(1)| \leq 1$$

Therefore, W_n: n = 4m - 1 : m ≥ 1 satisfies Neutrosophic Cordial Labeling.

Theorem 2

Closed Helm CH_n admits Neutrosophic Cordial Labeling Graph

Proof:

Closed Helm graph is obtained by taking a Helm H_n graph and by adding edges between the pendant vertices. It contains three type of vertex. An Apex of degree n, n number of inner rim vertices of degree 4 and n number of outer rim vertices of degree n-1 . The apex vertex is denoted as v, n vertices of degree 4 is denoted as $v_i: 1 \leq i \leq n$ and outer rim vertices is denoted as $u_i: 1 \leq i \leq n$. Edges adjacent to the apex vertex is denoted as $e_i: 1 \leq i \leq n$, the edges of the wheels inner rim is denoted as $g_i: 1 \leq i \leq n$ and the edges incident with the outer rim of the wheel is denoted as $h_i: 1 \leq i \leq n$ and the edges of the outer rim of the wheel is denoted as $w_i: 1 \leq i \leq n$

Let G = (V, σ, μ) be a neutrosophic graph where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$, $V = [v, \{v_i: i \geq 1\}, \{u_i: i \geq 1\}]$ such that $T_1: V \ \ \rightarrow \{0,1\}, I_1: V \ \ \rightarrow \{0, 1\}\ \ and\ \ F_1: V \ \ \rightarrow \{0, 1\}$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the vertex

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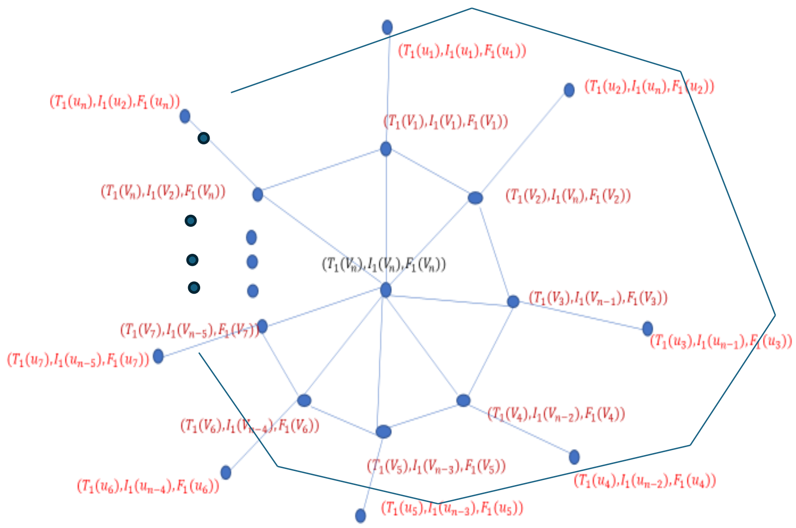
$v, v_i, u_i \in V$ and $T_2: V \to \{0,1\}, I_2: V \to \{0,1\} and\ F_2: V \to \{0,1\}$ where $T_2(e_i), I_2(e_i), F_2(e_i)$, $T_2(g_i), I_2(g_i), F_2(g_i)$, $T_2(h_i), I_2(h_i), F_2(h_i)$, $T_2(w_i), I_2(w_i), F_2(w_i)$ denote the degree of truth – membership function, indeterminacy – membership function and falsity – membership function of the edge.

Vertices are denoted as follows

$T_1(v_i)$ denote the truth – membership function and are labeled in the clockwise direction, $I_1(v_i)$ denote an indeterminacy – membership function and are labeled in the anticlockwise direction and $F_1(v_i)$ denote the falsity – membership function and are labeled in the clockwise direction. Similarly, $T_1(u_i)$ denote the truth – membership function and are labeled in the clockwise direction, $I_1(u_i)$ denote an indeterminacy – membership function and are labeled in the clockwise direction and $F_1(u_i)$ denote the falsity – membership function and are labeled in the anticlockwise direction and

Truth – membership function, indeterminacy – membership function and falsity – membership function of the vertices and edges are distinct.



Then,

Total number of vertices is 2n+1 and for edges 4n.

Define Vertex labeling as follows

Truth Membership Function of the Apex vertices is labeled as 0 and indeterminacy membership function as 1 and Falsity membership function as 0. Apex vertices are labeled as $T_1(v) = 0,\ I_1(v) = 1\ and\ F_1(v) = 0$. Truth membership function of the inner rim vertices is labeled as 1, inderterminancy membership function as 0 and Falsity membership function as 1. Vertices of the inner rim are denoted as $T_1(v_i) = 1, I_1(v_i) = 0\ and\ F_1(v_i) = 1$. For outer rim vertices, truth membership function is labeled as 0, indeterminacy membership function as 1 and falsity membership function as 0. i.e., $T_1(u_i) = 0,\ I_1(u_i) = 1\ and\ F_1(u_i) = 0$

Then the edges are labeled as follows

$$T_2(e_i) = \begin{Bmatrix} \max\ \{T_1(v), T_1(v_i)\}\ if\ \ v > v_i\ \ or\ \ v_i > v \\ 0\ \ if\ \ v_i = v \end{Bmatrix}$$

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$$I_2(e_i) = \begin{Bmatrix} \max\ \{I_1(v), I_1(v_j)\}\ \ if\ \ v > v_i\ \ or\ \ v_i > v \\ 0\ \ if\ \ v_i = v \end{Bmatrix}$$

$$F_2(e_i) = \begin{Bmatrix} \max\ \{F_1(v), F_1(v_j)\}\ \ if\ \ v > v_i\ \ or\ \ v_i > v \\ 0\ \ if\ \ v_i = v \end{Bmatrix}$$

$$T_2(g_i) = \begin{Bmatrix} \max\ \{T_1(v_i), T_1(v_j)\}\ \ if\ \ v_i > v_j\ \ or\ \ v_j > v_i \\ 0\ \ if\ \ v_i = v_j \end{Bmatrix}$$

$$I_2(g_i) = \begin{Bmatrix} \max\ \{I_1(v_i), I_1(v_j)\}\ \ if\ \ v_i > v_j\ \ or\ \ v_j > v_i \\ 0\ \ if\ \ v_i = v_j \end{Bmatrix}$$

$$F_2(g_i) = \begin{Bmatrix} \max\ \{F_1(v_i), F_1(v_j)\}\ \ if\ \ v_i > v_j\ \ or\ \ v_j > v_i \\ 0\ \ if\ \ v_i = v_j \end{Bmatrix}$$

$$T_2(h_i) = \begin{Bmatrix} \max\ \{T_1(u_i), T_1(v_j)\}\ \ if\ \ u_i > v_j\ \ or\ \ v_j > u_i \\ 0\ \ if\ \ u_i = v_j \end{Bmatrix}$$

$$I_2(h_i) = \begin{Bmatrix} \max\ \{I_1(u_i), I_1(v_j)\}\ \ if\ \ u_i > v_j\ \ or\ \ v_j > u_i \\ 0\ \ if\ \ u_i = v_j \end{Bmatrix}$$

$$F_2(h_i) = \begin{Bmatrix} \max\ \{F_1(u_i), F_1(v_j)\}\ \ if\ \ u_i > v_j\ \ or\ \ v_j > u_i \\ 0\ \ if\ \ u_i = v_j \end{Bmatrix}$$

Then for truth membership function

$$T_1\ \ [1] = \frac{n}{2}$$

In other words, for $T_1(v_i)$ the number of vertices labeled one are *n+1*

$$T_1\ \ [0] = \frac{n}{2} + 1$$

In other words, for $T_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |T_1(0) - T_1(1)| \leq 1$$

In other words, for $T_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for indeterminancy membership function

$$I_1\ \ [1] = \frac{n}{2} + 1$$

In other words, for $I_1(v_i)$ the number of vertices labeled one are $n + 1$

$$I_1\ \ [0] = \frac{n}{2}$$

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In other words, for $I_1(v_i)$ the number of vertices labeled Zero are n

$$\Rightarrow |I_1(0) - I_1(1)| \leq 1$$

In other words, for $I_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

Then for falsity membership function

$$F_1 \ [1] = \frac{n}{2}$$

In other words, for $F_1(v_i)$ the number of vertices labeled one are n

$$F_1 \ [0] = \frac{n}{2} + 1$$

In other words, for $F_1(v_i)$ the number of vertices labeled Zero are n+1

$$\Rightarrow |F_1(0) - F_1(1)| \leq 1$$

In other words, for $F_1(v_i)$ the difference of number of vertices labeled one and zero is less than or equal to 1.

For edges (v_i, v_j)

$$T_2 \ [1] = 2n$$

In other words, for $T_2 \ (v_i, v_j)$ the number of edges labeled one are 2n

$$T_2 \ [0] = 2n$$

In other words, for $T_2 \ (v_i, v_j)$ the number of edges labeled zero are 2n

$$\Rightarrow |T_2(0) - T_2(1)| \leq 1$$

Similarly,

$$I_2 \ [1] = 2n$$

In other words, for $I_2 \ (v_i, v_j)$ the number of edges labeled one are 2n

$$I_2 \ [0] = 2n$$

In other words, for $I_2 \ (v_i, v_j)$ the number of edges labeled zero are 2n

$$\Rightarrow |I_2(0) - I_2(1)| \leq 1$$

And

$$F_2 \ [1] = 2n$$

In other words, for $F_2 \ (v_i, v_j)$ the number of edges labeled one are 2n

$$F_2 \ [0] = 2n$$

In other words, for $F_2 \ (v_i, v_j)$ the number of edges labeled zero are 2n

$$\Rightarrow |F_2(0) - F_2(1)| \leq 1$$

Therefore, CH_n satisfies Neutrosophic Cordial Labeling.

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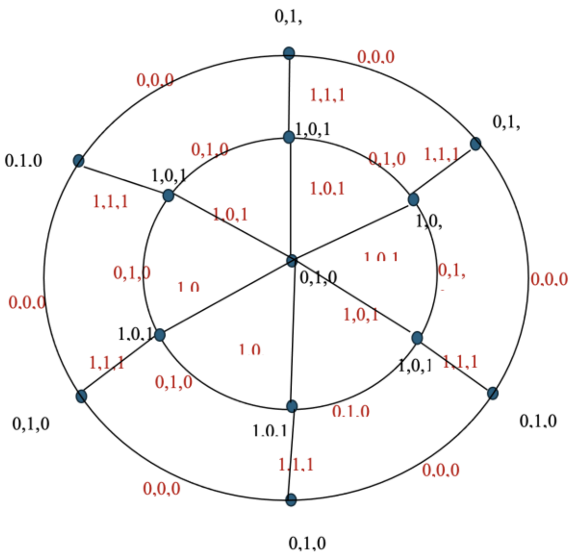

Closed Helm

Lemma 1

Every Neutrosophic Cordial Labeling graph is a Cordial Labeling but the converse is not always true

Proof:

Since every Neutrosophic Cordial Labeling graph satisfies the property of Cordial Labeling, it is true that every neutrosophic cordial labelling graph are also cordial labeling graph. Converse is not always true since we have three-membership function in a neutrosophic graph, not all cordial graphs will satisfy the property of neutrosophic cordial labeling condition.

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