Neutrosophic Maxwell–Boltzmann Distribution: Properties and Application to Healthcare Data

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Abstract

In this work, we present and analyze new probability distribution by generalizing the classical Maxwell–Boltzmann model to neutrosophic structure. The generalized structure, known as the neutrosophic Maxwell (NMX) model that is designed to analyze data with imprecise or vague information. Closed-form expressions for cumulative distribution functions, probability density functions, survival functions, hazard functions, and moments, moment generating functions, mode, skewness, and kurtosis are derived as part of its detailed mathematical and statistical characteristics. The parameter estimation of the suggested model is carried out employing the maximum likelihood estimation (MLE) technique, and the statistical properties of the estimators are discussed in uncertain environments. The inverse cumulative distribution method is established to generate random samples from the proposed model and to evaluate the efficiency of the MLE method. Eventually, a real-world healthcare data set is used to show the efficacy of the proposed model. This research provides new knowledge in the field of neutrosophic statistics, laying a foundation for further exploration in this area.

Keywords: Estimation; Neutrosophic probability; Maximum likelihood; Neutrosophic simulation

1. Introduction

Probability distributions are used to describe random behaviour of many real-world systems [1]. They are fundamental tools for such diverse applications as modeling and analysis of systems in physics, engineering, biology, and the social sciences [2]. Depending on the nature of the random variable, there are two types of distributions: discrete and continuous. Discrete distributions (binomial and Poisson distributions) are used for random variables with finite or countable sets of values. Continuous distributions, such as the normal, exponential, and Maxwell–Boltzmann distributions, are used when the random variable can take any value in a range [3-5]. One important special case, known as the Maxwell distribution, which is a special case of the chi distribution with three degrees of freedom is relevant in statistical mechanics and thermodynamics [6]. It was first defined and used for describing particle speeds in idealized gases [7]. The Maxwell model law is based on the kinetic theory of gases and applies to an ideal gas whose particles are non-interacting and obey classical mechanics. Maxwell distribution is affected by temperature. At lower temperatures, the molecules have less energy [8]. The distribution of speeds is mathematically expressed in terms of its probability density function, a function of speed and temperature, describing how the speeds of molecules vary in a system in thermal equilibrium [9]. Its familiar bell shape stretches to its highest point at the most probable speed and falls symmetrically as speed increases or decreases.

The Maxwell distribution is interesting due to its simplicity and provides closed-form expressions of important statistical measures including mean, variance, skewness, and kurtosis that describes its spread and shape [10]. Such versatility makes it much more general than just physics, however, and it is also useful in reliability engineering, telecommunications, and anywhere you need to model the magnitude of vectors or non-negative random variables

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[11-12]. The previous studies focused on the mathematical approach of the Maxwell model but do not enlighten the neutrosophic features of the Maxwell probability function, particularly in the applied statistical study.

In our daily life, we come across continuous variables, and the observations are clearly defined numbers. Different distributions have been used to model data in various domains in the past few decades. However, not all data fits well to the common distributions in different conditions. Conversely, all measurements of any continuous variables are imprecise by some degree [13]. Indeed, we should careful about the words "accurate" and "exact" from science in the realm of measurements; there is no such thing as measurement of continuous process [14]. Moreover, besides continuous variables, there are also some conditions in which accurate measurement is not possible because of variables of interest that may be not necessarily continuous [15]. Thus, the measurement is not perfect; all individual observations carry a degree of fuzziness [16]. In a classical statistical framework, the variability among these data is modelled indifferently by disregarding the fuzziness. Neutrosophic logic, introduced by Smarandache [17], is the extended form of fuzzy logic discussed to deal with the imprecision of the studied factors. Neutrosophic logic has been extended to many fields including statistical methods. Smarandache [18], who introduced neutrosophic statistics based on generalization of classical statistical techniques. Neutrosophic statistics deals with methods that can handle data having imprecise information. In the case of zero indeterminacy, the neutrosophic statistical method reduces to the classical method [19].

Recently, neutrosophic statistics have been introduced in applied distributional theory and many distributions have been developed under neutrosophic logic [20-24]. To the best of our knowledge, no study on the neutrosophic development of the Maxwell model to handle possibly imprecisely data specifically in healthcare sector, which is the subject of the current investigation based on the existing literature.

The outline of this paper is as follows: In Section 2, notions of the proposed model with, some distributional properties, related theorems, and reliability characteristics are provided. Section 3 explains maximum-likelihood estimation in the neutrosophic environment. Section 4 presents a large amount of simulation work for validating the authenticity of the proposed model. The applicability of NMX is also demonstrated in Section 5 by applying it to a real dataset on Malaria dataset. Lastly, in Section 6, main findings of the work are concluded.

2. Preliminary

In this section, we describe the basic functions and properties of the Maxwell model. In probability, the probability density function (pdf) is a basic function to describe random nature of any event. This function is used to specify the probability of a continuous random variable in a particular range of values. It allows giving information about how the data is distributed, calculating probability, expectation, variance, and so on. However, pdf also play a integral role in modeling real-world phenomena, which makes them key for decision-making processes in a multitude of domains including engineering, healthcare, business, ecommerce, finance and the natural sciences. The pdf function of the Maxwell model with θ can be written as:

$$f(x;\theta) = \sqrt{\frac{2}{\pi}\frac{x^2}{\theta^3}}\exp\left(-\frac{x^2}{2\theta^2}\right), \quad x > 0,\, \theta > 0 \; (1)$$

The sketch of pdf with different values of θ can be seen as in Figure 1.

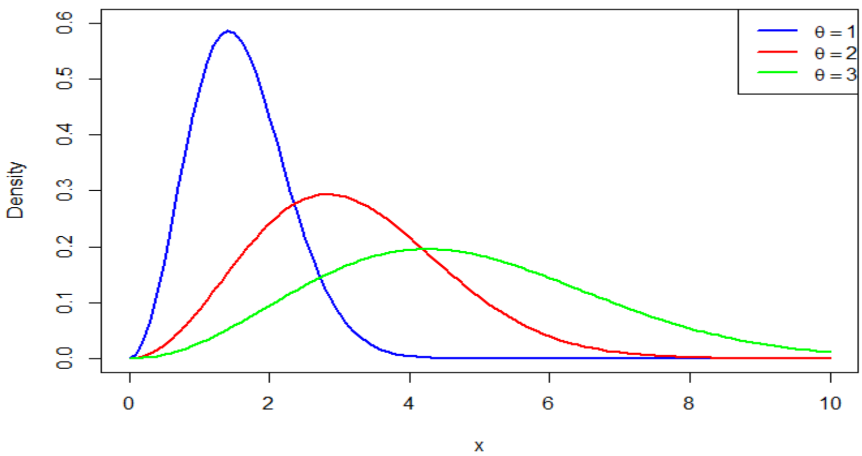

Figure 1. Pdf function of the Maxwell distribution with different values of parameter

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Figure 1 shows the pdf of the Maxwell distribution for different values of the parameter θ. These curves reveal how the shape of the distribution changes as a function of parameter θ, dictating the distribution's spread and location of its peak. A smaller parameter θ leads to sharper, taller peaks, while a larger parameter θ spreads out the distribution reflecting added noise in the data. The other function related to pdf is cumulative distribution function. The cdf function of the Maxwell distribution is defined as:

$$F(x;\theta) = \text{erf}\left(\frac{x}{\sqrt{2}\theta}\right) - \sqrt{\frac{2}{\pi}}\frac{x}{\theta}\exp\left(-\frac{x^2}{2\theta^2}\right), \quad x > 0,\ \theta > 0 \ (2)$$

The cdf function for different parameter setting can be seen in Figure 2.

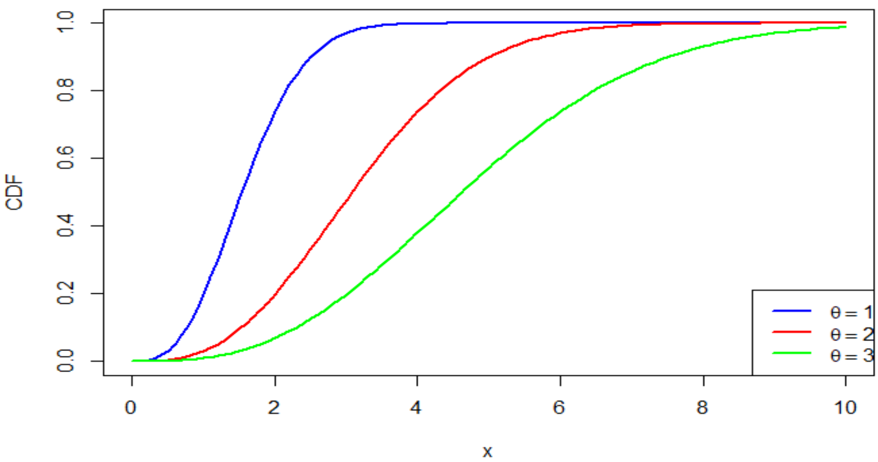

Figure 2. The cdf plot of the Maxwell distribution with various parameter setting

The cdf of the Maxwell distribution is shown for various θ values in Figure 2. The curves represent the cumulative probability up to a specific X. Low θ causes a narrower line, a sharper slope with an instantaneous probability accumulation rate and high θ yields a wider cover, slower slope, higher range distribution of the data.

Similarly, the other properties of the Maxwell can be established. For example, the rth moment of the Maxwell model can be written as:

$$\mu_r' = \theta^r \cdot 2^{r/2} \cdot \frac{\Gamma\left(\frac{r+3}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} \quad (3)$$

Based on Eq. (3) mean, variance of the Maxwell model can be written as:

$$\text{Var} = \left(3 - \frac{8}{\pi}\right)\theta^2 \quad (4)$$

$$\text{Mean} = 2\theta\sqrt{\frac{2}{\pi}} \quad (5)$$

The shape coefficients of the Maxwell model can be written as:

$$\text{Skewness} = \frac{2\sqrt{2}(\pi-3)}{\left(3-\frac{8}{\pi}\right)^{3/2}} \quad (6)$$

$$\text{Kurtosis} = \frac{4(\pi-2)}{3-\frac{8}{\pi}} \quad (7)$$

Skewness and kurtosis are statistical measures that assess the shape of the distribution. Skewness coefficient measures the asymmetry of the distribution. A positive skew shows a longer right tail, and a negative skew shows a longer left tail. It is one of the first things used to see if the data clusters more around one side of the mean than another. Kurtosis on the other hand measures the tailed density of the distribution. High kurtosis implies high tails and a sharp peak (more extreme values), while low kurtosis implies low tails and the distribution is flat. Both Skewness and Kurtosis gives information about the Shape and characteristics of the data, which are very important to understand the behavior of data. These coefficients also help to model and predict certain behaviors for data, which makes skewness and kurtosis important with respect to data analysis and decision-making.

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3. Proposed Distribution

In section Maxwell model under the neutrosophic framework is described. Important characteristics and key functions related to probability theory are presented for the Maxwell model. A random variable is said to follows the neutrosophic Maxwell (NXM) distribution if it has the following pdf:

$$\Phi_N(z) = \sqrt{\frac{2}{\pi}}\, \rho_n{}^{-3} z^2\, e^{-\frac{z^2}{2\rho_n{}^2}} \qquad z > 0 \; (8)$$

where $\rho_n \in [\rho_l, \rho_u]$ is the imprecise parameter value of the NXM distribution. The shape of pdf of under uncertain environment is shown in Figure 3.

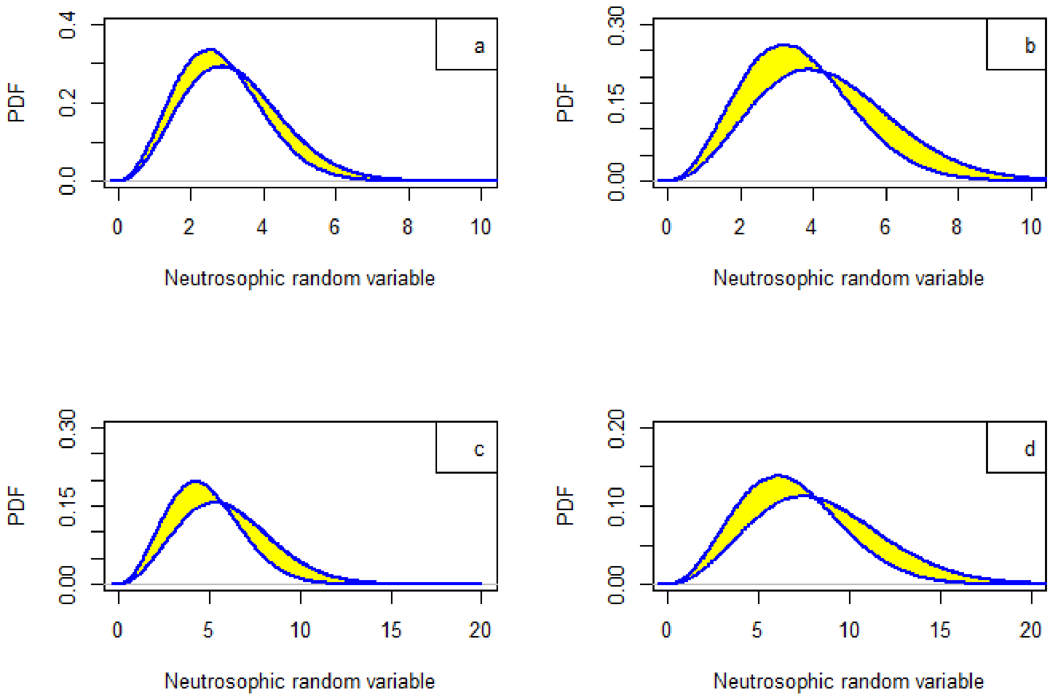

Figure 3. Density plot of the NXM distribution for different imprecise value of ρ_n

The neutrosophic version of Maxwell distribution pdf as shown in Figure 3, is obtained by combining Maxwell distribution with neutrosophic logic to deal with uncertainties and imprecisions in the data. The pdf expressed in neutrosophic parameters, including scale parameter (spread of the distribution) as interval or degree of truth, falsity, indeterminacy, integrated using the neutrosophic theory. Therefore, the generalization of Maxwell distribution can deal with such situations, especially for real problems where exact measurements are difficult. The neutrosophic version of this class makes it particularly useful and applicable in, among other fields, reliability engineering, quality control, healthcare and probabilistic modeling where uncertainties in data are greatly impactful.

The neutrosophic cumulative function is given by:

$$F(z) = P_N(Z \le z) = \frac{2}{\sqrt{\pi}} \gamma[\frac{3}{2}, \frac{z^2}{2\rho_N{}^2}] \qquad (9)$$

Expression given in Eq (9) can easily be established as:

$$F(z) = \left[\int_0^z F_l(z)\, dz, \int_0^z F(z)\, dz\right]$$

which further can be simplified as:

$$F(z) = \frac{2}{\sqrt{\pi}} \gamma\left[\left(\frac{3}{2}, \frac{z^2}{2\rho_l{}^2}\right), \left(\frac{3}{2}, \frac{z^2}{2\rho_u{}^2}\right)\right] = \frac{2}{\sqrt{\pi}} \gamma[\frac{3}{2}, \frac{z^2}{2\rho_N{}^2}]$$

The structure of neutrosophic cumulative of the proposed model is shown in Figure 4.

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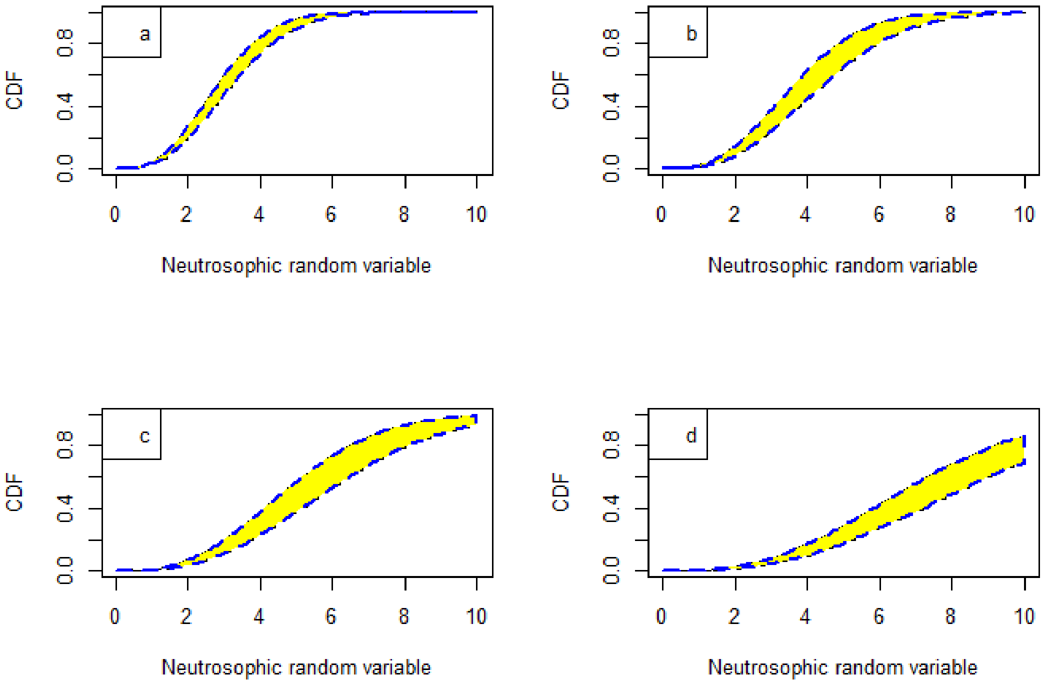

Figure 4. Neutrosophic cumulative function of NXM distribution for different imprecise value of ρ_n

It is observed that the neutrosophic cumulative distribution function (cdf) of the proposed neutrosophic distribution is shown in Figure 4 for different parameter values. The unique representation of the neutrosophic framework is illustrated by a thick curve (or band) representing the cdf. This band covers up the natural indeterminacies and vagueness in the data, where the range gives possible values of the distribution. The curves are monotonic non-decreasing and approaching unity, which is explained by the property of probabilistic completeness of the neutrosophic cdf. Now the other significant properties of the proposed model can be established in similar way.

The kth moment about the origin of NXM distribution can be found as:

$$\mu'_k = E(Z)^r = \int_0^\infty z^k F(z)dz$$

$$= \left[\int_0^\infty z^k F_l(z)dz, \int_0^\infty z^k F_u(z)dz\right]$$

$$= [\frac{2^{\frac{k+2}{2}}}{\sqrt{\pi}}\rho_l{}^k\Gamma\left(\frac{r+3}{2}\right), \frac{2^{\frac{k+2}{2}}}{\sqrt{\pi}}\rho_u{}^r\Gamma\left(\frac{k+3}{2}\right)]$$

$$\mu'_k = \frac{2^{\frac{k+2}{2}}}{\sqrt{\pi}}\rho_N{}^k\Gamma(\frac{k+3}{2}) \tag{10}$$

Eq (10) is important to generate different moments about zero of the proposed model. For example, the first and second can easily be derived from Eq (10) by taking $k = 1, 2$. These two moments then used to find mean and variance of the proposed NXM distribution. Mean of the proposed model can be defined as:

$$E(Z) = \int_0^\infty z\ \phi_N(z)dz \tag{11}$$

$$= \left[\int_0^\infty z\ F_l(z)dz, \int_0^\infty z\ F_u(z)dz\right]$$

$$= 2\rho_N\sqrt{\frac{2}{\pi}}$$

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$= [2\rho_l\sqrt{\frac{2}{\pi}}\ ,\ 2\rho_u\sqrt{\frac{2}{\pi}}]$ is mean of the NXM distribution which can be derived by sampling assuming $k = 1$ in Eq (10).

Similarly, variance of the NXM distribution is defined by:

$$\sigma^2{}_N(z) = E(Z^2) - \left(E(Z)\right)^2 \qquad (12)$$

where

$$E(Z^2) = \left[\int_0^\infty z^2\, F_l(z)dz, \int_0^\infty z^2\, F_u(z)dz\right]$$

$$E(W^2) = 3{\rho_N}^2$$

Thus Eq(12) can be simplified and written as:

Now $\sigma^2{}_N(z) = [3{\rho_l}^2,\ 3{\rho_u}^2] - ([2\rho_l\sqrt{\frac{2}{\pi}}\ ,\ 2\rho_u\sqrt{\frac{2}{\pi}}])^2$

$= (3\pi - 8)\frac{{\rho_N}^2}{\pi}$ is required variance of the proposed distribution.

Some key distributional properties of the proposed model are shape coefficients, which can easily be derived from the Eq (10).

$$\sqrt{\beta_1} = \frac{{\mu_3}^2}{{\mu_2}^3} = \sqrt{\frac{\frac{\frac{2\sqrt{2}}{3}{\rho_N}^3[(16-5\pi)]^2}{\pi^{\frac{3}{2}}}}{[\frac{{\rho_N}^2}{\pi}(3\pi-8)]^3}} \qquad (13)$$

Simplification of Eq (13) leads to:

$$\sqrt{\beta_1} = \frac{2(16-5\pi)\sqrt{2}}{(3\pi-8)^{\frac{3}{2}}} \qquad (14)$$

$\sqrt{\beta_1}$ is coefficient of skewness that measure the asymmetry of the proposed distribution.

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2} = \frac{\frac{{\rho_N}^4}{\pi^2}(15\pi^2+16\pi-192)}{[\frac{{\rho_N}^2}{\pi}(3\pi-8)]^2} \qquad (15)$$

Further simplification of Eq (15) yielded

$$= \frac{(15\pi^2+16\pi-192)}{[(3\pi-8)]^2} \qquad (16)$$

β_2 measure thickness of the tail of the distribution and commonly known as coefficient of kurtosis.

4. Maximum likelihood estimatio

This section describes the analytical procedure through which the most widely method of maximum likelihood (ML) is used; to estimate the parameter of the proposed model. In this part, we present a new method for estimating the parameter of NMD via neutrosophic statistics. Let $z_{1n}, z_{2n}, \ldots z_{pn}$ be the observed interval values sample from the proposed model with density function defined in Eq (4) . Consider the parameter is unknown in the defined distribution, we denote the joint probability of the observed sample by :

$$\zeta_n(z_{in}, \rho_n) = \frac{m}{2}\log\left(\frac{2}{\pi}\right) - 3m\log\rho_n + \log\prod_{i=1}^{m} {z_{in}}^2 - \frac{\sum_{i=1}^{m} {z_{in}}^2}{2{\rho_n}^2}$$

Unknown value of ρ_n can be obtained by maximizing $\zeta_n(z, \rho_n)$. This yielded:

$$\hat{\rho}_n = \max\left(\zeta_n(z_{in}, \rho_n)\right)$$

This ML estimate can be obtained by taking partial derivative with respect to unknown parameter.

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Table 1: Assessment of ML estimator of the proposed model

Sample	Mean bias	MSB
10	[0.009, 0.012]	[0.142 0.201]
20	[0.007, 0.010]	[0.101, 0.143]
40	[0.006, 0.008]	[0.069, 0.098]
80	[0.003, 0.004]	[0.049, 0.069]
120	[0.002, 0.003]	[0.040, 0.057]
160	[0.001, 0.001]	[0.034, 0.048]

$$\frac{\vartheta\zeta_n(z_{in},\rho_n)}{\vartheta\rho_n} = [\frac{\vartheta\zeta_l(z_{il},\rho_l)}{\vartheta\rho_u}, \frac{\vartheta\zeta_u(z_{iu},\rho_u)}{\vartheta\rho_l}] \quad (17)$$

where $\zeta_l(z,\rho_l) = \frac{m}{2}\log\left(\frac{2}{\pi}\right) - 3mlog\rho_l + log\prod_{i=1}^{m} z_{il}{}^2 - \frac{\sum_{i=1}^{m} z_{il}{}^2}{2\rho_l{}^2}$

and

$\zeta_u(z,\rho_u) = \frac{m}{2}\log\left(\frac{2}{\pi}\right) - 3mlog\rho_u + log\prod_{i=1}^{m} z_{iu}{}^2 - \frac{\sum_{i=1}^{m} z_{iu}{}^2}{2\rho_u{}^2}$.

Solving (17) and equating to zero resulted:

$$\frac{\vartheta\zeta_n(z,\rho_n)}{\vartheta\rho_n} = [\frac{-3m}{\rho_l} + \frac{\sum_{i=1}^{m} z_{il}{}^2}{\rho_l{}^3}, \frac{-3m}{\lambda_u} + \frac{\sum_{i=1}^{m} z_{iu}{}^2}{\rho_u{}^3}] \quad (18)$$

$$[\hat{\rho}_l, \hat{\rho}_u] = \left\{\sqrt{\frac{\sum_{i=1}^{m} z_{il}{}^2}{3m}}, \sqrt{\frac{\sum_{i=1}^{m} z_{iu}{}^2}{3m}}\right\},$$

Assessment of this parameter can be performed using well known metric which is defined as:

Root mean square bias $(MSB) = \sqrt{\frac{\sum_{i=1}^{n}(\hat{\rho}_n - \rho_n)^2}{n}}$

Now simulated data can be used to assess adequacy of the estimator and estimation method. For simulating data for the NXM distribution, inverse transformation method can be used. If you choose a value for distributional parameter, we can find a random number from that distribution by using a formula given below. The basic idea is to use the inverse cumulative function, a common tool used in applied statistics.

$$p = \frac{2}{\sqrt{\pi}}\gamma\left[\frac{3}{2}, \frac{z^2}{2\rho_N{}^2}\right] \quad (19)$$

where p is uniform random variable and γ is inverse gamma function.

A program written in R is used to evaluate the performance of proposed estimator via simulation. To see the impact of sample size and reliability of estimate, a Monte Carlo experiment has been carried out in simulation setup. There are 10000 simulated runs are completed, with varying sample sizes such as $n = 10, 20, 40, 80, 120$ and 160 and considering $\rho_n = [1.5, 3]$. Numerical outcomes of these simulated runs are shown in Table 1.

As seen from the results in Table 1, the biases lower as the size of the sample increases. This finding indicates that estimating better relies on larger sample sizes. Moreover, these results show that the neutrosophic estimation method produces more reliable and robust outcomes on larger data sets, carrying forward the method's capabilities with greater data variability.

5. Real Data Application

In this section, the proposed NXM distribution has been used to maternal mortality for Saudi Arabia obtained from the source [25]. Maternal mortality means the death of a woman while pregnant or within 42 days of termination of pregnancy, from any cause related to or aggravated by the pregnancy or its management, but not from accidental or incidental causes. Maternal mortality is a vital health marker that shows the quality of a health care system, especially maternal and reproductive health care. Monitoring maternal mortality is important to measure access to and quality of care and often reflects socio-economic, geographic and systemic inequities. Through the examination of rates and causes of maternal mortality, policymakers will be able to formulate targeted interventions to strengthen healthcare systems, facilitate access to prenatal and postnatal care, and combat systemic issues such as poverty, education, and gender inequality, ultimately preventing preventable deaths and advancing

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population health outcomes. Over time, Saudi Arabia has implemented a range of health care policies, initiatives, and interventions to successfully reduce maternal mortality significantly. To improve health services, maternal health is one of the components the Ministry of Health focuses on to implement at the national level as outlined in Vision 2030. Policies are about making sure high-quality care through prenatal, childbirth and postnatal care is accessible throughout the country, backed by adequate hospitals and trained professionals. Public awareness campaigns promote the need for early antenatal care and digital health systems track maternal health outcomes. The country's investment in universal healthcare coverage, as well as advanced care for obstetric emergencies, has resulted in one of the lowest maternal mortality rates in the region. The data on maternal mortality are not precisely measured. More generally, it is impossible to find exact numbers, as many specific urban areas lack confirmed cases or reporting of numbers. Thus, identifying maternal mortality is often difficult in the absence of clear exposure time point information. Therefore, the global maternal mortality period from the source is presented with uncertainties rather than crisp numbers, as seen in Table 2. The framework was developed in [26] and produces uncertainties in maternal mortality.

Table 2: Maternal mortality rates for the period 2001-2020

Imprecise mortality rates per 1000 cases				
[1.71, 2.29]	[27.26, 28.84]	[26.62, 27.44]	[24.13, 25.89]	[22.66, 24.54]
[23.55, 23.65]	[23.33, 24.39]	[21.58, 23.36]	[22.65, 23.75]	[20.52, 21.44]
[19.93, 21.85]	[20.24, 21.14]	[18.78, 20.14]	[19.88, 21.02]	[21.45, 21.65]
[21.16, 22.96]	[22.14, 22.64]	[22.86, 22.94]	[24.03, 24.69]	[21.44, 23.34]

Maxwell model here works well to study the mortality rate per10000 cases. However, conventional modeling approaches fail to produce an analysis of mortality data that often has uncertainties involved, as shown in Table 2. To overcome this, the NXM distribution has been used to provide a more complete statistical interpretation of the complex data. In Table 3, we present an informational rich statistical characterization of mortality rates from the data using the NXM density function and its shortcomings in accommodating uncertainty in the data, allowing for more accurate and insightful analysis.

Table 3: Estimated values for maternal mortality rates for the period

Statistical characteristics	Estimated value
Scale parameter	[159.37, 175.76]
mean	[0.120, 0.126]
variance	[72.28, 79.71]

We report the fitted estimates from maternal mortality rates data as point intervals for the mean, variance, estimated parameter under uncertainty term in Table 4. Since the proposed model can handle data with uncertainty, the proposed distribution offers the data, which can give more effective insights as well as more valuable information for applications and decision-making.

6. Conclusion

We have presented, and studied, the neutrosophic Maxwell–Boltzmann (NMX) model, a new probability distribution that accommodates data containing intrinsic indetermination or imprecision. The proposed model generalizes the classical Maxwell–Boltzmann distribution to a neutrosophic structure and then derives closed-form expressions for some essential statistical and mathematical properties such as cumulative distribution functions, probability density function, survival function, hazard functions, moments, mode, skewness, and kurtosis.MLE has been employed for parameter estimation, alongside comprehensive investigations into the statistical properties of the estimators in uncertain scenarios. Moreover, the inverse cumulative distribution is constructed for generating random samples, demonstrating the competency of the MLE method. Simulation results indicated that reliable estimation could be obtained with larger sample sizes. For a practical utility, we have applied the proposed model on a real-world dataset of maternal mortality in Saudi Arabia from 2001-2020. Empirical findings show that NMX model presents a comprehensive framework to effectively handle uncertain and incomplete data. The proposed model provides a solid groundwork for the implementation and exploration of other neutrosophic models in both healthcare and other sectors dealing with fuzzy information.

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