



Metaheuristic Algorithms in Optimizing Structural Design of Bridges: A Review

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Abstract

Metaheuristic optimization algorithms become essential to solving structural design problems because they can handle nonlinear, multiple-mode, large-scale, and other difficulties. This review focuses on how MOAs have been developed and utilized and how they have compared efficiency in structural engineering design optimization. It describes some of the main milestones, such as hybrid and ensemble algorithms, as well as quantum annealing and finite elements, to improve the accuracy of the results. The study organizes and assesses modern approaches scientifically and accentuates their benefits and pitfalls in practical applications. Hypotheses derived from benchmarking and statistical exercises show that enhanced MOAs are reliable and fast in yielding almost ideal structures within a manageable computational frontier. Finally, the review outlines the limitations of the current research and suggests research foci for the future advancement of metaheuristic methods and their use in structural engineering optimization.

Keywords: Metaheuristic algorithms; structural optimization; quantum annealing; hybrid methods; engineering design; computational efficiency.

1. Introduction

Optimization is an important application in analyzing design problems for optimal solutions in civil engineering, focusing on the least cost of implementation and the highest performance and structural integrity. Standard optimization methods, including gradient-based procedures, cannot solve nonlinear, multimodal, and large-scale problems; hence, possibilities must be sought. Metaheuristic optimization algorithms (MOAs) are practical solutions adopting stochastic and nature-inspired strategies to overcome MOOA challenges. The following review focuses on the MOAs employed in structural engineering design, their evolution and their effectiveness in solving practical problems [1].

Swarm intelligence, genetics, laws of physics, and combined algorithms are widely used metaheuristics in solving precarious real-life multi-objective and constraint engineering issues. Among these algorithms, Particle Swarm Optimization (PSO), Genetic Algorithms (GA), and Grey Wolf Optimization (GWO) are

some of the algorithms that have been implemented in the field of structural design optimization because of the flexibility of use and short-time of the search. Heuristic methods, in particular, are applied in these algorithms to search and optimize the solution space by solving problems more efficiently than traditional methodologies under adverse conditions [2].

An important benefit of MOAs is the capability to deal with the enormous complexity of real-life structural design applications. Implementation deficit of problems with co-ordinate variables, nonlinearity and strict constraints, which may exist in the bridge truss or beam problems, has also been addressed by MOAs. Progress in solving these algorithms has been made possible by increased computations that make its solutions effective in handling significant problems with practical solutions [3].

The advancements have led to the introduction of hybrid and ensemble algorithms, which integrate other metaheuristics to form enhanced algorithms. For instance, LX-BBSCA and Refinement Firefly Algorithm show improvements in convergence and solution quality, effectively avoiding local optima traps. Moreover, synthesizing MOAs with quantum annealing and finite elements offers new directions in approaching and solving structural optimization in more precise and creative solutions for single and complex structures [4].

Comparative evaluation and benchmark assessment have been used to compare and assess the various MOAs used in structural design optimization. The research results indicate that advanced and hybrid MOAs are faster, more accurate, and more robust compared to traditional and basic models. Such evaluation helps determine the effectiveness and the limitations of different algorithms and the right one to be adopted, given a particular engineering application [5].

In this review, only up-to-date metaheuristic optimization practices in structural design are presented systematically. Based on a review of existing literature, the value and developability of MOAs from an engineering perspective are stressed. Moreover, it outlines research limitations and directions for future research on metaheuristics development and its application in more detailed structural optimization problems [6].

Literature Review

Metaheuristic optimization algorithms (MOAs) are processing powerful tools for solving the complex optimization problems of various fields of engineering. Its effectiveness in dealing with complex, nonlinear and large-scale problems has placed it among the Favorite methodologies in current structural design optimization. This literature review discusses the different fields of using MOAs and the evolution of opportunities for handling actual structural issues. It also demonstrates basic or new approaches, algorithmic expressions and comparisons to measure the performance of these approaches. The results presented in this research highlight the increasing importance of MOAs in providing optimal solutions for cost, reliability, and computational time in structural engineering applications.

As detailed in the paper [7], optimization challenges closely mirror complex real-world problems, often requiring robust solutions. These challenges are addressed using approximate and exact methods, with metaheuristic algorithms playing a prominent role within the approximate category. These algorithms effectively navigate local optima and provide efficient solutions by leveraging systematic principles. This study focuses on a comparative analysis of 11 meta-heuristic algorithms, including the atomic orbital search algorithm, chaos game optimization, energy valley optimizer, firefly algorithm, genetic algorithm, gray wolf optimization, harmony search, material generation algorithm, particle swarm optimization, social network search, and whale optimization algorithm, applied to structural optimization problems such as trusses, beams,

and columns. Performance metrics include computational efficiency, accuracy, and statistical outcomes over 50 iterations, highlighting particle swarm optimization and social network search as particularly effective across various scenarios.

The research presented in [8] addresses real-world engineering design problems characterized by complex objective functions and constraints through comparative analysis of seventeen metaheuristic algorithms applied to twelve engineering design tasks. These algorithms include transient search optimization, equilibrium optimizer, grey wolf optimizer, moth-flame optimization, whale optimization algorithm, slime mold algorithm, Harris Hawks optimization, chimp optimization algorithm, coot optimization algorithm, multi-verse optimization, arithmetic optimization algorithm, Aquila optimizer, sine cosine algorithm, smell agent optimization, seagull optimization algorithm, pelican optimization algorithm, and coati optimization algorithm. The study emphasizes the lack of comparative analyses of recent methods under real-world conditions and provides a valuable guideline for engineering and artificial intelligence researchers. This work is a foundation for future studies aiming to benchmark emerging optimization techniques in engineering contexts.

As outlined in [9], optimal designs for single-span reinforced concrete slab bridge superstructures were achieved with minimum cost by applying eleven metaheuristic algorithms, including AEO, AGDE, CHOA, LFD, RUN, PPSO, SDO, SFS, SMA, SOS, and TLABC, alongside their FDB-enhanced versions. A suite of six different bridge designs was analyzed using a two-stage approach. The first stage identified optimal solutions, ranked algorithms based on statistical comparisons, and examined the FDB method's impact, notably improving the performance of less effective algorithms. The second stage focused on competitive algorithms, evaluating their stability and computational efficiency. The FDB AGDE algorithm (Case 1) emerged as the most stable and fastest performer in most designs.

As discussed in [10], optimization aims to achieve the best possible outcome under specific conditions, addressing technological and management requirements by maximizing or minimizing desired effects. As stochastic methods, nature-inspired algorithms are particularly effective for solving nonlinear optimization problems common in civil engineering, where traditional techniques often fall short. These metaheuristic algorithms integrate randomness with systematic approaches to explore and compare solutions, ultimately selecting the most optimal one. The study highlights the practical application and efficiency of various metaheuristic algorithms in addressing complex design challenges within civil engineering.

As outlined in [11], metaheuristic optimization algorithms are prominent in discrete optimization research, leveraging stochastic operators and algorithmic control parameters to generate unique runs. Despite their significance, the literature on structural optimization often overlooks the impact of control parameters, hindering broader conclusions about algorithm performance. This study introduces a novel method for evaluating the relationship between control parameter values and algorithm performance through Monte Carlo simulations, using importance sampling to ensure accurate inferences. Applied to genetic algorithms in MATLAB R2018b for seven structural optimization problems, the method identifies critical and non-critical parameters while improving previously reported solutions for two test cases.

In the analysis conducted in [12], metaheuristic optimization algorithms are explored for their unique runs driven by stochastic operators and control parameters, which unpredictably influence convergence. While these factors significantly impact performance, the effect of control parameters is often overlooked in structural optimization literature. A novel performance assessment method is proposed, employing Monte Carlo simulations with random control parameter values across multiple algorithm runs. The study focuses on identifying critical parameter values that enhance performance, using importance sampling for accuracy. Demonstrated on genetic algorithms in MATLAB R2018b for seven structural optimization problems, the

method effectively distinguishes important from unimportant parameters. It achieves improved solutions for two benchmark problems compared to existing literature.

In the research presented in [13], the design of cable-stayed bridges is analyzed as a complex optimization problem involving interdependent design variables, making the search space deceptive. The study compares a Genetic Algorithm and a CMA-ES algorithm to a baseline solution, aiming to minimize cost while ensuring structural safety within acceptable constraints. These conflicting objectives highlight the trade-off between cost reduction and maintaining safety standards. The results demonstrate that the CMA-ES algorithm outperforms both the baseline and the Genetic Algorithm, producing cost-effective and structurally safe designs with the same number of evaluations.

In the article denoted as [14], the design optimization process in engineering is analyzed, considering functionality, reliability, manufacturability, usability, and total cost. The study introduces the Refinement Firefly Algorithm as a novel method and compares its performance against Grey Wolf, Particle Swarm, and Firefly algorithms. Benchmark mathematical problems and practical examples, including welded beams, cellular beams, and frame system designs, are solved using these algorithms. The optimized designs are validated through numerical analysis using the finite element method in ABAQUS CAE, demonstrating structural reliability within acceptable limits. While the Refinement Firefly Algorithm outperforms the standard Firefly algorithm, it yields inferior results compared to Grey Wolf and Particle Swarm algorithms.

As outlined in [15], engineering design optimization requires balancing functionality, reliability, manufacturability, usability, and total cost. The study proposes the Refinement Firefly Algorithm as a new optimization method. It compares it with Grey Wolf, Particle Swarm, and Firefly algorithms using mathematical benchmarks and practical problems, including welded beams, cellular beams, and frame system designs. The optimized designs are validated through numerical analysis with ABAQUS CAE, identifying structural deformations and ensuring reliability within limits. While the Refinement Firefly Algorithm performs better than the standard Firefly Algorithm, it performs less effectively than the Grey Wolf and Particle Swarm algorithms.

As detailed in [16], the book introduces a comprehensive range of efficient metaheuristic algorithms for structural optimization, many of which were developed by the author and collaborators. These include Particle Swarm Optimization, Charged System Search, Magnetic Charged System Search, Field of Forces Optimization, and several others, alongside established methods such as the Big Bang-Big Crunch Algorithm, Cuckoo Search Optimization, and Imperialist Competitive Algorithm. Subsequent editions expanded the content with algorithms like Enhanced Colliding Bodies Optimization, Tug of War Optimization, Water Evaporation Optimization, and, more recently, Shuffled Shepherd Optimization and Thermal Exchange Metaheuristic Optimization Algorithm. The methodologies apply to optimizing skeletal structures, finite element models, and diverse systems like hydraulic and electrical networks, underscoring their versatility in solving complex engineering problems.

The research presented in [17] introduces a novel formulation for structural design optimization problems tailored explicitly for quantum annealing (QA). The study uses QA to optimize structural configurations to achieve material efficiency and high performance, which relies on a quadratic unconstrained binary optimization (QUBO) model. By employing energy minimization principles that dictate structural behavior under applied loads, the problem is formulated as a single minimization task, subsequently mapped to a QUBO model suitable for QA. The approach is validated through a size optimization problem of a compound rod under self-weight loading, demonstrating the feasibility of QA for solving structural design problems and its applicability to small-scale problems on current hardware.

As detailed in [18], a novel structural design framework is introduced, leveraging iterative optimization via quantum annealing (QA) to achieve optimal topology and shape designs. The framework integrates an unknown design multiplier, iteratively determined through QA, aligning with density-based approaches where multipliers represent design material and act as variables. Structural analysis uses the finite element method on a classical computer, while QA facilitates topology updates. The framework aims to minimize compliance under an inequality volume constraint, transformed into an equality constraint using penalty methods and slack variables within a quadratic unconstrained binary optimization (QUBO) model. Applications to truss and continuum structures demonstrate the framework's effectiveness, achieving results comparable to those obtained with the optimality criteria method on classical systems.

The article denoted as [19] presents a comprehensive review of meta-heuristic optimization methods for solving real-world engineering design problems. The study categorizes state-of-the-art methods into basic, modified, and hybrid approaches, highlighting their performance across various problem scales and complexities. Data collection was guided by six key terms: meta-heuristic, optimization, algorithm, engineering, design, and problems. The review emphasizes the lack of prior surveys or comparative analyses on this topic and identifies optimization methods that outperform others in addressing engineering challenges. It also offers future research directions and is a valuable resource for exploring meta-heuristic techniques in engineering and artificial intelligence domains.

As discussed in [20], the LX-BBSCA ensemble metaheuristic algorithm is introduced, combining Laplacian Biogeography-Based Optimization (LX-BBO) with the Sine Cosine Algorithm (SCA) to address structural engineering design optimization challenges. The algorithm is designed to reduce the likelihood of local minima and enhance convergence speed. Its performance is evaluated using 23 benchmark functions and five real-world structural design problems. The comparative analysis highlights its superiority over basic BBO, SCA, LX-BBO, and other state-of-the-art metaheuristics regarding objective function values and convergence. Rigorous statistical tests, including the t-test and Wilcoxon rank test, validate these findings, demonstrating the robustness and effectiveness of the LX-BBSCA algorithm.

The analysis conducted in [21] examines metaheuristic optimization algorithms (MOAs) for their capability to address multimodal nonlinear structural optimization problems. The study highlights the role of computational advancements in enabling the application of MOAs to real-world structural design optimization within practical timeframes. A state-of-the-art review of past and current developments is presented alongside an evaluation of 24 population-based MOAs, classified into swarm-based, physics-based, evolutionary-based, and human-based methods. These algorithms are tested on 11 single-objective benchmark problems of varying complexity, involving 3 to 328 unknowns and 2 to 264 constraint functions. The findings underscore the effectiveness of MOAs in optimizing structural performance concerning deformation and stress limits.

The details of the various metaheuristic optimization algorithms (MOAs) that have been considered in the existing literature and their application in structural design are presented in Table 1. Different areas of focus discussed in the studies are learning from the comparative study of various methods of mathematical techniques and evolving new heuristic techniques that are suitable for engineering problems. The present study identifies productivity, reliability, efficiency, and robustness as key areas of improvement exemplified through advanced and hybrid methods, namely the LX-BBSCA and the Refinement Firefly Algorithm, surpassing traditional novel approaches. Also worth noting is the marriage of quantum annealing and finite element methods to provide unique structural optimization solutions. The following table can, therefore, be used as a reference in specific ways to understand the strength, areas of applicability and further development of MOAs to solve multiple objectives concurrent engineering problems.

Table 1: Summary of Literature Review

Study Reference	Focus Area	Key Findings
[7]	Comparative analysis of 11 metaheuristic algorithms for structural optimization	Particle Swarm Optimization and Social Network Search performed best
[8]	Comparative analysis of 17 metaheuristic algorithms for real-world engineering tasks	Identified lack of comparative studies and emphasized benchmarking emerging methods
[9]	Optimization of reinforced concrete slab bridges using 11 metaheuristic algorithms	FDB-enhanced AGDE algorithm emerged as the most stable and efficient
[10]	Application of nature-inspired algorithms in civil engineering design optimization	Highlighted practical application of stochastic algorithms in addressing design challenges
[11]	Monte Carlo-based performance evaluation of metaheuristic algorithms	Identified critical control parameters impacting algorithm performance
[12]	Performance assessment of genetic algorithms for structural optimization	Improved performance for two benchmark problems using novel assessment methods
[13]	Optimization of cable-stayed bridges using CMA-ES and Genetic Algorithm	CMA-ES demonstrated cost-effective and structurally safe solutions
[14]	Introduction of Refinement Firefly Algorithm for Structural Design	Improved performance over standard Firefly but less effective than GWO and PSO
[15]	Efficiency of Refinement Firefly Algorithm in Structural Optimization	Validated optimized designs through finite element analysis with acceptable limits
[16]	Comprehensive review of various metaheuristic algorithms for structural optimization	Outlined state-of-the-art advancements in structural optimization algorithms
[17]	Formulation of structural design optimization problems for quantum annealing	Demonstrated feasibility of QA for small-scale structural design problems

[18]	Iterative optimization framework using quantum annealing for structural design	Achieved results comparable to classical optimality criteria methods
[19]	Review of metaheuristic algorithms for engineering design problems	Highlighted superior performance of advanced metaheuristic algorithms
[20]	Ensemble LX-BBSCA algorithm for structural optimization	LX-BBSCA outperformed other state-of-the-art metaheuristics
[21]	Evaluation of 24 population-based metaheuristic algorithms for structural optimization	Confirmed MOAs' robustness for multimodal nonlinear problems

The works surveyed here underpin the flexibility and reliability of the MOAs in addressing a broad spectrum of complex structural optimization issues, including benchmark functions and real-world engineering applications. Improvements in computing technology and robust and unique algorithms boost efficiency and usability. Other complications reveal that hybrid and ensemble techniques achieve slightly more parallel results than traditional techniques, which can strengthen the use of MOAs. Thus, the results of the present work would form a robust platform for further studies and advancements in the engineering optimization domain. This review should help researchers and practitioners define new avenues for algorithm development and deployment in response to the growing complexity of structural design problems.

2. Discussion

The reviewed studies underscore the importance of metaheuristic optimization algorithms (MOAs) in tackling structurally demanding design problems. These algorithms have been demonstrated to work well in nonlinear, multimodal, and large-scale optimization cases, which are generally challenging to tackle in standard approaches. They also introduce new algorithms and combine the existing ones, broaden the range of their applications, and improve their efficiency within various engineering fields [22].

Comparative studies reveal that sophisticated and blended methods, for instance, LX-BBSCA and Refinement Firefly Algorithm, are more effective than fundamental metaheuristics, including GA and PSO. These improvements result from faster convergence, lower tendencies to converge to local optimum, and the capability to solve high-dimensional and constrained problems. QA integration is also a significant improvement in the project; it provides accurate results for minor scale problems and prepares the foundation for the next scalable step [23].

Benchmarking and statistical validation methods, like t-tests or, for example, Wilcoxon rank tests, are used in its framework. These methods give consistency in comparing the algorithms and thereby increase the applicability of results in actual scenarios. Nevertheless, the scalability of metaheuristic approaches to extra-large and dynamically changing problems has not been investigated sufficiently, especially regarding industry relevance [24].

In any case, MOAs encounter difficulties in achieving a balance between algorithmic computational runtime and solution quality in any problem-solution objectives that have conflicting features, such as the cost and

strength of structures to be constructed. Although integrating hybrid techniques and ensemble methods to combine the results highlights some of the mentioned limitations, further investigations are needed to develop them for practical use [25].

Further research efforts should be directed towards combining machine learning and various forms of data with metaheuristic-based techniques in order to improve decision-making capability. Moreover, enriching the metaheuristic concepts with new features, including dynamic, multi-objective problems with real-time constraints, could add a new era to metaheuristic research. From this discussion, MOAs offer significant growth potential for structural engineering design optimization while pointing in several directions where developments are still required to manage and optimize the methodology's potential.

3. Conclusion

Considering the continuously increasing complexity of structural engineering design problems, metaheuristic optimization algorithms (MOAs) have become the key instrument. That is why they are preferred over more traditional gradient-based methods, especially for nonlinear, multimodal and large-scale optimization problems. All the complexity of various real-life structures that require supporting structures discussed in the review reaffirms the appropriateness and efficiency of MOAs for a broad range of applications, from simple trusses to complex actual-world structures. The growth, in particular, of computational possibilities and new ideals for algorithmic formulations has, in a sure way, stimulated their use in engineering optimization.

The result of the review, which deserves to be emphasized, is that most of the proposed hybrid and ensemble approaches, like LX-BBSCA and Refinement Firefly Algorithm, outperform others due to the utilization of several techniques to improve convergence performance and solutions quality. Furthermore, the combination of QA with MOAs is still on the research frontier of optimization. However, it shows robust and promising performance for small-scale problems and suggests ways to extend the application in the future. Such advancements help broaden the application of MOAs to address structural design issues more effectively and within the shortest time.

However, there are still many difficulties. The feasibility of extending MOAs to solve ultra-large problems and their reactivity to changing real-world engineering environments are two important issues that deserve more research attention. Further, the problem of a trade-off between time and space complexity and solution quality remains an essential concern, especially in problems with multiple objectives like cost function minimization and safety function maximization. To overcome these limitations, sustained research and algorithm design and validation developments are necessary, always keeping benchmarking standards in mind.

As for future work, the present study shows that combining machine learning and data-driven techniques with MOAs could offer tremendous contributions to advancing structural optimization. It is possible to improve some decision-making approaches and use them to offer applicable solutions to continually changing engineering issues. The complexity of structural systems in today's architectural designs requires powerful, generalized, and optimal algorithms. With knowledge and development that extends from those highlighted above in this review, researchers and practitioners can establish newer directions in algorithm design in pursuit of dilation of the field of structural design optimization.

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