Algebraic structures such as distributive, associativity and boundedness properties via tangent neutrosophic set acting generalized weighted averaging and geometric

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Abstract

A novel technique to produce complicated tangent trigonometric $(\zeta, \partial, \epsilon)$ neutrosophic sets is presented in this study. Complex tangent trigonometric $(\zeta, \partial, \epsilon)$ neutrosophic weighted averaging, geometric, generalized weighted averaging, and generalized weighted geometric will all be discussed in this article. We calculated the weighted average and geometric using an aggregating model. The following algebraic methods will be used to further study several sets having significant properties.

Keywords: $(\zeta, \partial, \epsilon)$NS; WA; WG; GWA; GWG

1 Introduction

Numerous ideas have been put out to explain uncertainty, including fuzzy sets (FS),[1] which have membership grades (MG) ranging from zero to one. Atanassov.[2] For $\lambda, \upsilon \in [0, 1]$ created an intuitionistic FS (IFS) in which each element has two MGs: positive λ and negative υ, and $0 \leq \lambda + \upsilon \leq 1$. The Pythagorean FSs (PFS) idea was developed by Yager[3] and is distinguished by its MG and non-MG (NMG) with $\lambda + \upsilon \geq 1$ to $\lambda^2 + \upsilon^2 \leq 1$. The use of IFSs and PFSs in several domains has been the subject of numerous research. They still have a limited capacity to transmit information. As a result, the experts continued to struggle to interpret the data in these sets and the related data. The idea of complex IFS with DOMBI prioritized AOs and its use for reliable green supplier selection were examined by Wang et al.[4] The three main concepts of the picture FS are positive MG (λ), neutral MG (ζ), and negative MG (υ), as stated by Cuong et al.[5] It also provides more advantages than PFS and IFS. It has been observed that the picture FS is an improvement of the IFS that may manage more inconsistency and $0 \leq \lambda + \zeta + \upsilon \leq 1$ since $\lambda, \zeta, \upsilon \in [0, 1]$. Expert comments such as "yes," "abstain," "no," and "refusal" will be sent, in accordance with the image FS description.

Shahzaib et al.[6] used MADM to define the SFS for certain AOs. Instead of $0 \leq \lambda + \zeta + \upsilon \leq 1$, SFS demands that $0 \leq \lambda^2 + \zeta^2 + \upsilon^2 \leq 1$. The idea of an intelligent decision support system for SFS was initially put out by Hussain et al.[7] Rafiq et al.[8] were the first to introduce SFSs and their uses in DM. with example, $\lambda^2 + \upsilon^2 \geq 1$ is a DM issue with a property. Fermatean FS (FFS) was introduced by Senapati et al.[9] in 2019 with the restriction that $0 \leq \lambda^3 + \upsilon^3 \leq 1$. Yager was the first to suggest the idea of generalized orthopair FSs.[10] Both the MG and the NMG have power q in the q-rung orthogonal pair FS (q-ROFS), but their sum can never be more than one. The MADM technique for Pythagorean neutrosophic normal interval-valued fuzzy AOs was presented by Palanikumar et al.[11] Xu et al. developed geometric operators, including weighted, ordered weighted, and hybrid operators, that were derived from IFSs.[12] Generalized ordered weighted averaging operators (GOWs) were suggested by Li et al.[13] in 2002. Zeng et al.[14] explained how to compute ordered weighted distances using AOs and distance measurements. Based on the features of AOs, Peng et al. investigated a simple PFS.[15] Spherical fuzzy Ashraf and colleagues created Dombi AOs.[16] Ullah et al.[17,18] provide further information on SFSs and T-SFSs. Ibraheem et al.[19] discussed the concept of complex NSSs using various AOs. Al-Husban et al.[20] deals that Type-I extension Diophantine IVNS. Various algebraic structures and aggregation techniques with applications were studied by Palanikumar et al.[21–23]

I will follow the format listed below for the remainder of this task. See section 1 for an introduction. PFS and NS were discussed in Section 2. Several procedures on $(\zeta, \partial, \epsilon)$ NNs are described in Section 3. The AOs based on CTT $(\zeta, \partial, \epsilon)$ NN are discussed in Section 4. In the section 5, the conclusion is discussed.

2 Background

Many important definitions that we should review for future learning are included in this section.

Definition 2.1. Let $\mathcal{A}$ be a universal. The PFS $\Xi = \Big\{\hbar, \big\langle \mathcal{L}^{\urcorner}[\hbar], \mathcal{L}^{\digamma}[\hbar] \big\rangle \big| \hbar \in \mathcal{A} \Big\}$, $\mathcal{L}^{\urcorner} : \mathcal{A} \to [0,1]$ and $\mathcal{L}^{\digamma} : \mathcal{A} \to [0,1]$ called the MG and NMG of $\hbar \in \mathcal{A}$ to Ξ, respectively and $0 \preceq [\mathcal{L}^{\urcorner}[\hbar]]^2 + [\mathcal{L}^{\digamma}[\hbar]]^2 \preceq 1$. For $\Xi = \big\langle \mathcal{L}^{\urcorner}, \mathcal{L}^{\digamma} \big\rangle$ is called a Pythagorean fuzzy number [PFN].

Definition 2.2. The NS $\Xi = \Big\{\hbar, \big\langle \mathcal{L}^{\urcorner}[\hbar], \mathcal{L}^{\lrcorner}[\hbar], \mathcal{L}^{\digamma}[\hbar] \big\rangle \big| \hbar \in \mathcal{A} \Big\}$, where $\mathcal{L}^{\urcorner}, \mathcal{L}^{\lrcorner}, \mathcal{L}^{\digamma} : \mathcal{A} \to [0,1]$ is denote the MG, IMG and NMG of $\hbar \in \mathcal{A}$, respectively and $0 \leq [\mathcal{L}^{\urcorner}[\hbar]] + [\mathcal{L}^{\lrcorner}[\hbar]] + [\mathcal{L}^{\digamma}[\hbar]] \leq 2$. For $M = \big\langle \mathcal{L}^{\urcorner}, \mathcal{L}^{\lrcorner}, \mathcal{L}^{\digamma} \big\rangle$ is called a neutrosophic number [NN].

Definition 2.3. The Pythagorean NS $\Xi = \Big\{\hbar, \big\langle \mathcal{L}^{\urcorner}[\hbar], \mathcal{L}^{\lrcorner}[\hbar], \mathcal{L}^{\digamma}[\hbar] \big\rangle \big| \hbar \in \mathcal{A} \Big\}$, where $\mathcal{L}^{\urcorner}, \mathcal{L}^{\lrcorner}, \mathcal{L}^{\digamma} : \mathcal{A} \to [0,1]$ is called the MG, IMG and NMG of $\hbar \in \mathcal{A}$, respectively and $0 \leq [\mathcal{L}^{\urcorner}[\hbar]]^2 + [\mathcal{L}^{\lrcorner}[\hbar]]^2 + [\mathcal{L}^{\digamma}[\hbar]]^2 \leq 2$. For $M = \big\langle \mathcal{L}^{\urcorner}, \mathcal{L}^{\lrcorner}, \mathcal{L}^{\digamma} \big\rangle$ is called a Pythagorean neutrosophic number [PyNN].

Definition 2.4. Let $\Xi_1 = [a_1, b_1] \in N$and $\Xi_2 = [a_2, b_2] \in N$. Then the distance between Ξ_1 and Ξ_2 is defined as $\nabla[\Xi_1, \Xi_2] = \sqrt{[a_1 - a_2]^2 + \frac{1}{2}[b_1 - b_2]^2}$, where N is a natural number.

3 Operations for CTT $(\zeta, \partial, \epsilon)$ NN

We present the concept of the $(\zeta, \partial, \epsilon)$ NN, which is a complex tangent trigonometric. As a consequence, the CTT $(\zeta, \partial, \epsilon)$ NN and its operations were established and $\tan \pi/2 = \mho$

Definition 3.1. The $(\zeta, \partial, \epsilon)$ NS $\Xi = \Big\{\hbar, \Big\langle \Big[[\mho * \mathcal{R}^{\urcorner}][\hbar] * e^{[\mho * \mathcal{I}^{\urcorner}][\hbar]}, [\mho * \mathcal{R}^{\lrcorner}][\hbar] * e^{[\mho * \mathcal{I}^{\lrcorner}][\hbar]}, [\mho * \mathcal{R}^{\digamma}][\hbar] * e^{[\mho * \mathcal{I}^{\digamma}][\hbar]}\Big] \Big\rangle \Big| \hbar \in \mathcal{A} \Big\}$, where $[\mho * \mathcal{R}^{\urcorner}], [\mho * \mathcal{R}^{\lrcorner}], [\mho * \mathcal{R}^{\digamma}] : \mathcal{A} \to [0,1]$ denote the MG, IMG and NMG of $\hbar \in \mathcal{A}$ to Ξ, respectively and $0 \leq [[\mho * \mathcal{R}^{\urcorner}][\hbar]]^{\zeta} + [[\mho * \mathcal{R}^{\lrcorner}][\hbar]]^{\partial} + [[\mho * \mathcal{R}^{\digamma}][\hbar]]^{\epsilon} \leq 1$ and $0 \leq [[\mho * \mathcal{I}^{\urcorner}][\hbar]]^{\zeta} + [[\mho * \mathcal{I}^{\lrcorner}][\hbar]]^{\partial} + [[\mho * \mathcal{I}^{\digamma}][\hbar]]^{\epsilon} \leq 1$. For convenience, $\Xi = \Big\langle \Big[[\mho * \mathcal{R}^{\urcorner}] * e^{[\mho * \mathcal{I}^{\urcorner}]}, [\mho * \mathcal{R}^{\lrcorner}] * e^{[\mho * \mathcal{I}^{\lrcorner}]}, [\mho * \mathcal{R}^{\digamma}] * e^{[\mho * \mathcal{I}^{\digamma}]}\Big] \Big\rangle$ is represent a CTT $(\zeta, \partial, \epsilon)$ NN.

Definition 3.2. Let $\Xi = \langle [[\mho * \mathcal{R}^{\urcorner}] * e^{[\mho * \mathcal{I}^{\urcorner}]}, [\mho * \mathcal{R}^{\lrcorner}] * e^{[\mho * \mathcal{I}^{\lrcorner}]}, [[\mho * \mathcal{R}^{\digamma}]] * e^{[\mho * \mathcal{I}^{\digamma}]}] \rangle$, $\Xi_1 = \langle [[\mho * \mathcal{R}_1^{\urcorner}] * e^{[\mho * \mathcal{I}_1^{\urcorner}]}, [\mho * \mathcal{R}_1^{\lrcorner}] * e^{[\mho * \mathcal{I}_1^{\lrcorner}]}, [\mho * \mathcal{R}_1^{\digamma}] * e^{[\mho * \mathcal{I}_1^{\digamma}]}] \rangle$, $\Xi_2 = \langle [[\mho * \mathcal{R}_2^{\urcorner}] * e^{[\mho * \mathcal{I}_2^{\urcorner}]}, [\mho * \mathcal{R}_2^{\lrcorner}] * e^{[\mho * \mathcal{I}_2^{\lrcorner}]}, [\mho * \mathcal{R}_2^{\digamma}] * e^{[\mho * \mathcal{I}_2^{\digamma}]}] \rangle$ be any three CTT $(\zeta, \partial, \epsilon)$ NNs, and $(\zeta, \partial, \epsilon) > 0$. Then

$$1.\ \Xi_1 \sqcup \Xi_2 = \begin{pmatrix} \sqrt[\zeta]{\begin{matrix}[[\mho * \mathcal{R}_1^{\urcorner}]]^{\zeta} + [[\mho * \mathcal{R}_2^{\urcorner}]]^{\zeta} \\ -[[\mho * \mathcal{R}_1^{\urcorner}]]^{\zeta} * [[\mho * \mathcal{R}_2^{\urcorner}]]^{\zeta}\end{matrix}} * e^{\sqrt[\zeta]{\begin{matrix}[[\mho * \mathcal{I}_1^{\urcorner}]]^{\zeta} + [[\mho * \mathcal{I}_2^{\urcorner}]]^{\zeta} \\ -[[\mho * \mathcal{I}_1^{\urcorner}]]^{\zeta} * [[\mho * \mathcal{I}_2^{\urcorner}]]^{\zeta}\end{matrix}}}, \\ \sqrt[\partial]{\begin{matrix}[[\mho * \mathcal{R}_1^{\lrcorner}]]^{\partial} + [[\mho * \mathcal{R}_2^{\lrcorner}]]^{\partial} \\ -[[\mho * \mathcal{R}_1^{\lrcorner}]]^{\partial} * [[\mho * \mathcal{R}_2^{\lrcorner}]]^{\partial}\end{matrix}} * e^{\sqrt[\partial]{\begin{matrix}[[\mho * \mathcal{I}_1^{\lrcorner}]]^{\partial} + [[\mho * \mathcal{I}_2^{\lrcorner}]]^{\partial} \\ -[[\mho * \mathcal{I}_1^{\lrcorner}]]^{\partial} * [[\mho * \mathcal{I}_2^{\lrcorner}]]^{\partial}\end{matrix}}}, \\ [[\mho * \mathcal{R}_1^{F}]]^{\epsilon}[[\mho * \mathcal{R}_2^{F}]]^{\epsilon} * e^{[[\mho * \mathcal{I}_1^{F}]]^{\epsilon}[[\mho * \mathcal{I}_2^{F}]]^{\epsilon}} \end{pmatrix},$$

$$2.\ \Xi_1 \oslash \Xi_2 = \begin{pmatrix} [[\mho * \mathcal{R}_1^{\urcorner}]]^{\zeta}[[\mho * \mathcal{R}_2^{\urcorner}]]^{\zeta} * e^{[[\mho * \mathcal{I}_1^{\urcorner}]]^{\zeta}[[\mho * \mathcal{I}_2^{\urcorner}]]^{\zeta}}, \\ \sqrt[\partial]{\begin{matrix}[[\mho * \mathcal{R}_1^{\lrcorner}]]^{\partial} + [[\mho * \mathcal{R}_2^{\lrcorner}]]^{\partial} \\ -[[\mho * \mathcal{R}_1^{\lrcorner}]]^{\partial} * [[\mho * \mathcal{R}_2^{\lrcorner}]]^{\partial}\end{matrix}} * e^{\sqrt[\partial]{\begin{matrix}[[\mho * \mathcal{I}_1^{\lrcorner}]]^{\partial} + [[\mho * \mathcal{I}_2^{\lrcorner}]]^{\partial} \\ -[[\mho * \mathcal{I}_1^{\lrcorner}]]^{\partial} * [[\mho * \mathcal{I}_2^{\lrcorner}]]^{\partial}\end{matrix}}} \\ \sqrt[\epsilon]{\begin{matrix}[[\mho * \mathcal{R}_1^{F}]]^{\epsilon} + [[\mho * \mathcal{R}_2^{F}]]^{\epsilon} \\ -[[\mho * \mathcal{R}_1^{F}]]^{\epsilon} * [[\mho * \mathcal{R}_2^{F}]]^{\epsilon}\end{matrix}} * e^{\sqrt[\epsilon]{\begin{matrix}[[\mho * \mathcal{R}_1^{F}]]^{\epsilon} + [[\mho * \mathcal{I}_2^{F}]]^{\epsilon} \\ -[[\mho * \mathcal{R}_1^{F}]]^{\epsilon} * [[\mho * \mathcal{I}_2^{F}]]^{\epsilon}\end{matrix}}} \end{pmatrix}$$

$$3.\ \wp * \Xi = \begin{pmatrix} \sqrt[\zeta]{1 - \left[1 - [\mho * [\mathcal{R}^{\urcorner}]^{\zeta}\right]^{\wp}} * e^{\sqrt[\zeta]{1 - \left[1 - [\mho * [\mathcal{I}^{\urcorner}]^{\zeta}\right]^{\wp}}}, \\ \sqrt[\partial]{1 - \left[1 - [\mho * [\mathcal{R}^{\lrcorner}]^{\partial}\right]^{\wp}} * e^{\sqrt[\partial]{1 - \left[1 - [\mho * [\mathcal{I}^{\lrcorner}]^{\partial}\right]^{\wp}}}, \\ [[\mho * [\mathcal{R}^{F}]^{\epsilon}]^{\wp} * e^{[[\mho * [\mathcal{I}^{F}]^{\epsilon}]^{\wp}} \end{pmatrix}$$

$$4.\ \Xi^{\wp} = \begin{pmatrix} [[\mho * [\mathcal{R}^{\urcorner}]^{\zeta}]^{\wp} * e^{[[\mho * [\mathcal{I}^{\urcorner}]^{\zeta}]^{\wp}}, \\ \sqrt[\partial]{1 - \left[1 - [\mho * [\mathcal{R}^{\lrcorner}]^{\partial}\right]^{\wp}} * e^{\sqrt[\partial]{1 - \left[1 - [\mho * [\mathcal{I}^{\lrcorner}]^{\partial}\right]^{\wp}}}, \\ \sqrt[\epsilon]{1 - \left[1 - [\mho * [\mathcal{R}^{F}]^{\epsilon}\right]^{\wp}} * e^{\sqrt[\epsilon]{1 - \left[1 - [\mho * [\mathcal{I}^{F}]^{\epsilon}\right]^{\wp}}} \end{pmatrix}.$$

We present ED and HD measures for CTT $(\zeta, \partial, \epsilon)$ NNs and investigate their mathematical characteristics.

Definition 3.3. For any two CTT $(\zeta, \partial, \epsilon)$ NNs $\Xi_1 = \langle[[[\mho * \mathcal{R}_1^{\urcorner}], [\mho * \mathcal{R}_1^{\lrcorner}], [\mho * \mathcal{R}_1^{F}]]]\rangle$ and $\Xi_2 = \langle[[[\mho * \mathcal{R}_2^{\urcorner}], [\mho * \mathcal{R}_2^{\lrcorner}], [\mho * \mathcal{R}_2^{F}]]]\rangle$. Then

$$\nabla_E[\Xi_1, \Xi_2] = \sqrt{\frac{1}{2}\begin{pmatrix} \begin{pmatrix} 1 + [[\mho * \mathcal{R}_1^{\urcorner}]]^2 - [[\mho * \mathcal{R}_1^{\lrcorner}]]^2 - [[\mho * \mathcal{R}_1^{F}]]^2 \\ -\left[1 + [[\mho * \mathcal{R}_2^{\urcorner}]]^2 - [[\mho * \mathcal{R}_2^{\lrcorner}]]^2 - [[\mho * \mathcal{R}_2^{F}]]^2\right] \end{pmatrix}^2 \\ + \begin{pmatrix} [[\mho * \mathcal{I}_1^{\urcorner}]]^2 - [[\mho * \mathcal{I}_1^{\lrcorner}]]^2 - [[\mho * \mathcal{I}_1^{F}]]^2 \\ -\left[[[\mho * \mathcal{I}_2^{\urcorner}]]^2 - [[\mho * \mathcal{I}_2^{\lrcorner}]]^2 - [[\mho * \mathcal{I}_2^{F}]]^2\right] \end{pmatrix}^2 \end{pmatrix}}$$

where $\nabla_E[\Xi_1, \Xi_2]$ is called the ED between Ξ_1 and Ξ_2.

$$\nabla_H[\Xi_1, \Xi_2] = \frac{1}{2}\begin{pmatrix} \left| \begin{matrix} 1 + [[\mho * \mathcal{R}_1^{\urcorner}]]^2 - [[\mho * \mathcal{R}_1^{\lrcorner}]]^2 - [[\mho * \mathcal{R}_1^{F}]]^2 \\ -\left[1 + [[\mho * \mathcal{R}_2^{\urcorner}]]^2 - [[\mho * \mathcal{R}_2^{\lrcorner}]]^2 - [[\mho * \mathcal{R}_2^{F}]]^2\right] \end{matrix} \right| \\ + \left| \begin{matrix} [[\mho * \mathcal{I}_1^{\urcorner}]]^2 - [[\mho * \mathcal{I}_1^{\lrcorner}]]^2 - [[\mho * \mathcal{I}_1^{F}]]^2 \\ \left[-[[\mho * \mathcal{I}_2^{\urcorner}]]^2 - [[\mho * \mathcal{I}_2^{\lrcorner}]]^2 - [[\mho * \mathcal{I}_2^{F}]]^2\right] \end{matrix} \right| \end{pmatrix}$$

where $\nabla_H[\Xi_1, \Xi_2]$ is called the HD between Ξ_1 and Ξ_2.

4 AOs based on CTT $(\zeta, \partial, \epsilon)$ NN

We use CTT $(\zeta, \partial, \epsilon)$ NWA, CTT $(\zeta, \partial, \epsilon)$ NWG, GCTT $(\zeta, \partial, \epsilon)$ NWA, and GCTT $(\zeta, \partial, \epsilon)$ NWG to describe the AOs.

4.1 CTT $(\zeta, \partial, \epsilon) NWA$

Definition 4.1. Let $\Xi_i = \langle [[[\mho * \mathcal{R}_i^{\daleth}] * e^{[\mho * \mathcal{I}_i^{\daleth}]}, [\mho * \mathcal{R}_i^{\gimel}] * e^{[\mho * \mathcal{I}_i^{\gimel}]}, [\mho * \mathcal{R}_i^{\digamma}] * e^{[\mho * \mathcal{I}_i^{\digamma}]}]] \rangle$ be the CTT $(\zeta, \partial, \epsilon)$ NNs, $W = [\varkappa_1, \varkappa_2, ..., \varkappa_n]$ be the weight of Ξ_i, $\varkappa_i \geq 0$ and $\sqcup_{i=1}^n \varkappa_i = 1$. Then CTT $(\zeta, \partial, \epsilon)$ NWA $[\Xi_1, \Xi_2, ..., \Xi_n] = \sqcup_{i=1}^n \varkappa_i \Xi_i$.

Theorem 4.2. *Let* $\Xi_i = \left\langle [[[\mho * \mathcal{R}_i^{\daleth}] * e^{[\mho * \mathcal{I}_i^{\daleth}]}, [\mho * \mathcal{R}_i^{\gimel}] * e^{[\mho * \mathcal{I}_i^{\gimel}]}, [\mho * \mathcal{R}_i^{\digamma}] * e^{[\mho * \mathcal{I}_i^{\digamma}]}]] \right\rangle$ *be the CTT* $(\zeta, \partial, \epsilon)$ *NNs. Then CTT* $(\zeta, \partial, \epsilon) NWA[\Xi_1, \Xi_2, ..., \Xi_n]$

$$= \left(\begin{array}{c} \sqrt[\zeta]{1 - \sqcap_{i=1}^n \left[1 - [[\mho * \mathcal{R}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1 - \sqcap_{i=1}^n \left[1 - [[\mho * \mathcal{I}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}}, \\ \sqrt[\partial]{1 - \sqcap_{i=1}^n \left[1 - [[\mho * \mathcal{R}_i^{\gimel}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1 - \sqcap_{i=1}^n \left[1 - [[\mho * \mathcal{I}_i^{\gimel}]]^{\partial}\right]^{\varkappa_i}}}, \\ \sqcap_{i=1}^n [[[\mho * \mathcal{R}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i} * e^{\sqcap_{i=1}^n [[[\mho * \mathcal{I}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i}} \end{array} \right).$$

Proof. If $n = 2$, then CTT $(\zeta, \partial, \epsilon)$ NWA$[\Xi_1, \Xi_2] = \varkappa_1 \Xi_1 \sqcup \varkappa_2 \Xi_2$, where

$$\varkappa_1 \Xi_1 = \left(\begin{array}{c} \sqrt[\zeta]{1 - \left[1 - [[\mho * \mathcal{R}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}} * e^{\sqrt[\zeta]{1 - \left[1 - [[\mho * \mathcal{I}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}}}, \\ \sqrt[\partial]{1 - \left[1 - [[\mho * \mathcal{R}_1^{\gimel}]]^{\partial}\right]^{\varkappa_1}} * e^{\sqrt[\partial]{1 - \left[1 - [[\mho * \mathcal{I}_1^{\gimel}]]^{\partial}\right]^{\varkappa_1}}}, \\ [[[\mho * \mathcal{R}_1^{\digamma}]]^{\epsilon}]^{\varkappa_1} * e^{[[[\mho * \mathcal{I}_1^{\digamma}]]^{\epsilon}]^{\varkappa_1}} \end{array} \right)$$

$$\varkappa_2 \Xi_2 = \left(\begin{array}{c} \sqrt[\zeta]{1 - \left[1 - [[\mho * \mathcal{R}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}} * e^{\sqrt[\zeta]{1 - \left[1 - [[\mho * \mathcal{I}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}}}, \\ \sqrt[\partial]{1 - \left[1 - [[\mho * \mathcal{R}_2^{\gimel}]]^{\partial}\right]^{\varkappa_2}} * e^{\sqrt[\partial]{1 - \left[1 - [[\mho * \mathcal{I}_2^{\gimel}]]^{\partial}\right]^{\varkappa_2}}}, \\ [[[\mho * \mathcal{R}_2^{\digamma}]]^{\epsilon}]^{\varkappa_2} * e^{[[[\mho * \mathcal{I}_2^{\digamma}]]^{\epsilon}]^{\varkappa_2}} \end{array} \right).$$

Now, $\varkappa_1 \Xi_1 \sqcup \varkappa_2 \Xi_2$

$$= \left(\begin{array}{c} \sqrt[\zeta]{\begin{array}{c} \left[1 - \left[1 - [[\mho * \mathcal{R}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}\right] + \\ \left[1 - \left[1 - [[\mho * \mathcal{R}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}\right] \\ -\left[1 - \left[1 - [[\mho * \mathcal{R}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}\right] * \\ \left[1 - \left[1 - [[\mho * \mathcal{R}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}\right], \end{array}} * e^{\sqrt[\zeta]{\begin{array}{c} \left[1 - \left[1 - [[\mho * \mathcal{I}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}\right] + \\ \left[1 - \left[1 - [[\mho * \mathcal{I}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}\right] \\ -\left[1 - \left[1 - [[\mho * \mathcal{I}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}\right] * \\ \left[1 - \left[1 - [[\mho * \mathcal{I}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}\right], \end{array}}} \\ \sqrt[\partial]{\begin{array}{c} \left[1 - \left[1 - [[\mho * \mathcal{R}_1^{\gimel}]]^{\partial}\right]^{\varkappa_1}\right] + \\ \left[1 - \left[1 - [[\mho * \mathcal{R}_2^{\gimel}]]^{\partial}\right]^{\varkappa_2}\right] \\ -\left[1 - \left[1 - [[\mho * \mathcal{R}_1^{\gimel}]]^{\partial}\right]^{\varkappa_1}\right] * \\ \left[1 - \left[1 - [[\mho * \mathcal{R}_2^{\gimel}]]^{\partial}\right]^{\varkappa_2}\right], \end{array}} * e^{\sqrt[\partial]{\begin{array}{c} \left[1 - \left[1 - [[\mho * \mathcal{I}_1^{\gimel}]]^{\partial}\right]^{\varkappa_1}\right] + \\ \left[1 - \left[1 - [[\mho * \mathcal{I}_2^{\gimel}]]^{\partial}\right]^{\varkappa_2}\right] \\ -\left[1 - \left[1 - [[\mho * \mathcal{I}_1^{\gimel}]]^{\partial}\right]^{\varkappa_1}\right] * \\ \left[1 - \left[1 - [[\mho * \mathcal{I}_2^{\gimel}]]^{\partial}\right]^{\varkappa_2}\right], \end{array}}} \\ [[[\mho * \mathcal{R}_1^{\digamma}]]^{\epsilon}]^{\varkappa_1} * [[[\mho * \mathcal{R}_2^{\digamma}]]^{\epsilon}]^{\varkappa_2} * e^{[[[\mho * \mathcal{I}_1^{\digamma}]]^{\epsilon}]^{\varkappa_1} * [[[\mho * \mathcal{I}_2^{\digamma}]]^{\epsilon}]^{\varkappa_2}} \end{array} \right)$$

$$= \begin{pmatrix} \sqrt[\zeta]{1-\left[1-[[\mho * \mathcal{R}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}\left[1-[[\mho * \mathcal{R}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}} * \\ e^{\sqrt[\zeta]{1-\left[1-[[\mho * \mathcal{I}_1^{\daleth}]]^{\zeta}\right]^{\varkappa_1}\left[1-[[\mho * \mathcal{I}_2^{\daleth}]]^{\zeta}\right]^{\varkappa_2}}}, \\ \sqrt[\zeta]{1-\left[1-[[\mho * \mathcal{R}_1^{\beth}]]^{\partial}\right]^{\varkappa_1}\left[1-[[\mho * \mathcal{R}_2^{\beth}]]^{\partial}\right]^{\varkappa_2}} * \\ e^{\sqrt[\zeta]{1-\left[1-[[\mho * \mathcal{I}_1^{\beth}]]^{\partial}\right]^{\varkappa_1}\left[1-[[\mho * \mathcal{I}_2^{\beth}]]^{\partial}\right]^{\varkappa_2}}}, \\ [[[\mho * \mathcal{R}_1^{\digamma}]]^{\epsilon}]^{\varkappa_1} * [[[\mho * \mathcal{R}_2^{\digamma}]]^{\epsilon}]^{\varkappa_2} * \\ e^{[[[\mho * \mathcal{I}_1^{\digamma}]]^{\epsilon}]^{\varkappa_1} * [[[\mho * \mathcal{I}_2^{\digamma}]]^{\epsilon}]^{\varkappa_2}} \end{pmatrix}$$

Hence, CTT $(\zeta, \partial, \epsilon) NWA[\Xi_1, \Xi_2]$

$$= \begin{pmatrix} \sqrt[\zeta]{1-\sqcap_{i=1}^{2}\left[1-[[\mho * \mathcal{R}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{2}\left[1-[[\mho * \mathcal{I}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}}, \\ \sqrt[\partial]{1-\sqcap_{i=1}^{2}\left[1-[[\mho * \mathcal{R}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1-\sqcap_{i=1}^{2}\left[1-[[\mho * \mathcal{I}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}}}, \\ \sqcap_{i=1}^{2}[[[\mho * \mathcal{R}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i} * e^{\sqcap_{i=1}^{2}[[[\mho * \mathcal{I}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i}} \end{pmatrix}.$$

It valid for $n \geq 3$,
Thus, CTT $(\zeta, \partial, \epsilon) NWA[\Xi_1, \Xi_2, ..., \Xi_l]$

$$= \begin{pmatrix} \sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-[[\mho * \mathcal{R}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-[[\mho * \mathcal{I}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}}, \\ \sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-[[\mho * \mathcal{R}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-[[\mho * \mathcal{I}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}}}, \\ \sqcap_{i=1}^{l}[[[\mho * \mathcal{R}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i} * e^{\sqcap_{i=1}^{l}[[[\mho * \mathcal{I}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i}} \end{pmatrix}.$$

If $n = l + 1$, then CTT $(\zeta, \partial, \epsilon)$ NWA $[\Xi_1, \Xi_2, ..., \Xi_l, \Xi_{l+1}]$

$$= \begin{pmatrix} \sqrt[\zeta]{\begin{array}{c} \sqcup_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{R}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}\right] + \left[1-\left[1-[\mathcal{R}_{l+1}^{\daleth}]^{\zeta}\right]^{\varkappa_{l+1}}\right] \\ -\sqcap_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{R}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}\right] * \left[1-\left[1-[\mathcal{R}_{l+1}^{\daleth}]^{\zeta}\right]^{\varkappa_{l+1}}\right] \end{array}} * \\ e^{\sqrt[\zeta]{\begin{array}{c} \sqcup_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{I}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}\right] + \left[1-\left[1-[\mathcal{I}_{l+1}^{\daleth}]^{\zeta}\right]^{\varkappa_{l+1}}\right] \\ -\sqcap_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{I}_i^{\daleth}]]^{\zeta}\right]^{\varkappa_i}\right] * \left[1-\left[1-[\mathcal{I}_{l+1}^{\daleth}]^{\zeta}\right]^{\varkappa_{l+1}}\right] \end{array}}}, \\ \sqrt[\partial]{\begin{array}{c} \sqcup_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{R}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}\right] + \left[1-\left[1-[\mathcal{R}_{l+1}^{\beth}]^{\partial}\right]^{\varkappa_{l+1}}\right] \\ -\sqcap_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{R}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}\right] * \left[1-\left[1-[\mathcal{R}_{l+1}^{\beth}]^{\partial}\right]^{\varkappa_{l+1}}\right] \end{array}} * \\ e^{\sqrt[\partial]{\begin{array}{c} \sqcup_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{I}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}\right] + \left[1-\left[1-[\mathcal{I}_{l+1}^{\beth}]^{\partial}\right]^{\varkappa_{l+1}}\right] \\ -\sqcap_{i=1}^{l}\left[1-\left[1-[[\mho * \mathcal{I}_i^{\beth}]]^{\partial}\right]^{\varkappa_i}\right] * \left[1-\left[1-[\mathcal{I}_{l+1}^{\beth}]^{\partial}\right]^{\varkappa_{l+1}}\right] \end{array}}}, \\ \sqcap_{i=1}^{l}[[[\mho * \mathcal{R}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i} * [[\mathcal{R}_{l+1}^{\digamma}]^{\epsilon}]^{\varkappa_{l+1}} * e^{\sqcap_{i=1}^{l}[[[\mho * \mathcal{I}_i^{\digamma}]]^{\epsilon}]^{\varkappa_i} * [[\mathcal{I}_{l+1}^{\digamma}]^{\epsilon}]^{\varkappa_{l+1}}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\zeta]{1-\sqcap_{i=1}^{l+1}\left[1-[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{l+1}\left[1-[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\varkappa_i}}}, \\ \sqrt[\partial]{1-\sqcap_{i=1}^{l+1}\left[1-[[\mho*\mathcal{R}_i^{\lrcorner}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1-\sqcap_{i=1}^{l+1}\left[1-[[\mho*\mathcal{I}_i^{\lrcorner}]]^{\partial}\right]^{\varkappa_i}}}, \\ \sqcap_{i=1}^{l+1}[[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}]^{\varkappa_i} * e^{\sqcap_{i=1}^{l+1}[[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}]^{\varkappa_i}} \end{pmatrix}.$$

□

Theorem 4.3. *Let* $\Xi_i = \left\langle [[[\mho*\mathcal{R}_i^{\urcorner}] * e^{[\mho*\mathcal{I}_i^{\urcorner}]}, [\mho*\mathcal{R}_i^{\lrcorner}] * e^{[\mho*\mathcal{I}_i^{\lrcorner}]}, [\mho*\mathcal{R}_i^{F}] * e^{[\mho*\mathcal{I}_i^{F}]}]] \right\rangle$ *be the CTT* $(\zeta,\partial,\epsilon)$ *NNs. Then CTT* $(\zeta,\partial,\epsilon)$ *NWA* $[\Xi_1,\Xi_2,...,\Xi_n] = \Xi$ *[idempotency property].*

Proof. Since $[\mho*\mathcal{R}_i^{\urcorner}] = [\mho*\mathcal{R}^{\urcorner}]$, $[\mho*\mathcal{R}_i^{\lrcorner}] = [\mho*\mathcal{R}^{\lrcorner}]$ and $[\mho*\mathcal{R}_i^{F}] = [\mho*\mathcal{R}^{F}]$ and $[\mho*\mathcal{I}_i^{\urcorner}] = [\mho*\mathcal{I}^{\urcorner}]$, $[\mho*\mathcal{I}_i^{\lrcorner}] = [\mho*\mathcal{I}^{\lrcorner}]$ and $[\mho*\mathcal{I}_i^{F}] = [\mho*\mathcal{I}^{F}]$ and $\sqcup_{i=1}^{n}\varkappa_i = 1$. Now, CTT $(\zeta,\partial,\epsilon)NWA[\Xi_1,\Xi_2,...,\Xi_n]$

$$\begin{aligned} &= \begin{pmatrix} \sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\varkappa_i}}}, \\ \sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{R}_i^{\lrcorner}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{I}_i^{\lrcorner}]]^{\partial}\right]^{\varkappa_i}}}, \\ \sqcap_{i=1}^{n}[[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}]^{\varkappa_i} * e^{\sqcap_{i=1}^{n}[[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}]^{\varkappa_i}} \end{pmatrix} \\ &= \begin{pmatrix} \sqrt[\zeta]{1-\left[1-[\mho*[\mathcal{R}^{\urcorner}]^{\zeta}\right]^{\sqcup_{i=1}^{n}\varkappa_i}} * e^{\sqrt[\zeta]{1-\left[1-[\mho*[\mathcal{I}^{\urcorner}]^{\zeta}\right]^{\sqcup_{i=1}^{n}\varkappa_i}}}, \\ \sqrt[\partial]{1-\left[1-[\mho*[\mathcal{R}^{\lrcorner}]^{\partial}\right]^{\sqcup_{i=1}^{n}\varkappa_i}} * e^{\sqrt[\partial]{1-\left[1-[\mho*[\mathcal{I}^{\lrcorner}]^{\partial}\right]^{\sqcup_{i=1}^{n}\varkappa_i}}}, \\ [[\mho*[\mathcal{R}^{F}]^{\epsilon}]^{\sqcup_{i=1}^{n}\varkappa_i} * e^{[[\mho*[\mathcal{I}^{F}]^{\epsilon}]^{\sqcup_{i=1}^{n}\varkappa_i}} \end{pmatrix} \\ &= \begin{pmatrix} \sqrt[\zeta]{1-\left[1-[\mho*[\mathcal{R}^{\urcorner}]^{\zeta}\right]} * e^{\sqrt[\zeta]{1-\left[1-[\mho*[\mathcal{I}^{\urcorner}]^{\zeta}\right]}}, \\ \sqrt[\partial]{1-\left[1-[\mho*[\mathcal{R}^{\lrcorner}]^{\partial}\right]} * e^{\sqrt[\partial]{1-\left[1-[\mho*[\mathcal{I}^{\lrcorner}]^{\partial}\right]}}, \\ [\mho*[\mathcal{R}^{F}]^{\epsilon} * e^{[\mho*[\mathcal{I}^{F}]^{\epsilon}} \end{pmatrix} \\ &= \Xi. \end{aligned}$$

□

Theorem 4.4. *Let* $\Xi_i = \left\langle [[[\mho*\mathcal{R}_i^{\urcorner}] * e^{[\mho*\mathcal{I}_i^{\urcorner}]}, [\mho*\mathcal{R}_i^{\lrcorner}] * e^{[\mho*\mathcal{I}_i^{\lrcorner}]}, [\mho*\mathcal{R}_i^{F}] * e^{[\mho*\mathcal{I}_i^{F}]}]] \right\rangle$ *be the CTT* $(\zeta,\partial,\epsilon)$ *NNs. Then CTT* $(\zeta,\partial,\epsilon)$ *NWA*$[\Xi_1,\Xi_2,...,\Xi_n]$
where $\underbrace{[\mho*\mathcal{R}^{\urcorner}]} = \min[\mho*\mathcal{R}_{ij}^{\urcorner}]$, $\overbrace{[\mho*\mathcal{R}^{\urcorner}]} = \max[\mho*\mathcal{R}_{ij}^{\urcorner}]$, $\underbrace{[\mho*\mathcal{R}^{\lrcorner}]} = \min[\mho*\mathcal{R}_{ij}^{\lrcorner}]$, $\overbrace{[\mho*\mathcal{R}^{\lrcorner}]} = \max[\mho*\mathcal{R}_{ij}^{\lrcorner}]$, $\underbrace{[\mho*\mathcal{R}^{F}]} = \min[\mho*\mathcal{R}_{ij}^{F}]$, $\overbrace{[\mho*\mathcal{R}^{F}]} = \max[\mho*\mathcal{R}_{ij}^{F}]$
and $\underbrace{[\mho*\mathcal{I}^{\urcorner}]} = \min[\mho*\mathcal{I}_{ij}^{\urcorner}]$, $\overbrace{[\mho*\mathcal{I}^{\urcorner}]} = \max[\mho*\mathcal{I}_{ij}^{\urcorner}]$, $\underbrace{[\mho*\mathcal{I}^{\lrcorner}]} = \min[\mho*\mathcal{I}_{ij}^{\lrcorner}]$, $\overbrace{[\mho*\mathcal{I}^{\lrcorner}]} = \max[\mho*\mathcal{I}_{ij}^{\lrcorner}]$, $\underbrace{[\mho*\mathcal{I}^{F}]} = \min[\mho*\mathcal{I}_{ij}^{F}]$, $\overbrace{[\mho*\mathcal{I}^{F}]} = \max[\mho*\mathcal{I}_{ij}^{F}]$ *and where* $1 \le i \le n,\ j = 1,2,...,i_j$. *Then,*

$$\begin{aligned} &\left\langle \underbrace{[\mho*\mathcal{R}^{\urcorner}] * e^{[\mho*\mathcal{I}^{\urcorner}]}}, \underbrace{[\mho*\mathcal{R}^{\lrcorner}] * e^{[\mho*\mathcal{I}^{\lrcorner}]}}, \overbrace{[\mho*\mathcal{R}^{F}] * e^{[\mho*\mathcal{I}^{F}]}} \right\rangle \\ &\qquad \le \quad CTT(\zeta,\partial,\epsilon)NWA[\Xi_1,\Xi_2,...,\Xi_n] \\ &\qquad \le \quad \left\langle \overbrace{[\mho*\mathcal{R}^{\urcorner}] * e^{[\mho*\mathcal{I}^{\urcorner}]}}, \overbrace{[\mho*\mathcal{R}^{\lrcorner}] * e^{[\mho*\mathcal{I}^{\lrcorner}]}}, \underbrace{[\mho*\mathcal{R}^{F}] * e^{[\mho*\mathcal{I}^{F}]}} \right\rangle. \end{aligned}$$

[Boundedness property].

Proof. Since, $\underbrace{[\mho * \mathcal{R}^{\urcorner}]} = \min[\mho * \mathcal{R}^{\urcorner}_{ij}]$, $\overbrace{[\mho * \mathcal{R}^{\urcorner}]} = \max[\mho * \mathcal{R}^{\urcorner}_{ij}]$ and $\underbrace{[\mho * \mathcal{R}^{\urcorner}]} \leq [\mho * \mathcal{R}^{\urcorner}_{ij}] \leq \overbrace{[\mho * \mathcal{R}^{\urcorner}]}$ and $\underbrace{[\mho * \mathcal{I}^{\urcorner}]} = \min[\mho * \mathcal{I}^{\urcorner}_{ij}]$, $\overbrace{[\mho * \mathcal{I}^{\urcorner}]} = \max[\mho * \mathcal{I}^{\urcorner}_{ij}]$ and $\underbrace{[\mho * \mathcal{I}^{\urcorner}]} \leq [\mho * \mathcal{I}^{\urcorner}_{ij}] \leq \overbrace{[\mho * \mathcal{I}^{\urcorner}]}$.

Now $\underbrace{[\mho * \mathcal{R}^{\urcorner}]} * e^{\underbrace{[\mho * \mathcal{I}^{\urcorner}]}}$

$$\begin{aligned}
&= \sqrt[\varsigma]{1 - \sqcap_{i=1}^{n}\left[1 - [\underbrace{[\mho * \mathcal{R}^{\urcorner}]}]^{\varsigma}\right]^{\varkappa_i}} * e^{\sqrt[\varsigma]{1 - \sqcap_{i=1}^{n}\left[1 - [\underbrace{[\mho * \mathcal{I}^{\urcorner}]}]^{\varsigma}\right]^{\varkappa_i}}} \\
&\leq \sqrt[\varsigma]{1 - \sqcap_{i=1}^{n}\left[1 - [[[\mho * \mathcal{R}^{\urcorner}_{ij}]]]^{\varsigma}\right]^{\varkappa_i}} * e^{\sqrt[\varsigma]{1 - \sqcap_{i=1}^{n}\left[1 - [[[\mho * \mathcal{I}^{\urcorner}_{ij}]]]^{\varsigma}\right]^{\varkappa_i}}} \\
&\leq \sqrt[\varsigma]{1 - \sqcap_{i=1}^{n}\left[1 - [\overbrace{[\mho * \mathcal{R}^{\urcorner}]}]^{\varsigma}\right]^{\varkappa_i}} * e^{\sqrt[\varsigma]{1 - \sqcap_{i=1}^{n}\left[1 - [\overbrace{[\mho * \mathcal{I}^{\urcorner}]}]^{\varsigma}\right]^{\varkappa_i}}} \\
&= \overbrace{[\mho * \mathcal{R}^{\urcorner}]}.
\end{aligned}$$

Since, $\underbrace{[\mho * \mathcal{R}^{\lrcorner}]} = \min[\mho * \mathcal{R}^{\lrcorner}_{ij}]$, $\overbrace{[\mho * \mathcal{R}^{\lrcorner}]} = \max[\mho * \mathcal{R}^{\lrcorner}_{ij}]$ and $\underbrace{[\mho * \mathcal{R}^{\lrcorner}]} \leq [\mho * \mathcal{R}^{\lrcorner}_{ij}] \leq \overbrace{[\mho * \mathcal{R}^{\lrcorner}]}$ and $\underbrace{[\mho * \mathcal{I}^{\lrcorner}]} = \min[\mho * \mathcal{I}^{\lrcorner}_{ij}]$, $\overbrace{[\mho * \mathcal{I}^{\lrcorner}]} = \max[\mho * \mathcal{I}^{\lrcorner}_{ij}]$ and $\underbrace{[\mho * \mathcal{I}^{\lrcorner}]} \leq [\mho * \mathcal{I}^{\lrcorner}_{ij}] \leq \overbrace{[\mho * \mathcal{I}^{\lrcorner}]}$.

Now, $\underbrace{[\mho * \mathcal{R}^{\lrcorner}]} * e^{\underbrace{[\mho * \mathcal{I}^{\lrcorner}]}}$

$$\begin{aligned}
&= \sqrt[\partial]{1 - \sqcap_{i=1}^{n}\left[1 - [\underbrace{[\mho * \mathcal{R}^{\lrcorner}]}]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1 - \sqcap_{i=1}^{n}\left[1 - [\underbrace{[\mho * \mathcal{I}^{\lrcorner}]}]^{\partial}\right]^{\varkappa_i}}} \\
&\leq \sqrt[\partial]{1 - \sqcap_{i=1}^{n}\left[1 - [[\mho * \mathcal{R}^{\lrcorner}_{ij}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1 - \sqcap_{i=1}^{n}\left[1 - [[\mho * \mathcal{I}^{\lrcorner}_{ij}]]^{\partial}\right]^{\varkappa_i}}} \\
&\leq \sqrt[\partial]{1 - \sqcap_{i=1}^{n}\left[1 - [\overbrace{[\mho * \mathcal{R}^{\lrcorner}]}]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1 - \sqcap_{i=1}^{n}\left[1 - [\overbrace{[\mho * \mathcal{I}^{\lrcorner}]}]^{\partial}\right]^{\varkappa_i}}} \\
&= \overbrace{[\mho * \mathcal{R}^{\lrcorner}]} * e^{\overbrace{[\mho * \mathcal{I}^{\lrcorner}]}}.
\end{aligned}$$

Since, $\underbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]} = \min[[\mho * \mathcal{R}^{F}_{ij}]]^{\epsilon}$, $\overbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]} = \max[[\mho * \mathcal{R}^{F}_{ij}]]^{\epsilon}$ and $\underbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]} \leq [[\mho * \mathcal{R}^{F}_{ij}]]^{\epsilon} \leq \overbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]}$ and $\underbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]} = \min[[\mho * \mathcal{I}^{F}_{ij}]]^{\epsilon}$, $\overbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]} = \max[[\mho * \mathcal{I}^{F}_{ij}]]^{\epsilon}$ and $\underbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]} \leq [[\mho * \mathcal{I}^{F}_{ij}]]^{\epsilon} \leq \overbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]}$.

We have,

$$\begin{aligned}\underbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}} &= \sqcap_{i=1}^{n} \overbrace{\underbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]}}^{\varkappa_i} * e^{\sqcap_{i=1}^{n} \overbrace{\underbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]}}^{\varkappa_i}} \\ &\leq \sqcap_{i=1}^{n}[[[\mho * \mathcal{R}_{ij}^{F}]]^{\epsilon}]^{\varkappa_i} * e^{\sqcap_{i=1}^{n}[[[\mho * \mathcal{I}_{ij}^{F}]]^{\epsilon}]^{\varkappa_i}} \\ &\leq \sqcap_{i=1}^{n} \overbrace{\overbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]}}^{\varkappa_i} * e^{\sqcap_{i=1}^{n} \overbrace{\overbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]}}^{\varkappa_i}} \\ &= \overbrace{[\mho * [\mathcal{R}^{F}]^{\epsilon}]} * e^{\overbrace{[\mho * [\mathcal{I}^{F}]^{\epsilon}]}}.\end{aligned}$$

Therefore,

$$\begin{aligned} &\frac{1}{2} \times \left(\begin{array}{c} \left(\begin{array}{c} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - [\underbrace{[\mho * \mathcal{R}^{\urcorner}]}]^{\zeta} \right]^{\varkappa_i}} \right]^2 - \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [\underbrace{[\mho * \mathcal{R}^{\lrcorner}]}]^{\partial} \right]^{\varkappa_i}} \right]^2 \\ +1 - \left[\sqcap_{i=1}^{n} [[\overbrace{[\mho * \mathcal{R}^{F}]}]^{\epsilon}]^{\varkappa_i} \right]^2 \end{array} \right) \\ + \left(\begin{array}{c} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - [\underbrace{[\mho * \mathcal{I}^{\urcorner}]}]^{\zeta} \right]^{\varkappa_i}} \right]^2 - \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [\underbrace{[\mho * \mathcal{I}^{\lrcorner}]}]^{\partial} \right]^{\varkappa_i}} \right]^2 \\ - \left[\sqcap_{i=1}^{n} [[\overbrace{[\mho * \mathcal{I}^{F}]}]^{\epsilon}]^{\varkappa_i} \right]^2 \end{array} \right) \end{array} \right) \\ \leq\ &\frac{1}{2} \times \left(\begin{array}{c} \left(\begin{array}{c} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * [\mho * \mathcal{R}_{ij}^{\urcorner}]]]^{\zeta} \right]^{\varkappa_i}} \right]^2 - \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * \mathcal{R}_{ij}^{\lrcorner}]]^{\partial} \right]^{\varkappa_i}} \right]^2 + \\ +1 - \left[\sqcap_{i=1}^{n} [[[\mho * \mathcal{R}_{ij}^{F}]]^{\epsilon}]^{\varkappa_i} \right]^2 \end{array} \right) \\ + \left(\begin{array}{c} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * [\mho * \mathcal{I}_{ij}^{\urcorner}]]]^{\zeta} \right]^{\varkappa_i}} \right]^2 - \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * \mathcal{I}_{ij}^{\lrcorner}]]^{\partial} \right]^{\varkappa_i}} \right]^2 + \\ - \left[\sqcap_{i=1}^{n} [[[\mho * \mathcal{I}_{ij}^{F}]]^{\epsilon}]^{\varkappa_i} \right]^2 \end{array} \right) \end{array} \right) \\ \leq\ &\frac{1}{2} \times \left(\begin{array}{c} \left(\begin{array}{c} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - [\overbrace{[\mho * \mathcal{R}^{\urcorner}]}]^{\zeta} \right]^{\varkappa_i}} \right]^2 - \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [\overbrace{[\mho * \mathcal{R}^{\lrcorner}]}]^{\partial} \right]^{\varkappa_i}} \right]^2 + \\ +1 - \left[\sqcap_{i=1}^{n} [\underbrace{[\mho * \mathcal{R}^{F}]}]^{\epsilon}]^{\varkappa_i} \right]^2 \end{array} \right) \\ + \left(\begin{array}{c} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - [\overbrace{[\mho * \mathcal{I}^{\urcorner}]}]^{\zeta} \right]^{\varkappa_i}} \right]^2 - \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [\overbrace{[\mho * \mathcal{I}^{\lrcorner}]}]^{\partial} \right]^{\varkappa_i}} \right]^2 + \\ - \left[\sqcap_{i=1}^{n} [\underbrace{[\mho * \mathcal{I}^{F}]}]^{\epsilon}]^{\varkappa_i} \right]^2 \end{array} \right) \end{array} \right). \end{aligned}$$

Hence,

$$\left\langle \underbrace{[\mho * \mathcal{R}^{\urcorner}] * e^{[\mho * \mathcal{I}^{\urcorner}]}}, \underbrace{[\mho * \mathcal{R}^{\lrcorner}] * e^{[\mho * \mathcal{I}^{\lrcorner}]}}, \overbrace{[\mho * \mathcal{R}^{F}] * e^{[\mho * \mathcal{I}^{F}]}} \right\rangle$$
$$\leq CTT(\zeta, \partial, \epsilon)NWA[\Xi_1, \Xi_2, ..., \Xi_n]$$
$$\leq \langle \overbrace{[\mho * \mathcal{R}^{\urcorner}] * e^{[\mho * \mathcal{I}^{\urcorner}]}}, \overbrace{[\mho * \mathcal{R}^{\lrcorner}] * e^{[\mho * \mathcal{I}^{\lrcorner}]}}, \underbrace{[\mho * \mathcal{R}^{F}] * e^{[\mho * \mathcal{I}^{F}]}} \rangle. \qquad \square$$

Theorem 4.5. *Let* $\Xi_i = \langle [[\mho * \mathcal{R}_{t_{ij}}^{\urcorner}] * e^{[\mho * \mathcal{I}_{t_{ij}}^{\urcorner}]}, [\mho * \mathcal{R}_{t_{ij}}^{\lrcorner}] * e^{[\mho * \mathcal{I}_{t_{ij}}^{\lrcorner}]}, [\mho * \mathcal{R}_{t_{ij}}^{F}] * e^{[\mho * \mathcal{I}_{t_{ij}}^{F}]}] \rangle$ *and* $W_i = \langle [[\mho * \mathcal{R}_{h_{ij}}^{\urcorner}] * e^{[\mho * \mathcal{I}_{h_{ij}}^{\urcorner}]}, [\mho * \mathcal{R}_{h_{ij}}^{\lrcorner}] * e^{[\mho * \mathcal{I}_{h_{ij}}^{\lrcorner}]}, [\mho * \mathcal{R}_{h_{ij}}^{F}] * e^{[\mho * \mathcal{I}_{h_{ij}}^{F}]}] \rangle$*, be the CTT* $(\zeta, \partial, \epsilon)$ *NWAs. For any i, if there is* $[\mho * \mathcal{R}_{t_{ij}}^{\urcorner}]^2 \leq [\mho * \mathcal{R}_{h_{ij}}^{\urcorner}]^2$ *and* $[\mho * \mathcal{R}_{t_{ij}}^{\lrcorner}]^2 \leq [\mho * \mathcal{R}_{h_{ij}}^{\lrcorner}]^2$ *and* $[\mho * \mathcal{R}_{t_{ij}}^{F}]^2 \geq [\mho * \mathcal{R}_{h_{ij}}^{F}]^2$ *and* $[\mho * \mathcal{I}_{t_{ij}}^{\urcorner}]^2 \leq [\mho * \mathcal{I}_{h_{ij}}^{\urcorner}]^2$ *and* $[\mho * \mathcal{I}_{t_{ij}}^{\lrcorner}]^2 \leq [\mho * \mathcal{I}_{h_{ij}}^{\lrcorner}]^2$ *and* $[\mho * \mathcal{I}_{t_{ij}}^{F}]^2 \geq [\mho * \mathcal{I}_{h_{ij}}^{F}]^2$*or* $\Xi_i \leq W_i$*. Prove that CTT* $(\zeta, \partial, \epsilon)NWA[\Xi_1, \Xi_2, ..., \Xi_n] \leq CTT(\zeta, \partial, \epsilon)NWA[W_1, W_2, ..., W_n]$*, where* $[i = 1, 2, ..., n]; [j = 1, 2, ..., i_j]$ *[monotonicity property].*

Proof. For any i, $[\mho * \mathcal{R}_{t_{ij}}^{\urcorner}]^2 \leq [\mho * \mathcal{R}_{h_{ij}}^{\urcorner}]^2$.
Therefore, $1 - [[\mho * \mathcal{R}_{t_i}^{\urcorner}]]^2 \geq 1 - [[\mho * \mathcal{R}_{h_i}^{\urcorner}]]^2$.

Hence, $\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{t_i}^{\daleth}]\right]^2\right]^{\varkappa_i}\geq\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{h_i}^{\daleth}]\right]^2\right]^{\varkappa_i}$
and $\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{t_i}^{\daleth}]\right]^{\zeta}\right]^{\varkappa_i}}\leq\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{h_i}^{\daleth}]\right]^{\zeta}\right]^{\varkappa_i}}$.
Similarly, $[\mho*\mathcal{I}_{t_{ij}}^{\daleth}]^2\leq[\mho*\mathcal{I}_{h_{ij}}^{\daleth}]^2$.
Therefore, $1-[[\mho*\mathcal{I}_{t_i}^{\daleth}]]^2\geq1-[[\mho*\mathcal{I}_{h_i}^{\daleth}]]^2$.
Hence, $\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{t_i}^{\daleth}]\right]^2\right]^{\varkappa_i}\geq\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{h_i}^{\daleth}]\right]^2\right]^{\varkappa_i}$
and $\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{t_i}^{\daleth}]\right]^{\zeta}\right]^{\varkappa_i}}\leq\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{h_i}^{\daleth}]\right]^{\zeta}\right]^{\varkappa_i}}$.
For any i, $[\mho*\mathcal{R}_{t_{ij}}^{\beth}]^{\partial}\leq\left[[\mho*\mathcal{R}_{h_{ij}}^{\beth}]\right]^{\partial}$.
Therefore, $1-\left[[\mho*\mathcal{R}_{t_i}^{\beth}]\right]^{\partial}\geq1-\left[[\mho*\mathcal{R}_{h_i}^{\beth}]\right]^{\partial}$.
Hence, $\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{t_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}\geq\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{h_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}$.
This implies that $\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{t_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}}\leq\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{R}_{h_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}}$.

Similarly, for any i, $[\mho*\mathcal{I}_{t_{ij}}^{\beth}]^{\partial}\leq\left[[\mho*\mathcal{I}_{h_{ij}}^{\beth}]\right]^{\partial}$.
Therefore, $1-\left[[\mho*\mathcal{I}_{t_i}^{\beth}]\right]^{\partial}\geq1-\left[[\mho*\mathcal{I}_{h_i}^{\beth}]\right]^{\partial}$.
Hence, $\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{t_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}\geq\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{h_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}$.
This implies that $\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{t_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}}\leq\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[\mho*\mathcal{I}_{h_i}^{\beth}]\right]^{\partial}\right]^{\varkappa_i}}$.

For any i, $\left[[\mho*\mathcal{R}_{t_{ij}}^{F}]\right]^2\geq\left[[\mho*\mathcal{R}_{h_{ij}}^{F}]\right]^2$ and $\left[[\mho*\mathcal{R}_{t_{ij}}^{F}]\right]^{\epsilon}\geq\left[[\mho*\mathcal{R}_{h_{ij}}^{F}]\right]^{\epsilon}$.
Therefore, $1-\frac{\left[\sqcap_{i=1}^{n}[\mho*\mathcal{R}_{t_{ij}}^{F}]\right]^{\epsilon}}{2}\leq1-\frac{\left[\sqcap_{i=1}^{n}[\mho*\mathcal{R}_{h_{ij}}^{F}]\right]^{\epsilon}}{2}$.
Similarly, for any i,
$\left[[\mho*\mathcal{I}_{t_{ij}}^{F}]\right]^2\geq\left[[\mho*\mathcal{I}_{h_{ij}}^{F}]\right]^2$ and $\left[[\mho*\mathcal{I}_{t_{ij}}^{F}]\right]^{\epsilon}\geq\left[[\mho*\mathcal{I}_{h_{ij}}^{F}]\right]^{\epsilon}$.
Therefore, $-\left[\sqcap_{i=1}^{n}[\mho*\mathcal{I}_{t_{ij}}^{F}]\right]^{\epsilon}\leq-\left[\sqcap_{i=1}^{n}[\mho*\mathcal{I}_{h_{ij}}^{F}]\right]^{\epsilon}$.

Hence,

$$\frac{1}{2}\times\left(\begin{array}{c}\left(\begin{array}{c}\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{R}_{ti}^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}\right]^2-\left[\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{R}_{t_i}^{\beth}]]^{\partial}\right]^{\varkappa_i}}\right]^2\\+1-\left[\sqcap_{i=1}^{n}[[\mho*\mathcal{R}_{t_i}^{F}]]^{\epsilon}\right]^2\end{array}\right)\\+\left(\begin{array}{c}\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{I}_{ti}^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}\right]^2-\left[\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{I}_{t_i}^{\beth}]]^{\partial}\right]^{\varkappa_i}}\right]^2\\-\left[\sqcap_{i=1}^{n}[[\mho*\mathcal{I}_{t_i}^{F}]]^{\epsilon}\right]^2\end{array}\right)\end{array}\right)$$
$$\leq\quad\frac{1}{2}\times\left(\begin{array}{c}\left(\begin{array}{c}\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{R}_{hi}^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}\right]^2-\left[\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{R}_{h_i}^{\beth}]]^{\partial}\right]^{\varkappa_i}}\right]^2\\+1-\left[\sqcap_{i=1}^{n}[[\mho*\mathcal{R}_{h_i}^{F}]]^{\epsilon}\right]^2\end{array}\right)\\+\left(\begin{array}{c}\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{I}_{hi}^{\daleth}]]^{\zeta}\right]^{\varkappa_i}}\right]^2-\left[\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-[[\mho*\mathcal{I}_{h_i}^{\beth}]]^{\partial}\right]^{\varkappa_i}}\right]^2\\-\left[\sqcap_{i=1}^{n}[[\mho*\mathcal{I}_{h_i}^{F}]]^{\epsilon}\right]^2\end{array}\right)\end{array}\right).$$

Hence, CTT $(\zeta,\partial,\epsilon)NWA\,[\Xi_1,\Xi_2,...,\Xi_n]\leq CTT(\zeta,\partial,\epsilon)NWA\,[W_1,W_2,...,W_n]$. □

4.2 CTT $(\zeta, \partial, \epsilon)$ NWG

Definition 4.6. Let $\Xi_i = \left\langle \left[[[\mho * \mathcal{R}_i^{\urcorner}] * e^{[\mho * \mathcal{I}_i^{\urcorner}]}, [\mho * \mathcal{R}_i^{\lrcorner}] * e^{[\mho * \mathcal{I}_i^{\lrcorner}]}, [\mho * \mathcal{R}_i^{\ulcorner}] * e^{[\mho * \mathcal{I}_i^{\ulcorner}]}] \right] \right\rangle$ be the CTT $(\zeta, \partial, \epsilon)$ NNs. Then $(\zeta, \partial, \epsilon)$NWG $[\Xi_1, \Xi_2, ..., \Xi_n] = \sqcap_{i=1}^{n} \Xi_i^{\varkappa_i}$.

Corollary 4.7. *Let* $\Xi_i = \left\langle \left[[[\mho * \mathcal{R}_i^{\urcorner}] * e^{[\mho * \mathcal{I}_i^{\urcorner}]}, [\mho * \mathcal{R}_i^{\lrcorner}] * e^{[\mho * \mathcal{I}_i^{\lrcorner}]}, [\mho * \mathcal{R}_i^{\ulcorner}] * e^{[\mho * \mathcal{I}_i^{\ulcorner}]}] \right] \right\rangle$ *be the CTT* $(\zeta, \partial, \epsilon)$ *NNs. Then CTT* $(\zeta, \partial, \epsilon)$ *NWG* $[\Xi_1, \Xi_2, ..., \Xi_n]$

$$= \begin{pmatrix} \sqcap_{i=1}^{n} [[[\mho * \mathcal{R}_i^{\urcorner}]]^{\zeta}]^{\varkappa_i} * e^{\sqcap_{i=1}^{n} [[[\mho * \mathcal{I}_i^{\urcorner}]]^{\zeta}]^{\varkappa_i}} \\ \sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * \mathcal{R}_i^{\lrcorner}]]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * \mathcal{I}_i^{\lrcorner}]]^{\partial}\right]^{\varkappa_i}}} \\ \sqrt[\epsilon]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * \mathcal{R}_i^{\ulcorner}]]^{\epsilon}\right]^{\varkappa_i}} * e^{\sqrt[\epsilon]{1 - \sqcap_{i=1}^{n} \left[1 - [[\mho * \mathcal{I}_i^{\ulcorner}]]^{\epsilon}\right]^{\varkappa_i}}} \end{pmatrix}.$$

Corollary 4.8. *Let* $\Xi_i = \left\langle \left[[[\mho * \mathcal{R}_i^{\urcorner}] * e^{[\mho * \mathcal{I}_i^{\urcorner}]}, [\mho * \mathcal{R}_i^{\lrcorner}] * e^{[\mho * \mathcal{I}_i^{\lrcorner}]}, [\mho * \mathcal{R}_i^{\ulcorner}] * e^{[\mho * \mathcal{I}_i^{\ulcorner}]}] \right] \right\rangle$ *be the CTT* $(\zeta, \partial, \epsilon)$ *NNs and all are equal. Then* $(\zeta, \partial, \epsilon)$*NWG*$[\Xi_1, \Xi_2, ..., \Xi_n] = \Xi$.

Remark 4.9. It has other properties, including boundedness and monotonicity, as well as having $(\zeta, \partial, \epsilon)$NWG.

4.3 Generalized CTT $(\zeta, \partial, \epsilon)$ NWA [GCTT$(\zeta, \partial, \epsilon)$NWA]

Definition 4.10. Let $\Xi_i = \left\langle \ [[[\mho * \mathcal{R}_i^{\urcorner}], [\mho * \mathcal{R}_i^{\lrcorner}], [\mho * \mathcal{R}_i^{\ulcorner}]]] \ \right\rangle$ be the CTT $(\zeta, \partial, \epsilon)$ NN. Then GCTT $(\zeta, \partial, \epsilon)$ NWA $[\Xi_1, \Xi_2, ..., \Xi_n] = \left[\sqcup_{i=1}^{n} \varkappa_i \Xi_i^{\wp} \right]^{1/\wp}$.

Theorem 4.11. *Let* $\Xi_i = \left\langle \ [[[\mho * \mathcal{R}_i^{\urcorner}], [\mho * \mathcal{R}_i^{\lrcorner}], [\mho * \mathcal{R}_i^{\ulcorner}]]] \ \right\rangle$ *be the CTT* $(\zeta, \partial, \epsilon)$ *NNs. Then GCTT* $(\zeta, \partial, \epsilon)$ *NWA* $[\Xi_1, \Xi_2, ..., \Xi_n]$

$$= \begin{pmatrix} \left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - \left[[[\mho * \mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}} \right]^{1/\zeta} * e^{\left[\sqrt[\zeta]{1 - \sqcap_{i=1}^{n} \left[1 - \left[[[\mho * \mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}} \right]^{1/\zeta}}, \\ \left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - \left[[[\mho * \mathcal{R}_i^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}} \right]^{1/\partial} * e^{\left[\sqrt[\partial]{1 - \sqcap_{i=1}^{n} \left[1 - \left[[[\mho * \mathcal{I}_i^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}} \right]^{1/\partial}}, \\ \sqrt[\epsilon]{1 - \left[1 - \left[\sqcap_{i=1}^{n} \left[\sqrt[\epsilon]{1 - \left[1 - [[\mho * \mathcal{R}_i^{\ulcorner}]]^{\epsilon}\right]^{\epsilon}} \right]^{\varkappa_i} \right]^{\epsilon} \right]^{1/\epsilon}} * \\ e^{\sqrt[\epsilon]{1 - \left[1 - \left[\sqcap_{i=1}^{n} \left[\sqrt[\epsilon]{1 - \left[1 - [[\mho * \mathcal{I}_i^{\ulcorner}]]^{\epsilon}\right]^{\epsilon}} \right]^{\varkappa_i} \right]^{\epsilon} \right]^{1/\epsilon}}} \end{pmatrix}.$$

Proof. To illustrate this, we may first show that,

$$\sqcup_{i=1}^{n}\varkappa_i\Xi_i^{\zeta}=\begin{pmatrix}\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}*e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}}\\ \sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{R}_i^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}*e^{\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{I}_i^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}}\\ \sqcap_{i=1}^{n}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_i^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}*e^{\sqcap_{i=1}^{n}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_i^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}}\end{pmatrix}.$$

Put $n=2$, $\varkappa_1\Xi_1\sqcup\varkappa_2\Xi_2$

$$=\begin{pmatrix}\sqrt[\zeta]{\begin{matrix}\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{R}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}+\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{R}_2^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}\\-\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{R}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}*\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{R}_2^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}\end{matrix}}\\ *e^{\sqrt[\zeta]{\begin{matrix}\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{I}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}+\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{I}_2^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}\\-\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{I}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}*\left[\sqrt[\zeta]{1-\left[1-\left[[[\mho*\mathcal{I}_2^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}\end{matrix}}},\\ \sqrt[\partial]{\begin{matrix}\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{R}_1^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}+\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{R}_2^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}\\-\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{R}_1^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}*\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{R}_2^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}\end{matrix}}\\ *e^{\sqrt[\partial]{\begin{matrix}\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{I}_1^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}+\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{I}_2^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}\\-\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{I}_1^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}*\left[\sqrt[\partial]{1-\left[1-\left[[[\mho*\mathcal{I}_2^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}\end{matrix}}}\\ \left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_1^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_1}*\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_2^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_1}\\ *e^{\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_1^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_1}*\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_2^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_1}}\end{pmatrix}$$

$$=\begin{pmatrix}\sqrt[\zeta]{1-\sqcap_{i=1}^{2}\left[1-\left[[[\mho*\mathcal{R}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}*e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{2}\left[1-\left[[[\mho*\mathcal{I}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}}\\ \sqrt[\partial]{1-\sqcap_{i=1}^{2}\left[1-\left[[[\mho*\mathcal{R}_1^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}*e^{\sqrt[\partial]{1-\sqcap_{i=1}^{2}\left[1-\left[[[\mho*\mathcal{I}_1^{\lrcorner}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}}\\ \sqcap_{i=1}^{2}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_i^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}*\sqcap_{i=1}^{2}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_i^{\digamma}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}\end{pmatrix}.$$

Hence,

$$\sqcup_{i=1}^{l}\varkappa_i\Xi_i^{\wp} = \begin{pmatrix} \sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{R}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{I}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}} \\ \sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{R}_1^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{I}_1^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}} \\ \sqcap_{i=1}^{l}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i} * e^{\sqcap_{i=1}^{l}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}} \end{pmatrix}.$$

If $n = l+1$, then $\sqcup_{i=1}^{l}\varkappa_i\Xi_i^{\wp} + \varkappa_{l+1}\Xi_{l+1}^{\wp} = \sqcup_{i=1}^{l+1}\varkappa_i\Xi_i^{\wp}$.
Now, $\sqcup_{i=1}^{l}\varkappa_i\Xi_i^{\wp} + \varkappa_{l+1}\Xi_{l+1}^{\wp} = \varkappa_1\Xi_1^{\wp} \sqcup \varkappa_2\Xi_2^{\wp} \sqcup ... \sqcup \varkappa_l\Xi_l^{\wp} \sqcup \varkappa_{l+1}\Xi_{l+1}^{\wp}$

$$= \begin{pmatrix} \sqrt[\zeta]{\begin{matrix}\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}\right]^{\zeta} + \left[\sqrt[\zeta]{1-\left[1-\left[[\mathcal{R}_{l+1}^{\urcorner}]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta} \\ -\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}\right]^{\zeta} * \left[\sqrt[\zeta]{1-\left[1-\left[[\mathcal{R}_{l+1}^{\urcorner}]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}\end{matrix}} \\ * e^{\sqrt[\zeta]{\begin{matrix}\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}\right]^{\zeta} + \left[\sqrt[\zeta]{1-\left[1-\left[[\mathcal{I}_{l+1}^{\urcorner}]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta} \\ -\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}\right]^{\zeta} * \left[\sqrt[\zeta]{1-\left[1-\left[[\mathcal{I}_{l+1}^{\urcorner}]^{\zeta}\right]^{\zeta}\right]^{\varkappa_1}}\right]^{\zeta}\end{matrix}}} \\ \sqrt[\partial]{\begin{matrix}\left[\sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{R}_i^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{\partial} + \left[\sqrt[\partial]{1-\left[1-\left[[\mathcal{R}_{l+1}^{\gimel}]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial} \\ -\left[\sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{R}_i^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{\partial} * \left[\sqrt[\partial]{1-\left[1-\left[[\mathcal{R}_{l+1}^{\gimel}]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}\end{matrix}} \\ * e^{\sqrt[\partial]{\begin{matrix}\left[\sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{I}_i^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{\partial} + \left[\sqrt[\partial]{1-\left[1-\left[[\mathcal{I}_{l+1}^{\gimel}]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial} \\ -\left[\sqrt[\partial]{1-\sqcap_{i=1}^{l}\left[1-\left[[[\mho*\mathcal{I}_i^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{\partial} * \left[\sqrt[\partial]{1-\left[1-\left[[\mathcal{I}_{l+1}^{\gimel}]^{\partial}\right]^{\partial}\right]^{\varkappa_1}}\right]^{\partial}\end{matrix}}} \\ \sqcap_{i=1}^{l}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i} * \left[\sqrt[\epsilon]{1-\left[1-[\mathcal{R}_{l+1}^{F}]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_1} \\ * e^{\sqcap_{i=1}^{l}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i} * \left[\sqrt[\epsilon]{1-\left[1-[\mathcal{I}_{l+1}^{F}]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_1}} \end{pmatrix}$$

$$\sqcup_{i=1}^{l+1}\varkappa_i\Xi_i^{\zeta} = \begin{pmatrix} \sqrt[\zeta]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{R}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}} * e^{\sqrt[\zeta]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{I}_1^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}} \\ \sqrt[\partial]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{R}_1^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}} * e^{\sqrt[\partial]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{I}_1^{\gimel}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}} \\ \sqcap_{i=1}^{l+1}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i} * e^{\sqcap_{i=1}^{l+1}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}} \end{pmatrix}.$$

$$\left[\sqcup_{i=1}^{l+1}\varkappa_i\Xi_i^{\wp}\right]^{1/\wp} = \left(\begin{array}{c} \left[\sqrt[\zeta]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}\right]^{1/\zeta} * e^{\left[\sqrt[\zeta]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}\right]^{\varkappa_i}}\right]^{1/\zeta}} \\ \left[\sqrt[\partial]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{R}_i^{\daleth}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{1/\partial} * e^{\left[\sqrt[\partial]{1-\sqcap_{i=1}^{l+1}\left[1-\left[[[\mho*\mathcal{I}_i^{\daleth}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{1/\partial}} \\ \sqrt[\epsilon]{1-\left[1-\left[\sqcap_{i=1}^{l+1}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}\right]^{2}\right]^{1/\epsilon}} * \\ e^{\sqrt[\epsilon]{1-\left[1-\left[\sqcap_{i=1}^{l+1}\left[\sqrt[\epsilon]{1-\left[1-[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}\right]^{\epsilon}}\right]^{\varkappa_i}\right]^{2}\right]^{1/\epsilon}}} \end{array}\right)$$

□

Remark 4.12. If $(\zeta, \partial, \epsilon) = [1, 1, 1]$, then CTT $(\zeta, \partial, \epsilon)$ NWA operator is used instead of the GCTT $(\zeta, \partial, \epsilon)$ NWA operator.

Theorem 4.13. *If all* $\Xi_i = \left\langle\left[[[\mho*\mathcal{R}_i^{\urcorner}]*e^{[\mho*\mathcal{I}_i^{\urcorner}]}, [\mho*\mathcal{R}_i^{\daleth}]*e^{[\mho*\mathcal{I}_i^{\daleth}]}, [\mho*\mathcal{R}_i^{F}]*e^{[\mho*\mathcal{I}_i^{F}]}]\right]\right\rangle$ *and all are equal. Then GCTT* $(\zeta, \partial, \epsilon)$ *NWA*$[\Xi_1, \Xi_2, ..., \Xi_n] = \Xi$.

Remark 4.14. The GCTT $(\zeta, \partial, \epsilon)$ NWA operator meets both boundedness and monotonicity constraints.

4.4 Generalized CTT $(\zeta, \partial, \epsilon)$NWG [GCTT $(\zeta, \partial, \epsilon)$NWG]

Definition 4.15. Let $\Xi_i = \left\langle\left[[[\mho*\mathcal{R}_i^{\urcorner}]*e^{[\mho*\mathcal{I}_i^{\urcorner}]}, [\mho*\mathcal{R}_i^{\daleth}]*e^{[\mho*\mathcal{I}_i^{\daleth}]}, [\mho*\mathcal{R}_i^{F}]*e^{[\mho*\mathcal{I}_i^{F}]}]\right]\right\rangle$ be the CTT $(\zeta, \partial, \epsilon)$ NNs. Then GCTT$(\zeta, \partial, \epsilon)$NWG $[\Xi_1, \Xi_2, ..., \Xi_n] = \frac{1}{\wp}\left[\sqcap_{i=1}^{n}\left[\wp\Xi_i\right]^{\varkappa_i}\right]$.

Corollary 4.16. *Let* $\Xi_i = \left\langle\left[[[\mho*\mathcal{R}_i^{\urcorner}]*e^{[\mho*\mathcal{I}_i^{\urcorner}]}, [\mho*\mathcal{R}_i^{\daleth}]*e^{[\mho*\mathcal{I}_i^{\daleth}]}, [\mho*\mathcal{R}_i^{F}]*e^{[\mho*\mathcal{I}_i^{F}]}]\right]\right\rangle$ *be the CTT* $(\zeta, \partial, \epsilon)$ *NNs. Then GCTT*$(\zeta, \partial, \epsilon)$*NWG*$[\Xi_1, \Xi_2, ..., \Xi_n]$

$$= \left(\begin{array}{c} \sqrt[\zeta]{1-\left[1-\left[\sqcap_{i=1}^{n}\left[\sqrt[\zeta]{1-\left[1-[[\mho*\mathcal{R}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}}\right]^{\varkappa_i}\right]^{\zeta}\right]^{1/\zeta}} * \\ e^{\sqrt[\zeta]{1-\left[1-\left[\sqcap_{i=1}^{n}\left[\sqrt[\zeta]{1-\left[1-[[\mho*\mathcal{I}_i^{\urcorner}]]^{\zeta}\right]^{\zeta}}\right]^{\varkappa_i}\right]^{\zeta}\right]^{1/\zeta}}} \\ \left[\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{R}_i^{\daleth}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{1/\partial} * e^{\left[\sqrt[\partial]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{I}_i^{\daleth}]]^{\partial}\right]^{\partial}\right]^{\varkappa_i}}\right]^{1/\partial}} \\ \left[\sqrt[\epsilon]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{R}_i^{F}]]^{\epsilon}\right]^{\epsilon}\right]^{\varkappa_i}}\right]^{1/\epsilon} * e^{\left[\sqrt[\epsilon]{1-\sqcap_{i=1}^{n}\left[1-\left[[[\mho*\mathcal{I}_i^{F}]]^{\epsilon}\right]^{\epsilon}\right]^{\varkappa_i}}\right]^{1/\epsilon}} \end{array}\right).$$

Remark 4.17. When $\wp = 1$, the GCTT $(\zeta, \partial, \epsilon)$ NWG is converted to the $(\zeta, \partial, \epsilon)$NWG.

Remark 4.18. GCTT$(\zeta, \partial, \epsilon)$NWG operators satisfy the boundness and monotonicity characteristics.

Corollary 4.19. *If all* $\Xi_i = \left\langle\left[[[\mho*\mathcal{R}_i^{\urcorner}]*e^{[\mho*\mathcal{I}_i^{\urcorner}]}, [\mho*\mathcal{R}_i^{\daleth}]*e^{[\mho*\mathcal{I}_i^{\daleth}]}, [\mho*\mathcal{R}_i^{F}]*e^{[\mho*\mathcal{I}_i^{F}]}]\right]\right\rangle$ *are equal. Then GCTT*$(\zeta, \partial, \epsilon)$*NWG*$[\Xi_1, \Xi_2, ..., \Xi_n] = \Xi$.

5 Conclusion:

In this study, new weighted operators including averaging and geometric operators are introduced. These operators have several properties, such as monotonicity, idempotency, commutativity, associativity, and boundedness. We examined many standard metrics to characterize the weighted vector. Numerous criteria for aggregation operators have been examined. A few aggregating methods for such CTT $(\zeta, \partial, \epsilon)$ NNs have been studied, and some conclusions have been drawn about them.

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