# Epanechnikov-pareto Distribution with Application

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**Abstract**

In this article, we combined the Epanechnikov kernel function with the pareto distribution to produce the Epanechnikov-Pareto distribution (EPD). Some properties of this distribution are studied, like the moments, MLEs, reliability analysis functions, ordered statistics, and quintile function.

**Keywords:** Epanechnikov Pareto distribution; Epanechnikov distribution; Moments; Entropy; Order statistics; quintile function

## 1. Introduction

Kernel functions are used and applied in the theory of functions that achieve in a fixed domain, specific differential equation. One of these functions is the Epanechnikov Kernal Function (EKF) [1]. The Epanechnikov kernel function is a popular choice in kernel density estimation, which is a non-parametric way to estimate the probability density function of a random variable. The Epanechnikov kernel is defined as:

$$K(u) = \frac{3}{4}\, max(1 - u^2)$$

(EKF) is a continuous function and has the least mean square error (MSE).

EKF represents a probability density function.

In this paper, we will use the EKF and the Pareto distribution to generate a new distribution called the Epanechnikov-Pateto distribution (EPD).Many researchers used the distribution theory to generalize some distribution. [2] used the quadratic transmutation to suggest the transmuted Janardan distribution. [3] generalized two distributions: the transmuted Generalized Type-II Half-Logistics Distributions and the transmuted Gamma-Gompertz Distributions. [4] worked out the transmuted reciprocal and the transmuted two-parameter weighted exponential distributions. Used the quadratic transmutation map to generalize the power function and Type-I half-logistic distributions. [5] used Quadratic Transmutation Map to generalizations of Power Function and Type-I Half Logistic Distributions, EyobandShanker (2018) suggested a two–parameter weighted Garima distribution. EyobandShanker (2018) suggested a two–parameter weighted Garima distribution.

[6] suggested a two-parameter weighted Garima distribution.[20] used the quadratic transmutation map to propose the transmuted Aradhana distribution. [7] introduced the transmuted Gumbel-logistic distribution who used the inverse Weibull distribution as the kernel function. By applying a beta kernel function to a standard exponential distribution, [8] used the exponential distribution and the Epanechnikov kernel function to suggest a new distribution. [9] used the maximum likelihood estimator approach to estimate the reliability based on the Pareto distribution. In order to estimate the parameters and reliability, [10] presented a novel estimation method based on non-parametric kernel density estimation. [11] develop Mukherjee-Islam distribution into transmuted Mukherjee-Islam distribution. In this paper, we will use the EKF and the paretodistribution to generate a new distribution called the Epanechnikov-pareto distribution (EPD). Additional Work The key finding of the thesis is that, as research constantly generates new questions, we can use these new questions as seed ideas for further investigations. The work offered in this thesis offers the theoretical foundation for more research on the Epanechnikov kernel function with the pareto distribution and producing the Epanechnikov-Pareto distribution, as



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well as intriguing new research problems. We may suggest research issues to investigate further in the fields of fuzzy soft set theory and fixed point theory. For instance, we are hoping to compare our work with the works such as [12-18]. On the other hand, the huge works presented by many researchers [22-32] are considered important works in linking statistical tools and fuzzy tools [33-35].

## 2. Epanechnikov-pareto Distribution

The Pareto distribution is often described as the basis of the 80/20 rule. For example, 80% of customer complaints regarding a maker of vehicle typically arise from 20% of components. Other applications include the distribution of income and the classification of stock in a warehouse because of frequency of movement.

The probability density function of pareto is

$$f(x) = \theta x^{-\theta-1}\ x \geq 1$$

The cumulative distribution function of pareto is

$$F(x) = 1 - x^{-\theta}$$

The (EKF) density function is defined by [32] is given by

$$K(x) = \frac{3}{4}(1 - x^2\ )\ ,\quad |x| \leq 1$$

Probability density function of EP. Pareto distribution. EPD are given by the following theorem

Theorem (2.1).

A random variable is said to have an EPD if its CDF and pdf are respectively given by

$$G(x) = \frac{1}{2}(2 + x^{-3\theta} - 3x^{-2\theta})$$

And

$$g(x) = \frac{3\theta}{2}\ 2x^{-2\theta-1} - x^{-3\theta-1}\ )\ x \geq 1$$

Proof:

$$G(x) = 2\int_1^{F(x)} k(u)du$$

$$= 2\int_1^{1-x^{-\theta}} \frac{3}{4}(1 - u^2)du$$

$$G(x) = \frac{1}{2}\ (2 - 3x^{-2\theta} + x^{-3\theta}\ ) \ldots \ldots \ldots (2.1)$$

Hence, prove

By deriving (2.1) with respect to x we get

$$g(x) = \frac{3\theta}{2}\ 2x^{-2\theta-1} - x^{-3\theta-1}\ )\ x \geq 1 \quad \ldots \ldots (2.2)$$

$g(x)$ is probability density function where the integral of g(x) over the interval given one

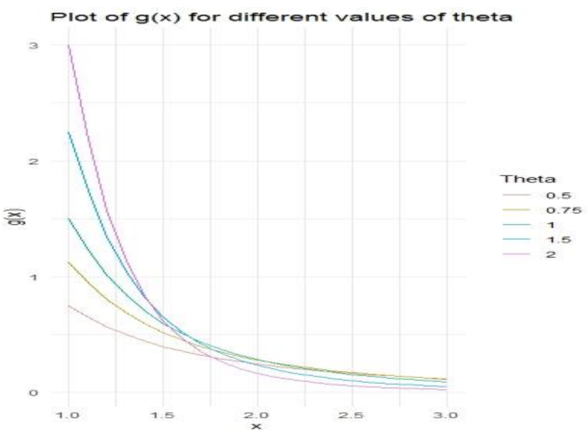


**Figure 1.** Plot of g(x) for different values of theta.



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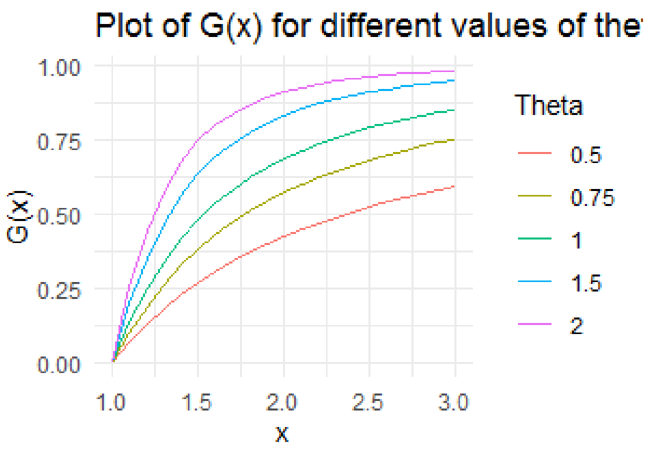


**Figure 2.** Plot of G(x) for different values of theta.

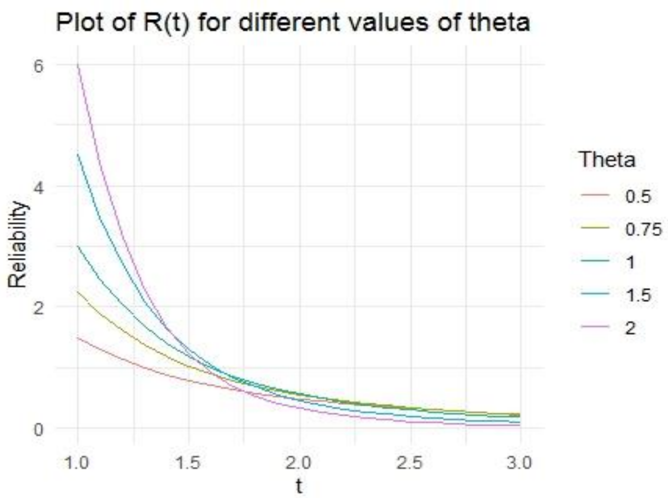


**Figure 3.** Plot of R(t) for different values of theta.

**3. The moment of the distribution:**

Let x be a random variable then the $r^{th}$ moment is given by

$$E(x^r) = \int_{-\infty}^{\infty} x^r g(x)dx$$

Theorem (3.1). Let x be a random variable belong to EPD then the $r^{th}$ moment is given by

$$E(x^r) = \frac{3\theta(4\theta + r)}{(2\theta + r)(3\theta + r)}$$

Proof.

$$E(x^r) = \frac{3\theta}{2}\int_{1}^{\infty}(x^r\, 2x^{-2\theta-1} - x^{-3\theta-1})dx$$

$$= \frac{3\theta}{2}\int_{1}^{\infty}(2x^{-2\theta+r-1} - x^{-3\theta+r-1})dx$$

$$= \frac{3\theta}{2}\left[\frac{-2}{2\theta + r}\, x^{-2\theta+r-1} + \frac{1}{3\theta + r}\, x^{-3\theta+r}\right]_1^{\infty}$$



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$$= \frac{3\theta}{2}\left(\frac{2}{2\theta + r} - \frac{1}{3\theta + r}\right)$$

$$E(x^r) = \frac{3\theta(4\theta + r)}{2(2\theta + r)(3\theta + r)}$$

Hence, prove

For $r = 1$

$$E(x) = \frac{3\theta}{2}\frac{4\theta + 1}{(2\theta + 1)(3\theta + 1)}$$

For $r = 2$

$$E(x\hat{}2) = \frac{3\theta}{2}\left[\frac{(4\theta + 2)}{(2\theta + 2)(3\theta + 2)}\right]$$

Theorem (3.2).

Let x be a random variable belong to EPD then the variance of the random variable is given by

$$v(x) = \frac{3\theta}{2}\left[\frac{2\theta}{(\theta + 1)(3\theta + 2)} - \frac{3\theta(4\theta + 1)^2(3\theta + 1)^2}{2(2\theta + 1)^2}\right)$$

Proof.

$$v(x) = E(x^2) - (E(x))^2$$

$$= \frac{3\theta}{2}\left[\frac{(4\theta + 2)}{(2\theta + 2)(3\theta + 2)}\right] - \frac{3\theta}{2}\left(\frac{4\theta + 1}{(2\theta + 1)(3\theta + 1)}\right)^2$$

By simplification we get

$$v(x) = \frac{3\theta}{2}\left[\frac{4\theta + 2}{2(\theta + 1)(3\theta + 2)} - \frac{3\theta(4\theta + 1)^2}{2(2\theta + 1)^2(3\theta + 1)^2}\right]$$

The values of the Mean and standard deviation of the EPD random variable X for different values of the parameter θ were computed and listed in Table (1). They are calculated using Excel. The table shows that the values of the mean and the standard deviation of the EPD random variable decrease by increasing the value of the parameter, (θ).

**Table 1:** The mean, standard deviation of EPD for different values of θ

| θ | mean | s.d | θ | mean | s.d |
|---|---|---|---|---|---|
| 0.1 | 0.13462 | 0.23027 | 2.3 | 0.79543 | 0.15843 |
| 0.2 | 0.24107 | 0.27659 | 2.4 | 0.80235 | 0.15410 |
| 0.3 | 0.32566 | 0.29142 | 2.5 | 0.80882 | 0.14999 |
| 0.4 | 0.39394 | 0.29306 | 2.6 | 0.81488 | 0.14608 |
| 0.5 | 0.45000 | 0.28847 | 2.7 | 0.82057 | 0.14237 |
| 0.6 | 0.49675 | 0.28088 | 2.8 | 0.82592 | 0.13884 |
| 0.7 | 0.53629 | 0.27192 | 2.9 | 0.83096 | 0.13547 |
| 0.8 | 0.57014 | 0.26246 | 3 | 0.83571 | 0.13225 |
| 0.9 | 0.59942 | 0.25296 | 3.1 | 0.84021 | 0.12918 |
| 1.0 | 0.62500 | 0.24367 | 3.2 | 0.84447 | 0.12625 |
| 1.1 | 0.64753 | 0.23473 | 3.3 | 0.84850 | 0.12345 |
| 1.2 | 0.66752 | 0.22621 | 3.4 | 0.85234 | 0.12076 |
| 1.3 | 0.68537 | 0.21813 | 3.5 | 0.85598 | 0.11819 |
| 1.4 | 0.70142 | 0.21049 | 3.6 | 0.85945 | 0.11572 |
| 1.5 | 0.71591 | 0.20327 | 3.7 | 0.86275 | 0.11335 |



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| | | | | | |
|---|---|---|---|---|---|
| 1.6 | 0.72906 | 0.19647 | 3.8 | 0.86590 | 0.11107 |
| 1.7 | 0.74106 | 0.19006 | 3.9 | 0.86892 | 0.10889 |
| 1.8 | 0.75204 | 0.18401 | 4 | 0.87179 | 0.10678 |
| 1.9 | 0.76213 | 0.17830 | 4.1 | 0.87455 | 0.10476 |
| 2.0 | 0.77143 | 0.17291 | 4.2 | 0.87719 | 0.10281 |
| 2.1 | 0.78003 | 0.16781 | 4.3 | 0.87972 | 0.10092 |
| 2.2 | 0.78801 | 0.16299 | 4.4 | 0.88215 | 0.09911 |
| 4.5 | 0.88448 | 0.09736 | 6.7 | 0.91953 | 0.07006 |
| 4.6 | 0.88672 | 0.09567 | 6.8 | 0.92062 | 0.06918 |
| 4.7 | 0.88888 | 0.09404 | 6.9 | 0.92169 | 0.06831 |
| 4.8 | 0.89096 | 0.09246 | 7 | 0.92273 | 0.06747 |
| 4.9 | 0.89296 | 0.09093 | 7.1 | 0.92374 | 0.06665 |
| 5.0 | 0.89489 | 0.08945 | 7.2 | 0.92472 | 0.06585 |
| 5.1 | 0.89675 | 0.08802 | 7.3 | 0.92568 | 0.06507 |
| 5.2 | 0.89854 | 0.08664 | 7.4 | 0.92662 | 0.06431 |
| 5.3 | 0.90028 | 0.08529 | 7.5 | 0.92753 | 0.06356 |
| 5.4 | 0.90195 | 0.08399 | 7.6 | 0.92842 | 0.06283 |
| 5.5 | 0.90357 | 0.08273 | 7.7 | 0.92928 | 0.06212 |
| 5.6 | 0.90514 | 0.08150 | 7.8 | 0.93013 | 0.06143 |
| 5.7 | 0.90666 | 0.08031 | 7.9 | 0.93096 | 0.06074 |
| 5.8 | 0.90813 | 0.07915 | 8 | 0.93176 | 0.06008 |
| 5.9 | 0.90955 | 0.07803 | 8.1 | 0.93255 | 0.05943 |
| 6.0 | 0.91093 | 0.07693 | 8.2 | 0.93332 | 0.05879 |
| 6.1 | 0.91227 | 0.07587 | 8.3 | 0.93408 | 0.05816 |
| 6.2 | 0.91357 | 0.07484 | 8.4 | 0.93481 | 0.05755 |
| 6.3 | 0.91483 | 0.07383 | 8.5 | 0.93553 | 0.05696 |
| 6.4 | 0.91606 | 0.07285 | 8.6 | 0.93624 | 0.05637 |
| 6.5 | 0.91725 | 0.07189 | 8.7 | 0.93693 | 0.05580 |
| 6.6 | 0.91840 | 0.07096 | 8.8 | 0.93760 | 0.05523 |

**4. Moment generating function**

The moment generating function of EPD does not exist

**5. Reliability analysis:**

In reliability analysis, parito distribution is one of the statistical distributions. The problem of reliability is widespread in aircraft reliability engineering.

Therefore, the reliability function is defined to be the probability that an item′s life lasts longer than t units are. The reliability function, denoted by R(t), defined by Forbes et al [19] as follow:

$$R(t) = p(T \geq t) = 1 - G(t)$$

Thus, the reliability for EPD is defined as follows

$$R(t) = \frac{2 - t^{-3\theta} + 3t^{-2\theta}}{2}$$

**6. Hazard Rat Function**

The hazard rate function is also known as the failure rate function. Mathematically the hazard function, denoted by h(t) defined as follow:

$$h(t) = \frac{g(t)}{R(t)}$$



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$$= \frac{\frac{3\theta}{2}\left(2t^{-2\theta-1} - t^{-3\theta-1}\right)}{\frac{2 - t^{-3\theta} + 3t^{-2\theta}}{2}}$$

$$h(t) = \frac{3\theta\left(2t^{-2\theta-1} - t^{-3\theta-1}\right)}{2 - t^{-3\theta} + 3t^{-2\theta}}$$

**7. Median Absolute Deviation**

The median absolute deviation (MAD) is a robust measure of how spread out of set of data. The variance and standard deviation are also measures of spread, but they are more effected by extreme value and non-normality. If the data is not normal, the (MDA) is one statistic can used instead of variance. Derive as the median deviation for an EPD random variable x, with median mcan:

$$DM = E|X - m| = \int_1^\infty |X - m| g(x)dx$$

$$= \int_1^m (x - m)g(x)dx + \int_m^\infty (X - m)g(x)dx$$

$$= \frac{3\theta}{2}\int_1^m (x - m)\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)dx$$

$$+ \frac{3\theta}{2}\int_m^\infty (X - m)\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)dx$$

$$= \frac{3\theta}{2}(\int_1^m x\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)dx - \int_1^m m\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)dx$$

$$+3\theta/2(\int_m^\infty x\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)dx - \int_m^\infty m\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)dx$$

$$MD = \frac{\theta}{(2\theta + 1)(3\theta + 1)} + \frac{m}{6}$$

**8. Maximum Likelihood Estimates**

Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from EPD. The likelihood function L of (2.2) is given by

$$L(\theta) = \prod_{i=1}^{n} \frac{3\theta}{2}\left(2x_i^{-2\theta-1} - x_i^{-3\theta-1}\right)$$

$$= \left(\frac{3\theta}{2}\right)^n \prod_{i=1}^{n}\left(2x_i^{-2\theta-1} - x_i^{-3\theta-1}\right)$$

Therefore, log likelihood function is thus obtained as

$$logL(\theta) = n\, log\frac{2}{3} + n\, log(\theta) + \sum_{i=1}^{n} log\left(2x_i^{-2\theta-1} - x_i^{-3\theta-1}\right)$$

Now

$$\frac{d\, logL(\theta)}{d\theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \frac{2(-2\theta - 1)x_i^{-2\theta-2} + (3\theta + 1)x_i^{-3\theta-2}}{2x_i^{-2\theta-1} - x_i^{-3\theta-1}}$$

The Maximum Likelihood Estimates (MLE),$\theta\hat{}$ of $\theta$ is the solution of the equation$\frac{d\, logL(\theta)}{d\theta} = 0$ we get

$$n/\theta = \sum_{i=1}^{n} \frac{2(-2\,\theta - 1)x_i^{-2\,\theta-2} - (3\,\theta + 1)x_i^{-3\,\theta-2}}{2x_i^{-2\,\theta-1} - x_i^{-3\,\theta-1}}$$

There is no exact solution for this equation.



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**9. Order Statistics**

Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from EPD (2.2). And let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$, be an ordered statistic where

$$X_{(1)} = min(X_1, X_2, \cdots, X_n),$$

$X_{(i)}$ is the ith ordered statistic $(i = 1, \ldots, n)$, and

$$X_{(n)} = max(X_1, X_2, \cdots, X_n)$$

then, the pdf of $X_{(1)}, X_{(n)}$ and the general case $X_{(i)}$ , $1 < i < n$ are defined by

$$g_i(x) = \frac{n!}{(i-1)!\,(n-i)!} g(x)[G(x)]^{i-1}[1-G(x)]^{n-i}$$

$$g_1(x) = ng(x)[1-G(x)]^{n-1}$$

$$g_1(x) = n\frac{3\theta}{2}\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)[1 - \frac{1}{2}(2 - 3x^{-2\theta} + x^{-3\theta}]^{(n-1)}$$

$$g_n(x) = ng(x)[G(x)]^{n-1}$$

$$g_n(x) = n\frac{3\theta}{2}\left(2x^{-2\theta-1} - x^{-3\theta-1}\right)[\frac{1}{2}(2 - 3x^{-2\theta} + x^{-3\theta}]^{(n-1)}$$

**10. Quintile Function**

The quintile q of the random variable x is the solution of the equation

$$q = G(x_q) = p(x \leq x_q)$$

Thus for a random variable belong to EPD the quintile q

$$q = G(x\_q) = G(x) = \frac{1}{2}\left(2 - 3x^{-2\theta} + x^{-3\theta}\right)$$

$$x^{-3\theta} - 3x^{-2\theta} - 2(q-1) = 0$$

We have used online calculator to solve this equation for three values of $q = 1/4, 1/2, 3/4$ . In all cases we assumed $\theta = 1/2$.

For q = 1 /4, we have

$x\_q = 0.12664$ which is not in the range of X.

$x\_q = 1.44550$ which is an accepted root as it is in the range of the random variable.

For $q = 1/2$, we have

$x\_q = 0.12061$ which is not in the range of X.

$x\_q = 2.34729$ which is an accepted root as it is in the range of the random variable.

For $q = 3/4$, we have

$x\_q = 0.11551$ which is not in the range of X.

$x\_q = 5.11574$ which is an accepted root as it is in the range of the random variable.

**11. Applications**

Real data sets have been used to study the performance of the EPD in comparison with the Pareto using the log-likelihood, Akaike Information Criteria (AIC) and Bayesian information criteria (BIC).

The data represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark (1975, P. 105). The data are as follows:

1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2, 1.4, 3.0, 1.7, 2.3, 1.6, 2.0



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**Table 2:** Performance of the EPD using lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic

| Distribution | $\hat{\theta}$ | SE | LogLik | AIC | BIC |
|---|---|---|---|---|---|
| Epd | 0.8269 | 0.0548 | -19.96 | 40.92 | 42.22 |
| Pareto | 1.6970 | 0.315 | -21.97 | 44.94 | 46.24 |

## 12. Conclusion

A one-parameter lifetime distribution called "EPD" has been introduced in this research. Its various mathematical properties, including shape, moments and associated measures, hazard rate and mean residual life functions, median absolute deviations, and order statistics, have been discussed. Its parameter has been estimated using the maximum likelihood estimation approach. Lastly, real-world data applications demonstrate that the proposed distribution fits real-life data better and is more adaptable than the base distribution. For more future work, see [32-34].

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