Time Fuzzy Parameterized Fuzzy Soft Expert Sets

Anwar Bataihah[1,*], Ayman.A Hazaymeh[1]
[1]Department of Mathematics, Faculty of Science, Jadara University, Irbid, Jordan
E-mails: a.bataihah@jadara.edu.jo; aymanha@jadara.edu.jo

Abstract

Finally In this study, we define parameterized time fuzzy soft expert set ($\overline{PTFSES}$) as an extension of fuzzy soft set. Additionally, we will clarify and investigate the characteristics of its primary operation (complement, union intersection, "AND" and "OR"). , we'll apply this approach to decision-making difficulties.

Keywords: Soft set; Fuzzy soft set; Fuzzy parameterized; Time-fuzzy soft set

1 General Introduction

The majority of problems in engineering, medical research, economics, and the environment are fraught with uncertainty. Molodtsov[1]] introduced the notion of soft set theory as a tool in math for coping with such uncertainty. Following Molodtsov's work,,[2] Maji et al.[3] and researched several soft set operations and applications. Also Maji et al.[4,5] they presented the notion of fuzzy soft set as a more broad concept, as well as a combination of fuzzy set and soft set, and investigated its features. Roy and Maji[6] also applied this idea to handle decision-making challenges. Recently, various scholars have begun studying the properties and applications of soft set theory as in the research.[7] Furthermore, in 2010 Çağman et al.[7] established the notion of fuzzy parameterized fuzzy soft set and its operations.In addition, the fpfs-aggregation operator is used to create the fpfs-decision making technique, which allows for more efficient decision processes.[8] proposed the notion of soft expert sets and fuzzy soft expert sets, which allow users to get the views of all experts in one model without any procedures.Hazaymeh. A[9] discusses fuzzy parameterized fuzzy soft expert sets, which offer a membership value for each parameter in a collection of parameters and are an extension of fuzzy soft expert sets. Wang[10]showed that in many real situations, immediate sensory data is insufficient for decision making.[11] provide an overview of generalized fuzzy soft expert set. Recently, various scholars have begun studying the properties and applications of soft set theory. Some topics in algebraic structures are extended by fuzzy soft sets, neutrosophic, or even plithogenic logical sets, as in the research,[12],[13] and there are also studies in fuzzy tapology and neutrosophic fuzzy topology. For more details about neutrosophic topology, see.[14] Additionally, researchers introduced using a neutrosophic fuzzy soft set to solve decision-making problems, like in,[15],[16] other researchers introduced topics in complex fuzzy as,[17] we are looking to integrate time fuzzy soft set and fuzzy soft set with new concepts as in the works,[18],[19].[20] Within the sophisticated domain of neutrosophic fuzzy metric spaces, some academics presented fixed point theorems associated with neutrosophic fuzzy contractions in their research as,[21].[22] Through integrating our research with different disciplines, we may provide novel and significant subjects. For instance, we may incorporate the study of fuzzy soft sets with the findings of scholars like,[22],[23].[24] Enriching the state with knowledge about prior actions and events can help you distinguish between situations that might otherwise look identical, allowing you to make accurate judgments while also learning the proper options. Furthermore, knowledge of the past can eliminate the need for unrealistic sensors, such as knowing your exact location in a maze. Using historical information as part of the state representation provides us with important information to assist us in making better judgments in situations when temporal value is not taken into account, resulting in less accurate decision-making. If we want to take the views of more

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than one time (period), we must perform various operations such as union, intersection, etc. For a solution to this problem, we take a collection of time intervals, generalize it into what we call a time-fuzzy soft expert set (T-FSES) that is induced by,[26] Many academics used the time fuzzy soft set notion to decision-making issues by combining it with other ideas and introducing applications as investigate some of its features, and apply this notion to a decision-making problem as,[27].[28] It is critical to understand the history of the parameters under consideration in order to ensure the credibility of the information provided by specialists. The experts' previous experiences are gathered in the number of periods (years, months, etc.) in which they are involved in a certain decision-making circumstance, and by looking at the time component, individuals are more confident in the conclusion that they make. We must examine the influence of time on fuzzy soft set applications, not only for the present period, but also for the past and future periods (forecasting information). Based on the literature reviews, we found that the parameterized fuzzy soft set has been applied to solve a decision making problem which involve uncertainty or vagueness in data. The idea behind parameterized fuzzy soft set has a potential promising for giving an optimal solution for decision making problem. Sometimes it is justifiable to impose weights on the choice of parameters, to reach to the final optimal or suboptimal decisions. To apply this decision rule, the decision maker may demand greater "membership levels" on the parameters he may highlight. For example, while purchasing anything, one often wants a higher quality for the feature that he considers more significant. In previous study we showed the importance of the concept fuzzy soft expert set and that it is a suitable mathematical tool for dealing with uncertainty problems. Our goal in this research is to enrich these concepts by combining fuzzy parameterized fuzzy soft and fuzzy soft expert set. In this research we present the concept of time fuzzy parameterized fuzzy soft expert's sets, this will be more effective and valuable and is a hybrid of the fuzzy soft expert set and the parameterized time fuzzy soft sets. We also define and investigate the features of its fundamental operations, which include complement, union, intersection, AND, and OR. Finally, we present some possible applications of this notion to decision-making difficulties.

2 Preliminary

In this part, we cover several fundamental concepts in soft set theory. Molodtsov[?] defined soft sets over U as follows: Let U be a universe set and E set of parameters, $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.1. [1] consider this mapping

$$F : A \to P(U).$$

Any A pair (F, A) is considered a *soft set* over U. In other terms, a soft set over U is a parameterized collection of subsets of the universe set U. For $\varepsilon \in A, F(\varepsilon)$ can be viewed as the set of ε-approximate members of the soft set (F, A).

Definition 2.2. [2] Let U be the initial universal set, and E be the set of parameters. Let I^U be the power set of all fuzzy subsets of U. Let $A \subseteq E$, and F be the mapping

$$F : A \to I^U.$$

A pair (F, E) is known as a *fuzzy soft set* over U.

Definition 2.3. [3] Regarding two fuzzy soft sets (F, A) and (G, B) over U, (F, A) is known as a fuzzy soft subset of. (G, B) if

1. $A \subset B$ and
2. $\forall \varepsilon \in A, F(\varepsilon)$ is fuzzy subset of $G(\varepsilon)$.

The relationship is represented by $(F,\ A) \tilde{\subset} (G, B)$. In this situation, (G, B) is known as a fuzzy, soft superset of. (F, A).

Definition 2.4. [6] The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ And has been defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping provided by

$$F^c(\alpha) = c(F(\rceil\alpha)), \forall \alpha \in \rceil A.$$

c describes any fuzzy complement.

Definition 2.5. [6] If (F, A) and (G, B) are two fuzzy soft sets then "(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ is defined by

$$(F, A) \wedge (G, B) = (H, A \times B)$$

such that $H(\alpha, \beta) = t(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where t is any t-norm.

Definition 2.6. [6] If (F, A) and (G, B) are two fuzzy soft sets then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by

$$(F, A) \vee (G, B) = (O, A \times B)$$

such that $O(\alpha, \beta) = s(F(\alpha), G(\beta)), \forall (\alpha, \beta) \in A \times B$, where s is any s-norm.

Definition 2.7. [6] The union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if \quad \varepsilon \in A \cup B. \end{cases}$$

Where s is any s-norm.

Definition 2.8. [6] The intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if \quad \varepsilon \in A - B, \\ G(\varepsilon), & if \quad \varepsilon \in B - A, \\ s(F(\varepsilon), G(\varepsilon)), & if \quad \varepsilon \in A \cap B. \end{cases}$$

Definition 2.9. .[5] Let U be an initial universe, E the set of all parameters and X a fuzzy set over E with membership function

$$\mu_X : E \to [0, 1],$$

and let γ_X be a fuzzy set over U for all $x \in E$. Then an fpfs-set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping $\gamma_X : E \to F(U)$ such that

$$\gamma_X(x) = \emptyset \text{ if } \mu_X(x) = 0.$$

Here, γ_X is called a fuzzy approximate function of the fpfs-set Γ_X, and the value $\gamma_X(x)$ is a set called x-element of the fpfs-set for all $x \in E$. Thus, an fpfs-set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \ \gamma_X(x) \in F(U), \mu_X(x) \in [0, 1]\}.$$

Definition 2.10. [5] Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U, let $A \subseteq E$ and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$. A collection of pairs $(F, E)^t \, \forall \, t \in T$ is called a *time-fuzzy soft set* $\{T - FSS\}$ over U where F is a mapping given by

$$F^t : A \to I^U.$$

3 Time Fuzzy Parameterized Fuzzy Soft Expert Set

3.1 Main Definition

In this part, we define a time fuzzy parameterized fuzzy soft expert set ($\overline{TFPSES}$) and discuss its fundamental characteristics.

Let U be a universe set, E be a set of parameters, I^E represent all fuzzy subsets of E, X be a set of experts, and $O = \{1 = \text{agree}, 0 = \text{disagree}\}$ a set of two opinions and T be a set of time where $T = \{t_1, t_2, ..., t_n\}$ .. Let $Z = \Psi \times X \times O$ and $A \subseteq Z$ where $\Psi \subset I^E$.

Definition 3.1. Let F be mapping given by

$$F^t_\Psi : A \to I^U$$

, I^U represent all fuzzy subset of U. A pair $(F, A)^t_\Psi$ is known as *fuzzy parameterized fuzzy soft expert set* ($\overline{PTFSES}$) over U.

Example 3.2. Assume that a manufacturer distributes its products to three major regions of the country. Because of the geographical separation between both demand and supply areas, as well as the aim to meet consumer demand and ensure the availability of their goods, the factory's management has opted to open outlets near their current client bases. Now, let $W = \{u_1,\ u_2,\ u_3, u_4\}$ be a set of alternatives for stores location , $E = \left\{\dfrac{0.5}{e_1}, \dfrac{0.7}{e_2}, \dfrac{0.9}{e_3}\right\}$ be a set of decision parameters where $e_i\,(i = 1, 2, 3)$ represents the following parameters: land or building expenses associated with the location, the cost of traveling between the demand and supply areas, and "the continuing safety of the area," respectively. Let $\Psi = \{\frac{0.5}{e_1}, \frac{0.7}{e_2}, \frac{0.9}{e_3}\}$ be a fuzzy subset of I^P and $Y = \{m, n, r\}$ be a set of experts.These insights allow us to choose the best location for the new factory stores. Now suppose that

$$F_1\left(\frac{0.5}{e_1}, m, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.8}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.4}, \frac{{u_4}^{t_1}}{0.3}\right\}, F_1\left(\frac{0.5}{e_1}, n, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.9}, \frac{{u_3}^{t_1}}{0.6}, \frac{{u_4}^{t_1}}{0.5}\right\},$$

$$F_1\left(\frac{0.5}{e_1}, r, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.7}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.3}, \frac{{u_4}^{t_1}}{0.7}\right\}, F_1\left(\frac{0.7}{e_2}, m, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.1}, \frac{{u_3}^{t_1}}{0.4}, \frac{{u_4}^{t_1}}{0.8}\right\},$$

$$F_1\left(\frac{0.7}{e_2}, n, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.7}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.5}\right\}, F_1\left(\frac{0.7}{e_2}, r, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.2}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.7}, \frac{{u_4}^{t_1}}{0.3}\right\},$$

$$F_1\left(\frac{0.9}{e_3}, m, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.2}, \frac{{u_2}^{t_1}}{0.4}, \frac{{u_3}^{t_1}}{0.9}, \frac{{u_4}^{t_1}}{0.6}\right\}, F_1\left(\frac{0.9}{e_3}, n, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.6}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.6}, \frac{{u_4}^{t_1}}{0.7}\right\},$$

$$F_1\left(\frac{0.9}{e_3}, r, 1\right) = \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.7}, \frac{{u_3}^{t_1}}{0.1}, \frac{{u_4}^{t_1}}{0.5}\right\}, F_2\left(\frac{0.5}{e_1}, m, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.4}, \frac{{u_2}^{t_2}}{0.8}, \frac{{u_3}^{t_2}}{0.2}, \frac{{u_4}^{t_2}}{0.4}\right\},$$

$$F_2\left(\frac{0.5}{e_1}, n, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.6}, \frac{{u_2}^{t_2}}{0.9}, \frac{{u_3}^{t_2}}{0.4}, \frac{{u_4}^{t_2}}{0.8}\right\}, F_2\left(\frac{0.5}{e_1}, r, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.9}, \frac{{u_2}^{t_2}}{0.2}, \frac{{u_3}^{t_2}}{0.3}, \frac{{u_4}^{t_2}}{0.3}\right\},$$

$$F_2\left(\frac{0.7}{e_2}, m, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.2}, \frac{{u_2}^{t_2}}{0.9}, \frac{{u_3}^{t_2}}{0.5}, \frac{{u_4}^{t_2}}{0.5}\right\}, F_2\left(\frac{0.7}{e_2}, n, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.7}, \frac{{u_2}^{t_2}}{0.6}, \frac{{u_3}^{t_2}}{0.2}, \frac{{u_4}^{t_2}}{0.6}\right\},$$

$$F_2\left(\frac{0.7}{e_2}, r, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.4}, \frac{{u_3}^{t_2}}{0.7}, \frac{{u_4}^{t_2}}{0.4}\right\}, F_2\left(\frac{0.9}{e_3}, m, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.9}, \frac{{u_2}^{t_2}}{0.4}, \frac{{u_3}^{t_2}}{0.1}, \frac{{u_4}^{t_2}}{0.7}\right\},$$

$$F_2\left(\frac{0.9}{e_3}, n, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.8}, \frac{{u_2}^{t_2}}{0.6}, \frac{{u_3}^{t_2}}{0.1}, \frac{{u_4}^{t_2}}{0.5}\right\}, F_2\left(\frac{0.9}{e_3}, r, 1\right) = \left\{\frac{{u_1}^{t_2}}{0.2}, \frac{{u_2}^{t_2}}{0.9}, \frac{{u_3}^{t_2}}{0.6}, \frac{{u_4}^{t_2}}{0.7}\right\},$$

$$F_3\left(\frac{0.5}{e_1}, m, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.2}, \frac{{u_2}^{t_3}}{0.7}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.3}\right\}, F_3\left(\frac{0.5}{e_1}, n, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.4}, \frac{{u_2}^{t_3}}{0.9}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.5}\right\},$$

$$F_3\left(\frac{0.5}{e_1}, r, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.6}, \frac{{u_2}^{t_3}}{0.5}, \frac{{u_3}^{t_3}}{0.7}, \frac{{u_4}^{t_3}}{0.8}\right\}, F_3\left(\frac{0.7}{e_2}, m, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.5}, \frac{{u_2}^{t_3}}{0.4}, \frac{{u_3}^{t_3}}{0.7}, \frac{{u_4}^{t_3}}{0.9}\right\},$$

$$F_3\left(\frac{0.7}{e_2}, n, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.8}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.2}, \frac{{u_4}^{t_3}}{0.7}\right\}, F_3\left(\frac{0.7}{e_2}, r, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.1}, \frac{{u_2}^{t_3}}{0.8}, \frac{{u_3}^{t_3}}{0.5}, \frac{{u_4}^{t_3}}{0.8}\right\},$$

$$F_3\left(\frac{0.9}{e_3}, m, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.7}, \frac{{u_2}^{t_3}}{0.4}, \frac{{u_3}^{t_3}}{0.1}, \frac{{u_4}^{t_3}}{0.8}\right\}, F_3\left(\frac{0.9}{e_3}, n, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.6}, \frac{{u_2}^{t_3}}{0.5}, \frac{{u_3}^{t_3}}{0.3}, \frac{{u_4}^{t_3}}{0.9}\right\},$$

$$F_3\left(\frac{0.9}{e_3}, r, 1\right) = \left\{\frac{{u_1}^{t_3}}{0.4}, \frac{{u_2}^{t_3}}{0.7}, \frac{{u_3}^{t_3}}{0.5}, \frac{{u_4}^{t_3}}{0.6}\right\}, F_1\left(\frac{0.5}{e_1}, m, 0\right) = \left\{\frac{{u_1}^{t_1}}{0.3}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.7}, \frac{{u_4}^{t_1}}{0.8}\right\},$$

$$F_1\left(\frac{0.5}{e_1}, n, 0\right) = \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.5}, \frac{{u_4}^{t_1}}{0.4}\right\}, F_1\left(\frac{0.5}{e_1}, r, 0\right) = \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.8}, \frac{{u_4}^{t_1}}{0.3}\right\},$$

$$F_1\left(\frac{0.7}{e_2}, m, 0\right) = \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.7}, \frac{{u_3}^{t_1}}{0.5}, \frac{{u_4}^{t_1}}{0.3}\right\}, F_1\left(\frac{0.7}{e_2}, n, 0\right) = \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.4}, \frac{{u_3}^{t_1}}{0.9}, \frac{{u_4}^{t_1}}{0.6}\right\},$$

$$F_1\left(\frac{0.7}{e_2},r,0\right)=\left\{\tfrac{{u_1}^{t_1}}{0.6},\tfrac{{u_2}^{t_1}}{0.3},\tfrac{{u_3}^{t_1}}{0.4},\tfrac{{u_4}^{t_1}}{0.1}\right\},F_1\left(\frac{0.9}{e_3},m,0\right)=\left\{\tfrac{{u_1}^{t_1}}{0.7},\tfrac{{u_2}^{t_1}}{0.5},\tfrac{{u_3}^{t_1}}{0.2},\tfrac{{u_4}^{t_1}}{0.3}\right\},$$

$$F_1\left(\frac{0.9}{e_3},n,0\right)=\left\{\tfrac{{u_1}^{t_1}}{0.5},\tfrac{{u_2}^{t_1}}{0.4},\tfrac{{u_3}^{t_1}}{0.5},\tfrac{{u_4}^{t_1}}{0.2}\right\},F_1\left(\frac{0.9}{e_3},r,0\right)=\left\{\tfrac{{u_1}^{t_1}}{0.8},\tfrac{{u_2}^{t_1}}{0.2},\tfrac{{u_3}^{t_1}}{0.8},\tfrac{{u_4}^{t_1}}{0.4}\right\},$$

$$F_2\left(\frac{0.5}{e_1},m,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.7},\tfrac{{u_2}^{t_2}}{0.4},\tfrac{{u_3}^{t_2}}{0.7},\tfrac{{u_4}^{t_2}}{0.5}\right\},F_2\left(\frac{0.5}{e_1},n,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.5},\tfrac{{u_2}^{t_2}}{0.2},\tfrac{{u_3}^{t_2}}{0.5},\tfrac{{u_4}^{t_2}}{0.4}\right\},$$

$$F_2\left(\frac{0.5}{e_1},r,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.3},\tfrac{{u_2}^{t_2}}{0.7},\tfrac{{u_3}^{t_2}}{0.8},\tfrac{{u_4}^{t_2}}{0.8}\right\},F_2\left(\frac{0.7}{e_2},m,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.9},\tfrac{{u_2}^{t_2}}{0.3},\tfrac{{u_3}^{t_2}}{0.6},\tfrac{{u_4}^{t_2}}{0.6}\right\},$$

$$F_2\left(\frac{0.7}{e_2},n,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.4},\tfrac{{u_2}^{t_2}}{0.5},\tfrac{{u_3}^{t_2}}{0.9},\tfrac{{u_4}^{t_2}}{0.4}\right\},F_2\left(\frac{0.7}{e_2},r,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.6},\tfrac{{u_2}^{t_2}}{0.7},\tfrac{{u_3}^{t_2}}{0.4},\tfrac{{u_4}^{t_2}}{0.1}\right\},$$

$$F_2\left(\frac{0.9}{e_3},m,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.2},\tfrac{{u_2}^{t_2}}{0.7},\tfrac{{u_3}^{t_2}}{0.8},\tfrac{{u_4}^{t_2}}{0.2}\right\},F_2\left(\frac{0.9}{e_3},n,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.4},\tfrac{{u_2}^{t_2}}{0.5},\tfrac{{u_3}^{t_2}}{0.7},\tfrac{{u_4}^{t_2}}{0.6}\right\},$$

$$F_2\left(\frac{0.9}{e_3},r,0\right)=\left\{\tfrac{{u_1}^{t_2}}{0.7},\tfrac{{u_2}^{t_2}}{0.1},\tfrac{{u_3}^{t_2}}{0.3},\tfrac{{u_4}^{t_2}}{0.3}\right\},F_3\left(\frac{0.5}{e_1},m,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.6},\tfrac{{u_2}^{t_3}}{0.8},\tfrac{{u_3}^{t_3}}{0.2},\tfrac{{u_4}^{t_3}}{0.4}\right\},$$

$$F_3\left(\frac{0.5}{e_1},n,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.7},\tfrac{{u_2}^{t_3}}{0.5},\tfrac{{u_3}^{t_3}}{0.4},\tfrac{{u_4}^{t_3}}{0.6}\right\},F_3\left(\frac{0.5}{e_1},r,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.5},\tfrac{{u_2}^{t_3}}{0.7},\tfrac{{u_3}^{t_3}}{0.2},\tfrac{{u_4}^{t_3}}{0.5}\right\},$$

$$F_3\left(\frac{0.7}{e_2},m,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.3},\tfrac{{u_2}^{t_3}}{0.7},\tfrac{{u_3}^{t_3}}{0.4},\tfrac{{u_4}^{t_3}}{0.1}\right\},F_3\left(\frac{0.7}{e_2},n,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.1},\tfrac{{u_2}^{t_3}}{0.5},\tfrac{{u_3}^{t_3}}{0.6},\tfrac{{u_4}^{t_3}}{0.4}\right\},$$

$$F_3\left(\frac{0.7}{e_2},r,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.7},\tfrac{{u_2}^{t_3}}{0.3},\tfrac{{u_3}^{t_3}}{0.3},\tfrac{{u_4}^{t_3}}{0.2}\right\},F_3\left(\frac{0.9}{e_3},m,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.4},\tfrac{{u_2}^{t_3}}{0.6},\tfrac{{u_3}^{t_3}}{0.8},\tfrac{{u_4}^{t_3}}{0.3}\right\},$$

$$F_3\left(\frac{0.9}{e_3},n,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.3},\tfrac{{u_2}^{t_3}}{0.4},\tfrac{{u_3}^{t_3}}{0.7},\tfrac{{u_4}^{t_3}}{0.2}\right\},F_3\left(\frac{0.9}{e_3},r,0\right)=\left\{\tfrac{{u_1}^{t_3}}{0.7},\tfrac{{u_2}^{t_3}}{0.2},\tfrac{{u_3}^{t_3}}{0.6},\tfrac{{u_4}^{t_3}}{0.5}\right\}.$$

Then we can find the time-parameterized fuzzy soft expert sets $(F,E)^t$ as consisting of the following collection of approximations:

$$(F,E)^t_{\Psi}=\left\{\left(\left(\frac{0.5}{e_1},m,1\right),\left\{\tfrac{{u_1}^{t_1}}{0.8},\tfrac{{u_2}^{t_1}}{0.5},\tfrac{{u_3}^{t_1}}{0.4},\tfrac{{u_4}^{t_1}}{0.3}\right\}\right),\left(\left(\frac{0.5}{e_1},n,1\right),\left\{\tfrac{{u_1}^{t_1}}{0.6},\tfrac{{u_2}^{t_1}}{0.9},\tfrac{{u_3}^{t_1}}{0.6},\tfrac{{u_4}^{t_1}}{0.5}\right\}\right)\right.$$
$$\vdots \qquad \ddots$$
$$\left.\left(\left(\frac{0.9}{e_3},n,0\right),\left\{\tfrac{{u_1}^{t_3}}{0.3},\tfrac{{u_2}^{t_3}}{0.4},\tfrac{{u_3}^{t_3}}{0.7},\tfrac{{u_4}^{t_3}}{0.2}\right\}\right),\left(\left(\frac{0.9}{e_3},r,0\right),\left\{\tfrac{{u_1}^{t_3}}{0.7},\tfrac{{u_2}^{t_3}}{0.2},\tfrac{{u_3}^{t_3}}{0.6},\tfrac{{u_4}^{t_3}}{0.5}\right\}\right)\right\}.$$

Definition 3.3. For two ($\overline{PTFSES's}$) $(F,A)^t_{\Psi}$ and $(G,B)^t_{\Upsilon}$ over U, $(F,A)^t_{\Psi}$ is called a($\overline{PTFSES}$)subset of $(G,B)^t_{\Upsilon}$ if

1. $\Upsilon\subseteq\Psi$,
2. $\forall\varepsilon\in B, G(\varepsilon)$ is fuzzy soft expert subset of $J(\varepsilon)$.

Definition 3.4. Two ($\overline{PTFSES's}$) $(F,A)^t_{\Psi}$ and $(G,B)^t_{\Upsilon}$ over U, are said to be *equal* if $(F,A)^t_{\Psi}$ is a($\overline{PTFSES}$)subset of $(G,N)^t_{\Upsilon}$ and $(G,N)^t_{\Upsilon}$ is a($\overline{PTFSES}$)subset of $(F,A)^t_{\Psi}$.

Example 3.5. Consider Example 3.2, where

$$A=\left\{\left(\frac{0.5}{e_1},m,1\right)^{t_1},\left(\frac{0.7}{e_2},m,1\right)^{t_1},\left(\frac{0.7}{e_2},n,1\right)^{t_3},\left(\frac{0.7}{e_2},r,1\right)^{t_3},\left(\frac{0.7}{e_2},n,0\right)^{t_2},\left(\frac{0.7}{e_2},r,0\right)^{t_2},\left(\frac{0.7}{e_2},r,0\right)^{t_3},\right.$$

$$\left(\frac{0.9}{e_3}, m, 0\right)_{t_3}\Big\},$$

$$B=\Big\{\left(\frac{0.5}{e_1}, m, 1\right)^{t_1}, \left(\frac{0.7}{e_2}, m, 1\right)^{t_1}, \left(\frac{0.7}{e_2}, n, 1\right)^{t_3}, \left(\frac{0.7}{e_2}, r, 1\right)^{t_3}, \left(\frac{0.7}{e_2}, n, 0\right)_{t_2}\Big\}.$$

Clearly $B \subset A$. Now, let $(G,B)^t_\Upsilon$ and $(F,A)^t_\Psi$ be defined as follows:

$$(F,E)^t_\Psi=\Big\{\left(\left(\frac{0.5}{e_1}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(\frac{0.7}{e_2}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.1}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.8}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2}, n, 1\right), \left\{\frac{u_1{}^{t_3}}{0.8}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.7}\right\}\right), \left(\left(\frac{0.7}{e_2}, r, 1\right), \left\{\frac{u_1{}^{t_3}}{0.1}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.8}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2}, n, 0\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.9}, \frac{u_4{}^{t_2}}{0.4}\right\}\right), \left(\left(\frac{0.7}{e_2}, r, 0\right) \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.1}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2}, r, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.2}\right\}\right), \left(\left(\frac{0.9}{e_3}, m, 0\right), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.3}\right\}\right)\Big\},$$

$$(G,E)^t_\Upsilon=\Big\{\left(\left(\frac{0.5}{e_1}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.2}, \frac{u_3{}^{t_1}}{0.1}, \frac{u_4{}^{t_1}}{0.0}\right\}\right), \left(\left(\frac{0.4}{e_2}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.1}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left(\left(\frac{0.4}{e_2}, n, 1\right), \left\{\frac{u_1{}^{t_3}}{0.8}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.6}\right\}\right), \left(\left(\frac{0.4}{e_2}, r, 1\right), \left\{\frac{u_1{}^{t_3}}{0.0}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.3}\right\}\right),$$
$$\left(\left(\frac{0.3}{e_3}, m, 0\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.2}\right\}\right)\Big\}.$$

We can easily verify that $(G,E)^t_\Upsilon \subseteq (F,E)^t_\Psi$.

Definition 3.6. An *agree-PTFSES* $(F,A)^t_{\Psi_1}$ over U is a($\overline{PTFSES}$)subset of $(F,A)^t_\Psi$ be defined as outlined below::

$$(F,A)^t_{\Psi_1}=\left\{F^t_{\Psi_1}(\alpha) : \alpha \in \Psi \times X \times \{1\}\right\}.$$

Definition 3.7. A *disagree-PTFSES* $(F,A)^t_{\Psi_0}$ over U is a($\overline{PTFSES}$)subset of $(F,A)^t_\Psi$ be defined as outlined below:

$$(F,A)^t_{\Psi_0}=\left\{J_{\Psi_0}(\alpha) : \alpha \in \Psi \times X \times \{0\}\right\}.$$

Example 3.8. Think about this example: 3.2. Then the agree-PTFSES $(F,Z)^t_{\Psi_1}$ over U. Then the agree-time fuzzy soft expert set $\left((F,A)^t_\Psi\right)_1$ over U is

$$\left((F,A)^t_\Psi\right)_1=\Big\{\left(\left(\frac{0.5}{e_1}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.8}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(\frac{0.5}{e_1}, n, 1\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.9}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.5}\right\}\right),$$
$$\left(\left(\frac{0.5}{e_1}, r, 1\right), \left\{\frac{u_1{}^{t_1}}{0.7}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.3}, \frac{u_4{}^{t_1}}{0.7}\right\}\right), \left(\left(\frac{0.7}{e_2}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.1}, \frac{u_3{}^{t_1}}{0.4}, \frac{u_4{}^{t_1}}{0.8}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2}, n, 1\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left(\left(\frac{0.7}{e_2}, r, 1\right), \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left(\left(\frac{0.9}{e_3}, m, 1\right), \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.4}, \frac{u_3{}^{t_1}}{0.9}, \frac{u_4{}^{t_1}}{0.6}\right\}\right), \left(\left(\frac{0.9}{e_3}, n, 1\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left(\left(\frac{0.9}{e_3}, r, 1\right), \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.1}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left(\left(\frac{0.5}{e_1}, m, 1\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left(\left(\frac{0.5}{e_1}, n, 1\right), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.8}\right\}\right), \left(\left(\frac{0.5}{e_1}, r, 1\right), \left\{\frac{u_1{}^{t_2}}{0.9}, \frac{u_2{}^{t_2}}{0.2}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.3}\right\}\right),$$

$$
\begin{aligned}
&\left(\left(\frac{0.7}{e_2},m,1\right),\left\{\frac{{u_1}^{t_2}}{0.2},\frac{{u_2}^{t_2}}{0.9},\frac{{u_3}^{t_2}}{0.5},\frac{{u_4}^{t_2}}{0.5}\right\}\right),\left(\left(\frac{0.7}{e_2},n,1\right),\left\{\frac{{u_1}^{t_2}}{0.7},\frac{{u_2}^{t_2}}{0.6},\frac{{u_3}^{t_2}}{0.2},\frac{{u_4}^{t_2}}{0.6}\right\}\right),\\
&\left(\left(\frac{0.7}{e_2},r,1\right),\left\{\frac{{u_1}^{t_2}}{0.5},\frac{{u_2}^{t_2}}{0.4},\frac{{u_3}^{t_2}}{0.7},\frac{{u_4}^{t_2}}{0.4}\right\}\right),\left(\left(\frac{0.9}{e_3},m,1\right),\left\{\frac{{u_1}^{t_2}}{0.9},\frac{{u_2}^{t_2}}{0.4},\frac{{u_3}^{t_2}}{0.1},\frac{{u_4}^{t_2}}{0.7}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},n,1\right),\left\{\frac{{u_1}^{t_2}}{0.8},\frac{{u_2}^{t_2}}{0.6},\frac{{u_3}^{t_2}}{0.1},\frac{{u_4}^{t_2}}{0.5}\right\}\right),\left(\left(\frac{0.9}{e_3},r,1\right),\left\{\frac{{u_1}^{t_2}}{0.2},\frac{{u_2}^{t_2}}{0.9},\frac{{u_3}^{t_2}}{0.6},\frac{{u_4}^{t_2}}{0.7}\right\}\right),\\
&\left(\left(\frac{0.5}{e_1},m,1\right),\left\{\frac{{u_1}^{t_3}}{0.2},\frac{{u_2}^{t_3}}{0.7},\frac{{u_3}^{t_3}}{0.8},\frac{{u_4}^{t_3}}{0.3}\right\}\right),\left(\left(\frac{0.5}{e_1},n,1\right),\left\{\frac{{u_1}^{t_3}}{0.4},\frac{{u_2}^{t_3}}{0.9},\frac{{u_3}^{t_3}}{0.6},\frac{{u_4}^{t_3}}{0.5}\right\}\right),\\
&\left(\left(\frac{0.5}{e_1},r,1\right),\left\{\frac{{u_1}^{t_3}}{0.6},\frac{{u_2}^{t_3}}{0.5},\frac{{u_3}^{t_3}}{0.7},\frac{{u_4}^{t_3}}{0.8}\right\}\right),\left(\left(\frac{0.7}{e_2},m,1\right),\left\{\frac{{u_1}^{t_3}}{0.5},\frac{{u_2}^{t_3}}{0.4},\frac{{u_3}^{t_3}}{0.7},\frac{{u_4}^{t_3}}{0.9}\right\}\right),\\
&\left(\left(\frac{0.7}{e_2},n,1\right),\left\{\frac{{u_1}^{t_3}}{0.8},\frac{{u_2}^{t_3}}{0.6},\frac{{u_3}^{t_3}}{0.2},\frac{{u_4}^{t_3}}{0.7}\right\}\right),\left(\left(\frac{0.7}{e_2},r,1\right),\left\{\frac{{u_1}^{t_3}}{0.1},\frac{{u_2}^{t_3}}{0.8},\frac{{u_3}^{t_3}}{0.5},\frac{{u_4}^{t_3}}{0.8}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},m,1\right),\left\{\frac{{u_1}^{t_3}}{0.7},\frac{{u_2}^{t_3}}{0.4},\frac{{u_3}^{t_3}}{0.1},\frac{{u_4}^{t_3}}{0.8}\right\}\right),\left(\left(\frac{0.9}{e_3},n,1\right),\left\{\frac{{u_1}^{t_3}}{0.6},\frac{{u_2}^{t_3}}{0.5},\frac{{u_3}^{t_3}}{0.3},\frac{{u_4}^{t_3}}{0.9}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},r,1\right),\left\{\frac{{u_1}^{t_3}}{0.4},\frac{{u_2}^{t_3}}{0.7},\frac{{u_3}^{t_3}}{0.5},\frac{{u_4}^{t_3}}{0.6}\right\}\right)\Bigg\}.
\end{aligned}
$$

and the disagree- time fuzzy soft expert set $\left((F,A)^t_\Psi\right)_0$ over U is

$$
\begin{aligned}
\left((F,A)^t_\Psi\right)_0=\Bigg\{&\left(\left(\frac{0.5}{e_1},m,0\right),\left\{\frac{{u_1}^{t_1}}{0.3},\frac{{u_2}^{t_1}}{0.6},\frac{{u_3}^{t_1}}{0.7},\frac{{u_4}^{t_1}}{0.8}\right\}\right),\left(\left(\frac{0.5}{e_1},n,0\right),\left\{\frac{{u_1}^{t_1}}{0.5},\frac{{u_2}^{t_1}}{0.3},\frac{{u_3}^{t_1}}{0.5},\frac{{u_4}^{t_1}}{0.4}\right\}\right),\\
&\left(\left(\frac{0.5}{e_1},r,0\right),\left\{\frac{{u_1}^{t_1}}{0.4},\frac{{u_2}^{t_1}}{0.5},\frac{{u_3}^{t_1}}{0.8},\frac{{u_4}^{t_1}}{0.3}\right\}\right),\left(\left(\frac{0.7}{e_2},m,0\right),\left\{\frac{{u_1}^{t_1}}{0.5},\frac{{u_2}^{t_1}}{0.7},\frac{{u_3}^{t_1}}{0.5},\frac{{u_4}^{t_1}}{0.3}\right\}\right),\\
&\left(\left(\frac{0.7}{e_2},n,0\right),\left\{\frac{{u_1}^{t_1}}{0.4},\frac{{u_2}^{t_1}}{0.4},\frac{{u_3}^{t_1}}{0.9},\frac{{u_4}^{t_1}}{0.6}\right\}\right),\left(\left(\frac{0.7}{e_2},r,0\right),\left\{\frac{{u_1}^{t_1}}{0.6},\frac{{u_2}^{t_1}}{0.3},\frac{{u_3}^{t_1}}{0.4},\frac{{u_4}^{t_1}}{0.1}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},m,0\right),\left\{\frac{{u_1}^{t_1}}{0.7},\frac{{u_2}^{t_1}}{0.5},\frac{{u_3}^{t_1}}{0.2},\frac{{u_4}^{t_1}}{0.3}\right\}\right),\left(\left(\frac{0.9}{e_3},n,0\right),\left\{\frac{{u_1}^{t_1}}{0.5},\frac{{u_2}^{t_1}}{0.4},\frac{{u_3}^{t_1}}{0.5},\frac{{u_4}^{t_1}}{0.2}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},r,0\right),\left\{\frac{{u_1}^{t_1}}{0.8},\frac{{u_2}^{t_1}}{0.2},\frac{{u_3}^{t_1}}{0.8},\frac{{u_4}^{t_1}}{0.4}\right\}\right),\left(\left(\frac{0.5}{e_1},m,0\right),\left\{\frac{{u_1}^{t_2}}{0.7},\frac{{u_2}^{t_2}}{0.4},\frac{{u_3}^{t_2}}{0.7},\frac{{u_4}^{t_2}}{0.5}\right\}\right),\\
&\left(\left(\frac{0.5}{e_1},n,0\right),\left\{\frac{{u_1}^{t_2}}{0.5},\frac{{u_2}^{t_2}}{0.2},\frac{{u_3}^{t_2}}{0.5},\frac{{u_4}^{t_2}}{0.4}\right\}\right),\left(\left(\frac{0.5}{e_1},r,0\right),\left\{\frac{{u_1}^{t_2}}{0.3},\frac{{u_2}^{t_2}}{0.7},\frac{{u_3}^{t_2}}{0.8},\frac{{u_4}^{t_2}}{0.8}\right\}\right),\\
&\left(\left(\frac{0.7}{e_2},m,0\right),\left\{\frac{{u_1}^{t_2}}{0.9},\frac{{u_2}^{t_2}}{0.3},\frac{{u_3}^{t_2}}{0.6},\frac{{u_4}^{t_2}}{0.6}\right\}\right),\left(\left(\frac{0.7}{e_2},n,0\right),\left\{\frac{{u_1}^{t_2}}{0.4},\frac{{u_2}^{t_2}}{0.5},\frac{{u_3}^{t_2}}{0.9},\frac{{u_4}^{t_2}}{0.4}\right\}\right),\\
&\left(\left(\frac{0.7}{e_2},r,0\right)\left\{\frac{{u_1}^{t_2}}{0.6},\frac{{u_2}^{t_2}}{0.7},\frac{{u_3}^{t_2}}{0.4},\frac{{u_4}^{t_2}}{0.1}\right\}\right),\left(\left(\frac{0.9}{e_3},m,0\right),\left\{\frac{{u_1}^{t_2}}{0.2},\frac{{u_2}^{t_2}}{0.7},\frac{{u_3}^{t_2}}{0.8},\frac{{u_4}^{t_2}}{0.2}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},n,0\right),\left\{\frac{{u_1}^{t_2}}{0.4},\frac{{u_2}^{t_2}}{0.5},\frac{{u_3}^{t_2}}{0.7},\frac{{u_4}^{t_2}}{0.6}\right\}\right),\left(\left(\frac{0.9}{e_3},r,0\right),\left\{\frac{{u_1}^{t_2}}{0.7},\frac{{u_2}^{t_2}}{0.1},\frac{{u_3}^{t_2}}{0.3},\frac{{u_4}^{t_2}}{0.3}\right\}\right),\\
&\left(\left(\frac{0.5}{e_1},m,0\right),\left\{\frac{{u_1}^{t_3}}{0.6},\frac{{u_2}^{t_3}}{0.8},\frac{{u_3}^{t_3}}{0.2},\frac{{u_4}^{t_3}}{0.4}\right\}\right),\left(\left(\frac{0.5}{e_1},n,0\right),\left\{\frac{{u_1}^{t_3}}{0.7},\frac{{u_2}^{t_3}}{0.5},\frac{{u_3}^{t_3}}{0.4},\frac{{u_4}^{t_3}}{0.6}\right\}\right),\\
&\left(\left(\frac{0.5}{e_1},r,0\right),\left\{\frac{{u_1}^{t_3}}{0.5},\frac{{u_2}^{t_3}}{0.7},\frac{{u_3}^{t_3}}{0.2},\frac{{u_4}^{t_3}}{0.5}\right\}\right),\left(\left(\frac{0.7}{e_2},m,0\right),\left\{\frac{{u_1}^{t_3}}{0.3},\frac{{u_2}^{t_3}}{0.7},\frac{{u_3}^{t_3}}{0.4},\frac{{u_4}^{t_3}}{0.1}\right\}\right),\\
&\left(\left(\frac{0.7}{e_2},n,0\right),\left\{\frac{{u_1}^{t_3}}{0.1},\frac{{u_2}^{t_3}}{0.5},\frac{{u_3}^{t_3}}{0.6},\frac{{u_4}^{t_3}}{0.4}\right\}\right),\left(\left(\frac{0.7}{e_2},r,0\right),\left\{\frac{{u_1}^{t_3}}{0.7},\frac{{u_2}^{t_3}}{0.3},\frac{{u_3}^{t_3}}{0.3},\frac{{u_4}^{t_3}}{0.2}\right\}\right),\\
&\left(\left(\frac{0.9}{e_3},m,0\right),\left\{\frac{{u_1}^{t_3}}{0.4},\frac{{u_2}^{t_3}}{0.6},\frac{{u_3}^{t_3}}{0.8},\frac{{u_4}^{t_3}}{0.3}\right\}\right),\left(\left(\frac{0.9}{e_3},n,0\right),\left\{\frac{{u_1}^{t_3}}{0.3},\frac{{u_2}^{t_3}}{0.4},\frac{{u_3}^{t_3}}{0.7},\frac{{u_4}^{t_3}}{0.2}\right\}\right),
\end{aligned}
$$

$$\left(\left(\frac{0.9}{e_3},r,0\right),\left\{\frac{{u_1}^{t_3}}{0.7},\frac{{u_2}^{t_3}}{0.2},\frac{{u_3}^{t_3}}{0.6},\frac{{u_4}^{t_3}}{0.5}\right\}\right)\Bigg\}.$$

4 Basic Operations

In this part, we define the complement, union, and intersection of FPFSES, deduce several features, and provide illustrative instances.

4.1 Complement

Definition 4.1. The *complement* of a$(\overline{PTFSES})(F,A)^t_\Psi$ is indicated by $\widetilde{c}(F,A)^t_\Psi$, it can be expressed as $\widetilde{c}(F,A)^t_\Psi=(F^c,M)_\Psi$ where F^c is fuzzy soft expert complement and Ψ^c is a fuzzy complement.

Example 4.2. Let us examine Example 3.2. Utilizing the fundamental fuzzy complement, we have

$$\begin{aligned}
\widetilde{c}(F,A)^t_\Psi=\Bigg\{ & \left(\left(\frac{0.5}{e_1},m,1\right),\left\{\frac{{u_1}^{t_1}}{0.2},\frac{{u_2}^{t_1}}{0.5},\frac{{u_3}^{t_1}}{0.6},\frac{{u_4}^{t_1}}{0.7}\right\}\right),\left(\left(\frac{0.5}{e_1},n,1\right),\left\{\frac{{u_1}^{t_1}}{0.4},\frac{{u_2}^{t_1}}{0.1},\frac{{u_3}^{t_1}}{0.4},\frac{{u_4}^{t_1}}{0.5}\right\}\right)\\
& \left(\left(\frac{0.5}{e_1},r,1\right),\left\{\frac{{u_1}^{t_1}}{0.3},\frac{{u_2}^{t_1}}{0.4},\frac{{u_3}^{t_1}}{0.7},\frac{{u_4}^{t_1}}{0.3}\right\}\right),\left(\left(\frac{0.3}{e_2},m,1\right),\left\{\frac{{u_1}^{t_1}}{0.4},\frac{{u_2}^{t_1}}{0.9},\frac{{u_3}^{t_1}}{0.6},\frac{{u_4}^{t_1}}{0.2}\right\}\right),\\
& \left(\left(\frac{0.3}{e_2},n,1\right),\left\{\frac{{u_1}^{t_1}}{0.5},\frac{{u_2}^{t_1}}{0.3},\frac{{u_3}^{t_1}}{0.8},\frac{{u_4}^{t_1}}{0.5}\right\}\right),\left(\left(\frac{0.3}{e_2},r,1\right),\left\{\frac{{u_1}^{t_1}}{0.8},\frac{{u_2}^{t_1}}{0.4},\frac{{u_3}^{t_1}}{0.3},\frac{{u_4}^{t_1}}{0.7}\right\}\right),\\
& \left(\left(\frac{0.1}{e_3},m,1\right),\left\{\frac{{u_1}^{t_1}}{0.8},\frac{{u_2}^{t_1}}{0.6},\frac{{u_3}^{t_1}}{0.1},\frac{{u_4}^{t_1}}{0.4}\right\}\right),\left(\left(\frac{0.1}{e_3},n,1\right),\left\{\frac{{u_1}^{t_1}}{0.4},\frac{{u_2}^{t_1}}{0.5},\frac{{u_3}^{t_1}}{0.4},\frac{{u_4}^{t_1}}{0.3}\right\}\right),\\
& \left(\left(\frac{0.1}{e_3},r,1\right),\left\{\frac{{u_1}^{t_1}}{0.6},\frac{{u_2}^{t_1}}{0.3},\frac{{u_3}^{t_1}}{0.9},\frac{{u_4}^{t_1}}{0.5}\right\}\right),\left(\left(\frac{0.5}{e_1},m,1\right),\left\{\frac{{u_1}^{t_2}}{0.6},\frac{{u_2}^{t_2}}{0.2},\frac{{u_3}^{t_2}}{0.8},\frac{{u_4}^{t_2}}{0.6}\right\}\right),\\
& \left(\left(\frac{0.5}{e_1},n,1\right),\left\{\frac{{u_1}^{t_2}}{0.4},\frac{{u_2}^{t_2}}{0.1},\frac{{u_3}^{t_2}}{0.6},\frac{{u_4}^{t_2}}{0.2}\right\}\right),\left(\left(\frac{0.5}{e_1},r,1\right),\left\{\frac{{u_1}^{t_2}}{0.1},\frac{{u_2}^{t_2}}{0.8},\frac{{u_3}^{t_2}}{0.7},\frac{{u_4}^{t_2}}{0.7}\right\}\right),\\
& \left(\left(\frac{0.3}{e_2},m,1\right),\left\{\frac{{u_1}^{t_2}}{0.8},\frac{{u_2}^{t_2}}{0.1},\frac{{u_3}^{t_2}}{0.5},\frac{{u_4}^{t_2}}{0.5}\right\}\right),\left(\left(\frac{0.3}{e_2},n,1\right),\left\{\frac{{u_1}^{t_2}}{0.3},\frac{{u_2}^{t_2}}{0.4},\frac{{u_3}^{t_2}}{0.8},\frac{{u_4}^{t_2}}{0.4}\right\}\right),\\
& \left(\left(\frac{0.3}{e_2},r,1\right),\left\{\frac{{u_1}^{t_2}}{0.5},\frac{{u_2}^{t_2}}{0.6},\frac{{u_3}^{t_2}}{0.3},\frac{{u_4}^{t_2}}{0.6}\right\}\right),\left(\left(\frac{0.1}{e_3},m,1\right),\left\{\frac{{u_1}^{t_2}}{0.1},\frac{{u_2}^{t_2}}{0.6},\frac{{u_3}^{t_2}}{0.9},\frac{{u_4}^{t_2}}{0.3}\right\}\right),\\
& \left(\left(\frac{0.1}{e_3},n,1\right),\left\{\frac{{u_1}^{t_2}}{0.2},\frac{{u_2}^{t_2}}{0.4},\frac{{u_3}^{t_2}}{0.9},\frac{{u_4}^{t_2}}{0.5}\right\}\right),\left(\left(\frac{0.1}{e_3},r,1\right),\left\{\frac{{u_1}^{t_2}}{0.8},\frac{{u_2}^{t_2}}{0.1},\frac{{u_3}^{t_2}}{0.4},\frac{{u_4}^{t_2}}{0.3}\right\}\right),\\
& \left(\left(\frac{0.5}{e_1},m,1\right),\left\{\frac{{u_1}^{t_3}}{0.8},\frac{{u_2}^{t_3}}{0.3},\frac{{u_3}^{t_3}}{0.2},\frac{{u_4}^{t_3}}{0.7}\right\}\right),\left(\left(\frac{0.5}{e_1},n,1\right),\left\{\frac{{u_1}^{t_3}}{0.6},\frac{{u_2}^{t_3}}{0.1},\frac{{u_3}^{t_3}}{0.4},\frac{{u_4}^{t_3}}{0.5}\right\}\right),\\
& \left(\left(\frac{0.5}{e_1},r,1\right),\left\{\frac{{u_1}^{t_3}}{0.4},\frac{{u_2}^{t_3}}{0.5},\frac{{u_3}^{t_3}}{0.3},\frac{{u_4}^{t_3}}{0.2}\right\}\right),\left(\left(\frac{0.3}{e_2},m,1\right),\left\{\frac{{u_1}^{t_3}}{0.5},\frac{{u_2}^{t_3}}{0.6},\frac{{u_3}^{t_3}}{0.3},\frac{{u_4}^{t_3}}{0.1}\right\}\right),\\
& \left(\left(\frac{0.3}{e_2},n,1\right),\left\{\frac{{u_1}^{t_3}}{0.2},\frac{{u_2}^{t_3}}{0.4},\frac{{u_3}^{t_3}}{0.8},\frac{{u_4}^{t_3}}{0.3}\right\}\right),\left(\left(\frac{0.3}{e_2},r,1\right),\left\{\frac{{u_1}^{t_3}}{0.9},\frac{{u_2}^{t_3}}{0.2},\frac{{u_3}^{t_3}}{0.5},\frac{{u_4}^{t_3}}{0.2}\right\}\right),\\
& \left(\left(\frac{0.1}{e_3},m,1\right),\left\{\frac{{u_1}^{t_3}}{0.3},\frac{{u_2}^{t_3}}{0.6},\frac{{u_3}^{t_3}}{0.9},\frac{{u_4}^{t_3}}{0.2}\right\}\right),\left(\left(\frac{0.1}{e_3},n,1\right),\left\{\frac{{u_1}^{t_3}}{0.4},\frac{{u_2}^{t_3}}{0.5},\frac{{u_3}^{t_3}}{0.7},\frac{{u_4}^{t_3}}{0.1}\right\}\right),\\
& \left(\left(\frac{0.1}{e_3},r,1\right),\left\{\frac{{u_1}^{t_3}}{0.6},\frac{{u_2}^{t_3}}{0.3},\frac{{u_3}^{t_3}}{0.5},\frac{{u_4}^{t_3}}{0.4}\right\}\right),\left(\left(\frac{0.5}{e_1},m,0\right),\left\{\frac{{u_1}^{t_1}}{0.7},\frac{{u_2}^{t_1}}{0.4},\frac{{u_3}^{t_1}}{0.3},\frac{{u_4}^{t_1}}{0.2}\right\}\right),\\
& \left(\left(\frac{0.5}{e_1},n,0\right),\left\{\frac{{u_1}^{t_1}}{0.5},\frac{{u_2}^{t_1}}{0.7},\frac{{u_3}^{t_1}}{0.5},\frac{{u_4}^{t_1}}{0.6}\right\}\right),\left(\left(\frac{0.5}{e_1},r,0\right),\left\{\frac{{u_1}^{t_1}}{0.6},\frac{{u_2}^{t_1}}{0.5},\frac{{u_3}^{t_1}}{0.2},\frac{{u_4}^{t_1}}{0.7}\right\}\right),
\end{aligned}$$

$$\left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.3}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.7}\right\}\right), \left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{u_1{}^{t_1}}{0.6}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.1}, \frac{u_4{}^{t_1}}{0.4}\right\}\right),$$
$$\left(\left(\frac{0.3}{e_2}, r, 0\right), \left\{\frac{u_1{}^{t_1}}{0.4}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.6}, \frac{u_4{}^{t_1}}{0.9}\right\}\right), \left(\left(\frac{0.1}{e_3}, m, 0\right), \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.5}, \frac{u_3{}^{t_1}}{0.8}, \frac{u_4{}^{t_1}}{0.7}\right\}\right),$$
$$\left(\left(\frac{0.1}{e_3}, n, 0\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.5}, \frac{u_4{}^{t_1}}{0.8}\right\}\right), \left(\left(\frac{0.1}{e_3}, r, 0\right), \left\{\frac{u_1{}^{t_1}}{0.2}, \frac{u_2{}^{t_1}}{0.8}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.6}\right\}\right),$$
$$\left(\left(\frac{0.5}{e_1}, m, 0\right), \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.6}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.5}\right\}\right), \left(\left(\frac{0.5}{e_1}, n, 0\right), \left\{\frac{u_1{}^{t_2}}{0.5}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.5}, \frac{u_4{}^{t_2}}{0.6}\right\}\right),$$
$$\left(\left(\frac{0.5}{e_1}, r, 0\right), \left\{\frac{u_1{}^{t_2}}{0.7}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.2}\right\}\right), \left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{u_1{}^{t_2}}{0.1}, \frac{u_2{}^{t_2}}{0.7}, \frac{u_3{}^{t_2}}{0.4}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.1}, \frac{u_4{}^{t_2}}{0.6}\right\}\right), \left(\left(\frac{0.3}{e_2}, r, 0\right) \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.6}, \frac{u_4{}^{t_2}}{0.9}\right\}\right),$$
$$\left(\left(\frac{0.1}{e_3}, m, 0\right), \left\{\frac{u_1{}^{t_2}}{0.8}, \frac{u_2{}^{t_2}}{0.3}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.8}\right\}\right), \left(\left(\frac{0.1}{e_3}, n, 0\right), \left\{\frac{u_1{}^{t_2}}{0.6}, \frac{u_2{}^{t_2}}{0.5}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.4}\right\}\right),$$
$$\left(\left(\frac{0.1}{e_3}, r, 0\right), \left\{\frac{u_1{}^{t_2}}{0.3}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.7}, \frac{u_4{}^{t_2}}{0.7}\right\}\right), \left(\left(\frac{0.5}{e_1}, m, 0\right), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.2}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.6}\right\}\right),$$
$$\left(\left(\frac{0.5}{e_1}, n, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.4}\right\}\right), \left(\left(\frac{0.5}{e_1}, r, 0\right), \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.8}, \frac{u_4{}^{t_3}}{0.5}\right\}\right),$$
$$\left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.3}, \frac{u_3{}^{t_3}}{0.6}, \frac{u_4{}^{t_3}}{0.9}\right\}\right), \left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{u_1{}^{t_3}}{0.9}, \frac{u_2{}^{t_3}}{0.5}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.6}\right\}\right),$$
$$\left(\left(\frac{0.3}{e_2}, r, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.7}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left(\left(\frac{0.1}{e_3}, m, 0\right), \left\{\frac{u_1{}^{t_3}}{0.6}, \frac{u_2{}^{t_3}}{0.4}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.7}\right\}\right),$$
$$\left(\left(\frac{0.1}{e_3}, n, 0\right), \left\{\frac{u_1{}^{t_3}}{0.7}, \frac{u_2{}^{t_3}}{0.6}, \frac{u_3{}^{t_3}}{0.3}, \frac{u_4{}^{t_3}}{0.8}\right\}\right), \left(\left(\frac{0.1}{e_3}, r, 0\right), \left\{\frac{u_1{}^{t_3}}{0.3}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.4}, \frac{u_4{}^{t_3}}{0.5}\right\}\right)\Bigg\}.$$

Proposition 4.3. *If $(F, A)_{\Psi}^{t}$ is a($\overline{PTFSES}$)over U, then*

1. $\left((F, A)_{\Psi^c}^{t}\right)^c = (F, A)_{\Psi}^{t}$,

Proof. By employing Definition 4.1 we have $(F, A)_{\Psi}^{t\ c}$ is defined by
$c\left(\mu_{\Psi}\left(x\right)\right)$ and $\tilde{c}\left(F\left(x\right)\right), \forall x \in Z$.
Then $c\left(\mu_{\Psi}\left(x\right)\right)^c$ is defined by $c\left(c\left(\mu_{\Psi}\left(x\right)\right)\right)$ and $\tilde{c}\left(\tilde{c}\left(F\left(x\right)\right)\right), \forall x \in Z$.
But $c\left(c\left(\mu_{\Psi}\left(x\right)\right)\right) = \left(\mu_{\Psi}\left(x\right)\right)$, and $\tilde{c}\left(\tilde{c}\left(F\left(x\right)\right)\right) = \left(F\left(x\right)\right)$.

□

where c is parameterized time fuzzy soft set complement.

4.2 Union

Definition 4.4. The *union* of two ($\overline{PTFSES's}$) $(F, A)_{\Psi}^{t}$ and $(G, B)_{\Upsilon}^{t}$ over U, denoted by $(F, A)_{\Psi}^{t} \widetilde{\cup} (G, B)_{\Upsilon}^{t}$, is the($\overline{PTFSES}$)$(H, C)_{\Omega}^{t}$ such that $R = M \cup N$, $\Omega = max\left(\Psi \cup \Upsilon\right)$,$\forall\, \varepsilon \in R$,
$H\left(\varepsilon\right) = \left(F\left(\varepsilon\right) \widetilde{\cup} G\left(\varepsilon\right)\right)$ where $\widetilde{\cup}$ is an fuzzy soft expert union.

Example 4.5. Think about this example: 3.2. Let Suppose $(F, A)_{\Psi}^{t}$ and $(G, B)_{\Upsilon}^{t}$ are two time-fuzzy soft expert sets over U such that

$$(F,A)^t_\Psi = \Bigg\{ \left(\left(\frac{0.7}{e_2},n,1\right),\left\{\tfrac{u_1{}^{t_1}}{0.5},\tfrac{u_2{}^{t_1}}{0.7},\tfrac{u_3{}^{t_1}}{0.2},\tfrac{u_4{}^{t_1}}{0.5}\right\}\right),\left(\left(\frac{0.7}{e_2},r,1\right),\left\{\tfrac{u_1{}^{t_1}}{0.2},\tfrac{u_2{}^{t_1}}{0.6},\tfrac{u_3{}^{t_1}}{0.7},\tfrac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2},r,1\right),\left\{\tfrac{u_1{}^{t_2}}{0.5},\tfrac{u_2{}^{t_2}}{0.4},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.4}\right\}\right),\left(\left(\frac{0.9}{e_3},n,1\right),\left\{\tfrac{u_1{}^{t_2}}{0.8},\tfrac{u_2{}^{t_2}}{0.6},\tfrac{u_3{}^{t_2}}{0.1},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),$$
$$\left(\left(\frac{0.9}{e_3},m,1\right),\left\{\tfrac{u_1{}^{t_3}}{0.7},\tfrac{u_2{}^{t_3}}{0.4},\tfrac{u_3{}^{t_3}}{0.1},\tfrac{u_4{}^{t_3}}{0.8}\right\}\right),\left(\left(\frac{0.5}{e_1},m,0\right),\left\{\tfrac{u_1{}^{t_1}}{0.3},\tfrac{u_2{}^{t_1}}{0.6},\tfrac{u_3{}^{t_1}}{0.7},\tfrac{u_4{}^{t_1}}{0.8}\right\}\right),$$
$$\left(\left(\frac{0.9}{e_3},n,0\right),\left\{\tfrac{u_1{}^{t_1}}{0.5},\tfrac{u_2{}^{t_1}}{0.4},\tfrac{u_3{}^{t_1}}{0.5},\tfrac{u_4{}^{t_1}}{0.2}\right\}\right),\left(\left(\frac{0.9}{e_3},r,0\right),\left\{\tfrac{u_1{}^{t_1}}{0.8},\tfrac{u_2{}^{t_1}}{0.2},\tfrac{u_3{}^{t_1}}{0.8},\tfrac{u_4{}^{t_1}}{0.4}\right\}\right),$$
$$\left(\left(\frac{0.5}{e_1},m,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.4},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),\left(\left(\frac{0.9}{e_3},m,0\right),\left\{\tfrac{u_1{}^{t_3}}{0.4},\tfrac{u_2{}^{t_3}}{0.6},\tfrac{u_3{}^{t_3}}{0.8},\tfrac{u_4{}^{t_3}}{0.3}\right\}\right)\Bigg\}.$$

$$(G,B)^t_\Upsilon = \Bigg\{ \left(\left(\frac{0.4}{e_2},n,1\right),\left\{\tfrac{u_1{}^{t_1}}{0.7},\tfrac{u_2{}^{t_1}}{0.9},\tfrac{u_3{}^{t_1}}{0.1},\tfrac{u_4{}^{t_1}}{0.3}\right\}\right),\left(\left(\frac{0.4}{e_2},r,1\right),\left\{\tfrac{u_1{}^{t_1}}{0.6},\tfrac{u_2{}^{t_1}}{0.4},\tfrac{u_3{}^{t_1}}{0.8},\tfrac{u_4{}^{t_1}}{0.1}\right\}\right),$$
$$\left(\left(\frac{0.4}{e_2},r,1\right),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.3},\tfrac{u_3{}^{t_2}}{0.2},\tfrac{u_4{}^{t_2}}{0.6}\right\}\right),\left(\left(\frac{0.2}{e_3},n,1\right),\left\{\tfrac{u_1{}^{t_2}}{0.5},\tfrac{u_2{}^{t_2}}{0.8},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.3}\right\}\right),$$
$$\left(\left(\frac{0.8}{e_1},m,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.5},\tfrac{u_2{}^{t_2}}{0.6},\tfrac{u_3{}^{t_2}}{0.8},\tfrac{u_4{}^{t_2}}{0.4}\right\}\right),\left(\left(\frac{0.5}{e_1},r,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.5},\tfrac{u_3{}^{t_2}}{0.6},\tfrac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left(\left(\frac{0.4}{e_2},m,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.6},\tfrac{u_2{}^{t_2}}{0.7},\tfrac{u_3{}^{t_2}}{0.4},\tfrac{u_4{}^{t_2}}{0.9}\right\}\right),\left(\left(\frac{0.2}{e_3},m,0\right),\left\{\tfrac{u_1{}^{t_3}}{0.2},\tfrac{u_2{}^{t_3}}{0.8},\tfrac{u_3{}^{t_3}}{0.9},\tfrac{u_4{}^{t_3}}{0.1}\right\}\right)\Bigg\}.$$

Then $(F,A)^t_\Upsilon \,\widetilde{\cup}\, (G,B)^t_\Upsilon = (H,C)^t_\Omega$ where

$$(H,C)^t_\Omega = \Bigg\{ \left(\left(\frac{0.7}{e_2},n,1\right),\left\{\tfrac{u_1{}^{t_1}}{0.7},\tfrac{u_2{}^{t_1}}{0.9},\tfrac{u_3{}^{t_1}}{0.2},\tfrac{u_4{}^{t_1}}{0.5}\right\}\right),\left(\left(\frac{0.7}{e_2},r,1\right),\left\{\tfrac{u_1{}^{t_1}}{0.6},\tfrac{u_2{}^{t_1}}{0.6},\tfrac{u_3{}^{t_1}}{0.8},\tfrac{u_4{}^{t_1}}{0.3}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2},r,1\right),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.4},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.6}\right\}\right),\left(\left(\frac{0.9}{e_3},n,1\right),\left\{\tfrac{u_1{}^{t_2}}{0.8},\tfrac{u_2{}^{t_2}}{0.8},\tfrac{u_3{}^{t_2}}{0.7},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),$$
$$\left(\left(\frac{0.9}{e_3},m,1\right),\left\{\tfrac{u_1{}^{t_3}}{0.7},\tfrac{u_2{}^{t_3}}{0.4},\tfrac{u_3{}^{t_3}}{0.1},\tfrac{u_4{}^{t_3}}{0.8}\right\}\right),\left(\left(\frac{0.8}{e_1},m,0\right),\left\{\tfrac{u_1{}^{t_1}}{0.3},\tfrac{u_2{}^{t_1}}{0.6},\tfrac{u_3{}^{t_1}}{0.7},\tfrac{u_4{}^{t_1}}{0.8}\right\}\right),$$
$$\left(\left(\frac{0.9}{e_3},n,0\right),\left\{\tfrac{u_1{}^{t_1}}{0.5},\tfrac{u_2{}^{t_1}}{0.4},\tfrac{u_3{}^{t_1}}{0.5},\tfrac{u_4{}^{t_1}}{0.2}\right\}\right),\left(\left(\frac{0.9}{e_3},r,0\right),\left\{\tfrac{u_1{}^{t_1}}{0.8},\tfrac{u_2{}^{t_1}}{0.2},\tfrac{u_3{}^{t_1}}{0.8},\tfrac{u_4{}^{t_1}}{0.4}\right\}\right),$$
$$\left(\left(\frac{0.8}{e_1},m,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.6},\tfrac{u_3{}^{t_2}}{0.8},\tfrac{u_4{}^{t_2}}{0.5}\right\}\right),\left(\left(\frac{0.8}{e_1},r,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.7},\tfrac{u_2{}^{t_2}}{0.5},\tfrac{u_3{}^{t_2}}{0.6},\tfrac{u_4{}^{t_2}}{0.7}\right\}\right),$$
$$\left(\left(\frac{0.7}{e_2},m,0\right),\left\{\tfrac{u_1{}^{t_2}}{0.6},\tfrac{u_2{}^{t_2}}{0.7},\tfrac{u_3{}^{t_2}}{0.4},\tfrac{u_4{}^{t_2}}{0.9}\right\}\right),\left(\left(\frac{0.9}{e_3},m,0\right),\left\{\tfrac{u_1{}^{t_3}}{0.4},\tfrac{u_2{}^{t_3}}{0.8},\tfrac{u_3{}^{t_3}}{0.9},\tfrac{u_4{}^{t_3}}{0.3}\right\}\right)\Bigg\}.$$

Proposition 4.6. *If $(F,A)^t_\Psi$, $(G,B)^t_\Upsilon$ and $(H,C)^t_\Omega$ are three ($\overline{PTFSES's}$) over U, then*

1. $\big(F,A)^t_\Psi \,\widetilde{\cup}\, ((G,B)^t_\Upsilon \,\widetilde{\cup}\, (H,C)^t_\Omega)) = ((F,A)^t_\Psi \,\widetilde{\cup}\, (G,B))_\Upsilon \,\widetilde{\cup}\, (H,C)^t_\Omega\ )$,
2. $(F,A)^t_\Psi \,\widetilde{\cup}\, (F,A)^t_\Psi = (F,A)^t_\Psi$.

Proof. 1. By using definition 4.4 we want to prove that
$(F,A)^t_\Psi \,\widetilde{\cup}\, \Big((G,B)^t_\Upsilon \,\widetilde{\cup}\, (H,C)^t_\Omega\Big) = \Big((F,A)^t_\Psi \,\widetilde{\cup}\, (G,B)^t_\Upsilon\Big) \,\widetilde{\cup}\, (H,C)^t_\Omega$.

But we know that if M, N and R are three fuzzy sets then $(A \cup N) \cup C = A \cup (B \cup C)$ and this fact also hold if M, N and R are three fuzzy soft expert sets. By using these facts we have

$$(F,A)^t_\Psi \widetilde{\cup} \left((G,B)^t_\Upsilon \widetilde{\cup} (H,C)^t_\Omega\right) = \left(F^t_\Psi(\varepsilon) \widetilde{\cup} \left(G^t_\Upsilon(\varepsilon) \widetilde{\cup} H_\Omega(\varepsilon)\right), A\widetilde{\cup}(B\widetilde{\cup}C)\right).$$
$$= \left(\left(F^t_\Psi(\varepsilon) \widetilde{\cup} G^t_\Upsilon(\varepsilon)\right) \widetilde{\cup} H_\Omega(\varepsilon), (A\widetilde{\cup}B)\widetilde{\cup}C\right).$$
$$= \left((F,A)^t_\Psi \widetilde{\cup} (G,B)^t_\Upsilon\right) \widetilde{\cup} (H,C,)_\Omega.$$
$$= \left((F,A)^t_\Psi \widetilde{\cup} (G,B)^t_\Upsilon\right) \widetilde{\cup} (H,C)^t_\Omega.$$

2. Likewise, the proof of 2 can be shown in similar way.

□

4.3 Intersection

Definition 4.7. The *intersection* of two $(\overline{PTFSES's})$ $(F,A)^t_\Psi$ and $(G,B)^t_\Upsilon$ over U, denoted by $(F,A)^t_\Psi \widetilde{\cap} (G,B)^t_\Upsilon$, is the$(\overline{PTFSES})(H,C)^t_\Omega$ such that $C = M \cap N$, $\Omega = max\,(\Psi \cap \Upsilon)$,$\forall\, \varepsilon \in C$,
$H(\varepsilon) = \left(F(\varepsilon) \widetilde{\cap} G(\varepsilon)\right)$ where $\widetilde{\cap}$ is an fuzzy soft expert intersection.

Example 4.8. Consider Example 4.5. By using basic fuzzy intersection (minimum) we have $(F,A)^t_\Psi \widetilde{\cap}(G,B)^t_\Upsilon = (H,C)^t_\Omega$ Where

$$\begin{aligned}(H,C)^t_\Omega = \Bigg\{ & \left(\left(\frac{0.4}{e_2}, n, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.5}, \tfrac{u_2{}^{t_1}}{0.7}, \tfrac{u_3{}^{t_1}}{0.1}, \tfrac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(\frac{0.4}{e_2}, r, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.2}, \tfrac{u_2{}^{t_1}}{0.4}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.1}\right\}\right),\\
& \left(\left(\frac{0.4}{e_2}, r, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.5}, \tfrac{u_2{}^{t_2}}{0.3}, \tfrac{u_3{}^{t_2}}{0.2}, \tfrac{u_4{}^{t_2}}{0.4}\right\}\right), \left(\left(\frac{0.2}{e_3}, n, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.5}, \tfrac{u_2{}^{t_2}}{0.6}, \tfrac{u_3{}^{t_2}}{0.1}, \tfrac{u_4{}^{t_2}}{0.3}\right\}\right),\\
& \left(\left(\frac{0.2}{e_3}, m, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.7}, \tfrac{u_2{}^{t_3}}{0.4}, \tfrac{u_3{}^{t_3}}{0.1}, \tfrac{u_4{}^{t_3}}{0.8}\right\}\right), \left(\left(\frac{0.5}{e_1}, m, 0\right), \left\{\tfrac{u_1{}^{t_1}}{0.3}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.8}\right\}\right),\\
& \left(\left(\frac{0.2}{e_3}, n, 0\right), \left\{\tfrac{u_1{}^{t_1}}{0.5}, \tfrac{u_2{}^{t_1}}{0.4}, \tfrac{u_3{}^{t_1}}{0.5}, \tfrac{u_4{}^{t_1}}{0.2}\right\}\right), \left(\left(\frac{0.2}{e_3}, r, 0\right), \left\{\tfrac{u_1{}^{t_1}}{0.8}, \tfrac{u_2{}^{t_1}}{0.2}, \tfrac{u_3{}^{t_1}}{0.8}, \tfrac{u_4{}^{t_1}}{0.4}\right\}\right),\\
& \left(\left(\frac{0.5}{e_1}, m, 0\right), \left\{\tfrac{u_1{}^{t_2}}{0.5}, \tfrac{u_2{}^{t_2}}{0.4}, \tfrac{u_3{}^{t_2}}{0.7}, \tfrac{u_4{}^{t_2}}{0.4}\right\}\right), \left(\left(\frac{0.5}{e_1}, r, 0\right), \left\{\tfrac{u_1{}^{t_2}}{0.7}, \tfrac{u_2{}^{t_2}}{0.5}, \tfrac{u_3{}^{t_2}}{0.6}, \tfrac{u_4{}^{t_2}}{0.7}\right\}\right),\\
& \left(\left(\frac{0.4}{e_2}, m, 0\right), \left\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.7}, \tfrac{u_3{}^{t_2}}{0.4}, \tfrac{u_4{}^{t_2}}{0.9}\right\}\right), \left(\left(\frac{0.2}{e_3}, m, 0\right), \left\{\tfrac{u_1{}^{t_3}}{0.2}, \tfrac{u_2{}^{t_3}}{0.6}, \tfrac{u_3{}^{t_3}}{0.8}, \tfrac{u_4{}^{t_3}}{0.1}\right\}\right)\Bigg\}.\end{aligned}$$

Proposition 4.9. *If $(F,A)^t_\Psi$, $(G,B)^t_\Upsilon$ and $(H,C)^t_\Omega$ are three $(\overline{PTFSES's})$ over U, then*

1. $(F,A)^t_\Psi \widetilde{\cap} ((G,B)^t_\Upsilon \widetilde{\cap} (H,C)^t_\Omega = ((F,A)^t_\Psi \widetilde{\cap} (G,B))^t_\Upsilon \widetilde{\cap} (H,C)^t_\Omega$,
2. $(F,A)^t_\Psi \widetilde{\cap} (F,A)^t_\Psi = (F,A)^t_\Psi$.

Proof. 1. By using definition 4.7 we want to prove that
$(F,A)^t_\Psi \widetilde{\cap} \left((G,B)^t_\Upsilon \widetilde{\cap} (H,C)^t_\Omega\right) = \left((F,A)^t_\Psi \widetilde{\cap} (G,B)^t_\Upsilon\right) \widetilde{\cap} (H,C)^t_\Omega$.

But we know that if M, N and R are three fuzzy sets then $(A \cap N) \cap C = A \cap (B \cap C)$ and this fact also hold if M, N and R are three fuzzy soft expert sets. By using these facts we have

$$(F,A)^t_\Psi \widetilde{\cap} \left((G,B)^t_\Upsilon \widetilde{\cap} (H,C)^t_\Omega\right) = \left(F^t_\Psi(\varepsilon) \widetilde{\cap} \left(G^t_\Upsilon(\varepsilon) \widetilde{\cap} H_\Omega(\varepsilon)\right), A\widetilde{\cap}(B\widetilde{\cap}C)\right).$$
$$= \left(\left(F^t_\Psi(\varepsilon) \widetilde{\cap} G^t_\Upsilon(\varepsilon)\right) \widetilde{\cap} H_\Omega(\varepsilon), (A\widetilde{\cap}B)\widetilde{\cap}C\right).$$
$$= \left((F,A)^t_\Psi \widetilde{\cap} (G,B)^t_\Upsilon\right) \widetilde{\cap} (H,C,)_\Omega.$$
$$= \left((F,A)^t_\Psi \widetilde{\cap} (G,B)^t_\Upsilon\right) \widetilde{\cap} (H,C)^t_\Omega.$$

2. Likewise, the proof of b can be shown in similar way.

□

Proposition 4.10. *If $(F,A)^t_\Psi$, $(G,B)^t_\Upsilon$ and $(H,C)^t_\Omega$ are three ($\overline{PTFSES's}$) over U, then*

1. $(F,A)^t_\Psi \,\tilde{\cup}\, \left((G,B)^t_\Upsilon \,\tilde{\cap}\, (H,C)^t_\Omega\right) = \left((F,A)^t_\Psi \,\tilde{\cup}\, (G,B)^t_\Upsilon\right) \tilde{\cap} \left((F,A)^t_\Psi \,\tilde{\cup}\, (H,C)^t_\Omega\right)$,
2. $(F,A)^t_\Psi \,\tilde{\cap}\, \left((G,B)^t_\Upsilon \,\tilde{\cup}\, (H,C)^t_\Omega\right) = \left((F,A)^t_\Psi \,\tilde{\cap}\, (G,B)^t_\Upsilon\right) \tilde{\cup} \left((F,A)^t_\Psi \,\tilde{\cap}\, (H,C)^t_\Omega\right)$.

Proof. 1. We want to prove that
$(F,A)^t_\Psi \,\tilde{\cap}\, \left((G,B)^t_\Upsilon \,\tilde{\cup}\, (H,C)^t_\Omega\right) = \left((F,A)^t_\Psi \,\tilde{\cap}\, (G,B)^t_\Upsilon\right) \tilde{\cup} \left((F,A)^t_\Psi \,\tilde{\cap}\, (H,C)^t_\Omega\right)$,

But we know that if M, N and R are three fuzzy sets then $(A \cap N) \cup C = (A \cap N) \cup (A \cap C)$ and this fact also hold if M, N and R are three fuzzy soft expert sets. By using these facts we have

$$\begin{aligned}
(F,A)^t_\Psi \,\tilde{\cap}\, \left((G,B)^t_\Upsilon \,\tilde{\cup}\, (H,C)^t_\Omega\right) &= \left(F^t_\Psi(\varepsilon) \cup \left(G^t_\Upsilon(\varepsilon) \cap H_\Omega(\varepsilon)\right), A \cup (B \cup C)\right).\\
&= \left(\left(F^t_\Psi(\varepsilon) \cup G^t_\Upsilon(\varepsilon)\right), A \cup N\right) \cap \left(F(\varepsilon) \cup H_\Omega(\varepsilon), A \cup C\right).\\
&= \left((F,A)^t_\Psi \,\tilde{\cup}\, (G,B)^t_\Upsilon\right) \tilde{\cap} \left((F,A) \,\tilde{\cup}\, (H,C)^t_\Omega\right).\\
&= \left((F,A)^t_\Psi \,\tilde{\cap}\, (G,B)^t_\Upsilon\right) \tilde{\cup} \left((F,A)^t_\Psi \,\tilde{\cap}\, (H,C)^t_\Omega\right).
\end{aligned}$$

2. Likewise, the proof of b can be shown in similar way.

□

5 AND and OR Operations

This section defines the AND and OR operations for PTFSES, explains how they work, and provides some examples.

Definition 5.1. If $(F,A)^t_\Psi$ and $(G,B)^t_\Upsilon$ are two ($\overline{PTFSES's}$) over U then "$(F,A)^t_\Psi$ AND $(G,B)^t_\Upsilon$" represent by $(F,A)^t_\Psi \wedge (G,B)^t_\Upsilon$ is defined by

$$(F,A)^t_\Psi \wedge (G,B)^t_\Upsilon = (H, M \times N)_\Omega$$

$\Omega = \Psi \times \Upsilon$, and $H(\alpha,\beta) = F(\alpha) \tilde{\cap} G(\beta), \forall (\alpha,\beta) \in M \times N$.

Example 5.2. Consider Example 3.2. Let $(F,A)^t_\Psi$ and $(G,B)^t_\Upsilon$ are two time-fuzzy soft sets over U such that

$$(F,A)^t_\Psi = \left\{ \left(\left(\frac{0.5}{e_2}, n, 1\right), \left\{\frac{u_1{}^{t_1}}{0.5}, \frac{u_2{}^{t_1}}{0.7}, \frac{u_3{}^{t_1}}{0.2}, \frac{u_4{}^{t_1}}{0.5}\right\}\right), \left(\left(\frac{0.3}{e_1}, m, 1\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.8}, \frac{u_3{}^{t_2}}{0.2}, \frac{u_4{}^{t_2}}{0.4}\right\}\right), \right.$$

$$\left. \left(\left(\frac{0.9}{e_3}, r, 1\right), \left\{\frac{u_1{}^{t_3}}{0.4}, \frac{u_2{}^{t_3}}{0.7}, \frac{u_3{}^{t_3}}{0.5}, \frac{u_4{}^{t_3}}{0.6}\right\}\right), \left(\left(\frac{0.3}{e_1}, m, 0\right), \left\{\frac{u_1{}^{t_1}}{0.3}, \frac{u_2{}^{t_1}}{0.6}, \frac{u_3{}^{t_1}}{0.7}, \frac{u_4{}^{t_1}}{0.8}\right\}\right) \right\}.$$

$$(G,B)^t_\Upsilon = \left\{ \left(\left(\frac{0.6}{e_3}, n, 1\right), \left\{\frac{u_1{}^{t_2}}{0.4}, \frac{u_2{}^{t_2}}{0.9}, \frac{u_3{}^{t_2}}{0.3}, \frac{u_4{}^{t_2}}{0.1}\right\}\right), \left(\left(\frac{0.8}{e_2}, r, 0\right), \left\{\frac{u_1{}^{t_3}}{0.5}, \frac{u_2{}^{t_3}}{0.8}, \frac{u_3{}^{t_3}}{0.2}, \frac{u_4{}^{t_3}}{0.6}\right\}\right) \right\}.$$

Then $(F,A)^t_\Psi \wedge (G,B)^t_\Upsilon = (H, A \times B)^t_\Omega$ where

$$(H, A \times B)^t_\Omega = \Bigg\{ \left(\frac{0.5}{(e_2, e_3)}, (n, n), (1, 1) \right), \left\{ \frac{u_1{}^{t_{1,2}}}{0.4}, \frac{u_2{}^{t_{1,2}}}{0.7}, \frac{u_3{}^{t_{1,2}}}{0.2}, \frac{u_4{}^{t_{1,2}}}{0.1} \right\} \Bigg),$$

$$\left(\frac{0.5}{(e_2, e_2)}, (n, r), (1, 0) \right), \left\{ \frac{u_1{}^{t_{1,3}}}{0.5}, \frac{u_2{}^{t_{1,3}}}{0.7}, \frac{u_3{}^{t_{1,3}}}{0.2}, \frac{u_4{}^{t_{1,3}}}{0.5} \right\},$$

$$\left(\frac{0.3}{(e_1, e_3)}, (m, n), (1, 1) \right), \left\{ \frac{u_1{}^{t_{2,2}}}{0.4}, \frac{u_2{}^{t_{2,2}}}{0.8}, \frac{u_3{}^{t_{2,2}}}{0.2}, \frac{u_4{}^{t_{2,2}}}{0.1} \right\},$$

$$\left(\frac{0.3}{(e_1, e_2)}, (m, r), (1, 0) \right), \left\{ \frac{u_1{}^{t_{2,3}}}{0.4}, \frac{u_2{}^{t_{2,3}}}{0.8}, \frac{u_3{}^{t_{2,3}}}{0.2}, \frac{u_4{}^{t_{2,3}}}{0.4} \right\},$$

$$\left(\frac{0.6}{(e_3, e_3)}, (r, n), (1, 1) \right), \left\{ \frac{u_1{}^{t_{3,2}}}{0.4}, \frac{u_2{}^{t_{3,2}}}{0.7}, \frac{u_3{}^{t_{3,2}}}{0.3}, \frac{u_4{}^{t_{3,2}}}{0.1} \right\},$$

$$\left(\frac{0.8}{(e_3, e_2)}, (r, r), (1, 0) \right), \left\{ \frac{u_1{}^{t_{3,3}}}{0.4}, \frac{u_2{}^{t_{3,3}}}{0.7}, \frac{u_3{}^{t_{3,3}}}{0.2}, \frac{u_4{}^{t_{3,3}}}{0.6} \right\},$$

$$\left(\frac{0.3}{(e_1, e_3)}, (m, n), (0, 1) \right), \left\{ \frac{u_1{}^{t_{1,2}}}{0.3}, \frac{u_2{}^{t_{1,2}}}{0.6}, \frac{u_3{}^{t_{1,2}}}{0.3}, \frac{u_4{}^{t_{1,2}}}{0.1} \right\},$$

$$\left(\frac{0.3}{(e_1, e_2)}, (m, r), (0, 0) \right), \left\{ \frac{u_1{}^{t_{1,3}}}{0.3}, \frac{u_2{}^{t_{1,3}}}{0.6}, \frac{u_3{}^{t_{1,3}}}{0.2}, \frac{u_4{}^{t_{1,3}}}{0.6} \right\} \Bigg\}.$$

Definition 5.3. If $(F, A)^t_\Psi$ and $(G, B)^t_\Upsilon$ are two $(\overline{PTFSES's})$ over U then "$(F, A)^t_\Psi$ OR $(G, B)^t_\Upsilon$" represent by $(F, A)^t_\Psi \vee (G, B)^t_\Upsilon$ is defined by

$$(F, A)^t_\Psi \vee (G, B)^t_\Upsilon = (H, M \times N)_\Omega$$

$\Omega = \Psi \times \Upsilon$, and $H(\alpha, \beta) = F(\alpha) \widetilde{\cup} G(\beta), \forall (\alpha, \beta) \in M \times N$.

Example 5.4. Think about the example 5.2. Using the maximum basic fuzzy union, we have $(F, A)^t_\Psi \vee (G, B)^t_\Upsilon = (H, M \times N)_\Omega$ where

$(G, B)^t_\Upsilon$" denoted by $(H, C)^t_\Omega = (F, A)^t_\Psi \bigvee (G, B)^t_\Upsilon$ where

$$(H, C)^t_\Omega = \Bigg\{ \left(\frac{0.6}{(e_2, e_3)}, (n, n), (1, 1) \right), \left\{ \frac{u_1{}^{t_{1,2}}}{0.4}, \frac{u_2{}^{t_{1,2}}}{0.7}, \frac{u_3{}^{t_{1,2}}}{0.2}, \frac{u_4{}^{t_{1,2}}}{0.1} \right\},$$

$$\left(\frac{0.8}{(e_2, e_2)}, (n, r), (1, 0) \right), \left\{ \frac{u_1{}^{t_{1,3}}}{0.5}, \frac{u_2{}^{t_{1,3}}}{0.7}, \frac{u_3{}^{t_{1,3}}}{0.2}, \frac{u_4{}^{t_{1,3}}}{0.5} \right\},$$

$$\left(\frac{0.6}{(e_1, e_3)}, (m, n), (1, 1) \right), \left\{ \frac{u_1{}^{t_{2,2}}}{0.4}, \frac{u_2{}^{t_{2,2}}}{0.8}, \frac{u_3{}^{t_{2,2}}}{0.2}, \frac{u_4{}^{t_{2,2}}}{0.1} \right\},$$

$$\left(\frac{0.8}{(e_1, e_2)}, (m, r), (1, 0) \right), \left\{ \frac{u_1{}^{t_{2,3}}}{0.4}, \frac{u_2{}^{t_{2,3}}}{0.8}, \frac{u_3{}^{t_{2,3}}}{0.2}, \frac{u_4{}^{t_{2,3}}}{0.4} \right\},$$

$$\left(\frac{0.9}{(e_3, e_3)}, (r, n), (1, 1) \right), \left\{ \frac{u_1{}^{t_{3,2}}}{0.4}, \frac{u_2{}^{t_{3,2}}}{0.7}, \frac{u_3{}^{t_{3,2}}}{0.3}, \frac{u_4{}^{t_{3,2}}}{0.1} \right\},$$

$$\left(\frac{0.9}{(e_3, e_2)}, (r, r), (1, 0) \right), \left\{ \frac{u_1{}^{t_{3,3}}}{0.4}, \frac{u_2{}^{t_{3,3}}}{0.7}, \frac{u_3{}^{t_{3,3}}}{0.2}, \frac{u_4{}^{t_{3,3}}}{0.6} \right\},$$

$$\left(\frac{0.6}{(e_1, e_3)}, (m, n), (0, 1) \right), \left\{ \frac{u_1{}^{t_{1,2}}}{0.3}, \frac{u_2{}^{t_{1,2}}}{0.6}, \frac{u_3{}^{t_{1,2}}}{0.3}, \frac{u_4{}^{t_{1,2}}}{0.1} \right\},$$

$$\left(\frac{0.8}{(e_1,e_2)},(m,r),(0,0)\right),\left\{\frac{u_1{}^{t_{1,3}}}{0.3},\frac{u_2{}^{t_{1,3}}}{0.6},\frac{u_3{}^{t_{1,3}}}{0.2},\frac{u_4{}^{t_{1,3}}}{0.6}\right\}\Bigg\}.$$

Proposition 5.5. *If $(F,A)^t_\Psi$ and $(G,B)^t_\Upsilon$ are two fuzzy parameterized fuzzy soft expert sets over W, then*

1. $\left((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon\right)^c = (F,A)^t_{\Psi^c} \vee (G,B)^t_{\Upsilon^c}$
2. $\left((F,A)^t_\Psi \vee (G,B)^t_\Upsilon\right)^c = (F,A)^t_{\Psi^c} \wedge (G,B)^t_{\Upsilon^c}$

Proof. 1. We want to prove that $\left((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon\right)^c = (F,A)^t_{\Psi^c} \vee (G,B)^t_{\Upsilon^c}$

But we know that if M and N are two fuzzy sets then $(M \wedge N)^c = A^c \vee B^c$ and this fact also hold if M and N are two fuzzy soft expert sets. By using these facts we have:
Suppose that $(F,A)^t_\Psi \wedge (G,B)^t_\Upsilon = (O, M \times N)$.
Therefore, $((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon)^c = (O, M\times N)^c = (O^c, (M \times N))$. Now,
$\left((F,A)^t_\Psi \vee (G,B)^t_\Upsilon\right)^c = ((F^t{}_{\Psi^c}, M) \vee (G^t{}_{\Upsilon^c}, N))$
$= (J,(M\times N))$, where $J(x,y) = t\left(c(F^t_\Psi(\alpha)), c(G^t_\Upsilon(\beta))\right)$.
Now, take $(\alpha,\beta) \in (M \times N)$.
Then,
$O^c(\alpha,\beta) = \bar{1} - O(\alpha,\beta)$,
$= \bar{1} - [F^t_\Psi(\alpha) \bigcup G^t_\Upsilon(\beta)]$
$= [\bar{1} - F^t_\Psi(\alpha)] \bigcap [\bar{1} - G^t_\Upsilon(\beta)]$
$= t\left(c(F^t_\Psi(\alpha)), c(G^t_\Upsilon(\beta))\right)$
$= J(\alpha,\beta)$
Therefore O^c and F are the same. Hence, proved.

2. The proof is uncomplicated and straightforward .

□

Proposition 5.6. *If $(F,A)^t_\Psi$, $(G,N)^t_\Upsilon$ and $(H,C)^t_\Omega$ are three fuzzy parameterized fuzzy soft expert sets over W, then*

1. $(F,A)^t_\Psi \wedge ((G,B)^t_\Upsilon \wedge (H,C)^t_\Omega) = ((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon) \wedge (H,C)^t_\Omega\ )$,
2. $(F,A)^t_\Psi \vee ((G,B)^t_\Upsilon \vee (H,C)^t_\Omega) = ((F,A)^t_\Psi \vee (G,B)^t_\Upsilon) \vee (H,C)^t_\Omega\ )$,
3. $(F,A)^t_\Psi \vee ((G,B)^t_\Upsilon \wedge (H,C)^t_\Omega) = ((F,A)^t_\Psi \vee (G,B)^t_\Upsilon) \wedge ((F,A)^t_\Psi \vee (H,C)^t_\Omega)$,
4. $(F,A)^t_\Psi \wedge ((G,B)^t_\Upsilon \vee (H,C)^t_\Omega) = ((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon) \vee ((F,A)^t_\Psi \wedge (H,C)^t_\Omega)$.

Proof. 1. Our goal is showing that $(F,A)^t_\Psi \wedge ((G,B)^t_\Upsilon \wedge (H,C)^t_\Omega) = ((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon) \wedge (H,C)^t_\Omega$,

But we know that if M, N and R are three fuzzy sets then $(M \wedge N) \wedge C = M \wedge (B \wedge C)$ and this fact also hold if M, N and R are three fuzzy soft expert sets. By using these facts we have
Suppose that $(G,N)^t_\Upsilon \wedge (H,C)^t_\Omega = t\left(G^t_\Upsilon(\alpha), H_\Omega(\beta)\right), \forall (\alpha,\beta) \in B \times C$.

Then

$$\begin{aligned}(F,A)^t_\Psi \wedge ((G,B)^t_\Upsilon \wedge (H,C)^t_\Omega) &= t\left(F^t_\Psi(\gamma), t\left(G^t_\Upsilon(\alpha), H_\Omega(\beta)\right)\right), \forall (\gamma,(\alpha,\beta)) \in A \times (B\times C).\\ &= t\left(t\left(F^t_\Psi(\gamma), G^t_\Upsilon(\alpha)\right), H_\Omega(\beta)\right), \forall ((\gamma,\alpha),\beta) \in (A \times N) \times C.\\ &= \left((F,A)^t_\Psi \wedge (G,B)^t_\Upsilon\right) \wedge (H,C)^t_\Omega\ ).\end{aligned}$$

2. The proof is uncomplicated and straightforward .

3. The proof is uncomplicated and straightforward .
4. The proof is uncomplicated and straightforward .

□

6 A Use for Fuzzy Parameterized Time Fuzzy Soft Expert Sets for Making Decisions

In this section, we provide three hypothetical applications of the parameterized time fuzzy soft expert set theory in a decision making problem which demonstrate that this method can be successfully work and it can be applied to problems of many fields that contain uncertainty. The first application consists set of two opinions $\{\text{agree}, \text{disagree}\}$, and the second application gives more consideration to the expert weights. This means that the choice experts may not be of equal in importance degree.

6.1 A decision-making application of parameterized time fuzzy soft expert sets with two viewpoints

Example 6.1. Let us imagine that a chain of restaurants intends to invest in opening a new location in a city suburb. To finish their economic feasibility analysis of their chosen region, they conferred with a few specialists. The following standards were applied by the experts to arrive at their conclusions: The following are their four options: $W = \{u_1,\ u_2,\ u_3, u_4\}$,. There may be six parameters $E = \left\{\frac{0.1}{e_1}, \frac{0.3}{e_2}, \frac{0.5}{e_3}, \right\}$, to assess the restaurant's location . For i = 1, 2, 3, the parameters p_i $(i = 1, 2, 3)$ represent the quantity of comparable eateries that compete in the same chosen area, the percentage of the local population that eats out every day, the structural modifications that must be made to the chosen site, the anticipated number of weekly or monthly patrons, the restaurant's accessibility, including its ample parking options, and the reasonably priced menu options, all of which are somewhat significant in degree 0.1, 0.3, 0.5, respectively. That is, the subset of parameters is $Y = 0.1/e_1, 0.3/e_2, 0.5/e_3$,. And $Y = \{m, n\}$ exist a group of specialists (committee members). $T = \{t_1, t_2, t_3, t_4\}$ be a set of previous time periods. We are able to determine the best site for the chain's new restaurant by using those data. Following a thoughtful deliberation, the committee creates the fuzzy parameterized fuzzy soft expert set that follows.

$$
\begin{aligned}
(F, Z)^t = \Bigg\{ & \left(\left(\tfrac{0.1}{e_1}, m, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.3}, \tfrac{u_3{}^{t_1}}{0.5}, \tfrac{u_4{}^{t_1}}{0.6}\right\}\right), \left(\left(\tfrac{0.1}{e_1}, n, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.7}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.4}, \tfrac{u_4{}^{t_1}}{0.7}\right\}\right), \\
& \left(\left(\tfrac{0.3}{e_2}, m, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.7}, \tfrac{u_2{}^{t_1}}{0.4}, \tfrac{u_3{}^{t_1}}{0.2}, \tfrac{u_4{}^{t_1}}{0.7}\right\}\right), \left(\left(\tfrac{0.3}{e_2}, n, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.5}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.3}, \tfrac{u_4{}^{t_1}}{0.5}\right\}\right), \\
& \left(\left(\tfrac{0.5}{e_3}, m, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.4}, \tfrac{u_2{}^{t_1}}{0.6}, \tfrac{u_3{}^{t_1}}{0.7}, \tfrac{u_4{}^{t_1}}{0.3}\right\}\right), \left(\left(\tfrac{0.5}{e_3}, n, 1\right), \left\{\tfrac{u_1{}^{t_1}}{0.6}, \tfrac{u_2{}^{t_1}}{0.5}, \tfrac{u_3{}^{t_1}}{0.9}, \tfrac{u_4{}^{t_1}}{0.5}\right\}\right), \\
& \left(\left(\tfrac{0.1}{e_1}, m, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.4}, \tfrac{u_2{}^{t_2}}{0.8}, \tfrac{u_3{}^{t_2}}{0.2}, \tfrac{u_4{}^{t_2}}{0.4}\right\}\right), \left(\left(\tfrac{0.1}{e_1}, n, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.9}, \tfrac{u_3{}^{t_2}}{0.4}, \tfrac{u_4{}^{t_2}}{0.8}\right\}\right), \\
& \left(\left(\tfrac{0.3}{e_2}, m, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.4}, \tfrac{u_2{}^{t_2}}{0.9}, \tfrac{u_3{}^{t_2}}{0.7}, \tfrac{u_4{}^{t_2}}{0.5}\right\}\right), \left(\left(\tfrac{0.3}{e_2}, n, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.8}, \tfrac{u_3{}^{t_2}}{0.5}, \tfrac{u_4{}^{t_2}}{0.6}\right\}\right), \\
& \left(\left(\tfrac{0.5}{e_3}, m, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.6}, \tfrac{u_2{}^{t_2}}{0.4}, \tfrac{u_3{}^{t_2}}{0.1}, \tfrac{u_4{}^{t_2}}{0.8}\right\}\right), \left(\left(\tfrac{0.5}{e_3}, n, 1\right), \left\{\tfrac{u_1{}^{t_2}}{0.8}, \tfrac{u_2{}^{t_2}}{0.6}, \tfrac{u_3{}^{t_2}}{0.3}, \tfrac{u_4{}^{t_2}}{0.7}\right\}\right), \\
& \left(\left(\tfrac{0.1}{e_1}, m, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.2}, \tfrac{u_2{}^{t_3}}{0.7}, \tfrac{u_3{}^{t_3}}{0.8}, \tfrac{u_4{}^{t_3}}{0.3}\right\}\right), \left(\left(\tfrac{0.1}{e_1}, n, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.4}, \tfrac{u_2{}^{t_3}}{0.9}, \tfrac{u_3{}^{t_3}}{0.6}, \tfrac{u_4{}^{t_3}}{0.5}\right\}\right), \\
& \left(\left(\tfrac{0.3}{e_2}, m, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.5}, \tfrac{u_2{}^{t_3}}{0.4}, \tfrac{u_3{}^{t_3}}{0.7}, \tfrac{u_4{}^{t_3}}{0.9}\right\}\right), \left(\left(\tfrac{0.3}{e_2}, n, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.8}, \tfrac{u_2{}^{t_3}}{0.6}, \tfrac{u_3{}^{t_3}}{0.2}, \tfrac{u_4{}^{t_3}}{0.7}\right\}\right), \\
& \left(\left(\tfrac{0.5}{e_3}, m, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.7}, \tfrac{u_2{}^{t_3}}{0.6}, \tfrac{u_3{}^{t_3}}{0.3}, \tfrac{u_4{}^{t_3}}{0.7}\right\}\right), \left(\left(\tfrac{0.5}{e_3}, n, 1\right), \left\{\tfrac{u_1{}^{t_3}}{0.6}, \tfrac{u_2{}^{t_3}}{0.5}, \tfrac{u_3{}^{t_3}}{0.3}, \tfrac{u_4{}^{t_3}}{0.9}\right\}\right), \\
& \left(\left(\tfrac{0.1}{e_1}, m, 1\right), \left\{\tfrac{u_1{}^{t_4}}{0.2}, \tfrac{u_2{}^{t_4}}{0.9}, \tfrac{u_3{}^{t_4}}{0.6}, \tfrac{u_4{}^{t_4}}{0.7}\right\}\right), \left(\left(\tfrac{0.1}{e_1}, n, 1\right), \left\{\tfrac{u_1{}^{t_4}}{0.7}, \tfrac{u_2{}^{t_4}}{0.6}, \tfrac{u_3{}^{t_4}}{0.8}, \tfrac{u_4{}^{t_4}}{0.4}\right\}\right), \\
& \left(\left(\tfrac{0.3}{e_2}, m, 1\right), \left\{\tfrac{u_1{}^{t_4}}{0.8}, \tfrac{u_2{}^{t_4}}{0.6}, \tfrac{u_3{}^{t_4}}{0.2}, \tfrac{u_4{}^{t_4}}{0.7}\right\}\right), \left(\left(\tfrac{0.3}{e_2}, n, 1\right), \left\{\tfrac{u_1{}^{t_4}}{0.7}, \tfrac{u_2{}^{t_4}}{0.4}, \tfrac{u_3{}^{t_4}}{0.5}, \tfrac{u_4{}^{t_4}}{0.8}\right\}\right),
\end{aligned}
$$

$$\left(\left(\frac{0.5}{e_3}, m, 1\right), \left\{\frac{{u_1}^{t_4}}{0.7}, \frac{{u_2}^{t_4}}{0.5}, \frac{{u_3}^{t_4}}{0.3}, \frac{{u_4}^{t_4}}{0.9}\right\}\right), \left(\left(\frac{0.5}{e_3}, n, 1\right), \left\{\frac{{u_1}^{t_4}}{0.6}, \frac{{u_2}^{t_4}}{0.8}, \frac{{u_3}^{t_4}}{0.4}, \frac{{u_4}^{t_4}}{0.5}\right\}\right),$$

$$\left(\left(\frac{0.1}{e_1}, m, 0\right), \left\{\frac{{u_1}^{t_1}}{0.5}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.4}, \frac{{u_4}^{t_1}}{0.3}\right\}\right), \left(\left(\frac{0.1}{e_1}, n, 0\right), \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.7}, \frac{{u_4}^{t_1}}{0.2}\right\}\right),$$

$$\left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.7}, \frac{{u_4}^{t_1}}{0.4}\right\}\right), \left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.3}, \frac{{u_3}^{t_1}}{0.6}, \frac{{u_4}^{t_1}}{0.6}\right\}\right),$$

$$\left(\left(\frac{0.5}{e_3}, m, 0\right), \left\{\frac{{u_1}^{t_1}}{0.7}, \frac{{u_2}^{t_1}}{0.5}, \frac{{u_3}^{t_1}}{0.3}, \frac{{u_4}^{t_1}}{0.4}\right\}\right), \left(\left(\frac{0.5}{e_3}, n, 0\right), \left\{\frac{{u_1}^{t_1}}{0.4}, \frac{{u_2}^{t_1}}{0.6}, \frac{{u_3}^{t_1}}{0.2}, \frac{{u_4}^{t_1}}{0.6}\right\}\right),$$

$$\left(\left(\frac{0.1}{e_1}, m, 0\right), \left\{\frac{{u_1}^{t_2}}{0.7}, \frac{{u_2}^{t_2}}{0.4}, \frac{{u_3}^{t_2}}{0.7}, \frac{{u_4}^{t_2}}{0.5}\right\}\right), \left(\left(\frac{0.1}{e_1}, n, 0\right), \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.2}, \frac{{u_3}^{t_2}}{0.5}, \frac{{u_4}^{t_2}}{0.4}\right\}\right),$$

$$\left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{{u_1}^{t_2}}{0.2}, \frac{{u_2}^{t_2}}{0.3}, \frac{{u_3}^{t_2}}{0.4}, \frac{{u_4}^{t_2}}{0.6}\right\}\right), \left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.2}, \frac{{u_3}^{t_2}}{0.7}, \frac{{u_4}^{t_2}}{0.3}\right\}\right),$$

$$\left(\left(\frac{0.5}{e_3}, m, 0\right), \left\{\frac{{u_1}^{t_2}}{0.3}, \frac{{u_2}^{t_2}}{0.5}, \frac{{u_3}^{t_2}}{0.8}, \frac{{u_4}^{t_2}}{0.3}\right\}\right), \left(\left(\frac{0.5}{e_3}, n, 0\right), \left\{\frac{{u_1}^{t_2}}{0.5}, \frac{{u_2}^{t_2}}{0.4}, \frac{{u_3}^{t_2}}{0.7}, \frac{{u_4}^{t_2}}{0.2}\right\}\right),$$

$$\left(\left(\frac{0.1}{e_1}, m, 0\right), \left\{\frac{{u_1}^{t_3}}{0.7}, \frac{{u_2}^{t_3}}{0.2}, \frac{{u_3}^{t_3}}{0.1}, \frac{{u_4}^{t_3}}{0.6}\right\}\right), \left(\left(\frac{0.1}{e_1}, n, 0\right), \left\{\frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.4}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.4}\right\}\right),$$

$$\left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.7}, \frac{{u_3}^{t_3}}{0.4}, \frac{{u_4}^{t_3}}{0.1}\right\}\right), \left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{{u_1}^{t_3}}{0.1}, \frac{{u_2}^{t_3}}{0.5}, \frac{{u_3}^{t_3}}{0.6}, \frac{{u_4}^{t_3}}{0.5}\right\}\right),$$

$$\left(\left(\frac{0.5}{e_3}, m, 0\right), \left\{\frac{{u_1}^{t_3}}{0.4}, \frac{{u_2}^{t_3}}{0.6}, \frac{{u_3}^{t_3}}{0.8}, \frac{{u_4}^{t_3}}{0.3}\right\}\right), \left(\left(\frac{0.5}{e_3}, n, 0\right), \left\{\frac{{u_1}^{t_3}}{0.3}, \frac{{u_2}^{t_3}}{0.4}, \frac{{u_3}^{t_3}}{0.7}, \frac{{u_4}^{t_3}}{0.2}\right\}\right),$$

$$\left(\left(\frac{0.1}{e_1}, m, 0\right), \left\{\frac{{u_1}^{t_4}}{0.6}, \frac{{u_2}^{t_4}}{0.1}, \frac{{u_3}^{t_4}}{0.3}, \frac{{u_4}^{t_4}}{0.4}\right\}\right), \left(\left(\frac{0.1}{e_1}, n, 0\right), \left\{\frac{{u_1}^{t_4}}{0.4}, \frac{{u_2}^{t_4}}{0.4}, \frac{{u_3}^{t_4}}{0.3}, \frac{{u_4}^{t_4}}{0.7}\right\}\right),$$

$$\left(\left(\frac{0.3}{e_2}, m, 0\right), \left\{\frac{{u_1}^{t_4}}{0.3}, \frac{{u_2}^{t_4}}{0.3}, \frac{{u_3}^{t_4}}{0.7}, \frac{{u_4}^{t_4}}{0.2}\right\}\right), \left(\left(\frac{0.3}{e_2}, n, 0\right), \left\{\frac{{u_1}^{t_4}}{0.4}, \frac{{u_2}^{t_4}}{0.6}, \frac{{u_3}^{t_4}}{0.6}, \frac{{u_4}^{t_4}}{0.3}\right\}\right),$$

$$\left(\left(\frac{0.5}{e_3}, m, 0\right), \left\{\frac{{u_1}^{t_4}}{0.6}, \frac{{u_2}^{t_4}}{0.4}, \frac{{u_3}^{t_4}}{0.7}, \frac{{u_4}^{t_4}}{0.2}\right\}\right), \left(\left(\frac{0.5}{e_3}, n, 0\right), \left\{\frac{{u_1}^{t_4}}{0.5}, \frac{{u_2}^{t_4}}{0.3}, \frac{{u_3}^{t_4}}{0.7}, \frac{{u_4}^{t_4}}{0.5}\right\}\right)\Bigg\}.$$

The following algorithm may be followed by committee to evaluate their choice to find the most suitable choice for the decision.

1. Input the T-FPSES (F, Z).
2. Find an agree-time fuzzy parameterized soft expert set and a disagree-time fuzzy parameterized soft expert set, Tables (1, 2).
3. Find the tabular representation of $F(Z)$ as in Tables (3, 3), where $F(Z)$ defined as follows:

$$F(z) = \left\{ \frac{u}{\sum\limits_{i=1}^{n} t_i F_{t_i}(z) \big/ n \sum\limits_{i=1}^{n} F_{t_i}(z)} : u \in U, z \in Z \right\} \tag{1}$$

   where $n = |T|$.
4. Find $c_j = \sum\limits_{x \in X} \sum\limits_{i=1}^{3} (u_{ij})(\mu_E(e_i))$ for agree-TFPFSES, (Table 3).
5. Find $k_j = \sum\limits_{x \in X} \sum\limits_{i=1}^{3} (u_{ij})(\mu_E(e_i))$ for disagree-TFPFSES,(Table 4).
6. Find $s_j = c_j - k_j$, (Table 5).
7. Find m, for which $\mathrm{s}_m = \max s_j$. Then s_m is the optimal choice object. If m has more than one value, then any one of them could be chosen by the restaurant chain using its option.

   Then s_m is the optimal choice object If m has more than one value, then any one of them could be chosen by the restaurant chain using its option.

In Table 1 and Table 2 we present the agree-time fuzzy parameterized soft expert set and disagree-time fuzzy parameterized soft expert set, respectively.

Table 1: Agree time fuzzy parameterized soft expert set

U	u_1	u_2	u_3	u_4
$\left(\frac{0.1}{e_1}, m\right)_{t_1}$	0.6	0.3	0.5	0.6
$\left(\frac{0.1}{e_1}, m\right)_{t_2}$	0.4	0.8	0.2	0.4
$\left(\frac{0.1}{e_1}, m\right)_{t_3}$	0.2	0.7	0.8	0.3
$\left(\frac{0.1}{e_1}, m\right)_{t_4}$	0.2	0.9	0.6	0.7
$\left(\frac{0.1}{e_1}, n\right)_{t_1}$	0.7	0.5	0.4	0.7
$\left(\frac{0.1}{e_1}, n\right)_{t_2}$	0.6	0.9	0.4	0.8
$\left(\frac{0.1}{e_1}, n\right)_{t_3}$	0.4	0.9	0.6	0.5
$\left(\frac{0.1}{e_1}, n\right)_{t_4}$	0.7	0.6	0.8	0.4
$\left(\frac{0.3}{e_2}, m\right)_{t_1}$	0.7	0.4	0.2	0.7
$\left(\frac{0.3}{e_2}, m\right)_{t_2}$	0.4	0.9	0.7	0.5
$\left(\frac{0.3}{e_2}, m\right)_{t_3}$	0.5	0.4	0.7	0.9
$\left(\frac{0.3}{e_2}, m\right)_{t_4}$	0.8	0.6	0.2	0.7
$\left(\frac{0.3}{e_2}, n\right)_{t_1}$	0.5	0.6	0.3	0.5
$\left(\frac{0.3}{e_2}, n\right)_{t_2}$	0.6	0.8	0.5	0.6
$\left(\frac{0.3}{e_2}, n\right)_{t_3}$	0.8	0.6	0.2	0.7
$\left(\frac{0.3}{e_2}, n\right)_{t_4}$	0.7	0.4	0.5	0.8
$\left(\frac{0.5}{e_3}, m\right)_{t_1}$	0.4	0.6	0.7	0.3
$\left(\frac{0.5}{e_3}, m\right)_{t_2}$	0.6	0.4	0.1	0.8
$\left(\frac{0.5}{e_3}, m\right)_{t_3}$	0.7	0.6	0.3	0.7
$\left(\frac{0.5}{e_3}, m\right)_{t_4}$	0.7	0.5	0.3	0.9
$\left(\frac{0.5}{e_3}, n\right)_{t_1}$	0.6	0.5	0.9	0.5
$\left(\frac{0.5}{e_3}, n\right)_{t_2}$	0.8	0.6	0.3	0.7
$\left(\frac{0.5}{e_3}, n\right)_{t_3}$	0.6	0.5	0.3	0.9
$\left(\frac{0.5}{e_3}, n\right)_{t_4}$	0.6	0.8	0.4	0.8

Table 2: Disagree-time fuzzy parameterized soft expert set

U	u_1	u_2	u_3	u_4
$\left(\frac{0.1}{e_1}, m\right)_{t_1}$	0.5	0.6	0.4	0.3
$\left(\frac{0.1}{e_1}, m\right)_{t_2}$	0.7	0.4	0.7	0.5
$\left(\frac{0.1}{e_1}, m\right)_{t_3}$	0.7	0.2	0.1	0.6
$\left(\frac{0.1}{e_1}, m\right)_{t_4}$	0.6	0.1	0.3	0.4
$\left(\frac{0.1}{e_1}, n\right)_{t_1}$	0.4	0.5	0.7	0.2
$\left(\frac{0.1}{e_1}, n\right)_{t_2}$	0.5	0.2	0.5	0.4
$\left(\frac{0.1}{e_1}, n\right)_{t_3}$	0.3	0.4	0.6	0.5
$\left(\frac{0.1}{e_1}, n\right)_{t_4}$	0.4	0.4	0.3	0.7
$\left(\frac{0.3}{e_2}, m\right)_{t_1}$	0.4	0.6	0.7	0.4
$\left(\frac{0.3}{e_2}, m\right)_{t_2}$	0.2	0.3	0.4	0.6
$\left(\frac{0.3}{e_2}, m\right)_{t_3}$	0.3	0.7	0.4	0.1
$\left(\frac{0.3}{e_2}, m\right)_{t_4}$	0.3	0.3	0.7	0.2
$\left(\frac{0.3}{e_2}, n\right)_{t_1}$	0.4	0.3	0.6	0.6
$\left(\frac{0.3}{e_2}, n\right)_{t_2}$	0.5	0.2	0.7	0.3
$\left(\frac{0.3}{e_2}, n\right)_{t_3}$	0.1	0.5	0.6	0.5
$\left(\frac{0.3}{e_2}, n\right)_{t_4}$	0.4	0.6	0.6	0.3
$\left(\frac{0.5}{e_3}, m\right)_{t_1}$	0.7	0.5	0.3	0.4
$\left(\frac{0.5}{e_3}, m\right)_{t_2}$	0.3	0.5	0.8	0.3
$\left(\frac{0.5}{e_3}, m\right)_{t_3}$	0.4	0.6	0.8	0.3
$\left(\frac{0.5}{e_3}, m\right)_{t_4}$	0.6	0.4	0.7	0.2
$\left(\frac{0.5}{e_3}, n\right)_{t_1}$	0.4	0.6	0.2	0.6
$\left(\frac{0.5}{e_3}, n\right)_{t_2}$	0.5	0.4	0.7	0.2
$\left(\frac{0.5}{e_3}, n\right)_{t_3}$	0.3	0.4	0.7	0.2
$\left(\frac{0.5}{e_3}, n\right)_{t_4}$	0.5	0.3	0.7	0.5

Next by using relation (1) we compute $F(Z)$ to convert the agree time fuzzy soft expert set to agree fuzzy soft expert set , then we compute c_j, where c_j represent the column sum of an object u_i. The results are shown in Table 3.

Table 3: Converting agree-TFSES to agree-FSES

U	u_1	u_2	u_3	u_4
$\left(\frac{0.1}{e_1}, m\right)$	0.50	0.70	0.67	0.63
$\left(\frac{0.1}{e_1}, n\right)$	0.61	0.63	0.70	0.56
$\left(\frac{0.3}{e_2}, m\right)$	0.64	0.63	0.62	0.64

Continued on next page

Table 3 – continued

U	u_1	u_2	u_3	u_4
$\left(\frac{0.3}{e_2}, n\right)$	0.66	0.58	0.65	0.67
$\left(\frac{0.5}{e_3}, m\right)$	0.67	0.61	0.53	0.70
$\left(\frac{0.5}{e_3}, n\right)$	0.80	0.66	0.52	0.67
$c_j = \sum_{x\in X} \sum_{i=1}^{3} (u_{ij})(\mu_E(e_i))$	$c_1 = 1.236$	$c_2 = 1.131$	$c_3 = 1.043$	$c_4 = 1.197$

Likewise, by using relation (1) we compute $F(Z)$ to convert the disagree time fuzzy soft expert set to disagree fuzzy soft expert set, then we compute k_j, where k_j represent the column sum of an object u_i. The results are shown in Table 4.

Table 4: Converting disagree-TFSES to disagree-FSES

U	u_1	u_2	u_3	u_4
$\left(\frac{0.1}{e_1}, m\right)$	0.58	0.46	0.55	0.65
$\left(\frac{0.1}{e_1}, n\right)$	0.60	0.61	0.55	0.73
$\left(\frac{0.3}{e_2}, m\right)$	0.60	0.59	0.62	0.51
$\left(\frac{0.3}{e_2}, n\right)$	0.58	0.75	0.62	0.57
$\left(\frac{0.5}{e_3}, m\right)$	0.61	0.61	0.68	0.56
$\left(\frac{0.5}{e_3}, n\right)$	0.63	0.55	0.70	0.60
$k_j = \sum_{x\in X} \sum_{i=1}^{3} (u_{ij})(\mu_E(e_i))$	$k_1 = 1.092$	$k_2 = 1.089$	$k_3 = 1.172$	$k_4 = 1.042$

From Table 3 and Table 4 we calculate the final score (s_j) by subtracting the scores of agree-FPFSES from the scores of disagree-FPFSES.

Table 5: Calculating $s_j = c_j - k_j$

$c_j = \sum_{x\in X} \sum_{i} (u_{ij})(\mu_E(e_i))$	$k_j = \sum_{x\in X} \sum_{i} (u_{ij})(\mu_E(e_i))$	$s_j = c_j - k_j$
$c_1 = 1.236$	$k_1 = 1.092$	$s_1 = 0.144$
$c_2 = 1.131$	$k_2 = 1.089$	$s_2 = 0.042$
$c_3 = 1.043$	$k_3 = 1.172$	$s_3 = -0.127$
$c_4 = 1.197$	$k_4 = 1.042$	$s_4 = 0.155$

Then $\max s_j = s_4$, so the committee will choose location 4 for the restaurant chain depend on the above mentioned algorithm 6.1.

7 Conclusion

We have presented and examined some of the properties of the time fuzzy parameterized time fuzzy soft expert set in this study. The time fuzzy parameterized fuzzy soft expert set has been used to describe the complement, union, and intersection operations. An example of how this theory may be used to solve a decision-making dilemma is given. For future studies, these mathematical tools can be used effectively with other tools and techniques that can be observed through[29]–[31]

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