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FULL LENGTH ARTICLE

Leveraging Time Lag-Based Diffusion Models to Predict Innovation Adoption for Optimized Product Development

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ABSTRACT

The suggested models for the spread of technical breakthroughs make use of a phase structure to illustrate the steps involved in becoming familiar with the problem and making a choice. For it to portray genuine adopting conduct, a time-lag factor is included into the dispersion process. A two-step dissemination process is depicted by taking into account the reliance of adopting on the informed group of potential purchasers. Assuming that a prospective customer first becomes intrigued by an upcoming item's availability and then accepts the novel idea at a later point, a method of analysis for sales functions that incorporates time delay is proposed. The efficient propagation method for invention is shown using the various lag factors. Applying nonlinear regression modelling to worldwide shipping data of Acer PCs and Samsung smartphones experimentally validates the suggested mathematical models. Several comparison models are used to evaluate the predicting abilities of the suggested models. By integrating a distributed time-delay function into the implementation process, a theoretical intergenerational diffusion model is created. To measure how long it takes for innovation to be eventually accepted, the distributed time-lag function that follows the Erlang distribution is used. This framework incorporates switching and substitution, two forms of pragmatic shift behaviour. Using real shipping data of LCD (Liquid Crystal Display) computer monitors from consecutive generations, the predicted effectiveness of the suggested methods is examined and contrasted with well-established research. When contrasted with more conventional models, MGDM 1 achieves a 99.33% accuracy rate, MGDM 2 a 99.81% rate, and MGDM 3 a 99.91% accuracy rate.

Keywords: LCD ▪ MGDM ▪ OLS ▪ NLS ▪ OR ▪ MSE ▪ RMSE ▪ MAD

1. INTRODUCTION

Competition for consumers' attention is fierce in today's market because of the proliferation of functional advances that are functionally comparable and brought about by the fast development of technology. In today's dynamic economic climate, organizations that fail to meet client expectations will eventually fail. In light of the present state of the market, diffusion modelling has therefore evolved into the engine that propels advertising campaigns and execution processes [1, 2]. To improve the accuracy of sales forecasting by accurately representing real-world scenarios, diffusion equations are consistently recommended throughout the marketing literature. In this two-step adoption-of-innovation procedure, we can see how market awareness and knowledge penetration interact with one another. To focus on the real acceptance of an invention, the mathematical models in this study integrate the time-delay component. The time lag is the window of opportunity for prospective adopters to evaluate the technology and make a purchase decision, from the moment they become aware of its presence until the invention is finally adopted.

Additionally, there is a growing demand for precise and ver-

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satile forecasting systems due to the constant release of fresh product developments. Progress in technology and rising customer demands have prompted multiple generations of innovation to flood markets [3, 4]. Most of the new technologies that are now popular are upgraded versions of older ones that will be superseded by even better versions later. Sales of older versions of a product are cannibalized by sales of newer versions as long as both remain on the market. Upgrades from earlier generations are available to users whenever the latest version is released. By the same token, a prospective customer who was going to buy the previous generation may end up buying the newer one instead. Nevertheless, sales of the older model may continue for a while after the release of the newer one, as the majority of consumers do not immediately acquire the most recent features.

Practical models for the spread of new technologies are presented in this work, along with frameworks for predicting how these breakthroughs will sell throughout different generations. The two structures' dissemination processes are shown by various phases of their adoption. An integrated two-stage diffusion model is suggested [5]. This model takes into account the impact of knowledgeable persons on the development of adoption. The time difference between the two adoption phases is explained using various functional forms. Furthermore, the suggested paradigm presupposes that informed market potential has a beneficial effect on adoption.

The empirical investigation demonstrates how the suggested models may be used in the real world to predict interest in new technical goods. The parameters of the diffusion model are estimated using web-based sales data for Acer personal computers and Samsung smartphones [6]. To find out how well the constructed technique predicts the paradigm shift in technological advancement uptake, we examine several statistical metrics and compare the empirical findings with two other studies. The distributed time-lag function represents the adoption process as a two-stage intergenerational diffusion. The lag time between the two adoption phases is assumed to resemble an exponential or Erlang two-stage distribution. When simulating the spread of new generations of technology, the suggested research also distinguishes between two shifting phenomena: substitution and switching. Validation of the suggested statistical models is performed experimentally using sales data for two generations of LCD computer displays, and their predictive power is compared with that of established models.

When it comes to the success of a commercial enterprise, precise preparation for the creation of new products and the implementation of an efficient business plan are both necessary. In an effort to gain a competitive advantage, companies consistently introduce new products and services to the market [7]. On the other hand, new-product creation is one of the riskiest endeavours because it requires extensive financial expenditure and a well-planned commercial strategy. Marketplace economists and policymakers must therefore make prudent decisions using appropriate experience and skills to evaluate a novel item's performance.

It is also important to combine methods from fields such as statistics, marketing, mathematics, engineering, finance, and computer engineering to develop knowledge for resolving complicated business problems. One such field that uses scientific methods for decision-making is operational research (OR). OR provides the foundation on which management problems may be formulated and assists practitioners in arriving at the best possible conclusion. Over the recent past, OR methods have demonstrated their capacity to make pertinent judgments at all three levels of operation [8, 9]. Because of its multidisciplinary character, OR has applications across computer science, business administration, engineering, manufacturing, financial administration, medicine, marketing, epidemiology, supply-chain leadership, and many other areas. It uses quantitative methods and mathematical models to handle the challenges faced by businesses and industries in a consistent and effective manner.

The purpose of the study is to use OR approaches for making choices in order to predict the market response of new items under a given set of parameters [10]. Just as it is necessary to provide an acceptable quantity of materials at the right time and location, it is also necessary to accurately estimate demand for new products and the degree to which they will penetrate the market. In the suggested research, a fair estimate of the future market for innovation is determined by analysing real-life sales data and case-study results. This estimation is based on differential-equation statistical models. The present study also focuses on assessing marketing factors such as selling price, advertising expenditure, and warranty length for consumer goods and technical innovations. This assessment is carried out using optimization methodologies. The real-world applicability of the proposed optimization frameworks is identified by considering products that require significant capital [11, 12].

Mathematical frameworks offer a method for conceiving and resolving many kinds of real-world issues, ranging from daily numerical challenges to enormous, complicated problems affecting society and business [13]. They bring together expertise in mathematical information, statistics, computing, finance, and marketing. A conceptual framework is a generalization of a real-world structure, expressed through mathematical functions that explain the performance of the imagined system. Numerical models allow researchers to understand real-world behaviour, the influence of different features, and the likely conduct of entities [14]. The main objective of applying computational models to real-life problems is to provide a comprehensive understanding of the system and identify effective approaches to the problem. A major additional benefit is their effective use of modern computers.

Computational models are of utmost significance in marketing management. Statistical models are widely used to resolve management issues through assessing, planning, and implementing marketing strategies and ideas [15]. They provide analytical predictions of how new developments will perform in the market, enabling executives to make feasible and beneficial decisions. Measurement and decision-guidance systems are two categories of marketing models investigated in this research. These frameworks evaluate the consumption of new goods as a function of individual variables and assist managers in making decisions.

A better product, process, service, or technology with enhanced qualities and additional economic and social value is considered an innovation [16]. One of the most important factors determining the long-term success of a business is its

capacity to develop and introduce new goods and services. Innovations are new ideas, products, or services produced to meet the requirements of developing markets and provide economic value for an organization [17].

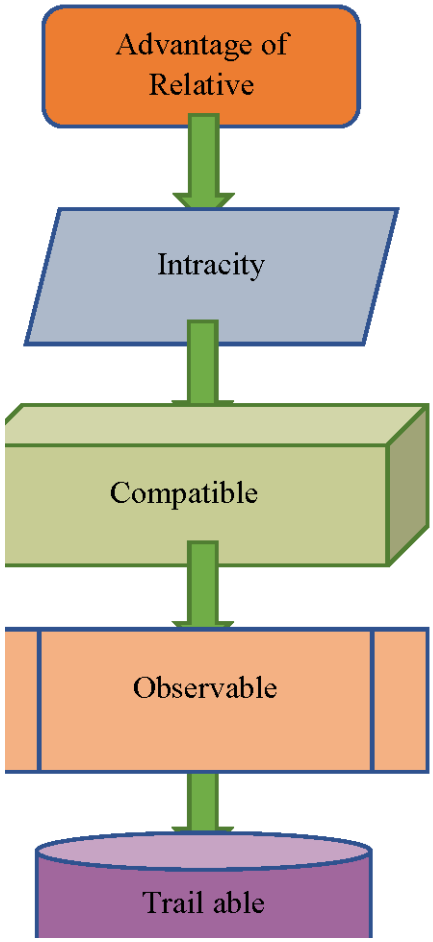


Figure 1. Features of innovation.

Innovation can be defined as the process of implementing modifications, both major and minor, revolutionary and incremental, to items, procedures, and offerings, resulting in the launch of a novel product for the company that provides value to consumers and helps build the organization's knowledge store. The five main features that the target audience views as crucial when evaluating new ideas are shown in Figure 1 [18, 19, 20]. The combination of a highly trained labour force and a productive commercial enterprise produces innovation, which is a prominent technological solution on a global scale. Researchers may learn more about an industry's traits and the process of technological adoption by examining its evolving pattern of innovation.

2. RELATED WORK

The Bass model laid the groundwork for innovation-diffusion research by offering an algebraic structure for analysing the pace of adoption of new ideas in relation to both internal word-of-mouth and external advertising factors [21]. The success of single-generation product market-growth predictions depended heavily on this model, which remains a standard. However, it could not handle time-dependent issues such as delayed diffusion or products spanning multiple generations. A simpler adoption model assumed a constant adoption rate, but its exponential form was later shown to be insufficient for explaining the S-shaped curves observed in actual circumstances. Further developments addressed these shortcomings by incorporating lag time during adoption. The time taken for an invention to move from awareness to use is known as the time lag. Erlang models were among the first to depict adoption as an ongoing procedure with varying delays.

Researchers extended the Bass model by including lagged effects to account for the review time prospective consumers undertake before acquiring an invention. Logistic growth, which is widely used in biological systems, was also employed to account for delayed S-shaped increases in adoption rates and to provide more flexibility when describing dynamics at the beginning and end of adoption [22].

Conventional diffusion models faced new challenges with the introduction of successive product generations such as mobile phones and computer displays. Multigenerational diffusion models show how different generations may coexist in the marketplace while newer products cannibalize the revenues of older ones. This structure laid the groundwork for further research that included phenomena such as switching, in which users of an older generation upgrade to a newer one, and substitution, in which prospective buyers move from one generation to another.

The incorporation of dependence and time-delay effects into diffusion models has been the subject of recent research aimed at enhancing accuracy. Two-stage diffusion models include awareness and adoption processes. Taking factors such as advertising and market trends into consideration, these frameworks postulate that prospective customers learn about a product and evaluate it before deciding to buy. Recognizing uncertainty in the adoption timescale, these models are made more realistic by including distributed time-lag functions such as exponential and Erlang-2 distributions. Logistic models also provide more nuanced control over the marketplace's awareness-spreading rate through their learning-rate parameters [23].

The proposed models have been tested using actual sales data for technical innovations. Nonlinear least-squares regression has, for example, been used to study the uptake of Acer personal computers and Samsung mobile phones and has outperformed more conventional models in predictive accuracy. When tested on datasets spanning several generations, such as the growth and decline of LCD-monitor sales, these models also outperform established structures. Such models are useful to marketing managers because they help forecast sales patterns, improve marketing methods, and analyse consumer behaviour. By clearly describing temporal delays and intergenerational dynamics, they can improve decision-making in production, distribution, and marketing.

A key component of modern diffusion models is the incorporation of time-lag and dependence variables to represent the complexities of actual adoption behaviour. By combining awareness and acceptance phases, a two-stage diffusion model emphasizes the lag from product discovery to uptake. Logistic and Erlang distributions provide flexible representations of different adoption periods. Applying this method to products spanning more than one generation makes it possible to address intergenerational switches and substitutions. These findings highlight the need for improved diffusion

Table 1. Summary of existing work

Method	Contribution	Case-study product
Ordinary least squares (OLS) [24]	Provided a model for the spread of long-lasting consumer goods by including innovation and imitation factors.	Durable appliances, such as clothes dryers and room air conditioners.
Ordinary least squares (OLS) [25]	Incorporated variations in prospective-adopter populations arising from demographic influences.	Durable consumer appliances, such as gas water heaters, dishwashers, and ranges.
Nonlinear least squares (NLS) [26]	Described possible market bases using diffusion models with various time-dependent market-growth factors.	Automobile sales in Thailand, including trucks.
Linear regression [27]	Investigated the beneficial effect of promotion on sales growth for technologies that are purchased infrequently.	Improvements to existing services, including a phone-banking system.
Nonlinear least squares (NLS) [28]	Enhanced the Bass diffusion approach by modelling price as a function of market size.	Consumer durable products.
Three-stage nonlinear least squares (3SLS) [29]	Proposed an intergenerational diffusion model incorporating substitution behaviour in the adoption process.	DRAM semiconductor technology and subsequent generations.
Nonlinear least squares (NLS) [30]	Developed a generalized Bass model reflecting the effects of pricing and advertising variables on innovation growth.	Home appliances such as air conditioners, colour televisions, and dryers.
Nonlinear least squares (NLS) [31]	Implemented leapfrogging behaviour in a multigenerational diffusion paradigm and investigated succeeding-generation launch timing.	Consumer electronics, including IBM mainframe machines.
Maximum likelihood (ML) [32]	Modelled uncertain future acceptance and market demand using stochastic considerations.	Energy use as a service product.
Maximum likelihood (ML) [33]	Devised a profit-maximization problem for finding the best warranty duration for new items.	Hypothetical data.
Nonlinear least squares (NLS) [34]	Presented a different expression of the Bass model by indicating the adoption rate per residual adopter using a logistic hazard factor.	Consumer durable products.
Nonlinear least squares (NLS) [35]	Added an autoregressive variable and diffusion-route uncertainties to expanded Bass diffusion models.	Consumer electronics, including compact discs.
Monte Carlo simulation	Established an optimal advertising strategy for an innovative item by expanding a general Bass model to show how promotional efforts affect diffusion.	Consumer durable products.

models that take internally generated dynamics, such as user evaluations of product value, and external effects, such as market-awareness efforts, into consideration. Compared with conventional techniques such as the Bass model, these algorithms have shown stronger forecasting performance when validated using real-world information such as LCD-monitor and smartphone sales.

The emergence of intergenerational diffusion models was an important change in innovation-adoption studies. Cannibalization of previous-product sales was one of the first issues discussed in relation to successive product generations and the influence each generation has on market structure. Contemporary research incorporates time-lag effects and distinguishes switching behaviour, in which current users upgrade, from substitution patterns, in which prospective customers replace an older choice with a newer one. These additions make it possible to simulate real consumption patterns more accurately, where future generations often overlap with older products in the market. Time-lag distributions such as the Erlang distribution improve the representation of delays between awareness and acceptance, particularly in highly competitive, rapidly growing technological industries.

Empirical evaluation has been essential in determining the importance and proper use of diffusion theories. Current models, including those proposed in this study, have been evaluated using real-world sales information from Samsung mobile phones and Acer laptops. The results demonstrate that time-lag-inclusive methods perform much better than typical models in predictive accuracy, as measured by indicators such as MSE and R^2. In addition, these models provide sales and development teams with useful knowledge by anticipating not only levels of acceptance but also probable delays in purchasing decisions.

3. OBJECTIVE OF THE RESEARCH WORK

The major goal of this study is to develop sophisticated multigenerational diffusion models (MGDMs) that adjust for complicated market dynamics, including substitution and switching behaviours, and add time-lag effects. To overcome the shortcomings of conventional diffusion models, these approaches aim to:

- increase the precision of sales and adoption projections across product generations;
- account for the actual lag between becoming aware of a product and making an adoption decision by including time-delay variables in the adoption process; and
- simulate the simultaneous presence of several product generations while considering relationships such as cannibalization and upgrading.

4. MOTIVATION OF THE RESEARCH WORK

Technology is advancing rapidly, and new product generations are introduced frequently. This makes it difficult to forecast how innovations will be adopted in markets that are always changing. The Bass model and other traditional diffusion models have several limitations in considering factors such as the lag between product knowledge and acceptance, the presence of different product generations, and the intricate relationships between them. In addition, precise modelling of substitution, where prospective consumers move from one generation to another, and switching, where current users upgrade to newer products, becomes more important as market circumstances change.

Improving the predictive accuracy of diffusion models through the incorporation of time delays and dependence variables is the driving force behind this study. Understanding real-world adoption patterns is essential, yet current models often fail to account for delays inherent in consumers' decision-making processes. Furthermore, when new product generations enter the marketplace, there is a growing need for approaches that can explicitly measure multigenerational dynamics, including the impact of current adopters and cannibalization.

The study seeks to fill these knowledge gaps and provide a better foundation for innovation-adoption modelling. To portray adoption delays accurately while taking changing customer habits into consideration, the suggested models use distributed time-lag functions, including exponential and Erlang distributions. Businesses can use the resulting practical information to plan manufacturing, marketing, and distribution; the improved simulation also enhances forecast accuracy. Ultimately, the study connects theoretical diffusion models with the real-world needs of technology-driven marketplaces.

5. PROPOSED METHOD

For the purpose of describing awareness and adoption phenomena, this section presents a two-stage model of the innovation-diffusion process. A potential buyer's choice is influenced by many elements, including selling cost, advertising in the popular press, and the buyer's financial resources. The diffusion models suggested here are therefore founded on the premise that prospective purchasers must initially become aware of an innovation before they can use it. Time-lag functions corresponding to different distributions are selected to describe the transition from product awareness to adoption. Innovation diffusion is assumed to follow an S-shaped growth structure.

Using two real-world sales datasets for technical innovations, the suggested models are evaluated in terms of estimation accuracy and future predictive performance. The empirical findings show that the time-delay effect has a significant influence on adoption. For every short interval $(v, v+\Delta v)$, the increase in the number of people aware of an innovation is assumed to be a function of the unused market potential:

$$M_\alpha(v+\Delta v) - M_\alpha(v) = c_k\left[n - M_\alpha(v)\right]\Delta v, \tag{1}$$

where c_k represents a time-dependent rate that raises awareness of the innovation.

Equation (1) may be rewritten as

$$c_k = \lim_{\Delta v \to 0} \frac{M_\alpha(v+\Delta v) - M_\alpha(v)}{\left[n - M_\alpha(v)\right]\Delta v}. \tag{2}$$

The following differential equation represents Equation (1) in continuous time:

$$\frac{dM_\alpha(v)}{dt} = c_k\left[n - M_\alpha(v)\right]. \tag{3}$$

With the initial condition $M_\alpha(0) = 0$, the number of people aware of the innovation by time v is

$$M_\alpha(v) = n\left(1 - e^{-c_k v}\right). \tag{4}$$

In the traditional model, it is presumed that people start using the product the moment they hear about it:

$$M_r(v) = M_\alpha(v) = n\left(1 - e^{-c_k v}\right). \tag{5}$$

This presumption may not be correct in actual situations. Customers often learn about a product, read reviews, and then decide whether to buy it based on their impressions of its features and functionality. The time taken for a product to move from awareness to use may therefore be long. The real adoption of innovation is described using a time-delay function $\phi(v)$:

$$M_r(v) = M_\alpha\left(v - \phi(v)\right) = n\left(1 - e^{-c_k(v-\phi(v))}\right). \tag{6}$$

Differentiating this expression gives the innovation sales-rate equation

$$\frac{dM_r(t)}{dt} = nc_k e^{-c_k v} e^{-c_k \phi(v)}\left(1 - \frac{d\phi(t)}{dt}\right), \qquad \frac{d\phi(t)}{dt} < 1. \tag{7}$$

A number of diffusion models may be obtained by defining the adoption rate and the time-lag function. This research presents diffusion models derived from three distinct forms of time-lag function. The first model assumes a constant innovation-adoption rate. Equation (8) gives the exponential diffusion model when $\phi(v) = 0$, meaning that there is no lag between product awareness and acquisition:

$$M_r(v) = M_\alpha\left(v - \phi(v)\right) = n\left(1 - e^{-c_k v}\right). \tag{8}$$

The delayed S-shaped, or Erlang two-stage, model represents a process in which the innovation-adoption rate rises initially and then falls. If $\phi(v) = \ln(1 + c_k v)/c_k$, then $d\phi(t)/dt = 1/(1 + c_k v) < 1$, and

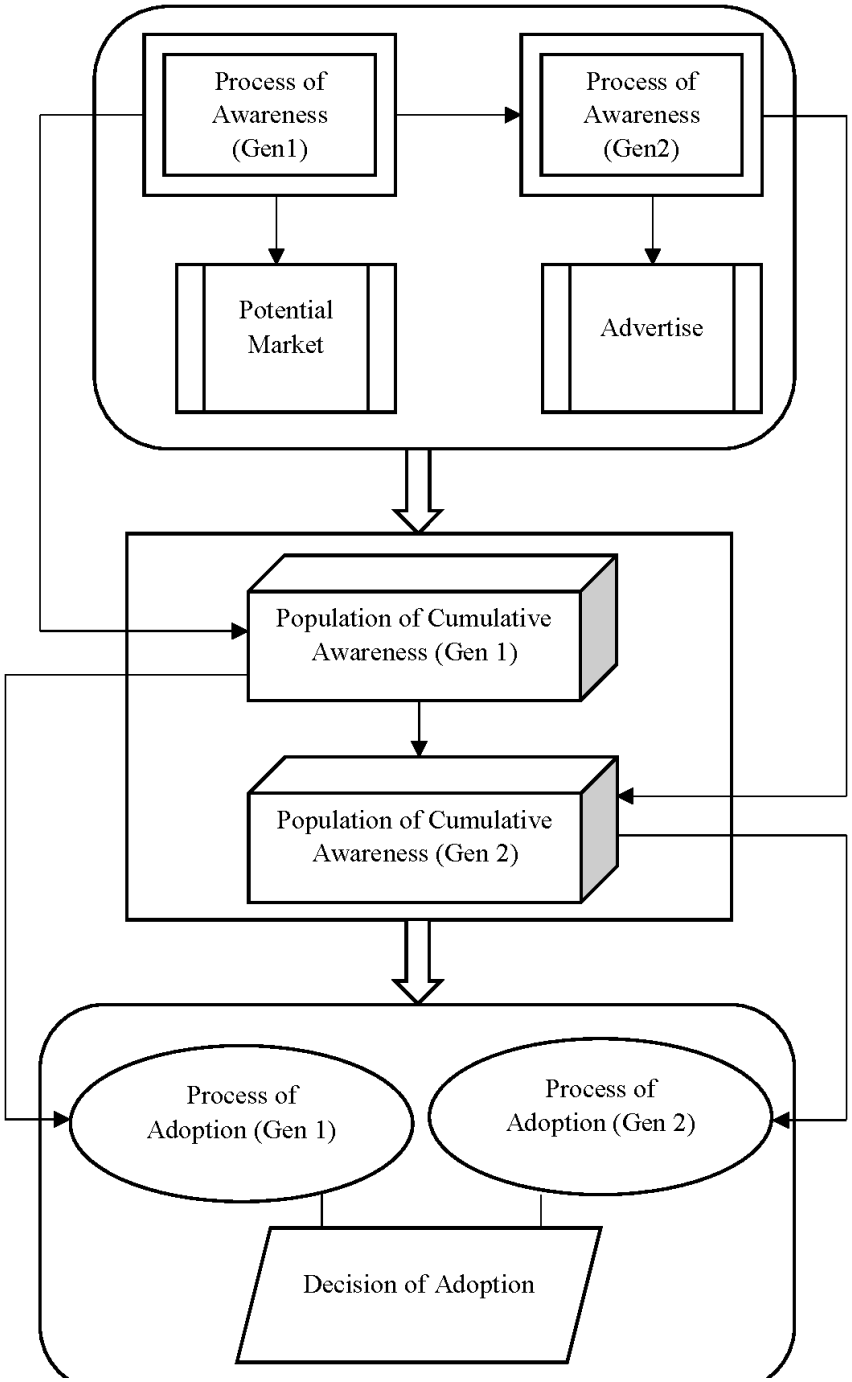


Figure 2. Architecture of the proposed method.

$$M_r(v) = M_\alpha\big(v-\phi(v)\big) = n\left[1-(1+c_k v)e^{-c_k v}\right]. \quad (9)$$

When innovation follows a logistic distribution or inflected S-shaped curve, the product starts slowly but expands quickly across the possible market after reaching the inflection point. The time-lag and diffusion functions are

$$\phi(v) = \frac{1}{c_k}\ln\left(\frac{1+\beta}{1+\beta e^{-c_k v}}\right), \quad (10)$$

$$M_r(v) = M_\alpha\big(v-\phi(v)\big) = n\left(\frac{1-e^{-c_k v}}{1+\beta e^{-c_k v}}\right). \quad (11)$$

We presume that adoption follows a learning phenomenon that eventually reaches saturation because market size is limited. The ratio of informed people to the total market size, as well as the number of untapped opportunities in the market, is assumed to be strongly related to the rate of adoption. The sales-rate equation is therefore

$$\frac{dM_r(t)}{dt} = c_2\frac{M_\alpha\big(v-\phi(v)\big)}{n}\big[n-M_r(t)\big], \quad (12)$$

where c_2 is the adoption coefficient and $M_\alpha(v-\phi(v))$ is the cumulative number of informed persons at time $v-\phi(v)$.

Two sets of real-world sales data are used to validate and analyse the developed models. The first dataset (DSI) contains Samsung smartphone sales information collected from an online source. The second dataset (DSII) contains quarterly sales figures, in millions, for Acer personal computers. For both datasets, nonlinear least-squares (NLS) estimation is

used to determine the diffusion parameters. A small awareness parameter indicates that the level of awareness is much lower for both datasets. In all three scenarios, the adoption coefficient is larger than the awareness coefficient because an informed population adopts the product more quickly.

The predictive performance of the proposed methods is evaluated against two benchmark models using goodness-of-fit measures. Mean squared error (MSE), root mean squared error (RMSE), and other statistical indicators are examined to compare the effectiveness of each diffusion model. The statistical measures show that the proposed models outperform the older models in predictive performance. For both datasets, all three proposed models provide a reasonable prediction of the actual sales curves.

Using the multi-stage structure of the adoption process as a foundation, the study next proposes multigenerational diffusion models (MGDMs). To predict demand for intergenerational products under conditions that better represent the real world, the mathematical model is expanded. The intergenerational adoption-growth function is calculated using a time-varying lag function so that the processes of product awareness and acceptance can be distinguished. The level of knowledge of prospective adopters is an important consideration. Differential equations are included in the suggested models to assess potential market awareness.

During the awareness process, the growth function is assumed to follow either exponential or logistic growth. Erlang distributions with different shape parameters illustrate the time-delay operation. The model distinguishes substitution, which occurs when a potential purchaser of an older generation decides to purchase a newer model instead, from switching, which occurs when users of an earlier generation move to an improved generation after it becomes available. The designs are tested by fitting the models to actual shipment data for LCD computer displays.

The proposed framework uses a phase structure to illustrate the innovation-diffusion process over several generations. Ultimate adoption is considered a step-by-step process in which a prospective buyer first becomes aware of the innovation and adopts it later. A two-stage architecture simplifies the mathematical evaluation. The diffusion dynamics of two generations are modelled simultaneously: the first product generation is launched at time $v = 0$, and the second enters the market at $v = \tau > 0$.

For a single product generation, product awareness refers to the level of information that the prospective market has about the innovation. Creating awareness among potential purchasers is the first stage in gaining acceptance. Knowledge of the characteristics, advantages, quality, and usefulness of a new product serves as the basis for the expansion of innovation in the market. The instantaneous diffusion rate of product awareness is

$$\frac{dM_\alpha(t)}{dt} = \frac{e_\alpha(v)}{1 - E_\alpha(v)}\left[n - M_\alpha(v)\right], \tag{13}$$

where $e_\alpha(v)$ is the adoption density, $E_\alpha(v)$ is the cumulative adoption function, $M_\alpha(v)$ is cumulative awareness among prospective consumers at time v, and n is the possible market size. With the condition $M_\alpha(0) = 0$, the closed-form solution is

$$M_\alpha(v) = nE_\alpha(v). \tag{14}$$

A consistent dissemination rate causes awareness of the innovation to expand exponentially over time. Equation (14) can therefore be written as

$$M_\alpha(v) = n\left(1 - e^{-cv}\right). \tag{15}$$

Here, n is the market size and c is the awareness parameter. Equation (15) describes the accumulation of potential customers who are aware of the innovation and may become inclined to buy it.

An informed person may take time to buy an innovation. Factors such as cost and purchasing power influence the prospective market's decision. The modelling approach therefore presupposes a distinct lag between product awareness and adoption. Product awareness is represented as a time-delay function, and product adoption is characterized accordingly.

Innovations often undergo iterative improvement as a result of scientific and technological progress; each new generation builds on the previous one. Users of subsequent generations are assumed not to return to prior generations, while users of a previous generation may upgrade if they become aware of a newer, better generation. Two product generations can coexist in the proposed architecture. Raising awareness and finally adopting a product are mathematically represented as a two-stage process for two generations operating at the same time, as shown in Figure 3.

Each generation has its own distinct prospective customer base when it first reaches the market. Potential adopters and users of previous generations may also be interested in products from the next generation. Accordingly, three categories make up the second generation's aware market potential: the new and unique market potential, knowledgeable potential adopters of the previous generation who could shift to the current one once they learn about it, and users of the previous generation who may switch to the new generation. Prior research classifies these as first-time buyers, potential substitutes, and potential switchers. The informed substitute and switcher populations for the second generation are

$$\mathrm{Subs}(v) = n_1\left[E_{\alpha 1}(v) - E_{r1}(v)\right]E_{\alpha 2}(v - \tau_1), \tag{16}$$

$$\mathrm{Sz}(v) = n_1 E_{r1}(v) E_{\alpha 2}(v - \tau_1), \tag{17}$$

where $\mathrm{Subs}(v)$ is the total number of first-generation consumers who know about the second generation and may replace the previous generation with the new one, and $\mathrm{Sz}(v)$ is the number of first-generation users who know about the second generation and may upgrade to it.

After substituting Equations (16) and (17), the total possible shift from generation one to generation two is

$$R(v) = \mathrm{Subs}(v) + \mathrm{Sz}(v). \tag{18}$$

The first-time purchasers, switchers, and substitutes of the second generation determine its prospective adopters. In a

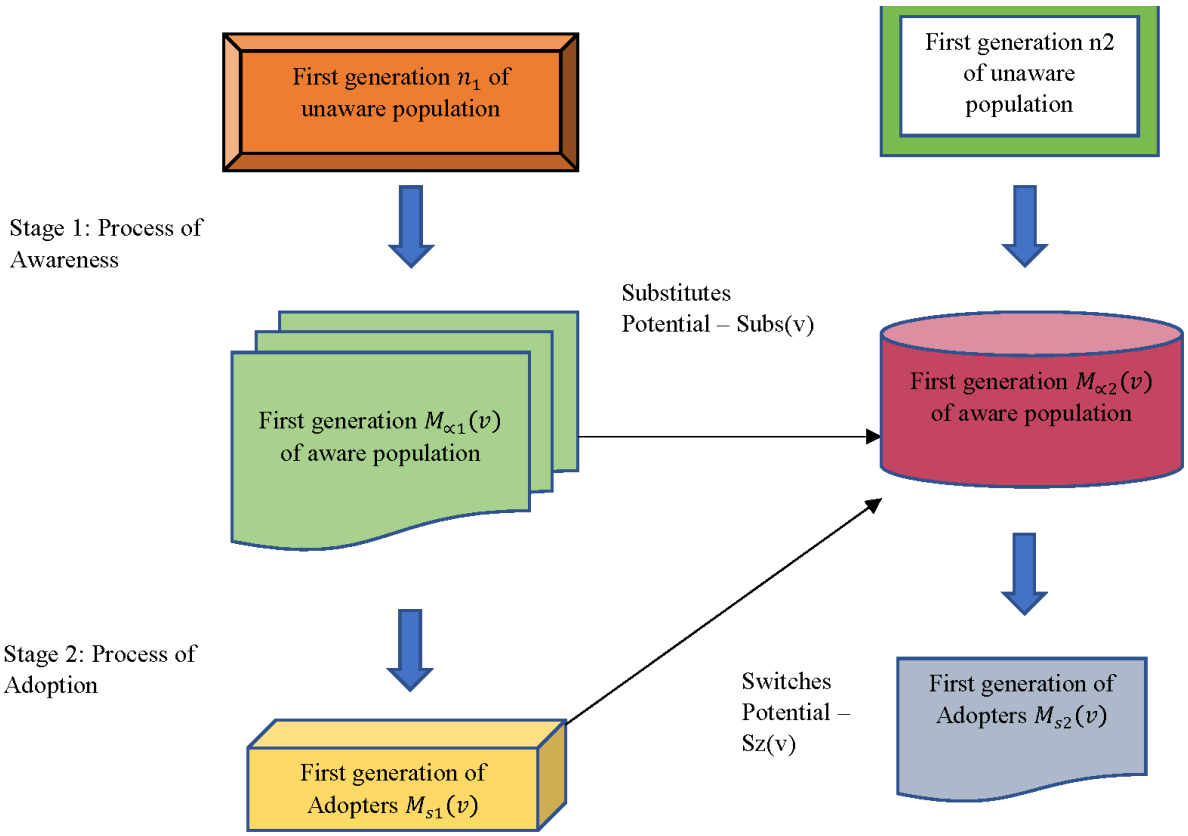


Figure 3. Adoption process of concurrent technological generations.

two-stage procedure, the concurrent adoption equations for two generations are

$$M_{r1}(v) = F\left[M_{r1}(v-\phi_1)\right] - F\left[\text{Subs}(v-\phi_2)\right] - F\left[\text{Sz}(v-\phi_2)\right], \quad (19)$$

$$M_{r2}(v) = F\left[M_{r2}(v-\phi_2)\right], \quad (20)$$

where ϕ_1 is the adoption delay of the first generation and ϕ_2 is the adoption delay of the second generation.

6. RESULTS AND DISCUSSION

Developing a sales strategy based on estimates of an invention's market potential and diffusion rate is one of the key capabilities associated with innovation-diffusion models. A sound sales strategy helps managers control production, finance, and marketing and prevent unexpected losses. Accurate information on diffusion patterns across generations enables important administrative decisions. The proposals in this study help marketing executives formulate effective manufacturing, development, and marketing strategies. The proposed approach fills a gap in earlier multigenerational research by predicting both the total population of modern-technology consumers and the number of informed potential buyers who have not yet purchased.

This information has substantial practical value. It helps business analysts determine which marketing and advertising strategies encourage informed prospective customers to make a timely purchase. The proposed intergenerational diffusion model makes an essential distinction between product awareness and acceptance. The pace of adoption depends on people's awareness of the innovation, which can be managed through promotional efforts. Estimating the rate of product awareness among prospective consumers allows managers to design promotion and marketing tactics that increase knowledge and understanding. The recommended framework can therefore guide management decisions.

Mean absolute deviation (MAD) measures the mean absolute discrepancy between expected and observed results. Without considering error direction, it measures the magnitude of forecast errors.

Mean squared error (MSE) measures the squared distance between predicted and actual values. Because it places greater weight on larger errors, it is sensitive to extreme values.

Root mean squared error (RMSE) expresses forecasting error in the same units as the actual data, making model accuracy easier to interpret.

Range (R^2)—also known as the coefficient of determination—quantifies the proportion of observed variation explained by the model and indicates goodness of fit.

Table 2. Outcome of statistical measures of MAD

Model of diffusion	MAD
Erlang distributions	1358.429
Bass diffusion model	3069.16
Proposed method (MGDM 1)	7302.478
Proposed method (MGDM 2)	6596.113
Proposed method (MGDM 3)	5652.571

Various degrees of predictive accuracy are seen when diffusion models are compared using MAD. A minimum MAD

of 1358.429 shows that the Erlang-distribution model is the most accurate in predicting adoption. The Bass diffusion model follows with a MAD of 3069.16. Among the proposed approaches, MGDM 3 has the lowest MAD at 5652.571, followed by MGDM 2 at 6596.113 and MGDM 1 at 7302.478. Although the MAD values of the proposed techniques are greater than those of the conventional models, the MGDMs provide a broader framework for studying multigenerational diffusion because they account for complicated real-world elements such as intergenerational interactions and temporal delays.

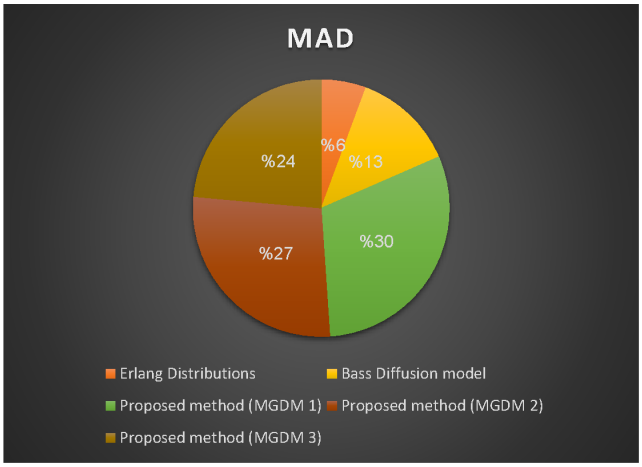


Figure 4. Evaluation of ML models in comparison to more traditional approaches.

Table 3. Results of RMSE statistical measures

Model of diffusion	RMSE
Erlang distributions	8182.9
Bass diffusion model	15509.9
Proposed method (MGDM 1)	1756.6
Proposed method (MGDM 2)	1657.6
Proposed method (MGDM 3)	1845.5

Significant disparities in predictive accuracy appear when diffusion models are compared using RMSE. Among the proposed approaches, MGDM 2 has the best RMSE at 1657.6, followed by MGDM 1 at 1756.6 and MGDM 3 at 1845.5. All three are better than the conventional approaches. The Erlang-distribution model has an RMSE of 8182.9 and the Bass diffusion model an RMSE of 15509.9, indicating less accurate predictions. The assessment highlights how the proposed multigenerational approaches capture real-world adoption trends and handle complexities such as intergenerational connections and time-lag effects more accurately.

Table 4. Statistical measures for MSE outcomes

Model of diffusion	MSE
Erlang distributions	9983377
Bass diffusion model	61593885
Proposed method (MGDM 1)	3457084
Proposed method (MGDM 2)	3892392
Proposed method (MGDM 3)	3994885

The analysis based on MSE demonstrates that the suggested approaches have a distinct advantage over the standard models. The Bass diffusion model has an MSE of 61593885, while the Erlang-distribution model has an MSE of 9983377. Among the proposed approaches, MGDM 1 has the lowest MSE at 3457084, followed by MGDM 2 at 3892392 and MGDM 3 at 3994885. The MGDMs provide a more complex strategy for intergenerational diffusion, reflecting market factors such as time-lag effects and intergenerational interconnections.

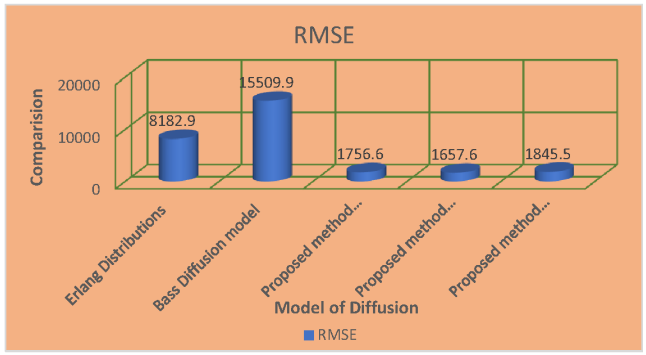


Figure 5. Comparison of ML models with conventional methods for evaluation.

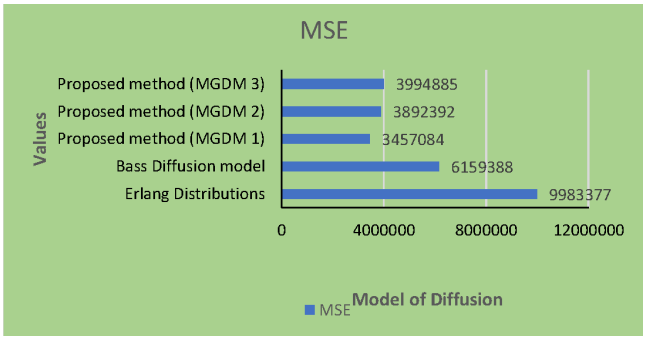


Figure 6. Efficacy of the different systems.

Table 5. Exploratory models in relation to the proposed methods

Model of diffusion	Range
Erlang distributions	0.9833
Bass diffusion model	0.9881
Proposed method (MGDM 1)	0.9933
Proposed method (MGDM 2)	0.9981
Proposed method (MGDM 3)	0.9991

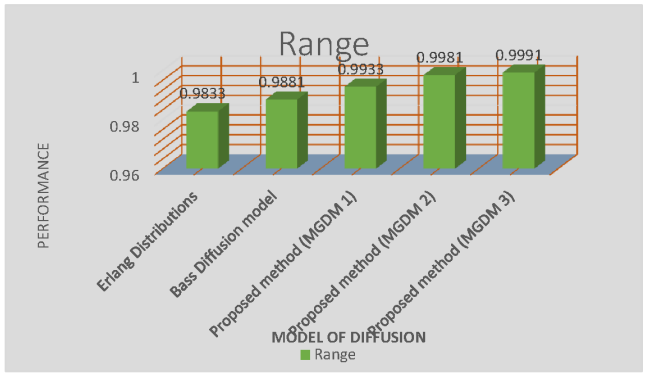


Figure 7. Effectiveness of the different models.

According to the R^2 results, the proposed techniques consistently provide a better fit than the traditional approaches.

The Bass diffusion model has an R^2 value of 0.9881, while the Erlang-distribution model has a slightly lower value of 0.9833. The suggested approaches provide progressively better fits: MGDM 1 reaches 0.9933, MGDM 2 reaches 0.9981, and MGDM 3 attains the highest value of 0.9991. These values demonstrate that the proposed models explain the observed variation more successfully, especially when intergenerational replication is considered.

7. CONCLUSION

This paper develops innovative product-diffusion models that take into account the lag between product awareness and uptake. In the first part, awareness-based diffusion models explicitly make adoption dependent on informed people. By using different functions to simulate awareness of market potential, an empirical solution for the diffusion process can be produced. Using nonlinear least-squares regression, the models are evaluated experimentally with worldwide shipment data for Acer personal computers and Samsung Galaxy smartphones. Their predictive effectiveness is determined by comparing statistical measures with those of two reference models.

In the second part, analytical multigenerational diffusion models are proposed by taking the adoption process's distributed time-delay function into account. The models use a differential equation to assess market awareness. Exponential and logistic growth structures are investigated for the development of the awareness process, and an Erlang distribution is used to examine future adoption. Both switching and substitution behaviours are included. Applying the proposed paradigm to real-world LCD-monitor sales data produces new findings. Of all the models examined, the proposed diffusion models show the best predictive capacity. The resulting framework is technically feasible and versatile and may be applied in different industries.

For optimal decisions, decision-makers should take several additional factors seriously. Future research may apply multi-criteria decision-making procedures to assess new-product marketing strategies.

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REFERENCES

[1] L. Osman, "A PSPICE fast model for the single electron transistor," *International Journal of Wireless and Ad Hoc Communication*, vol. 0, no. 1, pp. 8–23, 2019.

[2] A. A. Elngar and S.-d. Krit, "Performance analysis of machine-learning-based botnet detection and classification models for information security," *Journal of Cybersecurity and Information Management*, vol. 0, no. 1, pp. 44–53, 2019.

[3] M. Elsharkawy and A. N. Al Masri, "A novel image encryption with deep learning model for secure content-based image retrieval," *Journal of Cybersecurity and Information Management*, vol. 0, no. 2, pp. 54–64, 2019.

[4] F. M. Bass, "A new product growth model for consumer durables," *Management Science*, vol. 15, no. 5, pp. 215–227, 2020.

[5] P. K. Kapur, O. Singh, and R. Mittal, "Software reliability growth and innovation diffusion models: An interface," *International Journal of Reliability, Quality and Safety Engineering*, vol. 11, no. 4, pp. 339–364, Dec. 2004.

[6] K. Shankar, "Recent advances in sensing technologies for smart cities," *International Journal of Wireless and Ad Hoc Communication*, vol. 1, no. 1, pp. 5–15, 2020.

[7] C. Vivek, M. Indu, and N. Nandhini, "Speech recognition using artificial neural network," *Journal of Cognitive Human–Computer Interaction*, vol. 5, no. 2, pp. 8–14, 2023.

[8] V. Narayanan, Nithya P., and Sathya M., "Effective lung cancer detection using deep learning network," *Journal of Cognitive Human–Computer Interaction*, vol. 5, no. 2, pp. 15–23, 2023.

[9] S. Yang, X. Gou, M. Yang, Q. Shao, C. Bian, M. Jiang, and Y. Qiao, "Software bug number prediction based on complex network theory and panel data model," *IEEE Transactions on Reliability*, vol. 71, no. 1, pp. 162–177, Mar. 2022.

[10] P. Vizarreta, K. Trivedi, B. Helvik, P. Heegaard, A. Blenk, W. Kellerer, and C. Mas Machuca, "Assessing the maturity of SDN controllers with software reliability growth models," *IEEE Transactions on Network and Service Management*, vol. 15, no. 3, pp. 1090–1104, Sep. 2018.

[11] A. Sariga and J. Uthayakumar, "Type-2 fuzzy-logic-based unequal clustering algorithm for multi-hop wireless sensor networks," *International Journal of Wireless and Ad Hoc Communication*, vol. 1, no. 1, pp. 33–46, 2020.

[12] P. Sheela Rani, Harini M., Nandhitha N., and Teena A. Naahz G., "A decentralized and cooperative methodology for organ donation management based on Ethereum blockchain," *Journal of Cognitive Human–Computer Interaction*, vol. 6, no. 1, pp. 8–17, 2023.

[13] M. Srinivasan, J. Dineshan, G. Surender Dhanunjay, and S. Ramappa, "Measuring the coverage of assembly elections and COVID-19 during the pandemic in India," *Journal of Cognitive Human–Computer Interaction*, vol. 6, no. 1, pp. 18–31, 2023.

[14] K. Pandey, A. Mukherjee, P. Rai, and A. Kumar, "VAEs meet diffusion models: Efficient and high-fidelity generation," in *Proceedings of the NeurIPS Workshop on DGMs and Applications*, 2021.

[15] F. Bao, C. Li, J. Zhu, and B. Zhang, "Analytic-DPM: An analytic estimate of the optimal reverse variance in diffusion probabilistic models," in *Proceedings of ICLR*, 2022.

[16] T. Dockhorn, A. Vahdat, and K. Kreis, "Score-based generative modeling with critically-damped Langevin diffusion," in *Proceedings of ICLR*, 2022.

[17] N. Liu, S. Li, Y. Du, A. Torralba, and J. B. Tenenbaum, "Compositional visual generation with composable diffusion models," in *Proceedings of ECCV*, 2022.

[18] I. Oguz, J. D. Malone, Y. Atay, and Y. K. Tao, "Self-fusion for OCT noise reduction," in *Proceedings of SPIE Medical Imaging*, vol. 11313, p. 113130C, SPIE, 2020.

[19] K. Shazly, D. A. Salem, N. Hammami, and A. I. B. ElSeddawy, "A review on distributed denial-of-service detection in software-defined networks," *International Journal of Wireless and Ad Hoc Communication*, vol. 5, no. 2, pp. 8–18, 2022.

[20] Prabu S., Alekhya A., K. K. Chatragadda, and V. Lingala, "Secured authentication of node in mobile ad hoc network," *International Journal of Wireless and Ad Hoc Communication*, vol. 5, no. 2, pp. 19–29, 2022.

[21] R. Q. Charles, H. Su, M. Kaichun, and L. J. Guibas, "PointNet: Deep learning on point sets for 3D classification and segmentation," in *Proceedings of CVPR*, pp. 77–85, 2017.

[22] G. Balakrishnan, A. Zhao, M. R. Sabuncu, J. Guttag, and A. V. Dalca, "An unsupervised learning model for deformable medical image registration," in *Proceedings of CVPR*, pp. 9252–9260, 2018.

[23] X. Li, T.-K. L. Wong, R. T. Chen, and D. Duvenaud, "Scalable gradients for stochastic differential equations," in *Proceedings of AISTATS*, pp. 3870–3882, 2020.

[24] T. Park, M.-Y. Liu, T.-C. Wang, and J.-Y. Zhu, "Semantic image synthesis with spatially-adaptive normalization," in *Proceedings of CVPR*, pp. 2337–2346, 2019.

[25] I. A. Chaudhry and A. A. Khan, "A research survey: Review of flexible job shop scheduling techniques," *International Transactions in Operational Research*, vol. 23, no. 3, pp. 551–591, May 2016.

[26] H. Zhou, Y. Feng, and L. Han, "The hybrid heuristic genetic algorithm for job shop scheduling," *Computers & Industrial Engineering*, vol. 40, no. 3, pp. 191–200, Jul. 2001.

[27] J. Su, Y. Yang, and R. Duan, "A CA-based heterogeneous model for knowledge dissemination inside knowledge-based organizations," *Journal of Intelligent & Fuzzy Systems*, vol. 34, no. 4, pp. 2087–2097, 2018.

[28] A. V. Barenji, R. V. Barenji, D. Roudi, and M. Hashemipour, "A dynamic multi-agent-based scheduling approach for SMEs," *International Journal of Advanced Manufacturing Technology*, vol. 89, no. 9, pp. 3123–3137, Apr. 2017.

[29] D. Ouelhadj and S. Petrovic, "A survey of dynamic scheduling in manufacturing systems," *Journal of Scheduling*, vol. 12, no. 4, p. 417, 2009.

[30] A. H. Kashan, B. Karimi, and F. Jolai, "An effective hybrid multi-objective genetic algorithm for bi-criteria scheduling on a single batch-processing machine with non-identical job sizes," *Engineering Applications of Artificial Intelligence*, vol. 23, no. 6, pp. 911–922, Sep. 2010.

[31] J. Su, M. Wei, and A. Liu, "A robust predictive–reactive allocating approach considering random design change in complex product-design processes," *International Journal of Computational Intelligence Systems*, vol. 11, no. 1, pp. 1210–1228, 2018.

[32] S. H. H. Madni, M. S. A. Latiff, Y. Coulibaly, and S. M. Abdulhamid, "Recent advancements in resource-allocation techniques for cloud-computing environments: A systematic review," *Cluster Computing*, vol. 20, pp. 2489–2533, Sep. 2017.

[33] B. Guo, Y. Liu, L. Wang, V. O. Li, C. K. Jacqueline, and Z. Yu, "Task allocation in spatial crowdsourcing: Current state and future directions," *IEEE Internet of Things Journal*, vol. 5, no. 3, pp. 1749–1764, Jun. 2018.

[34] J. Yang, J. Su, and L. Song, "Selection of manufacturing enterprise innovation design project based on consumers' green preferences," *Sustainability*, vol. 11, no. 5, p. 1375, Mar. 2019.

[35] A. Morris, M. Halpern, R. Setchi, and P. Prickett, "Assessing the challenges of managing product design change throughlife," *Journal of Engineering Design*, vol. 27, nos. 1–3, pp. 25–49, 2016.