



# Leveraging Time Lag-Based Diffusion Models to Predict Innovation Adoption for Optimized Product Development

Muddassar Sarfraz<sup>1,\*</sup>

<sup>1</sup>School of Management, Zhejiang Shuren University, PR China

Email: [muddassar@zjsru.edu.cn](mailto:muddassar@zjsru.edu.cn)

## Abstract

The suggested models for the spread of technical breakthroughs make use of a phase structure to illustrate the steps involved in becoming familiar with the problem and making a choice. For it to portray genuine adopting conduct, a time-lag factor is included into the dispersion process. Depicts a two-step dissemination process by taking into account the reliance of adopting on the informed group of potential purchasers. Assuming that a prospective customer first becomes intrigued by an upcoming the item's availability and then accepts the novel idea at an ulterior point, a method of analysis for sales functions that incorporates time delay is proposed. The efficient propagation method for invention is shown using the various lag factors. Applying nonlinear regression modelling to worldwide shipping data of Acer PCs and Samsung smartphones experimentally validates the suggested models for mathematics. Several comparison models are used to evaluate the predicting abilities of the suggested models. By integrating a distributed time delay function into the implementation manage, a theoretical intergenerational diffusion model is created. To measure how long it takes for innovation to be eventually accepted, the distributed time lag function that follows the Erlang distributions is used. This framework incorporates switch and substituting, two forms of pragmatist shift behaviour. Using real shipping data of LCD (Liquid Crystal Display) computer monitors from consecutive generations, the predicted effectiveness of the suggested methods is examined and contrasted with well-established research. Here is the total accuracy of the approaches that have been proposed: When contrasted with more conventional models, MGDM 1 achieves a 99.33% accuracy rate, MGDM 2 a 99.81% rate, and MGDM 3 a 99.91% accuracy rate.

**Keywords:** LCD; MGDM; OLS; NLS; OR; MSE; RMSE; MAD

## 1. Introduction

Competition for consumers' attention is fierce in today's market because of the proliferation of functional advances that are functionally comparable and brought about by the fast development of technology. In today's dynamic economic climate, organizations who fail to meet client expectations will eventually fail. In light of the present state of the market, diffused modelling has therefore evolved into the engine that propels the advertising campaign and execution processes [1-2]. To improve the accuracy of sales forecasting by accurately representing real-world scenarios, diffusion equations are consistently recommended throughout marketing literature. In this two-step adoption of innovation procedure, we can see how market awareness and knowledge penetrating interact with one another. To focus on the real acceptance of an invention, the mathematical models in this study integrate the time delay component. The time lag is the window of opportunity for the prospective adopters to evaluate the technology and make a purchase decision, from the moment they become aware of its presence until the invention is finally adopted. Additionally, there is a growing demand for precise and versatile forecasting systems due to the constant release of fresh item developments. Progress in technology and rising customer demands have prompted multiple decades of innovation to flood markets [3-4]. So, most of the new technology that are now popular are really just upgraded versions of older ones that will be superseded by even better ones a little later on. There will be sales of older versions of a product eaten away by the sales of newer versions as long as they remain on the market. Upgrades from earlier generations are available to users whenever the latest version is released. By the

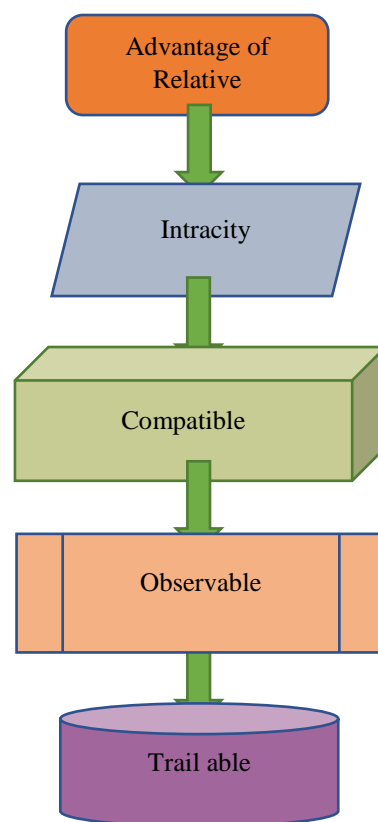
same token, the next generation that was going to buy the previous generation can end up buying older people instead. Nevertheless, the sales of the older model could continue for a while after the release of the newer one, as the majority of consumers do not immediately acquire the most recent features. Practical models for the spread of new technologies are presented in this section, along with frameworks for predicting how these breakthroughs will sell throughout different generations. The two structures' dissemination processes are shown by various phases of their adoption phase. This section is further divided into the two parts that follow. An incorporated two-stage diffusion model is suggested [5]. This model takes into account the impact of knowledgeable persons on the development of adoption. The time distinction among the two adopting phases is explained in this section using various function methods. Furthermore, the suggested paradigm presupposes that informed market opportunity has a beneficial effect on adoption. The purpose of the empirical investigation was to demonstrate how the suggested models may be used in the real world to predict the interest in new technical goods. The elements of the model of diffusion are estimated using the analysis of data that is conducted using web-based sources of sales data for Acer desktop computers and Samsung smartphones [6]. To find out how well the constructed technique predicts the paradigm shift in technological advancement uptake, we look at the various statistical metrics. We also compare our empirical findings to those of two other research. It suggests utilizing the distributed time lag function to represent the adoption process as a two-stage intergenerational diffusion. Presumption: the lag time between the two adoption phases resembles an exponential or an Erlang 2-stage distribution. When simulating the spread of new generations of technology, the suggested research would also distinguish between two distinct shifting phenomena: substitute and switch. Validation of the suggested statistical models is done experimentally using sales data of two different types of LCD computer displays. When comparing the predictive power of newly-developed models to that of more-established models, comparison studies additionally get taken into considerations.

When it comes to the success of a commercial enterprise, the precise preparation for the creation of new products and the implementation of an efficient business plan are both very necessary. In an effort to gain a competitive advantage over other businesses, the companies consistently introduce new products and services to the market [7]. On the other hand, the creation of new products represents one of the riskiest endeavours since it requires extensive financial expenditures and a well-planned commercial strategy. As a result, marketplace economists and policymakers are necessary to make prudent decisions by using the proper experience and skills in order to evaluate the performance of the novel item. In addition, it is of the highest significance to include methods from a variety of fields, like as statistical data, marketing, mathematical information, engineering, finance, and computer engineering, among others, in order to develop a new body of information specifically for the purpose of resolving complicated issues that arise in the business world. When it comes to making decisions, one of these fields that makes use of scientific methods is known as operational research (OR). It does this by providing the foundation upon which management problems may be formulated and by assisting practitioners in arriving at the best possible conclusion. Over the course of the most recent past, operating room methods have shown their capacity to make pertinent judgments at all three levels of operation [8-9]. Because of its multidisciplinary character, OR has uses across a wide range of industries, including computer science, business administration, engineering, manufacturing, financial administration, medical, marketing, epidemiological studies, supply chain leadership, and many others, for the purpose of resolving difficult and complicated issues that occur in the environment. It does this by using quantitative methods and mathematical models in order to handle the challenges that are faced by businesses and industries in a consistent and effective manner. The purpose of the study is to make use of OR approaches for making choices in order to predict the market response of new items under a certain set of parameters [10]. While it is necessary to provide an acceptable quantity of substances at the right time and in the correct location, it is also necessary to accurately estimate the demand for new products and the degree to which they will penetrate the market. In the suggested research, a fair estimate of the future market for innovation is determined by analysing real-life sales data as well as case study results. This estimation is based on differential equations equation-based statistical models, which are the foundation of the research that is suggested. The present study also focuses on the assessment of marketing factors such as selling price, advertising expenditures, and warranties length for consumer goods and technical innovation. This assessment is carried out with the use of optimization methodologies. The real-world applicability of the presented optimization frameworks is identified by taking into consideration the situation of items that need a significant amount of capital [11-12]. Within the context of this thesis, this subsection provides an introduction to the research that was given on theories of diffusion and efficiency challenges in marketing. A discussion of mathematical models related to marketing and the spread of creative thinking in the environment of consumer staples and high-tech goods is presented at the beginning of the article. The following part provides a concise summary of the essential marketing parameters that were evaluated in the current research. This then follows by a discussion of the effectiveness issues presented. Following that, the aims of the present investigation as well as the researching motivations surrounding it are discussed. After that, a discussion of the pertinent research on new product dissemination and models of optimization is presented.

In the next section, we will talk about the study's objectives, gaps that were found in previous studies, the relevance of the diffusing designs, the addition that the proposed research would make, and the design of the research paper. At long last, the whole list of authors has been expanded.

Mathematical frameworks offer a method for conceiving and resolving many kinds of real-world issues, ranging across the numeric challenges that arise on a daily basis to the enormous, complicated issues that plague both society and business [13]. They are able to do work that brings together expertise in a variety of academic subjects, including but not limited to mathematical information, statistics, computing, finance, and marketing, amongst others. The conceptual framework is a generalisation of the structure that exists in the actual world. The generalization is expressed in a way that is approachable as functions of mathematics that explain the efficiency of the imagined system. Researchers and investigators are able to better comprehend the motions of the actual world, the influence of various features, and produce projections regarding the conduct of the entities via the use of numerical models [14]. The primary objective in applying computational models to the issue that is being faced in real life is to offer a comprehensive knowledge of the framework and to identify the most effective approaches to the problem. Not only is that, but the most significant benefit of using mathematical frameworks the effective use of the features of contemporary computers. When it comes to marketing management, computational models are of utmost significance. Statistical models are thoroughly used in the field of marketing for the purpose of resolving management issues via the process of assessing, planning, and putting marketing strategies and ideas into action [15]. They are helpful tools that provide an analytic prediction of the success of new developments in the market, which enables executives to make choices that are both feasible and ideal for the firm. Measuring and guidance for decisions systems are the two kinds of marketing models that are investigated in this thesis. The purpose of these frameworks is to evaluate the consumption of new goods as a function of many individual variables and to provide assistance to managers in making management choices.

A better product (or process, service, technology, etc.) with increased qualities that have extra economic and social worth is considered to be an example of innovation. This procedure is considered to be a process related to innovation [16]. One of the most important factors that determines the long-term success of a business is its capacity to develop and introduce new goods and services. Innovations are defined as new ideas, products, or services that are produced to meet the requirements of developing markets and to provide economic value for the organization [17]. Products and services may also be considered innovations.



**Figure 1.** Features of Innovation

The definition of innovation is "the procedure that involves implementing modifications, both major and minor, revolutionary and incremental in nature, to items, procedures, and offerings resulting in the launch of a novel product for the company that provides value consumers and helps build the understanding store of the structure. The five main features that the target audience views as crucial when evaluating new ideas (refer to Figure 1) [18-20]. The combination of a highly trained labour and productive commercial enterprise produces innovation, which is a prominent technological solution on a global scale. Researchers may learn more about the industry's traits and the procedure of technological adoption by looking at its evolving pattern of innovation.

## 2. Related Work

The author laid the groundwork for innovation diffusion research by offering an algebraic structure for analysing the pace of adoption of new ideas in relation to both internal (word-of-mouth) and exterior (advertising) factors [21]. The success of single-generation goods market growth predictions was heavily dependent on this model, which is still used as a standard today. On the other hand, it wasn't perfect and couldn't handle time-dependent issues like delaying diffusion or items that span many generations. The investigator presented a more straightforward model for adopting that assumed a constant rate of the adoption process, their exponentially method was subsequently shown to be insufficient in explaining the S-shaped curves seen in actual circumstances. By factoring in lag time during adoption, further improvements were able to overcome the shortcomings of previous models. The time it takes for an invention to get from being known about to being used is known as the time lag. Erlang models, which depicted adoption as an ongoing procedure with varying delays, were the first to investigate this.

The review time that potential consumers undertake before acquiring an invention was taken into consideration by researchers who expanded the Bass model through including lagged effects. To account for delayed S-shaped rise in adoption rates, we also employed the method of logistic growth, which is widely used in biological products systems; this allowed us more leeway to describe dynamic at the beginning and end of adaptation [22]. Conventional diffusion models faced fresh obstacles with the introduction of novel product generation like cell phones and displays for computers. The investigators ground-breaking work presented multigenerational diffusion models, which show how various generations may survive in the marketplace and still eat into the revenues of previous ones. This structure laid the groundwork for further research that included phenomenon such as switch (users of older generational upgraded to newer ones) and substitute (moving from a particular generation to another). The incorporation of dependence and time-delay effects into diffusion models has been the subject of recent research aimed at enhancing their accuracy. As an example, two-stage diffusion models that include recognition and embrace processes were constructed by the researcher. Taking into consideration factors like advertisement and market trends, these frameworks postulate that prospective customers learn about a product and assess it before deciding to buy it.

Recognizing uncertainty in the adoption timescale, these models are further enhanced in their realism by include distribution time lag operations, such as exponentially and Erlang-2 distributions. Logistic models also provide more nuanced oversight over the marketplace's aware spreading rate via their learning rate settings [23]. The models suggested have been thoroughly tested with actual sales data of technical advancements. One example is the use of non-linear lowest-squares regression to study the uptake of Acer PCs and Samsung cell phones, which outperforms more conventional models in terms of predicted accuracy. When tested on datasets spanning many generations, such the growth or decline of LCD monitor sales, these models also beat the state-of-the-art structures. These types of models are useful for marketing managers because they help with things like forecasting sales patterns, improving methods for marketing, and analysing consumer behaviour. The decision-making process in production, delivery, and market may be improved with the help of these systems since they clearly describe temporal delays and intergenerational dynamics.

**Table 1:** Summary of Existing work

Method	Contribution	Case study product
Ordinary least squares (OLS) [24]	Provided a model for the spread of long-lasting consumer goods by including innovative and imitation factors.	Appliances that last a long time, such clothes dryers and room air conditioners,

Ordinary least squares (OLS) [25]	The model for the dissemination of new goods incorporates variations in the prospective adopter populations owing to demographics influences.	Appliances that last a long time and are purchased by consumers, such as gas water heaters, dishwashers, and ranges
Nonlinear least squares (NLS) [26]	Possible market bases described by models of diffusion using various time-dependent market growth factors	Sales of automobiles in Thailand, including trucks
Linear Regression [27]	Investigated the beneficial effects of promotion on the increase of sales for technologies that are not bought very often.	Improvements to existing services, including a phone banking system
Nonlinear least square (NLS) [28]	To enhance Bass's diffusion approach, we demonstrated pricing as a function of the market's size.	Consumer durable product
Three stage Nonlinear least square (3SLS) [29]	Put forward the intergenerational diffusion model that incorporates the adoption procedure's substitute behaviour.	The DRAM semiconductor technology and its subsequent generations
Nonlinear least squares (NLS) [30]	Developed a generalized Bass model that reflects the effect of both the pricing and advertising variables on the innovation growth structure	Home appliances like air conditioning, color TVs, and dryers are examples of consumer goods.
Nonlinear least squares (NLS) [31]	Implemented leapfrogging behaviour into the multigenerational diffusion modelling paradigm and investigated the succeeding generation launched time approach	Products for the general public's use in electronics, such as IBM mainframe machines
Maximum likelihood (ML) [32]	We considered that the underlying product's future acceptance is uncertain and modelled the market demand functional with stochastic considerations.	Energy use is an example of a service product.

Maximum likelihood (ML) [33]	To find the best warranty duration for new items, we devised a profit-maximizing issue. The retail cost goes up as the limit set by specification goes down.	Hypothetical data
Nonlinear least squares (NLS) [34]	Presented a different way of expressing the Bass model by indicating the adoption rate per residual adopter utilizing a logistical hazards factor.	Consumer durable products
Nonlinear least squares (NLS) [35]	Added an autoregressive variable and diffusion route uncertainties to the expanded bass diffusing models	Products for the general public's use in electronics, including compact discs
Monte Carlo simulation	Established the best advertising strategy for an innovative item by expanding a general Bass model to show how different promotional efforts affect the diffusion process.	Consumer durable products

A key component of modern diffusion models that aims to represent the complexities of actual adoption behaviour is the incorporation of time-lag and dependence variables. By combining the recognition and the acceptance phases, the author developed a two-stage diffusion model that emphasizes the lag time from product discovery and uptake. By using logistic & Erlang distributed to depict the lag, these models provide versatility in representing different adoption periods. By applying this method to items that span more than one generation, the investigators were able to solve problems like intergenerational switches and substitutions. Findings from this research highlight the need for improved diffusion models that take internally generated dynamics, such as user evaluations of product value, and external effects, like market awareness efforts, into consideration. Comparing to more conventional techniques like the Bass model, these algorithms have shown to be more effective forecasters by confirming their assumptions using real-world information like sales of LCD monitors and smartphones.

One of the most important changes that occurred in the field of innovation adopting study was the emergence of intergenerational diffusion models. Cannibalizing of previous product sales was one of the issues that the author was amongst the first to discuss in relation to the presence of consecutive item generation and the influence that each of them have on the structure of the market place. Using this concept as a foundation, contemporary research incorporates time-lag effects and differentiates among switching behaviours (current users upgrade to new generation) and substitution patterns (new customers replacing older technologies). The use of these additions makes it possible to more accurately simulate the consumption pattern that occur in the actual world, where future generations often crossover with older consumers in the market. These mathematical models are able to reflect lags among awareness and acceptance in an improved way because to the utilization of time-lag operations, such as Erlang distributed. This is particularly notable in technological industries that are heavily competitive and fast growing.

Evaluation of the diffusion theories via empirical research has been an essential component in determining the importance and proper use of these models. The most current models, among them those suggested by the author, have been evaluated through the use of real-world sales information from devices such as Samsung cell phones and Acer laptops. The results of these research demonstrate that time-lag-inclusive methods performed much better than typical models in regards to predicting accuracy. This is accomplished via the use of statistical measures such

as MSE and R-squared values. In addition, these models provide the sales and development teams with helpful knowledge by anticipating not just levels of acceptance but also probable delays in purchasing choices. This allows the teams to better understand the market and design products.

### 3. Objective of research work

Developing sophisticated multigenerational diffusion models (MGDM) which adjust for complicated market dynamics including substitution and switching behaviours and add time-lag effects is the major goal of the research study. To overcome the shortcomings of conventional diffusion models, these approaches attempt to do the following:

- Increasing the precision of items' sales and adoption projections over generations.
- To account for the actual lag time between becoming aware of a problem and selecting a choice, we must include time-delay variables into the adoption process.
- The process of simulating the presence of many product generations simultaneously while taking into consideration the relationships between them, such as the possibility of cannibalizing and upgrades.

### 4. Motivation of research work

Technologies is advancing at a quick pace, and new product generations are being introduced often. This makes it difficult to forecast how innovations will be adopted in markets that are always changing. The Bass model and other traditional diffusion models have several limitations when it comes to considering factors like the lag time among product knowledge and acceptance, the presence of different generations of products, and the intricate relationships between them. In addition, precise modelling of issues such as replacement (when consumers of one generation shift to another) and switching (when current users upgraded to newer goods) becomes more important as market circumstances change.

Improving the prediction accuracy of diffusion models via the incorporation of time delays and dependence variables is the driving force behind this study. It is essential to understand real-world adoption patterns, yet current models often fail to account for the delays that are inherent to consumer's decision-making processes. Furthermore, when new product generations hit the marketplace, there is a rising need for approaches that can specifically measure multi-generational dynamics including the impact of up-to-date adopters and cannibalizing impacts.

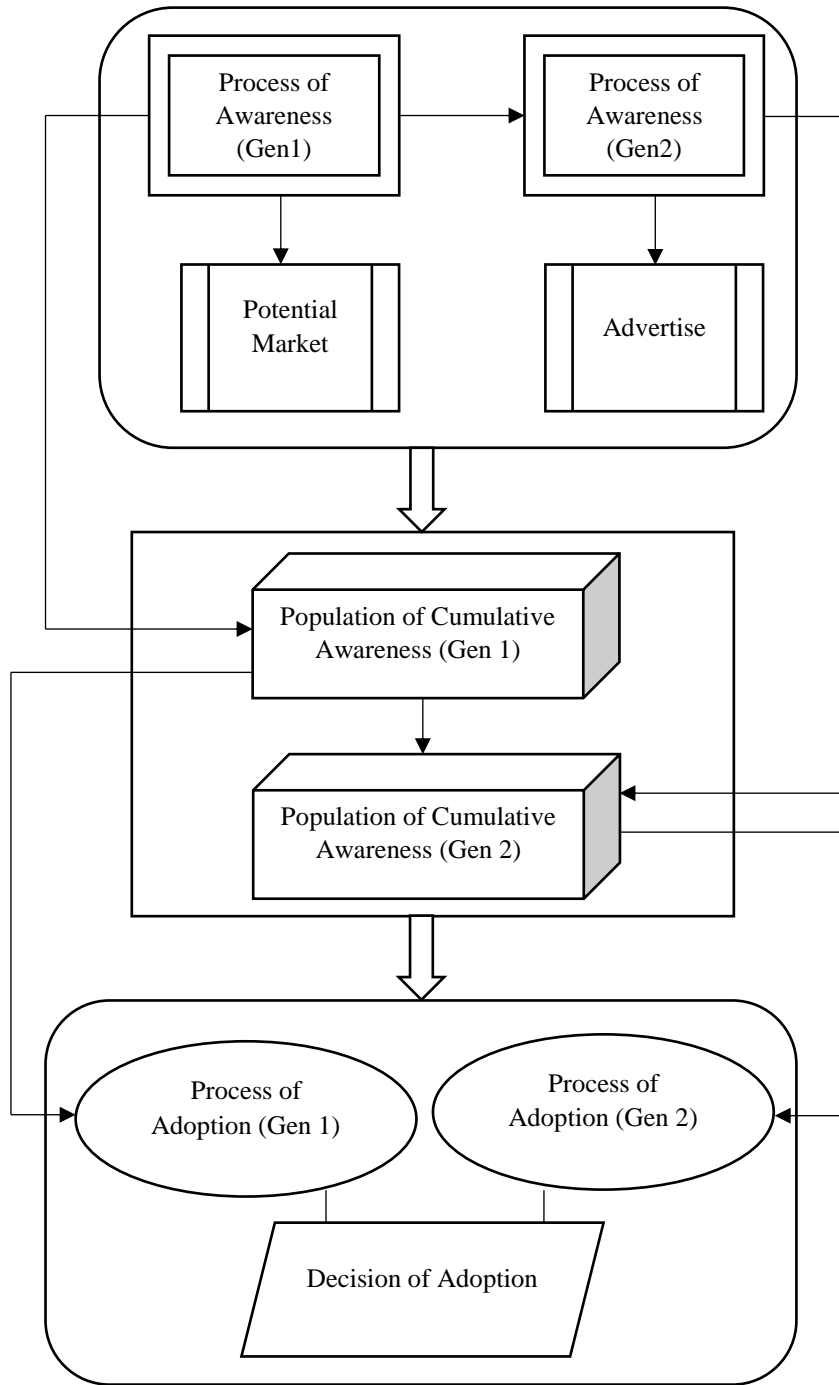
The goal of this study is to fill these knowledge gaps and provide a better foundation for adoption of innovation models. To accurately portray adoption delays while also changing customer habits into consideration, the suggested models use dispersed time-lag operations, including exponentially and Erlang distributions. Businesses can now plan their manufacturing, marketing, and distribution plans with the use of practical information provided by this improved simulation capabilities, which also enhances forecast accuracy. Finally, the objective of this study is to connect the dots amongst theoretical theories of dissemination and the real-world needs of today's technology-driven marketplaces.

### 5. Proposed Method

For the purpose of describing the recognition and adoption of phenomena, this section presents a model of the dissemination of innovation process, which is structured in two stages. The choice of a potential buyer to purchase anything is influenced by a number of different elements, including the selling cost, an advertising in the popular press, the financial resources of the buyer, and so on. As a result, the diffusion models that are suggested in this part are founded on the genuine premise that prospective purchasers must initially become aware of the existence of an invention before they can use it. For the purpose of attracting attention to the adoption of the goods in the market, timing lag functions that align with the different distribution functions have been selected. A growth structure that shapes like a S is thought to be followed by the dispersion of innovations. Using two real-world sales datasets of technical advancements, the suggested models were evaluated to see how accurate their estimations were and how well they projected future outcomes. During the adoption process, the time delay effect is shown to have a significant influence, as shown by the empirical findings of the information analysis. Taking into consideration the amount of time that elapses between the occurrence product recognition and acceptance of the item, this portion models a diffusion concept of an invention. For every short interval  $(v, v + \Delta v)$ , it is presumed that the amount of persons who are aware of the presence of an invention is a function of its unused possibilities because they are oblivious of the fact that it exists. Being the case,

$$M_{\alpha}(v + \Delta v) - M_{\alpha}(v) = c_k [n - M_{\alpha}(v)] \Delta v \quad (1)$$

where  $c_k$  represents a pace that is dependent on the passage of time and serves to raise knowledge about the breakthrough.



**Figure 2.** Architecture of proposed method

We may rewrite Equation (1) as:

$$c_k = \lim_{\Delta v \rightarrow 0} \frac{M_\alpha(v+\Delta v) - M_\alpha(v)}{[n - M_\alpha(v)]\Delta v} \quad (2)$$

The following differential equation  $\frac{dM_\alpha(v)}{dt}$  may be used to represent equation (1) in continuous-time:

$$\frac{dM_\alpha(v)}{dt} = c_k [n - M_\alpha(v)] \quad (3)$$

With the starting conditions  $M_{\alpha}(0) = 0$ , we can solve equation (3) and get the formula for the number of adopters by time  $v$ :

$$M_{\alpha}(v) = n(1 - f^{-c_k v}) \tag{4}$$

In the traditional model, it is presumed that people would start using the product the moment they hear about it, i.e.

$$M_r(v) = M_{\alpha}(v) = n(1 - f^{-c_k v}) \tag{5}$$

However, this presumption may not be correct when applied to actual situations. Customers often learn about a product, read reviews, and then decide whether or not to buy it based on their impressions of its features and functionality. The time it takes for a product to get from being known about to being used is, therefore, rather long. Hence, the real adoption of innovation is described in this section using the time-delayed formula ( $\phi(v)$ ). The real adoption is provided by equation (6) using assumptions

$$M_r(v) = M_{\alpha}(v - \phi(v)) = n(1 - f^{-c_k(v-\phi(v))}) \tag{6}$$

By separating equation (6), we may get the following innovation sales rate equation:

$$\frac{dM_r(t)}{dt} = nc f^{-c_k v} f^{-c_k \phi(v)} \left(1 - \frac{d\phi(t)}{dt}\right) \tag{7}$$

$$\text{Where } \frac{d\phi(t)}{dt} < 1$$

A number of diffusion models may be obtained by defining the rate of adoption as well as the time lag function. This research presents a variety of diffusion models derived from three distinct functional kinds of time lagged functions. The first research on product dispersion in marketing literature. It was suggested by them that the rate of innovation adaptation is constant, denoted as model is widely recognized to be among of the most effective models for predicting the popularity of new products. Equation (8) gives the formula for the exponentially diffusion model  $\frac{d\phi(t)}{dt} = 0 < 1$ , which is applicable when the time lag function  $\phi(v) = 0$ , meaning that there is no lag among the product's awareness and acquisition.

$$M_r(v) = M_{\alpha}(v - \phi(v)) = n(1 - f^{-c_k v}) \tag{8}$$

One way to see how innovations develop over time is via the delayed S-shaped or Erlang 2-stage model. The rate of embracing innovation rises at the outset and falls off later on, as shown in an S-shaped curve. Equation (2.1.6) is used to represent the Delayed S-shaped models as follows: if  $\phi(v) = km(1+ck)/ck$ , then  $\frac{d\phi(t)}{dt} = \frac{1}{1+bv} < 1$

$$M_r(v) = M_{\alpha}(v - \phi(v)) = n(1 - (1 + c_k v) f^{-c_k v}) \tag{9}$$

Whenever innovations follow a logistical distribution or an inflected S-shaped curve, the product starts off slowly but quickly expands across the possible market once it reaches the inflection point. Here we can define the diffusion model where adoption following a logistic distributional function as:

$$\phi(v) = \frac{1}{c_k} km \left( \frac{1+\beta}{1+\beta f^{-c_k v}} \right) \tag{10}$$

$$M_r(v) = M_{\alpha}(v - \phi(v)) = n \left( \frac{1-f^{-c_k v}}{1+\beta f^{-c_k v}} \right) \tag{11}$$

We presume that adoption follows a learning phenomenon that, in the end, reaches saturation owing to a limited market size. It is also believed that the ratio of educated people to the overall market size, as well as the number of untapped opportunities in the market, are strongly related to the rate of adoption. So, this is the equation for the sales rate:

$$\frac{dM_r(t)}{dt} = c_2 = \frac{M_{\alpha}(v-\phi(v))}{n} (n - M_r(t)) \tag{12}$$

where the adoptive coefficient is  $c_2$  and  $M_{\alpha}(v - \phi(v))$  is the cumulative number of conscious persons over time is denoted by  $v - \phi(v)$ .

We have utilized two sets of real-world sales data to validate and analyse the models that we have developed. Sales information for Samsung smartphones, collected from an internet source, is the initial dataset (DSI). The 2nd dataset, DSII, compiles quarterly sales figures (in millions) for Acer Personal Computers culled from an internet source. For both technical datasets, a non-linear least square (NLS) estimating approach is used to determine the diffusion variables. You can see the parameter estimate findings for both the DSI and DSII datasets. A smaller

value of 1 (0.06)  $b <$  indicates that the level of awareness component is much lower for both datasets. For all three scenarios, the degree of acceptance coefficient is larger than the awareness coefficient since the product will be adopted more quickly by an informed populace. We also evaluate the suggested methods' predicting effectiveness using goodness-of-fit metrics to two benchmarking models. In order to compare the efficacy of each diffusion model's predictions, three statistical indicators are examined: Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and others. We can get a summary of the system's performance indicators. The statistical metrics show that the suggested models outperform the old ones when it comes to making predictions. The congruence among the real and expected sales paradigm. Notably, for both datasets, all three suggested models have done a respectable job of predicting the actual curves.

Using the multi-stage structure of the adoption process as a foundation, the current part proposes innovative models of multigenerational diffusion models (MGDM). For the purpose of predicting the demand for intergenerational goods in settings that are more representative of the actual world, the mathematical model that the author developed has been expanded in this section. To be able to differentiate among the process of product awareness and acceptance in a strategic manner, the intergenerational adoption growing function is calculated using a time-varying lag function. When adopting a child, it is important to evaluate the adoptive parents' level of knowledge and understanding. Differential formulas are included into the suggested diffusion models in order to assess the market's potential awareness. In the process of becoming conscious of something, the growth function is thought to be looking for either an increase in speed or a logistical structure. For the purpose of illustrating the time delay operation, an Erlang distributed with a variety of shape variables. Substituting which occurs when a potential purchaser of an older generation decides to purchase an earlier model alternatively, and transitioning, which occurs when users of prior generations switch to an improved generations prior after it's made readily accessible: these are two distinct types of evolving behaviours that are differentiated by the current diffusion model. By fitting the suggested models to the actual shipping data of LCD computer displays, more testing is performed on the designs that have been presented. This section provides a model that utilizes the phase's structure to illustrate the process of invention dissemination over several generations. To show the patterns of adoption of multi-generational innovations, new diffusion models that include a time lag function have been presented. The ultimate adoption of an invention is considered to be a step-by-step process, first consisting of the prospective buyer being aware of the availability of the innovation, and then adopting the innovation in a later stage. A two-stage architecture is taken into consideration while modelling the diffusion process in order to simplify the procedure of mathematical evaluation. The diffusion dynamics of single and two simultaneously future generations are modelling whereby the first subsequent generations of the products is launched in the market at time  $t = 0$  and secondly generations enter the market at  $v = \tau > 0$ .

It is a circumstance that is taken into consideration when there is a single generation of their product that is offered for approval. The current subsection provides a mathematical expression of two phases of the adoption process. When we talk about product awareness, we are referring to the level of information that the prospective market has about the invention. Creating awareness among potential purchasers that an invention is available is the first stage in the process of gaining acceptability for an innovation. Understanding about the characteristics, advantages, qualities, and usefulness of a new product serves as the basis for the expansion of innovation in the market. Calculating the instantaneous diffusion rate of product awareness may be done using the following mathematical formula:

$$\frac{dM_{\alpha}(t)}{dt} = \frac{e_{\alpha}(v)}{1-E_{\alpha}(v)} [n - M_{\alpha}(v)] \quad (13)$$

where  $e_{\alpha}(v)$  is the cumulative adopting function,  $M_{\alpha}(v)$  is the cumulative awareness of prospective innovation consumers over time  $v$ , and a  $e_{\alpha}(v)$  is the intensity function of adoption. Possible market size is represented by  $n$ .

The subsequent closed-form solution is derived by analysing the differential equation given above, with the essential assumptions that at time  $v=0$ ,  $M_{\alpha}(v) = 0$ .

$$M_{\alpha}(v) = nE_{\alpha}(v) \quad (14)$$

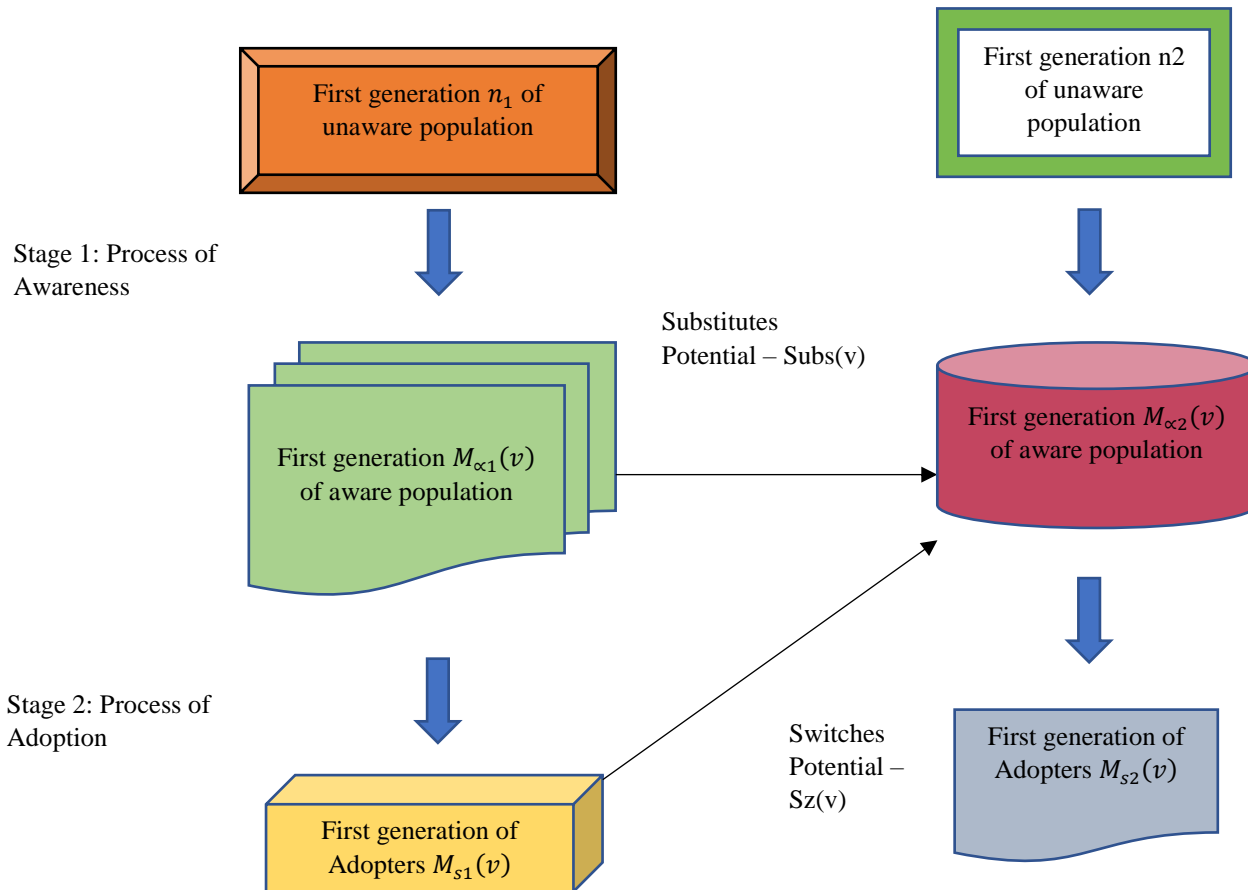
A consistent pace of dissemination of information about the invention among the prospective market causes the awareness of the innovation to expand exponential over time, i.e.  $(\cdot) = E t \exp b a$ . Therefore, we can rewrite equation (14) as:

$$M_{\alpha}(v) = n(1 - f^{-cv}) \quad (15)$$

The item's market size ( $n$ ) and the adoption variable ( $c$ ) are used here. An accumulation of potential customers who are aware of the innovation's appearance is expressed by Equation (15). Consequently, they provide a conscientious prospective consumer base that, given enough time, will be inclined to buy.

Someone who is well-informed about the invention may take their time buying it, as has been shown. Many factors, such the cost of the invention and people's purchasing power, play a role in the choice to adopt by prospective markets. Similarly, the current modelling approach presupposes a distinct lag between the stages of product awareness and adoption. Thus, the product awareness process may be represented as a time-delay function, and the product adoption process can be further characterized in the same way.

Innovations often undergo iterative improvements as a result of scientific and technological progress; each new generation builds upon the one before it. It is safe to assume that users of subsequent generations will not go back to using prior generations. Users of the previous generation may upgrade to the newer, better product generation if they are made aware of it. It is possible for two product generations to coexist in the current architecture. Similarly, raising awareness and finally adopting a practice may be mathematically represented as a two-stage process with two generations operating at the same time. The adoption process flow for two generations occurring at the same time is shown in Figure 3.



**Figure 3.** Adoption process of concurrent technological generations

Each generation has its own distinct prospective customer base when it first hits the shelves. Potential adopters and users of previous generations might additionally be interested in purchasing products from the next generation. Accordingly, there are three categories that make up the second generation's aware population: the new and unique market potential, the potential knowledgeable adopters of the generation before them who could shift to the current one once they learn about it, and those who use of the before subsequent generations who are now potential adopters of this generation's products. Prior research classifies potential customers into three categories: first, potential buyers; second, possible substitutes; and third, potential switchers. Here is the mathematical equation for second generation properly informed and possible substitutes and switchers:

$$\text{Subs}(v) = n_1[E_{\alpha_1}(v) - E_{r_1}(v)]E_{\alpha_2}(v - \tau_1) \tag{16}$$

$$\text{Sz}(v) = n_1E_{r_1}(v)E_{\alpha_2}(v - \tau_1) \tag{17}$$

where  $\text{Subs}(v)$  represents the total number of first-generation users who are knowledgeable regarding the second-generation and have the potential to replace the previous generation with the new one; and  $\text{Sz}(v)$  represents the

number of first-generation users who possess knowledge of the second-generation and could potentially enhance to it.

After plugging the values into equations (16 and 17), the possible total shift from generations one to generation two is stated as:

$$R(v) = \text{Subs}(v) + \text{Sz}(v) \tag{18}$$

The first-time purchasers, switches, and substitutes of the second generation determine the prospective adopters of the second generation. In a two-stage procedure, the concurrent formulas for two generations of consumers are represented as:

$$M_{r1}(v) = F[M_{r1}(v - \phi_1)] - F[\text{Subs}(v - \phi_2)] - F[\text{Sz}(v - \phi_2)] \tag{19}$$

$$M_{r2}(v) = F[M_{r2}(v - \phi_2)] \tag{20}$$

where the time delay in adoption of the very first generation is represented by  $\phi_1$  & the time delayed in adoption of this second generation is represented by  $\phi_2$ .

**4. Results and Discussion**

Development of a sales strategy based on estimations of the invention's market potential and diffusion pace is one of the key capabilities associated with innovation diffusion models. If managers have a solid sales strategy in place, they will be better able to control production, finance, and marketing strategies, as well as prevent unexpected cash losses. Administration benefits from having accurate information on diffusion patterns across generations since it allows them to make important choices. Marketing executives will benefit greatly from the research suggestions made in this part as they work to formulate the most effective strategies for manufacturing, advancement, and marketing. The proposed approach in this study fills a gap in the previous multi-generational research by specifically assisting in predicting both the overall population of modern technology consumers and the quantity of knowledgeable potential buyers who have not yet made the purchase.

This adds substantial practical value to the research being conducted. When it comes to marketing and advertisement strategies, this data is invaluable for helping business analysts determine what works best to encourage informed prospective customers to make a prompt purchase. This work fits well with the current mathematical frameworks for forecasting the development function of technical advances, and it is compatible with them. The suggested intergenerational diffusion model in this part makes an essential distinction between the stages of product awareness and acceptance. Several people's awareness of the invention, which is managed by promotional efforts, determines the pace of adoption. Utilizing the suggested mathematical approach to clearly estimate the rate of product awareness between prospective consumers allows managers to successfully design promotions and marketing tactics to boost business knowledge and comprehension. The management team may use the recommended research as an outline to make the best decisions possible.

Mean Absolute Deviation (MAD): The MAD is a metric that takes into account the median of the absolute discrepancies between the expected and observed results. Without taking their direction into account, it measures the size of forecast mistakes.

Mean squared error (MSE): The MSE measures how far off the mark the anticipated values are from the actual ones. Because it prioritizes bigger mistakes, it is easily influenced by extreme values.

Root-mean-squared error (RMSE): As a measure of the error in forecasting in the exact same units as the actual data, RMSE is useful. You may use it to understand how accurate the model is in the actual world.

Range ( $R^2$ ):  $R^2$  is also known as the coefficient of dedication, quantifies how much of the observed data variation can be clarified through the model. It shows how well a model matched the data.

**Table 2:** Outcome of Statistical Measures of MAD

Model of Diffusion	MAD
Erlang Distributions	1358.429
Bass Diffusion model	3069.16
Proposed method (MGDM 1)	7302.478
Proposed method (MGDM 2)	6596.113
Proposed method (MGDM 3)	5652.571

Various degrees of prediction accuracy are shown when diffusion models using mean absolute deviation (MAD) are compared. A minimum MAD value of 1358.429 shows that the Erlang Distributions model is the most accurate in predicting adoption, making it the top performer. The Bass Diffusion model comes next, displaying intermediate accuracy with an enhanced MAD of 3069.16. With a MAD of 5652.571, MGDM 3 outperforms the other suggested ways in terms of accuracy. Following closely behind are MGDM 2 with 6596.113 and the most deviating method, MGDM 1, with 7302.478. Although the MAD values of the suggested techniques are greater than those of conventional models, they provide a broader framework for studying multigenerational diffusion because they are built to take into consideration complicated real-world elements like intergenerational interactions and temporal delays.

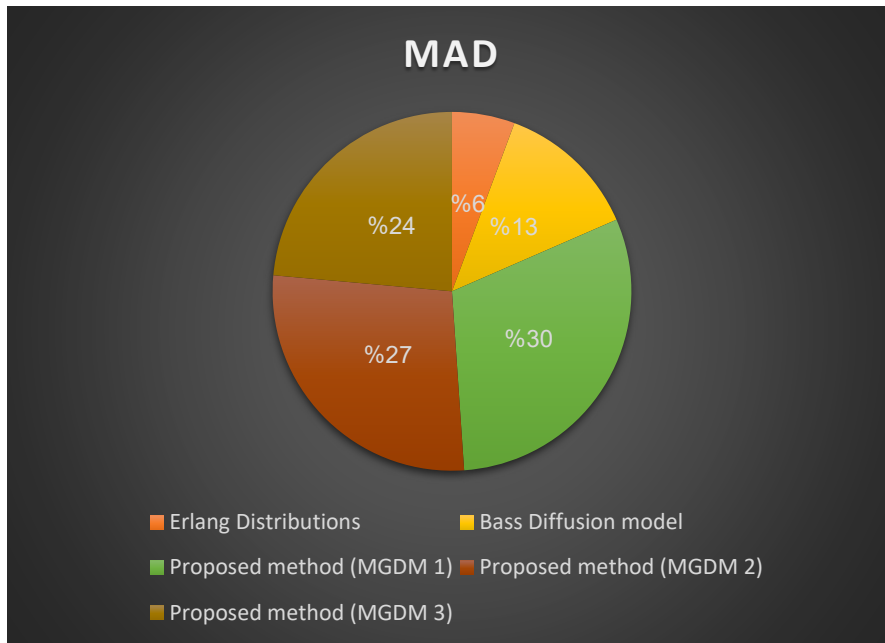


Figure 4. Evaluation of ML models in comparison to more traditional approaches.

Table 3: Results of RMSE Statistical Measures

Model of Diffusion	RMSE
Erlang Distributions	8182.9
Bass Diffusion model	15509.9
Proposed method (MGDM 1)	1756.6
Proposed method (MGDM 2)	1657.6
Proposed method (MGDM 3)	1845.5

Significant disparities in predicting accuracy are seen when diffusion models are compared using RMSE. Among the suggested approaches, MGDM 2 has the best RMSE at 1657.6, then follows MGDM 1 at 1756.6, and MGDM 3 at 1845.5, all of which are better than the conventional approaches. In comparison, the RMSE of 8182.9 for the Erlang Distributions model and 15509.9 for the Bass Diffusion model are significantly higher, suggesting less accurate predictions, respectively. The assessment highlights how the suggested multigenerational diffusion approaches capture real-world adoption trends and handle complexity like connections between generations and time-lag effects more accurately than the alternatives.

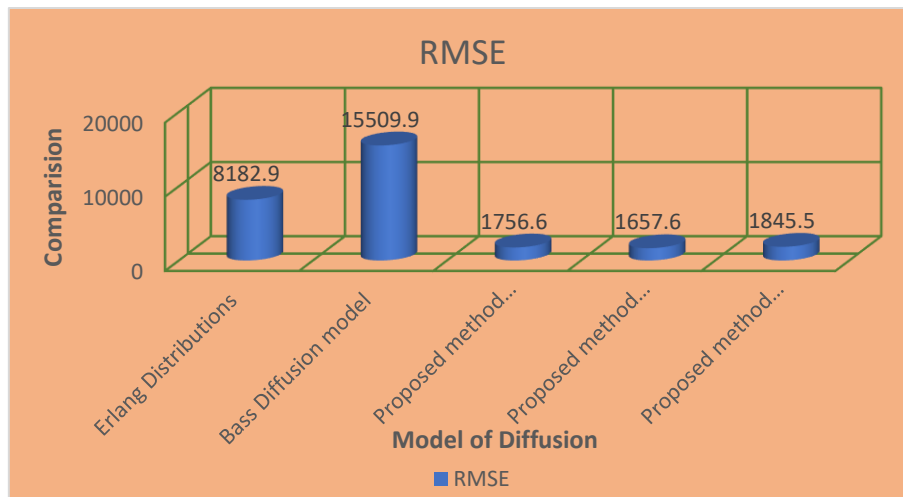


Figure 5. Comparing ML models to more conventional methods for evaluation.

Table 4: Statistical Measures for MAD Outcomes

Model of Diffusion	MSE
Erlang Distributions	9983377
Bass Diffusion model	61593885
Proposed method (MGDM 1)	3457084
Proposed method (MGDM 2)	3892392
Proposed method (MGDM 3)	3994885

The analysis of diffusion models that depends on MSE demonstrates that the suggested approaches have a distinct advantage over the standard models. When compared to the Bass Diffusion model, which has a much larger MSE of 61593885, the Erlang Distributions model has the lowest MSE, which is 9983377. This indicates that the Erlang Distributions technique is more accurate. MGDM 1 has the greatest efficiency among the offered approaches, with an MSE of 3457084. This is a major improvement over MGDM 2 (3892392) and MGDM 3 (3994885), which show an effectiveness that is much lower. The suggested approaches (MGDM 1, MGDM 2, and MGDM 3) nonetheless provide a more complex strategy for intergenerational diffusion, reflecting complicated market factors such as time-lag effects & inter-generational interconnections. This is despite the fact that the MSE values are larger when compared to Erlang Distributions.

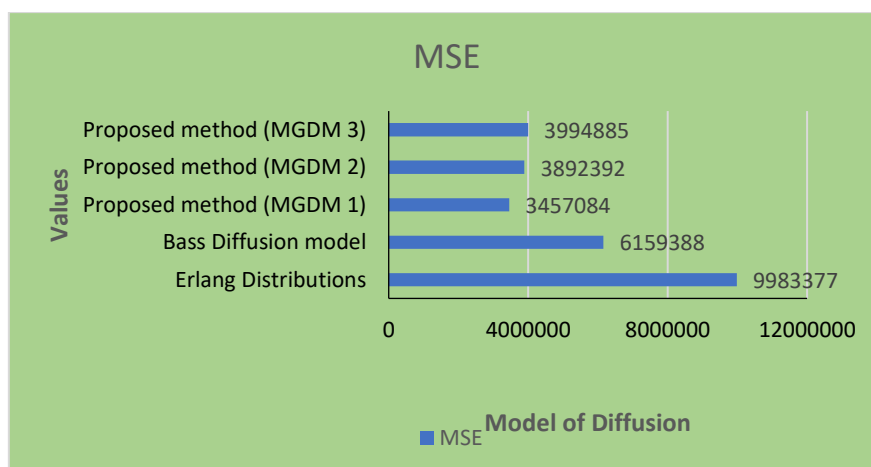
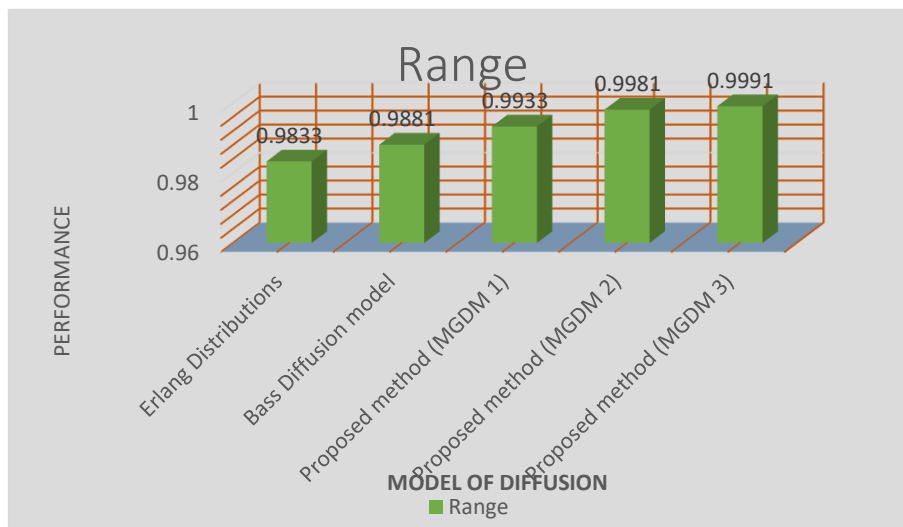


Figure 6. Efficacy of different systems

**Table 5:** Exploratory DL models in relation to proposed methods

Model of Diffusion	Range
Erlang Distributions	0.9833
Bass Diffusion model	0.9881
Proposed method (MGDM 1)	0.9933
Proposed method (MGDM 2)	0.9981
Proposed method (MGDM 3)	0.9991



**Figure 7.** Effectiveness of different models

According to the results of the analysis of diffusion models determined by the range (R2), the suggested techniques consistently perform better than traditional approaches in terms of how well they match the situation. The Bass Diffusion approach has an R2 value of 0.9881, which indicates that it provides a decent match; however, the Erlang Distributions approach has a somewhat lower R2 value of 0.9833, which sets it apart from the Bass Diffusion approach. On the other hand, the suggested approaches (MGDM 1, MGDM 2, and MGDM 3) provide a gradually better fit, with MGDM 3 attaining the highest R2 value of 0.9991, next to MGDM 2 at 0.9981 and MGDM 1 being at 0.9933. The fact that these R2 values are greater demonstrates that the suggested models are able to explicate the variation in information that has been seen more successfully, especially when considering the phenomenon of intergenerational replication.

**6. Conclusion**

Innovative models of product dispersion that take into account the lag period between product awareness and uptake. The two parts make up this section. In the first part, we provide awareness-based diffusion models that take into account the fact that adoption is explicitly dependent on enlightened people. By using various kinds of functions to simulate knowledge of market potential, an empirical answer for the method of diffusion may be produced. Using nonlinear least square statistical regression, the built models are evaluated experimentally using Acer Personal Computer and Galaxy Smartphone shipping sales data from throughout the world. Through comparing the outcomes of the statistical metrics with two reference models, we may determine the systems' prediction effectiveness. By taking the adoption process's distribution time delay function into account, analytically multigenerational diffusion models are proposed in the second part. The suggested models use a differential formula for assessing the market's level of awareness. Both exponentially and logistical growth structures are being investigated for the awareness process's development function. An Erlang distributive function is used to examine the future adoption. Both switching and substitutional forms of shifting behaviour are included into the suggested models. Here we provide fresh findings obtained by applying the suggested paradigm to real-world sales data of LCD computer displays. Out of each model that were examined, the results show that the novel diffusion models had the best prediction capacity. The created model is also technically feasible and versatile, thus it may be used in a variety of businesses.

For optimum judgments, decision-makers ought to take seriously various unique elements; future research might apply multi-criteria decision-making procedures to assess new product marketing tactics.

**Funding:** “This research received no external funding”

**Conflicts of Interest:** “The authors declare no conflict of interest.”

## References

- [1] Lobna Osman, A PSPICE Fast Model for the Single Electron Transistor, *International Journal of Wireless and Ad Hoc Communication*, Vol. 0 , No. 1 , (2019) : 8-23 (Doi : <https://doi.org/10.54216/IJWAC.000101>)
- [2] Ahmed A. Elngar , Salah-ddine KRIT, Performance Analysis of Machine Learning based Botnet Detection and Classification Models for Information Security, *Journal of Cybersecurity and Information Management*, Vol. 0 , No. 1 , (2019) : 44-53 (Doi : <https://doi.org/10.54216/JCIM.000104>)
- [3] Mohamed Elsharkawy , Ahmed N. Al Masri, A Novel Image Encryption with Deep Learning Model for Secure Content based Image Retrieval, *Journal of Cybersecurity and Information Management*, Vol. 0 , No. 2 , (2019) : 54-64 (Doi : <https://doi.org/10.54216/JCIM.000105>)
- [4] F. M. Bass, “A new product growth model for consumer durables,” *Manage. Sci.*, vol. 15, no. 5, pp. 215–227, 2020.
- [5] P. K. Kapur, O. Singh, and R. Mittal, “Software reliability growth and innovation diffusion models: An interface,” *Int. J. Rel., Quality Saf. Eng.*, vol. 11, no. 4, pp. 339–364, Dec. 2004.
- [6] K. Shankar, Recent Advances in Sensing Technologies for Smart Cities, *International Journal of Wireless and Ad Hoc Communication*, Vol. 1 , No. 1 , (2020) : 05-15 (Doi : <https://doi.org/10.54216/IJWAC.010101>)
- [7] C. Vivek,M. Indu,N. Nandhini, Speech Recognition Using Artificial Neural Network, *Journal of Journal of Cognitive Human-Computer Interaction*, Vol. 5 , No. 2 , (2023) : 08-14 (Doi : <https://doi.org/10.54216/JCHCI.050201>)
- [8] Vidyul Narayanan,Nithya P.,Sathya M., Effective lung cancer detection using deep learning network, *Journal of Journal of Cognitive Human-Computer Interaction*, Vol. 5 , No. 2 , (2023) : 15-23 (Doi : <https://doi.org/10.54216/JCHCI.050202>)
- [9] S. Yang, X. Gou, M. Yang, Q. Shao, C. Bian, M. Jiang, and Y. Qiao, “Software bug number prediction based on complex network theory and panel data model,” *IEEE Trans. Rel.*, vol. 71, no. 1, pp. 162–177, Mar. 2022.
- [10] P. Vizaretta, K. Trivedi, B. Helvik, P. Heegaard, A. Blenk, W. Kellerer, and C. Mas Machuca, “Assessing the maturity of SDN controllers with software reliability growth models,” *IEEE Trans. Netw. Service Manage.* vol. 15, no. 3, pp. 1090–1104, Sep. 2018.
- [11] A. Sariga , J. Uthayakumar, Type 2 Fuzzy Logic based Unequal Clustering algorithm for multi-hop wireless sensor networks, *International Journal of Wireless and Ad Hoc Communication*, Vol. 1 , No. 1 , (2020) : 33-46 (Doi : <https://doi.org/10.54216/IJWAC.010102>)
- [12] P. Sheela Rani,Harini M.,Nandhitha N.,Teena A. Naahz G., A Decentralized and Cooperative Methodology For Organ Donation Management Based on Ethereum Blockchain, *Journal of Cognitive Human-Computer Interaction*, Vol. 6 , No. 1 , (2023) : 08-17 (Doi : <https://doi.org/10.54216/JCHCI.060101>)
- [13] Malini Srinivasan,Jishnu Dineshan,Gondi Surender Dhanunjay,Shamala Ramappa, Measuring the Coverage of Assembly elections and Covid 19 during the pandemic in India, *Journal of Cognitive Human-Computer Interaction*, Vol. 6 , No. 1 , (2023) : 18-31 (Doi : <https://doi.org/10.54216/JCHCI.060102>)
- [14] K. Pandey, A. Mukherjee, P. Rai, and A. Kumar, “VAEs meetdiffusion models: Efficient and high-fidelity generation,” in *Pro-ceedings of NeurIPS Workshop on DGMs and Applications*, 2021.
- [15] F. Bao, C. Li, J. Zhu, and B. Zhang, “Analytic-DPM: an AnalyticEstimate of the Optimal Reverse Variance in Diffusion Probabilis-tic Models,” in *Proceedings of ICLR*, 2022.
- [16] T. Dockhorn, A. Vahdat, and K. Kreis, “Score-based generativemodeling with critically-damped Langevin diffusion,” in *Proceed-ings of ICLR*, 2022.
- [17] N. Liu, S. Li, Y. Du, A. Torralba, and J. B. Tenenbaum, “Composi-tional Visual Generation with Composible Diffusion Models,” in *Proceedings of ECCV*, 2022
- [18] I. Oguz, J. D. Malone, Y. Atay, and Y. K. Tao, “Self-fusion forOCT noise reduction,” in *Proceedings of SPIE Medical Imaging*,vol. 11313, p. 113130C, SPIE, 2020.
- [19] Khadija Shazly , Dina A. Salem , Nacereddine Hammami , Ahmed I. B. ElSeddawy, A Review on Distributed Denial of Service Detection in Software Defined Network, *International Journal of Wireless*

- and Ad Hoc Communication, Vol. 5 , No. 2 , (2022) : 08-18 (Doi : <https://doi.org/10.54216/IJWAC.050201>)
- [20] Prabu S. , Alekhya A. , Kiran K. Chatragadda , Venkateswarlu Lingala, Secured Authentication of Node in Mobile Adhoc Network, International Journal of Wireless and Ad Hoc Communication, Vol. 5 , No. 2 , (2022) : 19-29 (Doi : <https://doi.org/10.54216/IJWAC.050202>)
- [21] R. Q. Charles, H. Su, M. Kaichun, and L. J. Guibas, “PointNet:Deep Learning on Point Sets for 3D Classification and Segmenta-tion,” in Proceedings of CVPR, pp. 77–85, 2017.
- [22] G. Balakrishnan, A. Zhao, M. R. Sabuncu, J. Gutttag, and A. V.Dalca, “An unsupervised learning model for deformable medicalimage registration,” in Proceedings of CVPR, pp. 9252–9260, 2018.
- [23] X. Li, T.-K. L. Wong, R. T. Chen, and D. Duvenaud, “Scalablegradients for stochastic differential equations,” in Proceedings ofAISTATS, pp. 3870–3882, 2020.
- [24] T. Park, M.-Y. Liu, T.-C. Wang, and J.-Y. Zhu, “Semantic imagesynthesis with spatially-adaptive normalization,” in Proceedingsof CVPR, pp. 2337–2346, 2019
- [25] I. A. Chaudhry and A. A. Khan, “A research survey: Review of flexible job shop scheduling techniques,” Int. Trans. Oper. Res., vol. 23, no. 3, pp. 551–591, May 2016.
- [26] H. Zhou, Y. Feng, and L. Han, “the hybrid heuristic genetic algorithm for job shop scheduling,” Comput. Ind. Eng., vol. 40, no. 3, pp. 191–200, Jul. 2001.
- [27] J. Su, Y. Yang, and R. Duan, “A CA-based heterogeneous model for knowledge dissemination inside knowledge-based organizations,” J. Intell. Fuzzy Syst., vol. 34, no. 4, pp. 2087–2097, 2018.
- [28] A. V. Barenji, R. V. Barenji, D. Roudi, and M. Hashemipour, “A dynamic multi-agent-based scheduling approach for SMEs,” Int. J. Adv. Manuf. Technol., vol. 89, no. 9, pp. 3123–3137, Apr. 2017.
- [29] D. Ouelhadj and S. Petrovic, “A survey of dynamic scheduling in manufacturing systems,” J. Scheduling, vol. 12, no. 4, p. 417, 2009.
- [30] A. H. Kashan, B. Karimi, and F. Jolai, “an effective hybrid multi-objective genetic algorithm for bi-criteria scheduling on a single batch processing machine with non-identical job sizes,” Eng. Appl. Artif. Intell., vol. 23, no. 6, pp. 911–922, Sep. 2010.
- [31] J. Su, M. Wei, and A. Liu, “A robust predictive–reactive allocating approach, considering random design change in complex product design processes,” Int. J. Comput. Intell. Syst., vol. 11, no. 1, pp. 1210–1228, 2018.
- [32] S. H. H. Madni, M. S. A. Latiff, Y. Coulibaly, and S. M. Abdulhamid, “Recent advancements in resource allocation techniques for cloud computing environment: A systematic review,” Cluster Comput., vol. 20, pp. 2489–2533, Sep. 2017.
- [33] B. Guo, Y. Liu, L. Wang, V. O. Li, C. K. Jacqueline, and Z. Yu, “Task allocation in spatial crowdsourcing: Current state and future directions,” IEEE Internet Things J., vol. 5, no. 3, pp. 1749–1764, Jun. 2018.
- [34] J. Yang, J. Su, and L. Song, “Selection of manufacturing enterprise innovation design project based on consumer’s green preferences,” Sustainability, vol. 11, no. 5, p. 1375, Mar. 2019.
- [35] A. Morris, M. Halpern, R. Setchi, and P. Prikett, “Assessing the challenges of managing product design change through-life,” J. Eng. Des., vol. 27, nos. 1–3, pp. 25–49, 2016.