



The Mathematical Formulas of 2-Cyclic Refined Duplets and Triplets

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Abstract

This work is dedicated to studying the problem of computing 2-cyclic refined neutrosophic duplets and triplets in the 2-cyclic refined neutrosophic ring of real numbers, where we present four different formulas that describe all possible duplets in this extended ring. Also, we present four different formulas for the computation of related triplets in the same ring.

Keywords: Neutrosophic triplet; Neutrosophic duplet; 2-cyclic refined neutrosophic ring; Neutrosophic number

1. Introduction

The concept of neutrosophic sets and neutrosophic structures is considered one of the most recent mathematical concepts with applications in many different scientific fields [1]. The n-cyclic refined neutrosophic structures were first defined in [9], where algebraic structures related to this definition were studied, such as rings and modules. Also, the groups and spaces generated by these mathematical systems were studied in [9-12,14]. Then in [2-4], some famous algebraic problems were studied for the n-cyclic refined neutrosophic rings, where some conjectures were put forward that discuss the classification of the group of units such as generalized Von Shtawzen's conjecture [5].

The problem of the group of units for 3-cyclic and 4-cyclic refined neutrosophic rings of integers was studied by many researchers [7-8, 13], where it was classified in [6] as a proof of first and second Von Shtawzen's conjectures. In [15], the problem of finding idempotent elements in some n-cyclic refined neutrosophic rings was studied and handled. The concept of neutrosophic duplets and triplets has been defined and studied by many authors, see [16-18].

In this work, we present a solution for the duplet and triplet problem in the 2-cyclic refined neutrosophic rings, where we present four different formulas that describe all possible duplets in this extended ring. Also, we present four different formulas for the computation of related triplets in the same ring.

2. Main discussion

Definition:

Let $R_2(I) = \{a_0 + a_1I_1 + a_2I_2; a_i \in R, I_1I_1 = I_2, I_2I_2 = I_1, I_1I_2 = I_2I_1 = I_1\}$ be any 2-cyclic refined neutrosophic ring, then $(x, y) \in R_2(I)$ is called aduplet if and only if $xy = yx = x$ with y acts as an identity, and $x \neq 0$.

Discussion:

For a commutative 2-cyclic refined neutrosophic ring $R_2(I)$, we have:

For $x = x_0 + x_1I_1 + x_2I_2, y = y_0 + y_1I_1 + y_2I_2$ with y acts as an identity: $xy = x$ implies:

$$\begin{cases} x_0y_0 = x_0 & (1) \\ x_0y_1 + x_1y_0 + x_1y_2 + x_2y_1 = x_1 & (2) \\ x_0y_2 + x_2y_0 + x_1y_1 + x_2y_2 = x_2 & (3) \end{cases}$$

The previous system of equations can be formulated as:

$$\begin{cases} x_0y_0 = x_0 \\ (x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = x_0 + x_1 + x_2 \\ (x_0 - x_1 + x_2)(y_0 - y_1 + y_2) = x_0 - x_1 + x_2 \end{cases}$$

Thus $(x_0, y_0), (\sum_{i=0}^2 x_i, \sum_{i=0}^2 y_i), (x_0 - x_1 + x_2, y_0 - y_1 + y_2)$ are duplets in R with $y_0, \sum_{i=0}^2 y_i, y_0 - y_1 + y_2$ act as identities.

Remark:

For a ring with unity, we have to say that (x, y) is a non-trivial duplet if $xy = x$ and $y \neq 1, x \neq 0$.

Duplets in $Z_2(I)$:

Let $Z_2(I) = \{a_0 + a_1I_1 + a_2I_2 ; a_i \in \mathbb{Z}\}$ be the 2- cyclic refined neutrosophic ring of integers, then:

- 1) $x_0y_0 = x_0 \Leftrightarrow x_0 = 0 \text{ or } y_0 = 1$
- 2) $(x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = x_0 + x_1 + x_2 \Leftrightarrow x_0 + x_1 + x_2 = 0 \text{ or } y_0 + y_1 + y_2 = 1$
- 3) $(x_0 - x_1 + x_2)(y_0 - y_1 + y_2) = x_0 - x_1 + x_2 \Leftrightarrow x_0 - x_1 + x_2 = 0 \text{ or } y_0 - y_1 + y_2 = 1$

For $\begin{cases} x_0 = 0 \\ x_0 + x_1 + x_2 = 0 \\ x_0 - x_1 + x_2 = 0 \end{cases}$ or $\begin{cases} y_0 = 1 \\ y_0 + y_1 + y_2 = 1 \\ y_0 - y_1 + y_2 = 1 \end{cases}$, we get: $x = 0$ or $y = 1$ which are the trivial duplets.

We have to discuss the non-trivial cases:

For $x_0 = c \neq 0, x_0 + x_1 + x_2 = x_0 - x_1 + x_2 = 0$, we get:

$$x_1 = 0, x_2 = -c, \text{ then } X = c - cI_2 ; c \in \mathbb{Z}$$

$$\text{Also, } y_0 = 1, y_0 + y_1 + y_2 = b, y_0 - y_1 + y_2 = d$$

$$\text{Hence: } y_1 = \frac{1}{2}(b - d), y_2 = \frac{1}{2}(b + d) - 1 ; b - d \in 2\mathbb{Z}$$

$$\text{So that } Y = 1 + \frac{1}{2}(b - d)I_1 + (\frac{1}{2}(b + d) - 1)I_2$$

$$x \cdot y = c + \frac{1}{2}(bc - dc)I_1 + [\frac{1}{2}(bc + dc) - c]I_2 - cI_2 - \frac{1}{2}(bc - dc)I_1 + [\frac{-1}{2}(bc + dc) + c]I_2 = c - cI_2 = x,$$

thus, the first duplet is $(c - cI_2, 1 + \frac{1}{2}(b - d)I_1 + [\frac{1}{2}(b + d) - 1]I_2)$.

For $x_0 = c \neq 0, x_0 + x_1 + x_2 = 0, x_0 - x_1 + x_2 = t \neq 0$, we get: $x_1 = \frac{-1}{2}t, x_2 = \frac{1}{2}t - c ; t \in 2\mathbb{Z}$.

Also, $y_0 = 1, y_0 + y_1 + y_2 = d, y_0 - y_1 + y_2 = 1$, then: $y_1 = \frac{1}{2}(d - 1), y_2 = \frac{1}{2}(d + 1) - 1 ; d \notin 2\mathbb{Z}$.

Now, put $\begin{cases} X = c - \frac{1}{2}tI_1 + (\frac{1}{2}t - c)I_2 & t \in 2\mathbb{Z} \\ Y = 1 + \frac{1}{2}(d - 1)I_1 + [\frac{1}{2}(d + 1) - 1]I_2 & d \notin 2\mathbb{Z} \end{cases}$

$$\begin{aligned}
 x \cdot y &= c + \frac{1}{2}(dc - c)I_1 + \left[\frac{1}{2}(dc + c) - c \right] I_2 - \frac{1}{2}tI_1 - \frac{1}{4}(td - t)I_2 + \left[\frac{-1}{4}(td + t) + \frac{1}{2}t \right] I_1 + \left(\frac{1}{2}t - c \right) I_2 \\
 &\quad + \frac{1}{2}(d - 1) \left(\frac{1}{2}t - c \right) I_1 + \left[\frac{1}{4}(td + t) - \frac{1}{2}t - \frac{1}{2}(dc + c) + c \right] I_2 \\
 &= c + I_1 \left[\frac{dc}{2} - \frac{c}{2} - \frac{t}{2} - \frac{td}{4} - \frac{t}{4} + \frac{t}{2} + \frac{td}{4} - \frac{dc}{2} - \frac{t}{4} + \frac{c}{2} \right] \\
 &\quad + I_2 \left[\frac{dc}{2} + \frac{c}{2} - c - \frac{td}{4} + \frac{t}{4} + \frac{t}{2} - c + \frac{td}{4} + \frac{t}{4} - \frac{t}{2} - \frac{dc}{2} - \frac{c}{2} + c \right] \\
 &= c + I_1 \left(-\frac{1}{2}t \right) + I_2 \left(\frac{t}{2} - c \right) = x.
 \end{aligned}$$

The second duplet is:

$$\left(c, -\frac{1}{2}tI_1 + \left(-\frac{1}{2}t - c \right) I_2, 1 + \left(\frac{d-1}{2} \right) I_1 + \left(\frac{d+1}{2} - 1 \right) I_2 \right); t \in 2\mathbb{Z}, d \in 2\mathbb{Z}.$$

For $x_0 = c, x_0 + x_1 + x_2 = t, x_0 - x_1 + x_2 = 0$, we get: $x_1 = \frac{1}{2}t, x_2 = \frac{1}{2}t - c, y_0 = y_0 + y_1 + y_2 = 1, y_0 - y_1 + y_2 = d, y_1 = \frac{1-d}{2}, y_2 = \frac{1+d}{2} - 1$; $d \notin 2\mathbb{Z}, t \in 2\mathbb{Z}$

The third duplet is:

$$\left(c + \frac{1}{2}tI_1 + \left(\frac{1}{2}t - c \right) I_2, 1 + \frac{1-d}{2}I_1 + \left(\frac{1+d}{2} - 1 \right) I_2 \right); \begin{cases} d \notin 2\mathbb{Z} \\ t \in 2\mathbb{Z} \end{cases}$$

For $x_0 = 0, x_0 + x_1 + x_2 = t, x_0 - x_1 + x_2 = 0$, then: $x_1 = \frac{1}{2}t, x_2 = \frac{1}{2}t$; $t \in 2\mathbb{Z}, y_0 = a, y_0 + y_1 + y_2 = 1, y_0 - y_1 + y_2 = b$, thus: $y_1 = \frac{1-b}{2}, y_2 = \frac{1+b}{2} - a$, there for the duplet is:

$$\left(\frac{1}{2}tI_1 + \frac{1}{2}tI_2, a + \frac{1-b}{2}I_1 + \left(\frac{1+b}{2} - a \right) I_2 \right); \begin{matrix} t \in 2\mathbb{Z} \\ b \notin 2\mathbb{Z} \end{matrix}$$

For $x_0 = 0, x_0 + x_1 + x_2 = 0, x_0 - x_1 + x_2 = t$, we get: $x_1 = \frac{-1}{2}t, x_2 = \frac{1}{2}t, y_0 = a, y_0 + y_1 + y_2 = b, y_0 - y_1 + y_2 = 1$, so that: $y_1 = \frac{b-1}{2}, y_2 = \frac{b+1}{2} - a$, and the duplet is: $\left(\frac{-1}{2}tI_1 + \frac{1}{2}tI_2, a + \frac{b-1}{2}I_1 + \left(\frac{b+1}{2} - a \right) I_2 \right)$ with $t \in 2\mathbb{Z}, b \notin 2\mathbb{Z}$.

For $x_0 = 0, x_0 + x_1 + x_2 = t, x_0 - x_1 + x_2 = s$, we get: $x_1 = \frac{t-s}{2}, x_2 = \frac{t+s}{2}, y_0 = a, y_0 + y_1 + y_2 = y_0 - y_1 + y_2 = 1$, thus: $y_1 = 0, y_2 = 1 - a$, the duplet is: $\left(\frac{t-s}{2}I_1 + \frac{t+s}{2}I_2, a + (1-a)I_2 \right); t + s \in 2\mathbb{Z}$.

Definition:

A triple (x, y, z) is called a triplet if and only if:

$$\begin{cases} xy = yx = x \\ yz = zy = z \text{ with } y \text{ acts as an identity.} \\ xz = zx = y \end{cases}$$

Remark:

If (x, y, z) is a triplet, then $(x, y)(z, y)$ are duplets.

Remark:

In the 2-cyclic refined neutrosophic ring of integers $Z_2(I)$.

If $x = x_0 + x_1I_1 + x_2I_2, z = z_0 + z_1I_1 + z_2I_2$ such that: $x \cdot z = y$, then:

$$\begin{cases} x_0z_0 = y_0 & (1) \\ (x_0 + x_1 + x_2)(z_0 + z_1 + z_2) = y_0 + y_1 + y_2 & (2) \\ (x_0 - x_1 + x_2)(z_0 - z_1 + z_2) = y_0 - y_1 + y_2 & (3) \end{cases}$$

Discussion for triplets in $Z_2(I)$:

From equation (1), we can see:

$$x_0z_0y_0 = y_0^2 \implies x_0z_0 = y_0^2, \dots, x_0z_0 = y_0^n \text{ for all } n \in \mathbb{N}.$$

$$\begin{cases} (x_0 + x_1 + x_2)(z_0 + z_1 + z_2) = (y_0 + y_1 + y_2)^n \\ (x_0 - x_1 + x_2)(z_0 - z_1 + z_2) = (y_0 - y_1 + y_2)^n \end{cases} \text{ for } n \in \mathbb{N}.$$

The first formula for duplets was:

$$\begin{cases} X = c - cI_2 ; c \in \mathbb{Z} \\ Y = 1 + \frac{1}{2}(b - d)I_1 + \left(\frac{1}{2}(b + d) - 1\right)I_2 ; b - d \in 2\mathbb{Z} \end{cases}$$

Since $y_0 = 1$, then $z_0 = x_0^{-1} = \frac{1}{c} ; c \neq 0$

$$\begin{cases} y_0 + y_1 + y_2 = b \\ y_0 - y_1 + y_2 = d \end{cases} \text{ the only possible cases are: } \begin{cases} b = 1, d = 0 \\ b = 0, d = 0 \\ b = 0, d = 1 \\ b = d = 1 \end{cases}$$

For $b = d = 1, y = 1$, and $(z_0 + z_1 + z_2)(0) = 1, 0 = 1$

Which is a contradiction.

For $b = 1, d = 0$, then $0 \cdot (z_0 - z_1 + z_2) = 1$, another contradiction.

$$\text{For } b = d = 0, \text{ then } z_0 + z_1 + z_2 = a', z_0 - z_1 + z_2 = b' ; a', b' \in \mathbb{Z} \text{ and } Z = \frac{1}{c} + \left(\frac{a' - b'}{2}\right)I_1 + \left(\frac{a' + b'}{2} - \frac{1}{c}\right)I_2 ; c \in \{1, -1\} \\ a' - b' \in 2\mathbb{Z}$$

Thus $Z = 1 + \frac{1}{2}(a' - b')I_1 + \left(\frac{1}{2}(a' + b') - 1\right)I_2$ or $Z = -1 + \frac{1}{2}(a' - b')I_1 + \left(\frac{1}{2}(a' + b') + 1\right)I_2$ with $x = 1 - I_2$ or $x = -1 + I_2, y = 1 - I_2$

The triplets are: $\left(1 - I_2, 1 - I_2, 1 + \frac{a' - b'}{2}I_1 + \left(\frac{a' + b'}{2} - 1\right)I_2\right)$ and $\left(-1 + I_2, 1 - I_2, -1 + \frac{a' - b'}{2}I_1 + \left(\frac{a' + b'}{2} + 1\right)I_2\right), a' - b' \in 2\mathbb{Z}$

For $b = 0, d = 1$, we get a contradiction.

The second formula for duplets:

$$x = c - \frac{1}{2}tI_1 + \left(\frac{1}{2}t - c\right)I_2, y = 1 + \frac{d - 1}{2}I_1 + \left(\frac{d + 1}{2} - 1\right)I_2 ; \begin{cases} d \notin 2\mathbb{Z} \\ t \in 2\mathbb{Z} \end{cases}$$

Since $y_0 = 1$, then $c = \mp 1, z_0 = \mp 1$.

$$\begin{cases} y_0 + y_1 + y_2 = d \\ y_0 - y_1 + y_2 = 1 \\ x_0 + x_1 + x_2 = 0 \\ x_0 - x_1 + x_2 = t \end{cases} \Rightarrow \begin{cases} z_0 - z_1 + z_2 = \frac{1}{t} \in \{1, -1\} \\ z_0 + z_1 + z_2 = 0 \end{cases}$$

The only possible value for d is $d = 0$.

$$\begin{cases} z_0 - z_1 + z_2 = 1 \\ z_0 + z_1 + z_2 = 0 \end{cases} \Rightarrow \begin{cases} z_1 = \frac{-1}{2}, z_2 = \frac{1}{2} \mp 1 \in \left\{\frac{-1}{2}, \frac{3}{2}\right\} \end{cases} \text{ which is a contradiction}$$

$\begin{cases} z_0 - z_1 + z_2 = -1 \\ z_0 + z_1 + z_2 = 0 \end{cases}$ give us another contradiction.

The third formula for duplets:

$$x = c + \frac{1}{2}tI_1 + \left(\frac{1}{2}t - c\right)I_2, y = 1 + \frac{1 - d}{2}I_1 + \left(\frac{1 + d}{2} - 1\right)I_2 ; \begin{cases} d \notin 2\mathbb{Z} \\ t \in 2\mathbb{Z} \end{cases}$$

$y_0 = 1, x_0 = c, z_0 = \mp 1, c = \mp 1, x_0 + x_1 + x_2 = t, y_0 + y_1 + y_2 = 1$, then $z_0 + z_1 + z_2 = \mp 1, t = \mp 1$.

$y_0 - y_1 + y_2 = d, x_0 - x_1 + x_2 = 0$, thus:

$$\begin{cases} x_1 = \frac{1}{2} \text{ or } x_1 = \frac{-1}{2} \end{cases} \text{ it is a contradiction}$$

The fourth formula for duplets is:

$$x = \frac{1}{2}tI_1 + \frac{1}{2}tI_2, y = a + \frac{1-b}{2}I_1 + \left(\frac{1+b}{2} - a\right)I_2 ; \begin{cases} b \notin 2\mathbb{Z} \\ t \in 2\mathbb{Z} \end{cases}$$

$$x_0 = 0, y_0 = a, x_0z_0 = a, \text{ thus: } y_0 = 0, z_0 = s.$$

$$\begin{cases} x_0 + x_1 + x_2 = t & \text{and } (x_0 + x_1 + x_2)(z_0 + z_1 + z_2) = 1 \Rightarrow t = \bar{1} \\ y_0 + y_1 + y_2 = 1 & \text{and } z_0 + z_1 + z_2 = \bar{1} \end{cases}$$

$$\text{Also, } \begin{cases} x_0 - x_1 + x_2 = 0 \\ y_0 - y_1 + y_2 = b \end{cases} \text{ and } (x_0 - x_1 + x_2)(z_0 - z_1 + z_2) = b$$

$$\text{Hence, } b = 0, z_0 - z_1 + z_2 = l.$$

This means that:

$$\begin{cases} z_0 = s \\ z_1 + z_2 = 1 - s \\ z_2 - z_1 = l - s \end{cases} \text{ or } \begin{cases} z_0 = s \\ z_1 + z_2 = -1 - s \\ z_2 - z_1 = l - s \end{cases}$$

$$\text{Hence: } \begin{cases} z_0 = s \\ z_2 = \frac{1+l}{2} - s \\ z_1 = \frac{1-l}{2} \end{cases} \text{ or } \begin{cases} z_0 = s \\ z_2 = \frac{l-1}{2} - s \\ z_1 = \frac{-1-l}{2} \end{cases}$$

Since $t \notin 2\mathbb{Z}$, we get a contradiction.

The fifth formula for duplets:

$$x = \frac{-1}{2}tI_1 + \frac{1}{2}tI_2, y = a + \frac{b-1}{2}I_1 + \left(\frac{b+1}{2} - a\right)I_2$$

$$x_0 = 0, y_0 = a, z_0x_0 = a \Rightarrow z_0 = s, a = 0.$$

$$x_0 + x_1 + x_2 = 0, y_0 + y_1 + y_2 = b \Rightarrow b = 0, z_0 + z_1 + z_2 = l.$$

$$x_0 - x_1 + x_2 = t, y_0 - y_1 + y_2 = 1 \Rightarrow t = \bar{1}, z_0 - z_1 + z_2 = \bar{1}.$$

Which is a contradiction.

From the previous discussion, we get that the triplets in $Z_2(I)$ has two formulas (non-trivial) triplets:

$$\begin{pmatrix} -1 + I_2, 1 - I_2, -1 + \frac{a' - b'}{2}I_1 + \left(\frac{a' + b'}{2} + 1\right)I_2 \\ \left(1 - I_2, 1 - I_2, 1 + \frac{a' - b'}{2}I_1 + \left(\frac{a' + b'}{2} + 1\right)I_2 \right) \end{pmatrix} \quad a' - b' \in 2\mathbb{Z}.$$

3. Conclusion

In this work, we presented a solution for the duplet and triplet problem in the 2-cyclic refined neutrosophic rings, where we presented four different formulas that describe all possible duplets in this extended ring. Also, we present four different formulas for the computation of related triplets in the same ring. In future, we aim to solve the same problem for 3-cyclic refined and 4-cyclic refined neutrosophic rings.

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