Shape preserving monotonic and convex data interpolation using rational cubic ball functions

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Abstract

In this study, a rational cubic Ball function has been used to preserve the shape of monotonic and convex data. Conditions for shape preservation were drawn from the data and imposed on the free parameters of the interpolant function in such a way as to preserve the shape of the data. The interpolant is C^1, which is continuous and visually pleasant function. The outputs of a number of numerical examples are presented.

Keywords: Interpolation; Rational Ball function; Monotonic curve; Convexity preserving

1. Introduction

Interpolation is a fundamental process in scientific visualization and smooth curve representation to visualize the scientific data is of great significance in various areas of scientific research, including computer graphics, geometric modeling, numerical analysis, and the approximation theory.[1] Particularly when the data are obtained from some complex function or scientific phenomena, it becomes crucial to incorporate the innate features of the data in their visualization. Moreover, smoothness is one of the very important requirements for pleasant visual display.[2]

In recent years, there has been growing interest in using splines to address interpolation problems. In dealing with these problems, it is important that the solution preserves some shape properties such as positivity, monotonicity, and convexity because there are many physical situations in which entities or properties have meaning only when their values appear in positive, monotonic, or convex shape. Therefore, it is highly important to study shape-preserving problems to provide a computationally-economical and visually-pleasing solution to this sort of problems which are encountered in different scientific phenomena.[3]

Monotonic data are concomitant to many physical phenomena, engineering problems, and scientific applications.[4] The study of tensile strength of material in the engineering field is an example of an activity that produces monotonic data. The tensile strength of a material can be defined as the maximum force that this material can withstand before breaking. The applied force is called stress and is studied besides the stretch of the material, that is, its strain. The data on these two properties (stress and strain) are always monotonic data. Other examples of monotonic data are the erythrocyte sedimentation rates (ESRs) in cancer patients and blood uric acid concentrations in gout patients. The digital analog converter (DAC) is a good example of monotonic data generator. As the code input to this generator increases in value, the analog output must also increase if the device is monotonic.

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The problem of monotonicity preservation, that is, the need for the interpolant to preserve monotonicity when the data are monotonic, has been discussed by various researchers. As an example, Delbourgo and Gregory [5] developed a piecewise rational cubic interpolation function to preserve monotonicity of monotonic data. Tahat et.al [6] used a rational cubic Ball interpolant with four free parameters is used to construct a C^1 interpolant that preserve the shape of monotone data. Data dependent shape constraints derived on one of the shape parameters to guarantee the preservation of the monotonicity of the monotone data, while the other shape parameters remains free which provides extra freedom to the user to refine the shape of the curve. In another example, Abbas et al.[7] developed a C^1 piecewise rational cubic function with three shape parameters. Shape-specific conditions were derived from the data and imposed on the free parameters of this function to preserve the data shape. Tian [8] preserved the shape of monotonic data by using C^1 piecewise rational cubic spline function. However, Piah and Unsworth[9] introduced an improvement to the sufficient conditions for shape preservation that were identified by Wanga and Tana[10] in order to preserve monotonicity of the data when visualizing them. A rational cubic Ball interpolant was developed by Piah and Unsworth[11] with two free parameters that can be employed to generate the desired monotonic curves. However, there is no flexibility in this interpolant for the user to refine the curves further if needed. Accordingly, it is unsuitable for interactive curve design.

Convexity is an important shape property that is of particular relevance for different disciplines and applications. Non-linear programming in engineering and scientific applications such as design, optimal control, parameter estimation, and function approximation are few of them.[12] The problem of convexity preserving has been examined by many researchers.[1,3,5,13-20] For example, Brodlie and Butt [16] developed a C^1 convexity-preserving scheme for two-dimensional (2-D) data. They divided each interval in which the shape of data was lost into two sub-intervals by inserting an extra knot so that the shape of the data will be preserved. Then, this piecewise cubic interpolant was used to interpolate the data over each sub-interval. In a second example, Sarfraz [13] employed rational cubic Hermite interpolant with one shape parameter to develop a C^l convexity-preserving curve method. He extracted data-dependent constraints on the shape parameter so as to preserve the shape of the data. He also introduced a rational cubic interpolant with two families of shape parameters in an effort to obtain C^1 positivity-, and/or convexity-preserving interpolation curves.[19] We are looking to integrate our work with new concepts, as in the plication of [22-32] . Furthermore, a local shape-preserving interpolation method for 2-D data was proposed by Hussain and Irshad3 using piecewise rational (quadratic/linear) function.[3]

Sarfraz and Hussain1 utilized a rational cubic interpolant with two shape parameters in which the shape constraints are imposed on shape parameters to guarantee preservation of shape of the data. Abbas et al.[15] constructed a C^1 piecewise rational cubic function with three free parameters, one of which is constrained to ensure preservation of the data shape properties while the other two are left for designer's assessment in order for her/him to refine the curve as desired. Sarfraz et al.[18] presented a C^1 piecewise rational cubic function for preservation of the shapes of positive, monotonic, and convex data. This interpolant has four free parameters. Hussain et al.[14] presented a method for preservation of the shape of monotonic data based on a C^1 piecewise rational cubic function. Shape-specific conditions were derived from the data and inflicted on the free parameters to preserve the data shape.

In this paper, we use the same interpolant developed by Tahat, Piah, and Yahya[21] to solve the two problems of monotonicity and convexity preservation. The interpolation scheme proposed here improves the method suggested by Piah and Unsworth[11] by providing the user with the extra freedom to modify the shape of the curve.

The rest of this article is organized as follows. The rational cubic Ball function is first described. Then, a derivative approximation scheme is determined. After that, the problem of developing an interpolation scheme that preserves the shape of the monotonic data is discussed, followed by elaboration on outputs of a number of numerical experiments. Then, a convexity-preserving interpolation scheme is outlined. The sufficient conditions for convexity preservation have been derived from convex data and forced on the free parameters to generate a C^1 piecewise curve that preserves the shape of the convex data. Afterwards, the outputs of a number of numerical examples are presented. Then, conclusions drawn from the study results are highlighted.

2. Rational cubic ball function

Rational spline interpolation has an upper hand over polynomial spline interpolation as it can have more degrees of freedom in its description. This freedom can be utilized for achieving various purposes and objectives when dealing with diverse real-life problems that arise in different disciplines. In this section, I introduce the rational cubic function, which has four free parameters in its description that can be exploited to preserve the shape of the data.

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The rational cubic Ball function is described as follows:

Let $(x_i, f_i), i = 1, 2, \ldots, n$, be a given set of data points, where $x_1 < x_2 < \cdots < x_n$. Additionally, let $h_i = x_{i+1} - x_i$ and $\theta = \frac{x - x_i}{h_i}, 0 < \theta < 1$.

Consider the following interpolating curve $S(x)$ in the interval $[x_i, x_{i+1}]$:

$$S(x) \equiv S_i(x) = \frac{P_i(\theta)}{Q_i(\theta)} = \frac{\alpha_i f_i (1-\theta)^2 + V_i (1-\theta)^2 \theta + W_i (1-\theta)\theta^2 + \beta_i f_{i+1} \theta^2}{\alpha_i (1-\theta)^2 + a_i (1-\theta)^2 \theta + b_i (1-\theta)\theta^2 + \beta_i \theta^2},$$

$i = 1, \ldots, n-1$ (1)

with

$$\left.\begin{aligned} V_i &= a_i f_i + \alpha_i h_i d_i \\ W_i &= b_i f_{i+1} - \beta_i h_i d_{i+1} \end{aligned}\right\} \tag{2}$$

The rational function $S(x)$ has the following interpolation properties:

$$\begin{gathered} S(x_i) = f_i \text{ and } S(x_{i+1}) = f_{i+1} \\ S'(x_i) = d_i \text{ and } S'(x_{i+1}) = d_{i+1} \end{gathered} \tag{3}$$

where $S'(x)$ denotes derivative with respect to x and d_i indicates the derivative value (given or estimated) at the knot x_i. The function $S(x) \in C^1[x_0, x_n]$ has α_i, β_i, a_i, and b_i as free parameters in the interval $[x_i, x_{i+1}]$.

3. Derivative determination

In most applications, the derivative parameters $\{d_i\}$ are not given and, therefore, we have to determine them, either from the given data $(x_i, f_i), i = 1, 2, \ldots, n$, or by some other means. In this article, they were computed from the given data such that the C^1 smoothness of the interpolant rational function (Eq. 1) is maintained by using the arithmetic mean approximation. With reference to Hussain and Sarfraz[6], this method can be briefed as follows.

The arithmetic mean method for 2-D data is the three-point difference approximation:

$$d_i = \begin{cases} 0, \text{ if } \Delta_{i-1} = 0 \text{ or } \Delta_i = 0, \\ \frac{(\Delta_i + \Delta_{i-1})}{2}, \; i = 2, 3, \ldots, n-1 \end{cases} \tag{4}$$

and the end conditions are:

$$d_1 = \begin{cases} 0 \text{ if } \Delta_1 = 0 \text{ or } \operatorname{sgn}(d_1^*) \neq \operatorname{sgn}(\Delta_1) \\ d_1^* = \Delta_0 + (\Delta_0 - \Delta_1) \frac{h_0}{(h_0 + h_1)}, \text{otherwise.} \end{cases} \tag{5}$$

$$d_n = \begin{cases} 0 \text{ if } \Delta_{n-1} = 0 \text{ or } \operatorname{sgn}(d_n^*) \neq \operatorname{sgn}(\Delta_{n-1}) \\ d_n^* = \Delta_{n-1} + (\Delta_{n-1} - \Delta_{n-2}) \frac{h_{n-1}}{(h_{n-1} + h_{n-2})}, \text{otherwise.} \end{cases} \tag{6}$$

4. Monotonic data interpolation

In general, the rational function (Eq. 1) does not preserve the shape of the data. Thus, we have to define the sufficient conditions for the curve in order for this function to preserve the data shape. In this section, a monotonic dataset is considered and constraints are identified and inflected on the free parameters of the function to visualize these data. To this end,

Let $(x_i, f_i), i = 1, 2, \ldots, n$, be monotonic data defined over the interval $[a, b]$ such that

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$$f_i < f_{i+1}, \Delta_i = \frac{f_{i+1}-f_i}{h_i} > 0, \text{ and } \ d_i > 0\ , i = 1, 2, \ldots, n \tag{7}$$

The rational cubic function (Eq. 1) preserves the monotonicity if the following condition is satisfied:

$S_i'(x) > 0, \forall\ i = 1, \ldots, n.$

where

$$S_i'(x) = \frac{\sum_{i=0}^{5} V_i(1-\theta)^{5-i}\theta}{[Q_i(\theta)]^2} \tag{8}$$

with

$$A_0 = (\alpha_i{}^2 d_i)$$
$$A_1 = A_0 + (2\alpha_i b_i \Delta_i - 2\alpha_i \beta_i (d_{i+1} - \Delta_i)$$
$$A_2 = (\frac{3A_2}{2} + \frac{A_5}{2} - A_0 - A_5 + b_i(a_i\Delta_i - \alpha_i d_i) - \beta_i a_i d_{i+1} + 2\alpha_i\beta_i\Delta_i)$$
$$A_3 = (\frac{A_2}{2} + \frac{3A_5}{2} - A_0 - A_5 + +b_i(a_i\Delta_i - \alpha_i d_i) - \beta_i a_i d_{i+1} + 2\alpha_i\beta_i\Delta_i)$$
$$A_4 = A_5 + (2a_i\beta_i\Delta_i - 2\beta_i\alpha_i(d_i - \Delta_i)$$
$$A_5 = (\beta_i{}^2 d_{i+1})$$

Since the denominator of $S_i'(x)$ is positive, being a squared quantity, then the sufficient condition for monotonicity on $[x_i, x_{i+1}]$ is that $A_i > 0, i = 1, 2, \ldots, 6$. Now, $A_i > 0$ for $i = 1, 2, \ldots, 6$, if the following conditions are met:

$$a_i > \frac{\alpha_i(d_i - \Delta_i)}{\Delta_i}$$
$$b_i > max\left\{\frac{\beta_i(d_{i+1} - \Delta_i)}{\Delta_i}, \frac{\beta_i a_i d_{i+1}}{(a_i\Delta_i - \alpha_i d_i)}\right\}$$

The foregoing arguments can be summarized in following theorem.

Theorem 1: Let $(x_i, f_i), i = 1, 2, \ldots, n$, be monotonic data. The rational cubic polynomial given in Eq. 1 preserves monotonicity of the data if the free function parameters satisfy the following conditions:

$$\alpha_i > 0, \beta_i > 0,$$
$$a_i > \frac{\alpha_i(d_i - \Delta_i)}{\Delta_i}, \text{and}$$
$$b_i > max\left\{\frac{\beta_i(d_{i+1} - \Delta_i)}{\Delta_i}, \frac{\beta_i a_i d_{i+1}}{(a_i\Delta_i - \alpha_i d_i)}\right\}$$

5. Numerical Examples

In this section, I present some numerical examples to demonstrate the results that were obtained in the previous section.

Example 1: The monotonic data presented in Table 1 were taken from the paper of Hussain and Sarfraz.[6] The curve shown in Fig. 1 was produced by a rational cubic Ball function that does not preserves the monotonicity while the curve appearing in Fig. 2, which corresponds to a monotonic curve, was generated by the scheme developed in the previous section.

Table 1: Monotonic data taken from the paper of Hussain and Sarfraz[6]

i	1	2	3	4	5
x_i	0	6	10	29.5	30
f_i	0	15	15	25	30

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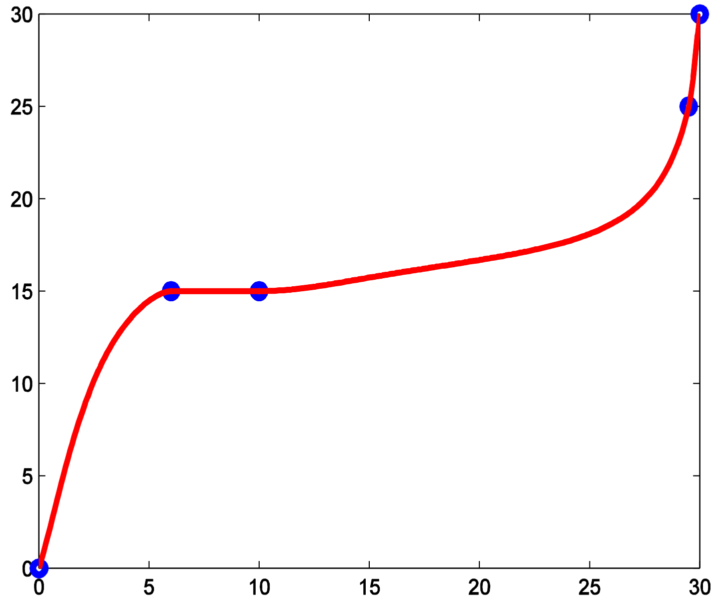

Figure 1. The Default Rational Cubic Ball Curve for the data listed in Table 1

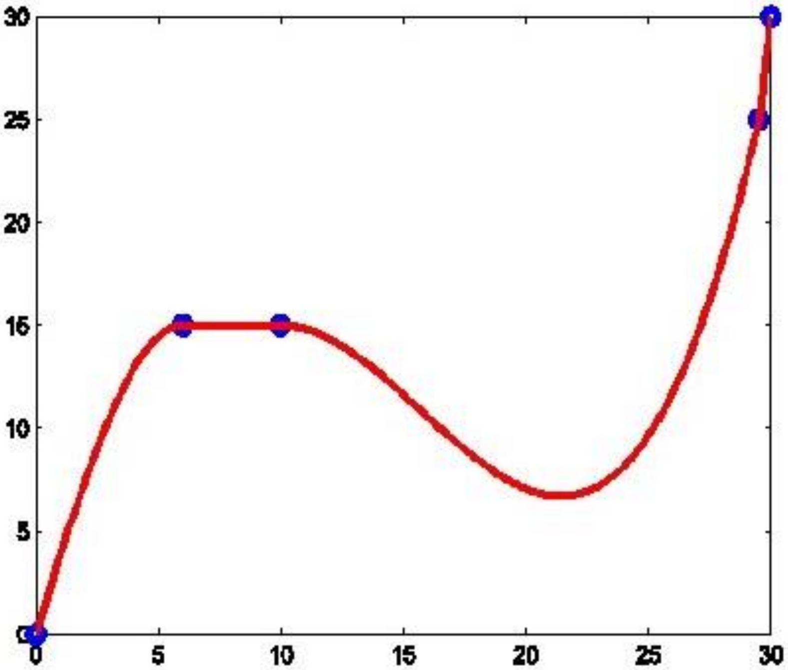

Figure 2. Monotonicity-preserving rational cubic Ball interpolant with $\alpha_i = \beta_i = 0.5$

Example 2: The monotonic data provided in Table 2 are presented in Fig. 3, which shows the curve obtained from using the rational cubic Ball function without any constraint. This curve does not monotone while the curve displayed in Fig. 4, which was produced by the proposed scheme and the same data, preserves the monotonicity.

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Table 2: Monotonic data taken from the paper of Hussain et al.[14]

i	1	2	3	4	5	6	7	8	9	10	11
x_i	0	2	3	5	6	8	9	11	12	14	15
f_i	10.01	10.02	10.03	10.04	10.05	10.06	10.50	15.00	50.00	60.00	85.00

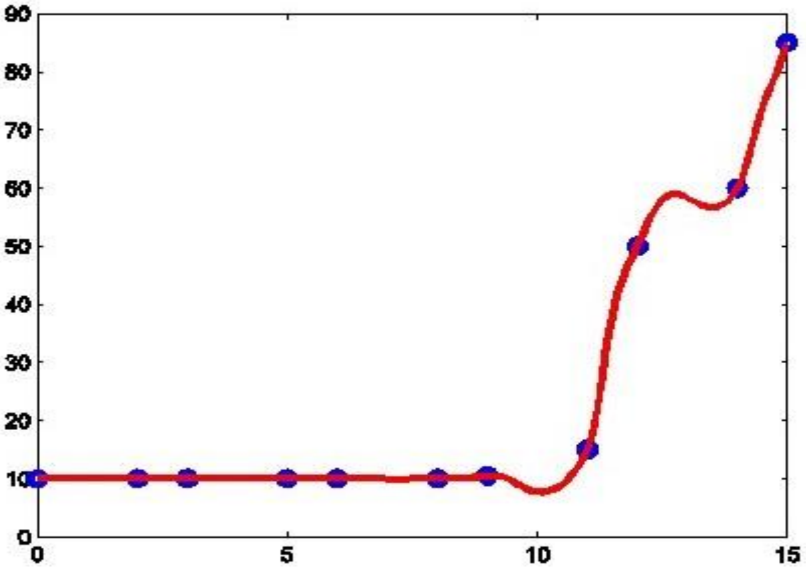

Figure 3. The Default Rational Cubic Ball Curve for the Data Given in Table 3

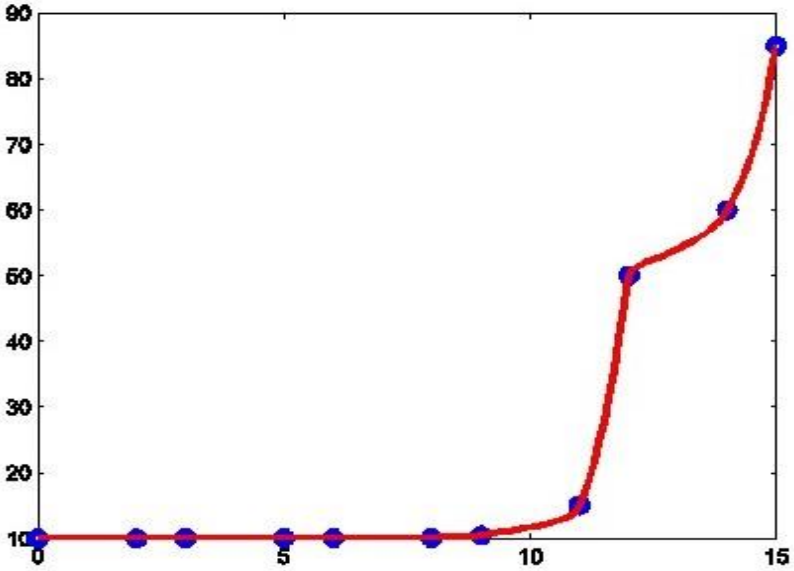

Figure 4. Monotonicity-preserving rational cubic Ball interpolant with $\boldsymbol{\alpha_i} = \boldsymbol{\beta_i} = 0.5$

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6. Convex data interpolation

The rational cubic Ball function described in the previous section does not preserve the shape of the convex data. This can be seen in Fig. 5, which shows a non-convex curve for the convex data given in Table 3.

Table 3: Convex Dataset

i	1	2	3	4	5	6	7	8	9	10
x_i	1.2	1.4	1.8	2.0	6.0	12.0	13.0	14.4	14.8	15.0
f_i	18	16	12	10	2.2	2.23	3.5	7.2	14	18

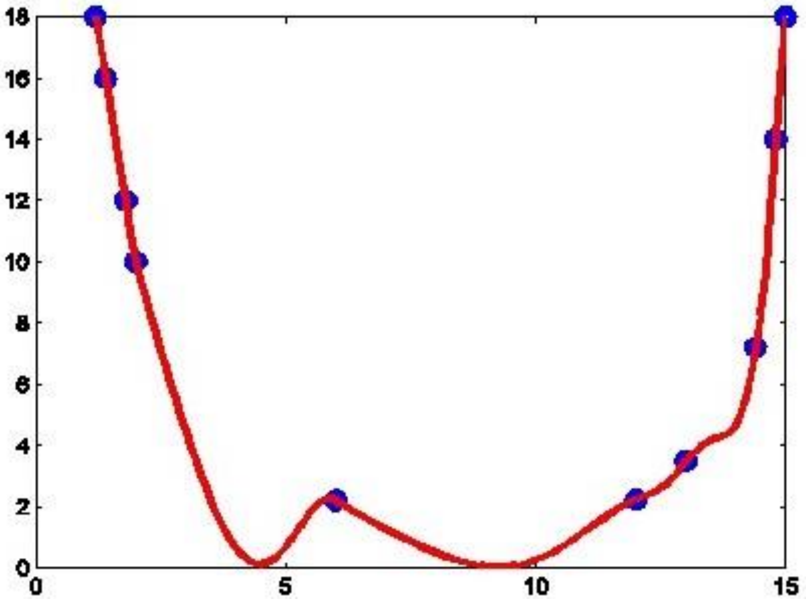

Figure 5. The Default Rational Cubic Ball Curve for the Data Provided in Table 3

Consequently, we have to obtain adequate information on the curve in order for the interpolant function to preserve the convex data shape. For this purpose,

Let $(x_i, f_i), i = 1, \dots, n,$ be a convex set of data defined over the interval $[a, b]$ such that

$$\Delta_1 < \Delta_2 < \cdots < \Delta_{n-1}. \quad (1)$$

For a convex interpolant $S(x)$, the necessary condition is

$$\mathrm{d}_1 < \Delta_1 < \cdots < \Delta_{i-1} < \mathrm{d}_i < \Delta_i < \cdots < \Delta_{n-1} < \mathrm{d}_n \quad (2)$$

Now, the rational cubic function (Eq. 1) preserves the data convexity if the following condition is fulfilled:

$S_i''(x) \geq 0, \forall\, i = 1, \dots, n.$

where

$$S_i''(x) = \frac{\sum_{i=0}^{7} V_i (1-\theta)^{7-i}\theta}{h_i [Q_i(\theta)]^3} \quad (3)$$

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The denominator of $S_i''(x)$ is positive iff the free parameters $\alpha_i, a_i, b_i,$ and β_i are positive. Then, the sufficient condition for convexity in $[x_i, x_{i+1}]$ is that $V_i > 0, i = 0, 1, \dots, 7$.

After some simplification, we get

$$V_0 = 2{\alpha_i}^2(b_i\Delta_i - (\beta_i d_{i+1} + a_i d_i) + \beta_i\Delta_i)$$

$$V_1 = V_0 + (6{\alpha_i}^2(b_i(\Delta_i - d_i) + \beta_i(\Delta_i - d_i) - \beta_i(d_{i+1} - \Delta_i))$$

$$V_2 = 2(V_1 - V_0) + (6\alpha_i(-{b_i}^2\Delta_i + b_i\beta_i(d_{i+1} - \Delta_i) + a_i b_i\Delta_i - a_i\beta_i(d_{i+1} - \Delta_i) - b_i\beta_i\Delta_i$$

$$+{\beta_i}^2(d_{i+1} - \Delta_i) + a_i\beta_i\Delta_i + 2\alpha_i\beta_i(\Delta_i - d_i))$$

$$V_3 = (V_2 + V_0 - V_1) + 2{a_i}^2\beta_i\Delta_i - 2a_i\alpha_i\beta_i(d_{i+1} - \Delta_i) + 2{\beta_i}^2 a_i(d_{i+1} - \Delta_i) - 2{\beta_i}^2\alpha_i(\Delta_i - d_i)$$

$$+2{\beta_i}^2\alpha_i(d_{i+1} - \Delta_i) + 2b_i a_i\beta_i(d_{i+1} - \Delta_i) - 2b_i\alpha_i\beta_i(\Delta_i - d_i) + 2a_i b_i\alpha_i(\Delta_i - d_i) - 2{b_i}^2\alpha_i\Delta_i$$

$$-2a_i\alpha_i\beta_i(d_{i+1} - \Delta_i) - 2{b_i}^2 a_i\Delta_i + 2{a_i}^2 b_i\Delta_i + 2{\alpha_i}^2 b_i(\Delta_i - d_i) - 2{\alpha_i}^2\beta_i(d_{i+1} - \Delta_i)$$

$$+16a_i\alpha_i\beta_i\Delta_i - 16{\alpha_i}^2\beta_i(\Delta_i - d_i) + 2b_i a_i\beta_i\Delta_i - 2b_i\alpha_i\beta_i(\Delta_i - d_i) + 2a_i\alpha_i\beta_i(\Delta_i - d_i)$$

$$-2{a_i}^2\beta_i(d_{i+1} - \Delta_i) - 16b_i\alpha_i\beta_i\Delta_i + 16{\beta_i}^2\alpha_i(d_{i+1} - \Delta_i) + 2{\alpha_i}^2\beta_i(\Delta_i - d_i) + 2a_i b_i\alpha_i\Delta_i$$

$$+2b_i\alpha_i\beta_i(d_{i+1} - \Delta_i) + 2b_i\alpha_i\beta_i(d_{i+1} - \Delta_i) - 2{b_i}^2\alpha_i(\Delta_i - d_i) + 2a_i\alpha_i\beta_i(\Delta_i - d_i))$$

$$V_4 = (V_5 + V_7 - V_6) + 2{a_i}^2\beta_i\Delta_i - 2a_i\alpha_i\beta_i(d_{i+1} - \Delta_i) + 2{\beta_i}^2 a_i(d_{i+1} - \Delta_i) - 2{\beta_i}^2\alpha_i(\Delta_i - d_i)$$

$$+2{\beta_i}^2\alpha_i(d_{i+1} - \Delta_i) + 2b_i a_i\beta_i(d_{i+1} - \Delta_i) - 2b_i\alpha_i\beta_i(\Delta_i - d_i) + 2a_i b_i\alpha_i(\Delta_i - d_i) + 2{a_i}^2 b_i\Delta_i$$

$$-2a_i\alpha_i\beta_i(d_{i+1} - \Delta_i) - 2{b_i}^2 a_i\Delta_i + 2{\alpha_i}^2 b_i(\Delta_i - d_i) - 2{\alpha_i}^2\beta_i(d_{i+1} - \Delta_i) + 16a_i\alpha_i\beta_i\Delta_i$$

$$-16{\alpha_i}^2\beta_i(\Delta_i - d_i) + 2b_i a_i\beta_i\Delta_i - 2b_i\alpha_i\beta_i(\Delta_i - d_i) + 2a_i\alpha_i\beta_i(\Delta_i - d_i) - 2{a_i}^2\beta_i(d_{i+1} - \Delta_i)$$

$$-16b_i\alpha_i\beta_i\Delta_i + 16{\beta_i}^2\alpha_i(d_{i+1} - \Delta_i) + 2{\alpha_i}^2\beta_i(\Delta_i - d_i) + 2a_i b_i\alpha_i\Delta_i + 2b_i\alpha_i\beta_i(d_{i+1} - \Delta_i)$$

$$-2{b_i}^2\alpha_i\Delta_i + 2b_i\alpha_i\beta_i(d_{i+1} - \Delta_i) - 2{b_i}^2\alpha_i(\Delta_i - d_i) + 2a_i\alpha_i\beta_i(\Delta_i - d_i))$$

$$V_5 = 2(V_6 - V_7) + (6\beta_i({a_i}^2\Delta_i + a_i\alpha_i(\Delta_i - d_i) - b_i a_i\Delta_i - b_i\alpha_i(\Delta_i - d_i) - b_i\alpha_i\Delta_i$$

$$+{\alpha_i}^2(\Delta_i - d_i) + a_i\alpha_i\Delta_i + 2\beta_i\alpha_i(d_{i+1} - \Delta_i))$$

$$V_6 = V_7 + 6{\beta_i}^2(\alpha_i(d_{i+1} - \Delta_i) + a_i(d_{i+1} - \Delta_i) - \alpha_i(\Delta_i - d_i)$$

$$V_7 = 2{\beta_i}^2(\mathrm{b}_i d_{i+1} - \Delta_i(a_i + \alpha_i) + \alpha_i d_i)$$

In V_5, $(\Delta_i - \mathrm{d}_i) > 0$ and $(\mathrm{d}_{i+1} - \Delta_i) > 0$. Accordingly, the following conditions will be sufficient for the rational cubic function (Eq. 1) to preserve the data convexity:

$$\left.\begin{aligned} a_i > max\left\{0, \frac{\alpha_i(d_i-\Delta_i)}{(d_{i+1}-\Delta_i)}, \frac{\alpha_i(d_{i+1}-\Delta_i)}{\Delta_i}\right\} \\ b_i > max\left\{0, \frac{\beta_i(d_{i+1}-\Delta_i)}{(d_i-\Delta_i)}, \frac{\Delta_i(a_i+\alpha_i)}{d_{i+1}}, \frac{(\beta_i d_{i+1}+a_i d_i)}{\Delta_i}\right\} \end{aligned}\right\} \qquad (4)$$

As Fig. 6 shows, the curve obtained using this rational cubic Ball interpolant function under the aforementioned conditions preserved convexity of the data shape.

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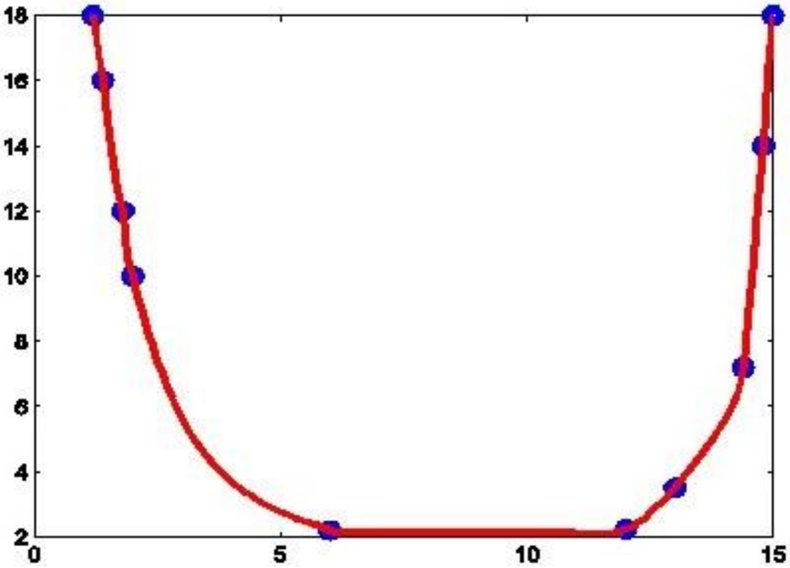

Figure 6. Convexity-preserving rational cubic Ball interpolant with $\alpha_i = 2.5$ and $\beta_i = 0.50$

The two conditions summarized by Eq. 12 can be coined in theorem as illustrated next.

Theorem 2: Let $(x_i, f_i), i = 1, 2, \dots, n,$ be convex data. The rational cubic function given in Eq. 1 preserves convexity of these data if the following conditions are satisfied:

$$\alpha_i > 0, \beta_i > 0$$

$$\left.\begin{aligned} a_i = l_i + \left\{0, \frac{\alpha_i(d_i - \Delta_i)}{(d_{i+1} - \Delta_i)}, \frac{\alpha_i(d_{i+1} - \Delta_i)}{\Delta_i}\right\}, l_i > 0 \\ b_i = m_i + \left\{0, \frac{\beta_i(d_{i+1} - \Delta_i)}{(d_i - \Delta_i)}, \frac{\Delta_i(a_i + \alpha_i)}{d_{i+1}}, \frac{(\beta_i d_{i+1} + a_i d_i)}{\Delta_i}\right\}, m_i > 0 \end{aligned}\right\} \tag{5}$$

7. Numerical Examples

This section presents two numerical examples that demonstrate the results that were obtained based on the arguments and equations provided in the previous section.

Example 3: Consider the convex data presented in Table 1. The curve appearing in Fig. 2 was produced by the proposed rational cubic Ball function. It is, actually, a convex curve.

Example 4: The convex data shown in Table 4 were displayed graphically first in Fig. 7, which depicts the curve generated by the rational cubic Ball function without any constraint. It is noticed in this figure that this curve does not preserve convexity of the data while the curve appearing in Fig. 8, which was produced by applying the proposed scheme on the same set of data (Table 4), preserves it.

Table 4: Convex Dataset II

I	1	2	3	4	5	6	7
x_i	0	2	4	10	28	30	32
f_i	20.8	8.8	4.2	0.5	3.9	6.2	9.6

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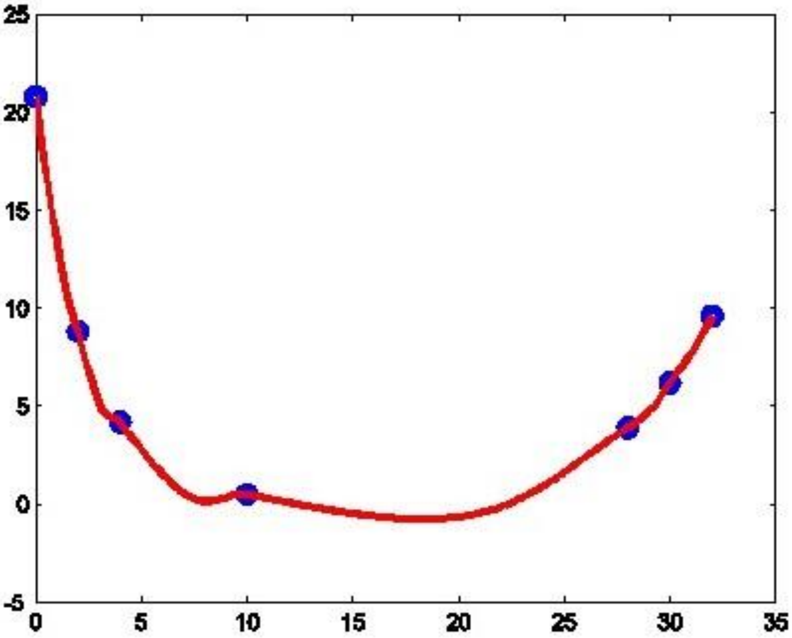

Figure 7. The Default Rational Cubic Ball Curve for the Data Shown in Table 4

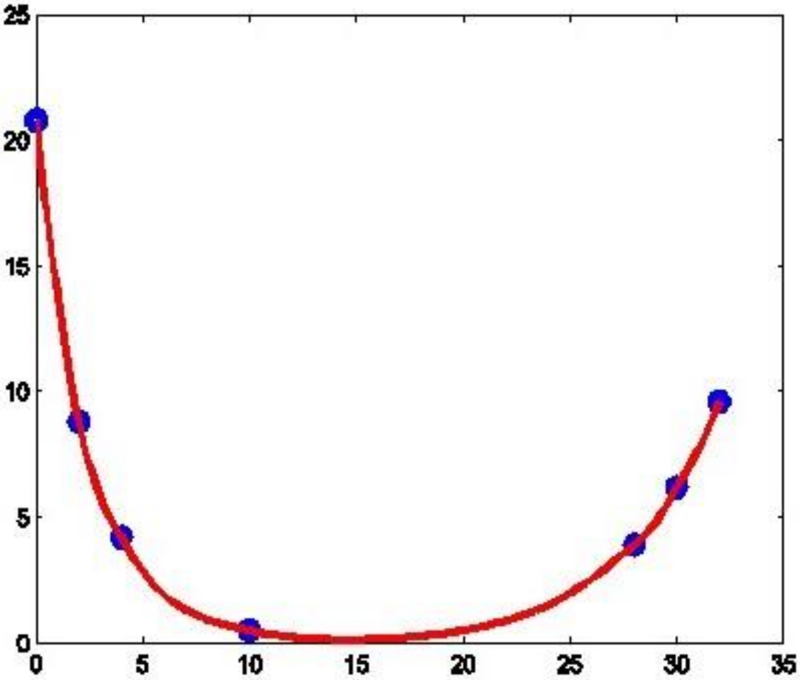

Figure 8. Convexity-preserving rational cubic Ball interpolant with $\boldsymbol{\alpha_i} = 1.5$ and $\boldsymbol{\beta_i} = 1.5$

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8. Conclusion

In this research paper, a rational cubic Ball function with four free parameters was used to construct a C^1 interpolant that preserves the shapes of monotonic and convex data. Shape-specific constraints were drawn from the data and imposed on two of the four free parameters to guarantee preservation of the monotonicity and convexity of the data. The remainder two free parameters can assume any positive values assigned to them by the user/designer. Leaving margins for the users to manipulate values of these two parameters enables them to refine the curve as necessary. In other respects, the three–point difference formula (arithmetic mean) was employed in this study to compute the values of the derivatives. The herein proposed scheme was tested on a number of data sets. The testing results demonstrate that this scheme produces good-looking and visually-pleasant curves that preserve monotonicity and convexity of the original data. This scheme can be generalized to the surface case. For future studies, these mathematical tools can be used effectively with other tools and techniques that can be observed through [33-43]

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