Fermatean Neutrosophic Soft Set

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Abstract

This paper aims to introduce a new concept which is Fermatean Neutrosophic Soft Set (FNSS), which is a combination of the Neutrosophic soft sets and Fermatean Fuzzy Sets. Some operations and properties of the new model, including complement, restricted union, and extended intersection are discussed. Further, an application of FNSS is modeled for multiple attribute decision-making and solved with the help of our newly launched algorithm, that is, the selection of the most attractive laptop based on a computer simulation report. Finally, a comparative analysis between the initiated FNSS model and some existing approaches is provided to show its reliability.

Keywords: Fuzzy Set; Soft Set; Fuzzy Soft Set; Fermatean Fuzzy Set; Fermatean Fuzzy Soft Set; Neutrosophic Set; Neutrosophic Soft Set; Fermatean Neutrosophic Set

1. Introduction

Fuzzy sets were developed by Zadeh [1] to solve problems that contain uncertain information. Some cases cannot deal with a fuzzy set, so Turksen [2] introduced an interval-valued fuzzy set. Atanassove [3] extended the fuzzy set to the Intuitionistic fuzzy set. Which is more general than a fuzzy set. Neutrosophy introduced by Smarandache [4] is a new tool for dealing with problems containing imprecise, indeterminacy, and inconsistent data. Neutrosophic sets which were introduced by Smarandach in 2005 [5] are a generalization of the Intuitionistic fuzzy set. Soft Set is defined by Molodtsov [6] as another commonly used method in handling uncertainties in the data. The concept of a fuzzy soft set was introduced by Maji [7]. Soft Set extended and introduced some of its operations and properties by Maji [8]. Fuzzy soft set extended to Generalized fuzzy soft sets by Majumdar and Samanta in 2010 [9].Sezgin et al. [10] were proved De Morgan s Law on Soft Set. Neutrosophic Soft Set NSS with basic operation and properties proposed by Maji [11]. The new concept of Generalized neutrosophic soft set GNSS which was introduced by Sahin [12], was the extension of the concept of NSS defined by Maji [8].NSS was developed by Broumi [13] as a Generalized Neutrosophic Soft Set with basic definitions and operations. He used this concept for solving decision-making problems.

This paper is structured as follows: Section 2, covers certain definitions and features of FNSS.In Section 3, we introduce the concepts of FNSS, FN-Soft subset, FN-Soft null set, Absolute FNSS, and FNSS operations. In Section 4 we discuss union and intersection on FNSSs. In Section 5, we present a MADM problem discussing the selection of the best model for the most attractive laptop based on the computer simulation report and solve it under FNSSs using a supporting algorithm. Finally, in Section 6, we provide the concluding remarks.

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2. Preliminary

In this section, we present some definitions required in this paper.

Definition 1: Fuzzy Set

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(X)$ is interpreted as the degree of membership of element X in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(x, \mu_A(x)) : x \in X\}$.

Definition 2: Intuitionistic Fuzzy Set

The intuitionistic fuzzy sets defined on a non-empty set X as objects having the form $A = \{\langle x, \alpha_A(x), \beta_A(x) : x \in X\}$,where the functions $\alpha_A(x) : X \to [0,1]$ and $\beta_A(x) : X \to [0,1]$, denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set *A* respectively, and $0 \le \alpha_A(x) + \beta_A(x) \le 1$, for all $x \in X$ .Clearly , when $\beta_A(x) = 1 - \alpha_A(x)$, for every $x \in X$,the set *A* becomes a fuzzy set.

Definition 3: Fermatean Fuzzy Set

Fermatean fuzzy set F in the universe set U is an object with the type $F = \{(u, \alpha F(u), \beta F(u)) : u \in U\}$ where $\alpha F : U \to [0,1]$ and $\beta F : U \to [0,1]$, with the condition $0 \le (\alpha F(u))^3 + (\beta F(u))^3 \le 1$ for all $u \in U$.

Definition 4: Neutrosophic Set

A Neutrosophic set A on the universe of discourse X is defined as $A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle ; x \in X\}$ where $T; I; X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 5: Fermatean Neutrosophic Set

Let X be a non-empty set (universe). A Fermatean neutrosophic set [FNS] $\{(x, T(x), I(x), F(x) : x \in X\}$, where $T(x), I(x), F(x) \in [0,1], 0 \le (T(x))^3 + (I(x))^3 + (F(x))^3 \le 2$. Then $0 \le (T(x))^3 + (I(x))^3 + (F(x))^3 \le 2$, for all x in X. $T(x)$ is the degree of membership, $I(x)$ is the degree of inderminancy and $F(x)$ is the degree of non-membership. Here $T(x)$ and $F(x)$ are dependent components and $I(x)$ is an independent component.

Definition 6: Soft Set

Let U be the universal set and E be the set of attributes with respect to U. Let $P(U)$ be the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U and its mapping is given as $F : A \to P(U)$. It is also defined as, $(F, A) = \{F(e) \in P(U) : e \in E, F(e) = \varnothing \ if \ e \notin A\}$.

Definition 7: Fuzzy Soft Set

Let U be the initial universal set and let E be the set of parameters. Let I^U denote the power set of all fuzzy subsets of U .Let $A \subseteq E$. A pair (F, E) is called a fuzzy soft set over U where F is a mapping given by: $F : A \to I^U$

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Definition 8: Generalized Fuzzy Soft Set

Let $U=\{u_1,u_2,...,u_n\}$ be the universal set of elements and $E=\{e_1,e_2,...,e_m\}$ be the universal set of parameters. The pair (U,E) is called a soft universe. Let $F:E\to I^U$ and μ be a fuzzy subset of E, *i.e*, $\mu:E\to I=[0,1]$, where I^U is the collection of all Fuzzy subset of U. Let F_μ be the mapping $F:E\to I^U\times I$ be a function defined as follows: $F_\mu(e)=(F(e),\mu(e))$,where $F_e\in I^U$.Then F_μ is called a Generalized fuzzy soft set (GFSS in short) over the soft universe (U,E). Here for each parameter $e_i, F_\mu(e_i)=(F(e_i),\mu(e_i))$,indicates not only the degree of belongingness of the elements of U in $F(e_i)$,but also the degree of possibility of such belongingness which is represented by μ_i. So, we can write this as follows:

$$F_\mu(e_i)=\left\{\left(\left(\frac{u_1}{F(e_i)(u_1)},\frac{u_2}{F(e_i)(u_2)},.....,\frac{u_n}{F(e_i)(u_n)}\right),\mu(e_i)\right)\right\},\forall u\in U,e\in E$$

Definition 9: Fermatean Fuzzy Soft Set

Let E be any set of deferent parameters, and let U be the universe, $A\subseteq E$ a Fermatean fuzzy soft set (FFSS) on U is defined as the pair (F,W) where F is mapping given by $F:W\to FFS(U)$, where $FFS(U)$ is the set of all Fermatean fuzzy sets over U. Here for any parameter $e\in A, F(e)$ is the Fermatean fuzzy set given as $F(e)=\{(u,\alpha F(e)(u),\ \beta F(e)(u)):u\in U\}$ where $\alpha F(e)(u)$ and $\beta F(e)(u)$ are corresponding degrees of membership and non-membership $0\le(\alpha F(e)(u))^3+(\beta F(e)(u))^3\le 1$. Hence $(F,A)=\{(e,\{(u,\alpha F(e)(u),\beta F(e)(u)\}):e\in A,u\in U\}$.

Definition 10: Neutrosophic Soft Set

Let U be an initial universe set and E be a set of parameters. Consider $A\subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F:A\to P(U)$.

Definition 11: Intuitionistic Fuzzy Soft Set

Consider U and E as a universe set and a set of parameters, respectively. Let $P(U)$ denotes the set of all Intuitionistic Fuzzy sets of U.Let $A\subseteq E$.A pair (F,A) is an intuitionistic fuzzy soft set over U. where F is a mapping given by $F:A\to P(U)$.

3. Fundamental of Fermatean Neutrosophic Soft Set

Definition 12: Fermatean Neutrosophic Soft Set

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Let $U=\{c_1,c_2,...,c_n\}$ be the universe set of elements and $E=\{e_1,e_2,...,e_m\}$ be the universal set of parameters. Then the pair (U,E) will called soft universe. Let $F:E\to FNS(U)$ where $FNS(U)$ is the collection of all Fermatean Neutrosophic subset of U. Let $F:E\to FNS(U)$ be a function defined as follows:

$$F=\left\{\left(e,\frac{c}{F(e)(c)}\right),\forall c\in U,e\in E\right\}$$, Where $F(e)\in FNS(U)$, then F is called FNSS.

Example 1: Let $U=\{c_1,c_2,c_3\}$ to be set of three houses consider some consideration. Assume $E=\{e_1,e_2,e_3\}$ be a set of adjectives where e_1 = cheap, e_2 = white, e_3 = location. We defined a function $F:E\to FNS(U)$ as follows:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\}.$$

Then F is $(FNSS)$ over (U,E).

In matrix form this can be expressed as:

$$F=\begin{pmatrix}(\langle 0.2,0.6,0.1\rangle) & (\langle 0.4,0.2,0.6\rangle) & (\langle 0.3,0.1,0.7\rangle)\\ (\langle 0.3,0.4,0.5\rangle) & (\langle 0.2,0.3,0.4\rangle) & (\langle 0.7,0.8,0.2\rangle)\\ (\langle 0.6,0.2,0.1\rangle) & (\langle 0.3,0.6,0.9\rangle) & (\langle 0.1,0.2,0.3\rangle)\end{pmatrix}.$$

Where i^{th} row vector represents $F(e_i)$, the i^{th} column vector represent c_i. It's clearly that $(0.2)^3+(0.1)^3\le 1$ and $(0.2)^3+(0.6)^3+(0.1)^3\le 2$ which satisfy the FN Condition.

Definition 13: Let F and G be two FNSSs on (U,E). Then F is said to be a Fermatean Neutrosophic soft subset of G, $F(e)$ is a Fermatean Neutrosophic subset of $G(e),\forall e\in E$, here we say that $F\subseteq G$.

Example 2: Let $U=\{c_1,c_2,c_3\}$ be a set of three cars, and let $E=\{e_1,e_2,e_3\}$ be a set of parameters where e_1 = cheap, e_2 = expensive and e_3 = red. Let F is FNSS over (U,E) defined as follows:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.2\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.6,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.6,0.1,0.9\rangle}\right)\right\};$$

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$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.5,0.3,0.8\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.3,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.6\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.3\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.5,0.8\rangle}\right),\left(\frac{c_3}{\langle 0.4,0.3,0.7\rangle}\right)\right\}.$$

Let $G:E\to FNS(U)$ be another FNSS over (U,E) defined as follows:

$$G(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.6,0.7,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.2,0.8\rangle}\right)\right\};$$

$$G(e_2)=\left\{\left(\frac{c_1}{\langle 0.6,0.4,0.7\rangle}\right),\left(\frac{c_2}{\langle 0.9,0.4,0.5\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.5,0.3\rangle}\right)\right\};$$

$$G(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.7,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.6,0.6,0.7\rangle}\right),\left(\frac{c_3}{\langle 0.5,0.4,0.5\rangle}\right)\right\}.$$

It's clear that F is a FNSS subset of G.

Definition 14: A FNSS is said to be a null FNSS, denoted by $F_{\varnothing}$ such that $F_{\varnothing}=\left\{\left(e,\{u,0,0,1\}\right):e\in E,c\in U\right\}$.

Example 3: Let $U=\{c_1,c_2,c_3\}$ be a set of three blouses and let $E=\{e_1,e_2,e_3\}$ be a set of qualities where e_1 = luminous, e_2 = colorful and e_3 = cheap. We defined a function $F:E\to FNS(U)$ which is a FNSS over (U,E) defined as follows:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0,0,1\rangle}\right),\left(\frac{c_2}{\langle 0,0,1\rangle}\right),\left(\frac{c_3}{\langle 0,0,1\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0,0,1\rangle}\right),\left(\frac{c_2}{\langle 0,0,1\rangle}\right),\left(\frac{c_3}{\langle 0,0,1\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0,0,1\rangle}\right),\left(\frac{c_2}{\langle 0,0,1\rangle}\right),\left(\frac{c_3}{\langle 0,0,1\rangle}\right)\right\};$$

$$F=\begin{pmatrix}(\langle 0,0,1\rangle) & (\langle 0,0,1\rangle) & (\langle 0,0,1\rangle)\\ (\langle 0,0,1\rangle) & (\langle 0,0,1\rangle) & (\langle 0,0,1\rangle)\\ (\langle 0,0,1\rangle) & (\langle 0,0,1\rangle) & (\langle 0,0,1\rangle)\end{pmatrix}.$$

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Then F is a null FNSS.

Definition 15: A FNSS is said to be absolute FNSS, denoted by A_1, such that $A_1 = \{(e, \{c, 1, 0, 0\}) : e \in E, c \in U\}$

Example 4: Let $U = \{c_1, c_2, c_3\}$ be a set of three buses. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters where e_1 = Yellow, e_2 = long and e_3 = Mercedes. Let F be a FNSS over (U, E).

$$F(e_1) = \left\{\left(\frac{c_1}{\langle 1,0,0\rangle}\right), \left(\frac{c_2}{\langle 1,0,0\rangle}\right), \left(\frac{c_3}{\langle 1,0,0\rangle}\right)\right\};$$

$$F(e_2) = \left\{\left(\frac{c_1}{\langle 1,0,0\rangle}\right), \left(\frac{c_2}{\langle 1,0,0\rangle}\right), \left(\frac{c_3}{\langle 1,0,0\rangle}\right)\right\};$$

$$F(e_3) = \left\{\left(\frac{c_1}{\langle 1,0,0\rangle}\right), \left(\frac{c_2}{\langle 1,0,0\rangle}\right), \left(\frac{c_3}{\langle 1,0,0\rangle}\right)\right\};$$

$$F = \begin{pmatrix} (\langle 1,0,0\rangle) & (\langle 1,0,0\rangle) & (\langle 1,0,0\rangle) \\ (\langle 1,0,0\rangle) & (\langle 1,0,0\rangle) & (\langle 1,0,0\rangle) \\ (\langle 1,0,0\rangle) & (\langle 1,0,0\rangle) & (\langle 1,0,0\rangle) \end{pmatrix}.$$

Then F is called absolute FNSS.

Definition 16: Let F be a FNSS over (U, E). Then the complement of F, denoted by F^c is defined by $F^c(e) = D(e), \forall e \in E$ where $D(e)$ denoted the Fermatean Neutrosophic complement, where

$$\mu^c(e) = 1 - \mu, T^c(e) = F(e), I^c(e) = I(e), F^c(e) = T(e)$$

Example 5: Consider the matrix notation in Example 1 :

$$F = \begin{pmatrix} (\langle 0.2,0.6,0.1\rangle) & (\langle 0.4,0.2,0.6\rangle) & (\langle 0.3,0.1,0.7\rangle) \\ (\langle 0.3,0.4,0.5\rangle) & (\langle 0.2,0.3,0.4\rangle) & (\langle 0.7,0.8,0.2\rangle) \\ (\langle 0.6,0.2,0.1\rangle) & (\langle 0.3,0.6,0.9\rangle) & (\langle 0.1,0.2,0.3\rangle) \end{pmatrix}.$$

By using the Fermatean Neutrosophic complement, we have $F^c = G$, where G is:

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$$G=\begin{pmatrix}(\langle 0.1,0.6,0.2\rangle) & (\langle 0.6,0.2,0.4\rangle) & (\langle 0.7,0.1,0.3\rangle)\\(\langle 0.5,0.4,0.3\rangle) & (\langle 0.4,0.3,0.2\rangle) & (\langle 0.2,0.8,0.7\rangle)\\(\langle 0.1,0.2,0.6\rangle) & (\langle 0.9,0.6,0.3\rangle) & (\langle 0.3,0.2,0.1\rangle)\end{pmatrix}.$$

4. Union and Intersection of FNSSs

Definition 17: The Union of two FNSSs F and G, denoted by $F\tilde{\bigcup}G$, is a FNSS $H:E\rightarrow FNS(U)$ defined by $H(e)=\left(F(e)\tilde{\bigcup}G(e)\right)$ $\tilde{\bigcup}$ is a Fermatean Neutrosophic union, where :

$$F\bigcup G=\left\{\left(x,\max\left(T_F\left(x\right),T_G\left(x\right)\right),\frac{I_F\left(x\right)+I_G\left(x\right)}{2},\min\left(F_F\left(x\right),F_G\left(x\right)\right)\right\}$$

Example 6: Let $U=\{c_1,c_2,c_3\}$ aid $E=\{e_1,e_2,e_3\}$. Let F be a FNSS know as follows:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\}.$$

Let $G:E\rightarrow FNS\left(U\right)$ be another FNSS over $\left(U,E\right)$ defined as follows:

$$G(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle}\right)\right\};$$

$$G(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle}\right)\right\}.$$

To find the union, we use the Fermatean Neutrosophic union method:

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$$H(e_1)=\left\{\left(\frac{c_1}{\max\langle 0.2,0.3\rangle,\langle\frac{0.6+0.2}{2}\rangle,\min\langle 0.1,0.1\rangle}\right),\right.$$

$$\left(\frac{c_2}{\max\langle 0.4,0.2\rangle,\langle\frac{0.2+0.4}{2}\rangle,\min\langle 0.6,0.3\rangle}\right),$$

$$\left.\left(\frac{c_3}{\max\langle 0.3,0.1\rangle,\langle\frac{0.1+0.7}{2}\rangle,\min\langle 0.7,0.2\rangle}\right)\right\}$$

$$=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.3,0.3\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.4,0.2\rangle}\right)\right\}.$$

Similarly, we get.

$$H(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.35,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.2,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.15,0.2\rangle}\right)\right\};$$

$$H(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.25,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.66,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.55,0.3\rangle}\right)\right\}.$$

In matrix notation, we write.

$$H(e)=\begin{pmatrix}(\langle 0.3,0.4,0.1\rangle) & (\langle 0.4,0.3,0.3\rangle) & (\langle 0.3,0.4,0.2\rangle)\\ (\langle 0.3,0.35,0.4\rangle) & (\langle 0.7,0.2,0.2\rangle) & (\langle 0.7,0.15,0.2\rangle)\\ (\langle 0.6,0.25,0.1\rangle) & (\langle 0.3,0.65,0.4\rangle) & (\langle 0.1,0.55,0.3\rangle)\end{pmatrix}.$$

Definition 17: The Intersection of two FNSSs F and G, denoted by $F\tilde{\cap}G$, is a FNSSs $H:E\rightarrow FNS(U)$ defined by $H(e)=\left(F(e)\tilde{\cap}G(e)\right)$,$\tilde{\cap}$ is a Fermatean Neutrosophic intersection, where

$$F\cap G=\left\{\left(x,\min\left(T_F(x),T_G(x)\right),\frac{I_F(x)+I_G(x)}{2},\max\left(F_F(x),F_G(x)\right)\right\}$$

Example (7): consider (6)

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$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\}.$$

Let $G:E\rightarrow FNS(U)$ be another FNSS over (U,E) defined as follows:

$$G(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle}\right)\right\};$$

$$G(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle}\right)\right\}.$$

To find the intersection, we use the FN intersection method:

$$H(e_1)=\left\{\left(\frac{c_1}{\min\langle 0.2,0.3\rangle,\langle\frac{0.6+0.2}{2}\rangle,\max\langle 0.1,0.1\rangle}\right),\right.$$

$$\left(\frac{c_2}{\min\langle 0.4,0.2\rangle,\langle\frac{0.2+0.4}{2}\rangle,\max\langle 0.6,0.3\rangle}\right),$$

$$\left.\left(\frac{c_3}{\min\langle 0.3,0.1\rangle,\langle\frac{0.1+0.7}{2}\rangle,\max\langle 0.7,0.2\rangle}\right)\right\}$$

$$=\left\{\left(\frac{c_1}{\langle 0.2,0.4,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.4,0.7\rangle}\right)\right\}.$$

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The Similarly, we got.

$$H(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.8,0.3\rangle}\right)\right\};$$

$$H(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.1,0.7,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle}\right)\right\};$$

Then:

$$H(e)=\begin{pmatrix}(\langle 0.2,0.4,0.1\rangle) & (\langle 0.2,0.3,0.6\rangle) & (\langle 0.1,0.4,0.7\rangle)\\ (\langle 0.2,0.35,0.5\rangle) & (\langle 0.2,0.2,0.4\rangle) & (\langle 0.1,0.15,0.3\rangle)\\ (\langle 0.4,0.25,0.5\rangle) & (\langle 0.1,0.65,0.9\rangle) & (\langle 0.1,0.55,0.3\rangle)\end{pmatrix}.$$

Proposition 1: F , G and H any three FNSSs over (U,E) then the following holds:

I. $F\tilde{\bigcup}G=G\tilde{\bigcup}F$

II. $F\tilde{\bigcap}G=G\tilde{\bigcap}F$

III. $F\tilde{\bigcup}\left(G\tilde{\bigcup}H\right)=\left(F\tilde{\bigcup}G\right)\tilde{\bigcup}H$

IV. $F\tilde{\bigcap}\left(G\tilde{\bigcap}H\right)=\left(F\tilde{\bigcap}G\right)\tilde{\bigcap}H$

Proof (I)

$$F\tilde{\bigcup}G=\left(\left(F\tilde{\bigcup}G\right)\right)$$

$=\left(\left(G\tilde{\bigcup}F\right)\right)$, since the FNS is commutative under union operation

$=G\tilde{\bigcup}F$.

Proof (II)

$$F\tilde{\bigcap}G=\left(\left(F\tilde{\bigcap}G\right)\right)$$

$=\left(\left(G\tilde{\bigcap}F\right)\right)$, since the FNS is commutative under intersection operation

$=G\tilde{\bigcap}F$.

Proof (III)

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$$F\tilde{\bigcup}\left(G\tilde{\bigcup}H\right)=\left(\left(F\tilde{\bigcup}\left(G\tilde{\bigcup}H\right)\right)\right)$$

$$=\left(\left(F\tilde{\bigcup}G\right)\tilde{\bigcup}H\right)\right)$$, since the FNS verify Demorgan laws and associative

$$=\left(F\tilde{\bigcup}G\right)\tilde{\bigcup}H.$$

Proof (IV)

$$F\tilde{\bigcap}\left(G\tilde{\bigcap}H\right)=\left(\left(F\tilde{\bigcap}\left(G\tilde{\bigcap}H\right)\right)\right)$$

$$=\left(\left(F\tilde{\bigcap}G\right)\tilde{\bigcap}H\right)\right)$$, since the FNS verify Demorgan laws and associative

$$=\left(F\tilde{\bigcap}G\right)\tilde{\bigcap}H.$$

Proposition 2: Let F and G are two FNSSs over (U,E) then following hold:

I. $\left(F\tilde{\bigcap}G\right)^{c}=\left(F^{c}\tilde{\bigcup}G^{c}\right)$

II. $\left(F\tilde{\bigcup}G\right)^{c}=\left(F^{c}\tilde{\bigcap}G^{c}\right)$

Proof (I)

$$\left(F\tilde{\bigcap}G\right)^{c}=\left(F^{c}\tilde{\bigcup}G^{c}\right)$$
$$=\left(F^{c}\tilde{\bigcup}G^{c}\right)$$
, since $\tilde{\bigcup}$ is FNS complement.

Proof (II)

$$\left(F\tilde{\bigcup}G\right)^{c}=\left(F^{c}\tilde{\bigcap}G^{c}\right)$$
$$=\left(F^{c}\tilde{\bigcap}G^{c}\right)$$
, since $\tilde{\bigcap}$ is FNS complement.

5. AND and OR Operations on FNSSs with Applications in Decision Making

In this section, we introduce the definitions of AND and OR operations on Fermatean Neutrosophic Soft Set, Applications of Fermatean Neutrosophic Soft Set in decision-making problem are given.

Definition 18: (F,A) and (G,B) are two FNSSs then "(F,A) AND (G,B)", defined by $(F,A)\wedge(G,B)$ is denoted by $(F,A)\wedge(G,B)=(H,A\times B)$, Where $H(\alpha,\beta)=\left(H(\alpha,\beta)(c)\right)$, for all $(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta)=\left(F(\alpha)\cap G(\beta)\right)$, $\forall(\alpha,\beta)\in A\times B$, where $\tilde{\bigcap}$ is Fermatean neutrosophic soft intersection.

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Example 8: Assume the universe consists of three cars that is, $U=\{c_1,c_2,c_3\}$, and there are three parameters $E=\{e_1,e_2,e_3\}$ which describe their performances according to certain specific tasks. Suppose Mr.X wants to buy one such car depending on either of the parameters only. Let there be two observations F and G by two experts defined as follows:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle}\right)\right\};$$

$$G(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle}\right)\right\}.$$

$$H(e_1,e_1)=\left\{\left(\frac{c_1}{\min\langle 0.2,0.3\rangle,\langle\frac{0.6+0.2}{2}\rangle,\max\langle 0.1,0.1\rangle}\right),\right.$$

$$\left(\frac{c_2}{\min\langle 0.4,0.2\rangle,\langle\frac{0.2+0.4}{2}\rangle,\max\langle 0.6,0.3\rangle}\right),$$

$$\left.\left(\frac{c_3}{\min\langle 0.3,0.1\rangle,\langle\frac{0.1+0.7}{2}\rangle,\max\langle 0.7,0.2\rangle}\right)\right\}.$$

$$H(e_1,e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.4,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.4,0.7\rangle}\right)\right\}.$$

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The Similarly, we get.

$$H(e_1,e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.45,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.15,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.15,0.7\rangle}\right)\right\};$$

$$H(e_1,e_3)=\left\{\left(\frac{c_1}{(0.2,0.45,0.5)}\right),\left(\frac{c_2}{(0.1,0.45,0.6)}\right),\left(\frac{c_3}{(0.1,0.5,0.7)}\right)\right\};$$

$$H(e_2,e_1)=\left\{\left(\frac{c_1}{(0.3,0.3,0.5)}\right),\left(\frac{c_2}{(0.2,0.35,0.4)}\right),\left(\frac{c_3}{(0.1,0.75,0.2)}\right)\right\};$$

$$H(e_2,e_2)=\left\{\left(\frac{c_1}{(0.2,0.35,0.5)}\right),\left(\frac{c_2}{(0.2,0.2,0.4)}\right),\left(\frac{c_3}{(0.1,0.5,0.3)}\right)\right\};$$

$$H(e_2,e_3)=\left\{\left(\frac{c_1}{(0.3,0.35,0.5)}\right),\left(\frac{c_2}{(0.1,0.5,0.4)}\right),\left(\frac{c_3}{(0.1,0.85,0.3)}\right)\right\};$$

$$H(e_3,e_1)=\left\{\left(\frac{c_1}{(0.3,0.2,0.1)}\right),\left(\frac{c_2}{(0.2,0.5,0.9)}\right),\left(\frac{c_3}{(0.1,0.45,0.3)}\right)\right\};$$

$$H(e_3,e_2)=\left\{\left(\frac{c_1}{(0.2,0.25,0.4)}\right),\left(\frac{c_2}{(0.3,0.35,0.9)}\right),\left(\frac{c_3}{(0.1,0.2,0.3)}\right)\right\};$$

$$H(e_3,e_3)=\left\{\left(\frac{c_1}{(0.4,0.25,0.5)}\right),\left(\frac{c_2}{(0.1,0.65,0.9)}\right),\left(\frac{c_3}{(0.1,0.55,0.3)}\right)\right\}.$$

In our matrix notation we get

$$H=\begin{pmatrix} (\langle 0.2,0.40,0.1\rangle) & (\langle 0.2,0.30,0.6\rangle) & (\langle 0.1,0.40,0.7\rangle) \\ (\langle 0.2,0.45,0.4\rangle) & (\langle 0.4,0.15,0.6\rangle) & (\langle 0.1,0.15,0.7\rangle) \\ (\langle 0.2,0.45,0.5\rangle) & (\langle 0.1,0.45,0.6\rangle) & (\langle 0.1,0.50,0.7\rangle) \\ (\langle 0.3,0.30,0.5\rangle) & (\langle 0.2,0.35,0.4\rangle) & (\langle 0.1,0.75,0.2\rangle) \\ (\langle 0.2,0.35,0.5\rangle) & (\langle 0.2,0.20,0.4\rangle) & (\langle 0.1,0.50,0.3\rangle) \\ (\langle 0.3,0.35,0.5\rangle) & (\langle 0.1,0.50,0.4\rangle) & (\langle 0.1,0.85,0.3\rangle) \\ (\langle 0.3,0.20,0.1\rangle) & (\langle 0.2,0.50,0.9\rangle) & (\langle 0.1,0.45,0.3\rangle) \\ (\langle 0.2,0.25,0.4\rangle) & (\langle 0.1,0.35,0.9\rangle) & (\langle 0.1,0.20,0.3\rangle) \\ (\langle 0.4,0.25,0.5\rangle) & (\langle 0.1,0.65,0.3\rangle) & (\langle 0.1,0.55,0.3\rangle) \end{pmatrix}$$

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Matrix representation of AND

Definition 19: If (F,A) and (G,B) are two FNSSs then "(F,A) OR (G,B)", denoted by $(F,A)\vee(G,B)$ is defined by $(F,A)\vee(G,B)=(H,A\times B)$, Where $H(\alpha,\beta)=(H(\alpha,\beta)(u))$, $\forall(\alpha,\beta)\in A\times B$, such that $H(\alpha,\beta)=(F(\alpha)\tilde{\bigcup}G(\beta))$,for all $(\alpha,\beta)\in A\times B$, where $\tilde{\bigcup}$ is Fermatean neutrosophic soft union.

Example 9: Consider Example 8 , where:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle}\right)\right\};$$

$$G(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle}\right)\right\}.$$

$$H(e_1,e_1)=\left\{\left(\left(\frac{c_1}{\left(\max(0.2,0.3),\left(\frac{0.6+0.2}{2}\right),\min(0.1,0.1)\right)}\right)\right),\right.$$

$$\left(\frac{c_2}{\left(\max(0.4,0.2),\left(\frac{0.2+0.4}{2}\right),\min(0.6,0.3)\right)}\right),$$

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$$\left(\frac{c_3}{\left(\max(0.3,0.1),\left(\frac{0.1+0.7}{2}\right),\min(0.7,0.2)\right)}\right)\Bigg\}.$$

$$H(e_1,e_1)=\left\{\left(\frac{c_1}{(0.3,0.4,0.1)}\right),\left(\frac{c_2}{(0.4,0.3,0.3)}\right),\left(\frac{c_3}{(0.3,0.4,0.2)}\right)\right\}.$$

The Similarly, we get.

$$H(e_1,e_2)=\left\{\left(\frac{c_1}{(0.2,0.45,0.1)}\right),\left(\frac{c_2}{(0.7,0.15,0.2)}\right),\left(\frac{c_3}{(0.3,0.15,0.3)}\right)\right\};$$

$$H(e_1,e_3)=\left\{\left(\frac{c_1}{(0.4,0.45,0.1)}\right),\left(\frac{c_2}{(0.4,0.45,0.4)}\right),\left(\frac{c_3}{(0.3,0.5,0.3)}\right)\right\};$$

$$H(e_2,e_1)=\left\{\left(\frac{c_1}{(0.3,0.3,0.1)}\right),\left(\frac{c_2}{(0.2,0.35,0.3)}\right),\left(\frac{c_3}{(0.7,0.75,0.2)}\right)\right\};$$

$$H(e_2,e_2)=\left\{\left(\frac{c_1}{(0.3,0.35,0.4)}\right),\left(\frac{c_2}{(0.7,0.2,0.2)}\right),\left(\frac{c_3}{(0.7,0.5,0.2)}\right)\right\};$$

$$H(e_2,e_3)=\left\{\left(\frac{c_1}{(0.4,0.35,0.5)}\right),\left(\frac{c_2}{(0.2,0.5,0.4)}\right),\left(\frac{c_3}{(0.7,0.85,0.2)}\right)\right\};$$

$$H(e_3,e_1)=\left\{\left(\frac{c_1}{(0.6,0.2,0.1)}\right),\left(\frac{c_2}{(0.3,0.5,0.3)}\right),\left(\frac{c_3}{(0.1,0.45,0.2)}\right)\right\};$$

$$H(e_3,e_2)=\left\{\left(\frac{c_1}{(0.6,0.25,0.1)}\right),\left(\frac{c_2}{(0.7,0.35,0.2)}\right),\left(\frac{c_3}{(0.1,0.2,0.3)}\right)\right\};$$

$$H(e_3,e_3)=\left\{\left(\frac{c_1}{(0.5,0.25,0.1)}\right),\left(\frac{c_2}{(0.3,0.65,0.4)}\right),\left(\frac{c_3}{(0.1,0.55,0.3)}\right)\right\}.$$

In our matrix notation we get

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$$H = \begin{pmatrix} (\langle 0.3,0.2,0.1\rangle) & (\langle 0.4,0.2,0.3\rangle) & (\langle 0.3,0.1,0.2\rangle) \\ (\langle 0.2,0.3,0.1\rangle) & (\langle 0.7,0.1,0.2\rangle) & (\langle 0.3,0.1,0.3\rangle) \\ (\langle 0.4,0.3,0.1\rangle) & (\langle 0.4,0.2,0.4\rangle) & (\langle 0.3,0.1,0.3\rangle) \\ (\langle 0.3,0.2,0.1\rangle) & (\langle 0.2,0.3,0.3\rangle) & (\langle 0.7,0.7,0.2\rangle) \\ (\langle 0.3,0.3,0.4\rangle) & (\langle 0.7,0.1,0.2\rangle) & (\langle 0.7,0.2,0.2\rangle) \\ (\langle 0.4,0.3,0.5\rangle) & (\langle 0.2,0.3,0.4\rangle) & (\langle 0.7,0.8,0.2\rangle) \\ (\langle 0.6,0.2,0.1\rangle) & (\langle 0.3,0.4,0.3\rangle) & (\langle 0.1,0.1,0.2\rangle) \\ (\langle 0.6,0.2,0.1\rangle) & (\langle 0.7,0.1,0.2\rangle) & (\langle 0.1,0.2,0.3\rangle) \\ (\langle 0.5,0.2,0.1\rangle) & (\langle 0.3,0.6,0.4\rangle) & (\langle 0.1,0.2,0.3\rangle) \end{pmatrix}$$

Matrix representation of OR

An Application of FNSS in Decision-Making Definition 20

Comparison Matrix is a matrix whose rows are labelled by the object names $h_1, h_2, ..., h_n$ and the columns are labelled by the parameters $e_1, e_2, ..., e_m$,The entries c_{ij} are calculated by $c_{ij} = a + b - c$ where ' a ' is the integer calculated as 'how many times $T_{hi}(e_i)$ exceeds or equal to $T_{hk}(e_j)$ for $h_i \neq h_k, \forall h_k \in U$ ', ' b ' is the integer calculated as 'how many times $I_{hi}(e_i)$ exceeds or equal to $I_{hk}(e_j)$ for $h_i \neq h_k, \forall h_k \in U$ and ' c ' is the integer calculated as 'how many times $F_{hi}(e_i)$ exceeds or equal to $F_{hk}(e_j)$ for $h_i \neq h_k, \forall h_k \in U$.

Definition 21: The score of an object h_i is S_i and is calculated as $S_i = \sum_j c_{ij}$.

Now we present an algorithm for the most appropriate selection of an object.

Algorithm:

(1) input the Fermatean Neutrosophic Soft Set.

(2) input *P*, the choice parameters of Mr. X.

(3) consider the FNSS and write it in tabular form.

(4) compute the comparison matrix of the FNSS.

(5) compute the score S_i of h_i , $\forall h_i$

(6) find $S_k = \max_i S_i$

(7) if k has more than one value then any one of h_i could be the preferable choice.

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Example 10: Assume $U=\{c_1,c_2,c_3\}$ is a set of three laptops and $E=\{e_1,e_2,e_3\}$ = {Price, Battery, CPU} is a set of parameters which is attractiveness of laptops. Suppose that Mr.X wants to buy the most suitable laptop.

Based on the choice parameters of Mr. X, FNSS, F and G constructed by two experts are as follows:

$$F(e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.6,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.2,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.3,0.1,0.7\rangle}\right)\right\};$$

$$F(e_2)=\left\{\left(\frac{c_1}{\langle 0.3,0.4,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.7,0.8,0.2\rangle}\right)\right\};$$

$$F(e_3)=\left\{\left(\frac{c_1}{\langle 0.6,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.3,0.6,0.9\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_1)=\left\{\left(\frac{c_1}{\langle 0.3,0.2,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.4,0.3\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.7,0.2\rangle}\right)\right\};$$

$$G(e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.3,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.7,0.1,0.2\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.2,0.3\rangle}\right)\right\};$$

$$G(e_3)=\left\{\left(\frac{c_1}{\langle 0.4,0.3,0.5\rangle}\right),\left(\frac{c_2}{\langle 0.1,0.7,0.4\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.9,0.3\rangle}\right)\right\};$$

$$H(e_1,e_1)=\left\{\left(\frac{c_1}{\min\langle 0.2,0.3\rangle,\langle\frac{0.6+0.2}{2}\rangle,\max\langle 0.1,0.1\rangle}\right),\right.$$

$$\left(\frac{c_2}{\min\langle 0.4,0.2\rangle,\langle\frac{0.2+0.4}{2}\rangle,\max\langle 0.6,0.3\rangle}\right),$$

$$\left.\left(\frac{c_3}{\min\langle 0.3,0.1\rangle,\langle\frac{0.1+0.7}{2}\rangle,\max\langle 0.7,0.2\rangle}\right)\right\}.$$

$$H(e_1,e_1)=\left\{\left(\frac{c_1}{\langle 0.2,0.4,0.1\rangle}\right),\left(\frac{c_2}{\langle 0.2,0.3,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.4,0.7\rangle}\right)\right\}.$$

The Similarly, we get.

$$H(e_1,e_2)=\left\{\left(\frac{c_1}{\langle 0.2,0.45,0.4\rangle}\right),\left(\frac{c_2}{\langle 0.4,0.15,0.6\rangle}\right),\left(\frac{c_3}{\langle 0.1,0.15,0.7\rangle}\right)\right\};$$

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$$H(e_1,e_3)=\left\{\left(\frac{c_1}{(0.2,0.45,0.5)}\right),\left(\frac{c_2}{(0.1,0.45,0.6)}\right),\left(\frac{c_3}{(0.1,0.5,0.7)}\right)\right\};$$

$$H(e_2,e_1)=\left\{\left(\frac{c_1}{(0.3,0.3,0.5)}\right),\left(\frac{c_2}{(0.2,0.35,0.4)}\right),\left(\frac{c_3}{(0.1,0.75,0.2)}\right)\right\};$$

$$H(e_2,e_2)=\left\{\left(\frac{c_1}{(0.2,0.35,0.5)}\right),\left(\frac{c_2}{(0.2,0.2,0.4)}\right),\left(\frac{c_3}{(0.1,0.5,0.3)}\right)\right\};$$

$$H(e_2,e_3)=\left\{\left(\frac{c_1}{(0.3,0.35,0.5)}\right),\left(\frac{c_2}{(0.1,0.5,0.4)}\right),\left(\frac{c_3}{(0.1,0.85,0.3)}\right)\right\};$$

$$H(e_3,e_1)=\left\{\left(\frac{c_1}{(0.3,0.2,0.1)}\right),\left(\frac{c_2}{(0.2,0.5,0.9)}\right),\left(\frac{c_3}{(0.1,0.45,0.3)}\right)\right\};$$

$$H(e_3,e_2)=\left\{\left(\frac{c_1}{(0.2,0.25,0.4)}\right),\left(\frac{c_2}{(0.3,0.35,0.9)}\right),\left(\frac{c_3}{(0.1,0.2,0.3)}\right)\right\};$$

$$H(e_3,e_3)=\left\{\left(\frac{c_1}{(0.4,0.25,0.5)}\right),\left(\frac{c_2}{(0.1,0.65,0.9)}\right),\left(\frac{c_3}{(0.1,0.55,0.3)}\right)\right\}.$$

In matrix notation we get

$$H=\begin{pmatrix}
(\langle 0.2,0.40,0.1\rangle) & (\langle 0.2,0.30,0.6\rangle) & (\langle 0.1,0.40,0.7\rangle)\\
(\langle 0.2,0.45,0.4\rangle) & (\langle 0.4,0.15,0.6\rangle) & (\langle 0.1,0.15,0.7\rangle)\\
(\langle 0.2,0.45,0.5\rangle) & (\langle 0.1,0.45,0.6\rangle) & (\langle 0.1,0.50,0.7\rangle)\\
(\langle 0.3,0.30,0.5\rangle) & (\langle 0.2,0.35,0.4\rangle) & (\langle 0.1,0.75,0.2\rangle)\\
(\langle 0.2,0.35,0.5\rangle) & (\langle 0.2,0.20,0.4\rangle) & (\langle 0.1,0.50,0.3\rangle)\\
(\langle 0.3,0.35,0.5\rangle) & (\langle 0.1,0.50,0.4\rangle) & (\langle 0.1,0.85,0.3\rangle)\\
(\langle 0.3,0.20,0.1\rangle) & (\langle 0.2,0.50,0.9\rangle) & (\langle 0.1,0.45,0.3\rangle)\\
(\langle 0.2,0.25,0.4\rangle) & (\langle 0.1,0.35,0.9\rangle) & (\langle 0.1,0.20,0.3\rangle)\\
(\langle 0.4,0.25,0.5\rangle) & (\langle 0.1,0.65,0.3\rangle) & (\langle 0.1,0.55,0.3\rangle)
\end{pmatrix}$$

Matrix representation of AND

Now, compute the comparison matrix of the FNSS by using Definition 20

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C	e_{11}	e_{12}	e_{13}	e_{21}	e_{22}	e_{23}	e_{31}	e_{32}	e_{33}
c_1	9	3	5	4	8	3	3	9	3
c_2	6	3	6	2	4	2	5	4	0
c_3	12	14	10	1	5	8	6	8	3

Comparsion Matrix of FNSS

Now, compute the score S_i of h_i , $\forall h_i$ by using Definition 21

C	$Score(S_i)$
c_1	47
c_2	32
c_3	60

It's clearly, the maximum score is 60, scored by the laptops.

Decision Mr. X will choose the laptop c_3. In any case, if he does not want to choose c_3 due to some reasons his second choice will be c_1.

6. Conclusion

In this paper, we have introduced the concept of Fermatean Neutrosophic Soft Set and studied some of its properties as a complement, union, intersection, AND and OR. Applications of this theory has been given to solve a decision-making problem.

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References

[1] Goguen, J. A. (1973). LA Zadeh. Fuzzy sets. Information and control, vol. 8 (1965), pp. 338–353.-LA Zadeh. Similarity relations and fuzzy orderings. Information sciences, vol. 3 (1971), pp. 177–200. The Journal of Symbolic Logic, 38(4), 656-657.
[2] Turksen, I. B. (1986). Interval valued fuzzy sets based on normal forms. Fuzzy sets and systems, 20(2), 191-210.
[3] Atanassov, K. T. (1999). Intuitionistic fuzzy sets. In Intuitionistic fuzzy sets (pp. 1-137). Physica, Heidelberg.
[4] Neutrosophy, S. F. (1998). Neutrosophic probability, set, and logic, ProQuest Information & Learning. Ann Arbor, Michigan, USA, 105, 118-123.
[5] Smarandache, F. (2002). Neutrosophic set–a generalization of the intuitionistic fuzzy set. In University of New Mexico.
[6] Molodtsov, D. (1999). Soft set theory—first results. Computers & mathematics with applications, 37(4-5), 19-31.
[7] Maji, P. K., Biswas, R. K., & Roy, A. (2001). Fuzzy soft sets.
[8] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. Computers & Mathematics with Applications, 45(4-5), 555-562.

DOI: https://doi.org/10.54216/IJNS.250339
Received: March 28, 2024 Revised: June 27, 2024 Accepted: November 12, 2024

[9] Majumdar, P., & Samanta, S. K. (2010). Generalised fuzzy soft sets. Computers & Mathematics with Applications, 59(4), 1425-1432.
[10] Sezgin, A., & Atagün, A. O. (2011). On operations of soft sets. Computers & Mathematics with Applications, 61(5), 1457-1467.
[11] Maji, P. K. (2013). Neutrosophic soft set. Infinite Study.
[12] Sahin, R., & Küçük, A. (2014). Generalised Neutrosophic Soft Set and its Integration to Decision Making Problem. Applied Mathematics & Information Sciences, 8(6).
[13] Broumi, S. (2013). Generalized neutrosophic soft set. Infinite Study.

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