



The neutrosophic-based analysis assessment framework: An assessment strategy to promote soft computing in educational contexts

Mohamad Ariffin Abu Bakar^{1,*}, Ahmad Termimi Ab Ghani^{1,*}, Mohd Lazim Abdullah¹

¹Faculty of Computer Science and Mathematics, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia.

Emails mohamad-ariffin6299@gmail.com; termimi@umt.edu.my; lazim_m@umt.edu.my

Abstract

Today's educational assessment strategies require innovation and digital transformation in order to overcome biases towards data orientation in uncertain, ambiguous, and fuzzy conditions. Neutrosophic-based analysis techniques in educational assessment provide a thoughtful and highly effective approach to calibrating student learning abilities. However, there is a lack of access to resources and complete guidance on implementing soft computing methods like neutrosophic sets, resulting in a gap in knowledge and practice. Research and development have shown that neutrosophic-based analysis techniques can explain the formulation and algorithms used in multi-criteria decision-making approaches. Since educational assessment also involves decision making, this paper proposes a neutrosophic-based analysis assessment framework aimed at transforming assessment strategies in education to promote soft computing. The development of this paper will begin by reviewing the literature and conducting a preliminary study, highlighting the benefits of the neutrosophic set, and then forming a framework and operational design for the assessment strategy in a systematic manner. Illustrations with numbered data will be used to explain the suitability and usability of this framework for real educational assessment. The implications of calibrating the factors that have the strongest influence on students' mathematics learning demonstrate that this assessment framework can be expanded as an innovative and flexible approach to assessment, capable of improving the efficiency of data analysis in real learning environments. This framework and initiative can be used synergistically to improve the quality of education by incorporating digital elements and providing strong support for the Sustainable Development Goal (SDG).

Keywords: Neutrosophic sets; Neutrosophic-based analysis; Soft computing; Assessment strategy; Decision making

1. Introduction

In today's contemporary education, the design and implementation of an effective assessment strategy is very important in the context of assessing student learning outcomes and also as a platform for obtaining decisions about teaching [1]. According to Sato-Ilic and Ilic [2], traditional assessment methods are faced with various limitations, especially in dealing with issues of complexity and uncertainty that exist in the context of digital education today. This situation has prompted various suggestions and opinions. Among other things, there are increasing suggestions about the need for an innovative and strategic approach [3]. Where can address those challenges and encourage more nuanced assessment practices, which can be adapted to the current assessment situation.

To improve and maintain quality assurance in educational assessment, one effective approach is integrating soft computing techniques. By leveraging principles from artificial intelligence and computational intelligence, these techniques address issues of inaccuracy, uncertainty, and ambiguity in data analysis [4]. Among these techniques, neutrosophic-based analysis offers a unique strategy and framework for representing and processing uncertain

data, making it particularly suitable for educational assessment [5-7]. The study demonstrates significant and effective results in implementing neutrosophic-based analysis, not only with basic neutrosophic sets but also with subclasses such as Single Valued Neutrosophic Sets (SVNS) and Interval Neutrosophic Sets (IVNS) [6, 8-11].

There have been numerous academic studies on the application, potential, and benefits of neutrosophic-based analysis in various fields, including educational assessment. However, significant issues and gaps still exist in the literature regarding the development of a comprehensive assessment strategy framework that utilizes neutrosophic-based analysis [5, 12]. While existing research has explored the theoretical basis and mathematical properties of neutrosophic sets, there remains a lack of empirical evidence demonstrating the conceptual and practical usefulness of neutrosophic-based analysis in the context of educational assessment [9, 10, 13]. This problem is especially evident in the design and implementation of assessment strategies that are appropriate for the complex, uncertain, and ambiguous learning environment. Sri Andayani *et al.* [1] and Kwok *et al.* [14] highlight confusion and issues in the assessment strategy, where there is no alignment between data orientation and analysis techniques. This problem results in data interpretation that does not effectively address problems or meet assessment objectives [15, 16]. If this weakness is not addressed, it will hinder the process of recovery, improvement, mentoring, and guidance in the context of management or learning and may even lead to incorrect actions [1].

Delays in analysing techniques can result in unclear data interpretation and difficulties in establishing connections between data sources, criteria, and dimensions, especially when dealing with various data orientations [1, 17, 18]. This is particularly complex in the context of educational assessment, which falls under the scope of humanistic system theory. As a result, analysis and interpretation techniques involve decision-making programs that fluctuate between uncertainty and probability, as well as handling large datasets and incorporating both quantitative and qualitative analysis components [8, 19]. Consequently, educational assessment strategies need to be modified, starting with setting measurable objectives, selecting appropriate instruments or tools, and ensuring the efficiency of analytical techniques to explain the impact, weighting, ranking, classification, and prediction of the data. Furthermore, an assessment strategy that can generate mathematical and statistical figures that hold significant meaning in data interpretation is also required.

To address this gap, this study proposes an innovative assessment strategy framework based on neutrosophic-based analysis. The aim is to increase the flexibility, adaptability, and effectiveness of assessment practices in the educational context. By utilizing the inherent ability of neutrosophic sets to accommodate uncertainty and ambiguity, this framework aims to provide educators with sustainable tools, methods, and techniques for accurately capturing and assessing diverse student learning outcomes. To explain this point of view, this paper is organized as follows: First, an overview of the concept of neutrosophic set and its potential application in educational assessment is provided. Next, the existing research landscape is discussed, specifically identifying research gaps that motivate this study. Following that, the theoretical and methodological framework used in the development of the proposed assessment strategy framework is outlined. The results of the empirical investigation are then presented, along with a discussion of their implications for theory and practice. Finally, the paper concludes with a summary of key findings, recommendations for future research, and implications for the broader field of educational evaluation.

2. Related Work

2.1 Assessment strategies in education

The assessment landscape in the context of education has grown significantly in recent years, driven by technological advances and awareness of traditional methods that often fail to address the complexity of student learning outcomes [20]. Research and development have emphasized the importance of adopting assessment strategies that are flexible, adaptable, and sensitive to the diverse needs of students [13]. Computer-based assessment in particular, has become famous for its ability to provide timely feedback and support the learning process [21]. However, challenges and difficulties still remain, especially in designing evaluation strategies that can reduce the issues and problems of uncertainty and ambiguity that exist in the context of educational data [13].

Learning assessment is generally used to identify students' weaknesses and strengths so that educators can provide academic support or appropriate intervention. Assessment of learning refers to a process that involves components such as, measuring progress continuously, assessing the strength of motivation, assessing the delivery approach and determining the level of student ability as well as including the assessment of the entire population [15]. Learning assessment can include evaluation for individuals and groups, such as students, teachers, lecturers, district administrators, universities, private companies, education departments, and can also be a combination of them.

In educational assessment strategies, educators play a crucial role in developing, managing, and analyzing data. They are also the primary users of the assessment system. Therefore, educators should be responsible for setting

assessment objectives, tasks, performance criteria, and standards. They should provide feedback and monitor results [16]. This requires educators to actively engage in assessment strategies. Sarala and Kavitha [15] support and recommend this idea, suggesting that every educator should master assessment methods appropriate to their responsibilities. Sri Andayani et al. [1] also emphasize that educators' priority should be mastering assessment strategies, including both testing and non-testing methods, as well as in-depth knowledge of analysis and interpretation techniques. Given the advancement of soft computing technology [3, 16], education must embrace and enhance educators' knowledge and skills accordingly. Soft computing methods, such as advanced data analysis techniques, should be utilized to improve the quality and sustainability of the assessment system.

2.2 Soft computing principles and application of Neutrosophic Sets (NS)

The main purpose of soft computing is to improve accuracy, efficiency and feasibility as well as address cost issues in solutions [9]. Soft computing is basically an outstanding mathematical analysis and calculation technique that can handle data that is in inaccuracy, uncertainty, and partial truth [9, 22]. The main components of soft computing consist of sets theory, fuzzy logic, neural networks, and computational algorithms [10, 18]. This analytical technique offers the best solution to traditional binary logic by accepting gradations of truth and also being able to construct approximate reasoning. Computing has been applied in various fields, including pattern recognition, decision making and optimization [4]. This technique is capable of modelling complex real-world phenomena. Rapidly growing research and development has led to the discovery of the concept of Neutrosophic Sets (NS).

NS was introduced and pioneered by F. Smarandache (1995) which is based on the theory of Neutrosophy which describes the need for fuzzy set generalization to handle uncertainty, inconsistency and incompleteness in data [5, 13, 23, 24]. Before these concepts and theorems were introduced, the string of fuzzy set generalizations started from the introduction of fuzzy set theory by Zadeh (1965), Atanassov (1986) formed intuitionistic fuzzy sets, then introduced interval-valued intuitionistic fuzzy sets as a connection to intuitionistic fuzzy sets [25]. When faced with real data, intuitionistic fuzzy sets still cannot deal with incomplete, ambiguous and inconsistent information, so there is a need for neutrosophic sets that are able to form an analytical system with higher accuracy [5, 26]. In neutrosophic sets, addition is to degrees of false membership independently in the normal interval $[0,1]$ which is still aligned with fuzzy sets and intuitionistic fuzzy sets [23, 26, 27]. The neutrosophic set is refined with the characteristics of truth-membership (T), uncertainty-membership (I) and false-membership (F) in the range $[0,3]$, compared to intuitionistic fuzzy sets consisting of membership degrees and non-membership degrees of something elements in the range $[0,1]$ [27].

This analytical framework is adaptable and can be flexibly applied in various fields including communication networks, social medicine, economics, agriculture and traffic circuit networks in the field of transportation [5, 21, 28]. This drives the development of new concepts and theorems that are more extensive including in today's educational context. The education assessment system also experienced a digitization revolution based on soft computing with the concept of NS that also coloured the change, and even the development and subclasses from NS were also applied. The following Table 1 shows some studies and algorithms of NS that are applied in the educational contexts.

Table 1: An analysis of application of NS in educational contexts

Author(s)	Objective(s)	Algorithm of NS applied
[5]	Assessing the group's mean performance of individual grades obtained by some (or all) students of the group	Sum of neutrosophic triplets
[13]	Analysing the mental abilities of a child on the basis of imaginative play	Single Valued Refined Neutrosophic Set (SVRNS)
[21]	Cross-examining students' performance in mathematics	Neutrosophic and AHP
[20]	To investigate the criteria affecting the sustainability of distance education (DE) in higher education institutions, and to understand the differences in several internal stakeholders' perspectives in DE	Neutrosophic AHP
[28]	To demonstrate some of the most fundamental aspects of the Internet of Things (IoT)	Neutrosophic AHP, Neutrosophic VIKOR

[8]	To rank and determine the best lecturers to be given priority in awards and promotion	TOPSIS and the single-valued neutrosophic set (SVNS)
[9]	Analyzing the impact of teaching strategies on the student performance and the time frame(phase) of the interaction	Complex neutrosophic sets (CNSs)
[6]	Exploring students' readiness to continue utilizing e-learning while measuring satisfaction levels with the e-learning system among higher education students	Neutrosophic AHP
[10]	Portraying the uncertain information during the higher vocational education quality evaluation in the new era	Triangular fuzzy neutrosophic sets (TFNSs)
[11]	Analyzing of how Chat GPT and social media can be used as tools for capturing real-time student feedback on teaching styles in higher education	Neutrosophic AHP

Although there is a significant body of literature on applications in education, there have been relatively few studies that explore the integration of NS into assessment frameworks. Existing research has mainly focused on the theoretical aspects and mathematical formulation of NS, with limited empirical evidence, particularly in terms of practical use in assessment [8, 11]. This indicates a gap in the literature when it comes to developing a comprehensive assessment strategy framework that leverages NS to address the complexities of data analysis in real-world learning environments. Consequently, further research is needed to strengthen the theory and practical application of neutrosophic-based analytical assessment strategies, especially through empirical studies that investigate their effectiveness. Additionally, it is crucial to introduce a systematic framework that can guide educators in designing and implementing assessment practices based on neutrosophical analysis. By developing a more comprehensive assessment strategy framework based on neutrosophic principles, it is hoped that educators can enhance the flexibility, adaptability, and effectiveness of assessment practices, ultimately improving the quality of learning.

3. Methodological framework

This section will further describe the development of the framework with a focus on the concept and formulation of Neutrosophic sets (NS). In fact, NS are characterized by three main components, namely truth membership (T), indeterminacy membership (I), and falsity membership (F) functions. These components allow for the representation of elements that are simultaneously true, indeterminate, and false to varying degrees, reflecting the inherent ambiguity and uncertainty present in educational data.

Definition 1: [23, 29] Let U be a universe of discourse, and a neutrosophic set M is defined as:

$M = \{ \langle m, T_M(m), I_M(m), F_M(m) \rangle / m \in U \}$, where $T_M(m)$, $I_M(m)$ and $F_M(m)$ are the truth membership function, the indeterminacy membership function and the falsity membership function, respectively, such that $0 \leq T_M(m) + I_M(m) + F_M(m) \leq 3$ and also in the state $T_M(m): U \rightarrow]0^-, 1^+[$, $I_M(m): U \rightarrow]0^-, 1^+[$, and $F_M(m): U \rightarrow]0^-, 1^+[$.

Definition 2: [26] The complement of a neutrosophic set M is denoted by $C_{(M)}$ for all m in M , and is defined by

$$T_{C(M)}(m) = \{1^+\} - T_M(m) \quad (1)$$

$$I_{C(M)}(m) = \{1^+\} - I_M(m) \quad (2) \quad F_{C(M)}(m) =$$

$$\{1^+\} - F_M(m) \quad (3)$$

Definition 3: [23,29] The containment of a neutrosophic set M in the other neutrosophic set N , $M \subseteq N$, based on conditions (if and only if)

$$\inf T_M(m) \leq \inf T_N(n), \sup T_M(m) \leq \sup T_N(n) \quad (4)$$

$$\inf F_M(m) \geq \inf F_N(n), \sup F_M(m) \geq \sup F_N(n) \quad (5)$$

Definition 4: [29] The union of two neutrosophic sets M and N is a neutrosophic set P , written as $P = M \cup N$, where the truth membership, indeterminacy membership and falsity membership functions for the membership of P are obtained

$$T_P(p) = T_M(m) + T_N(n) - T_M(m) \times T_N(n) \tag{6}$$

$$I_P(p) = I_M(m) + I_N(n) - I_M(m) \times I_N(n) \tag{7}$$

$$F_P(p) = F_M(m) + F_N(n) - F_M(m) \times F_N(n) \tag{8}$$

Definition 5: [29] The intersection of two neutrosophic sets M and N produces a neutrosophic set Q , can be expressed as $Q = M \cap N$, where truth membership, indeterminacy membership and falsity membership functions are obtained for membership Q is

$$T_Q(q) = T_M(m) \times T_N(n) \tag{9}$$

$$I_Q(q) = I_M(m) \times I_N(n) \tag{10}$$

$$F_Q(q) = F_M(m) \times F_N(n) \tag{11}$$

For more specific use, the parameters are first determined, using the concept of Single Valued Neutrosophic Sets (SVNS) (Wang et al., 2020), then:

Definition 6: [29] Let X be a space of points (objects), with a generic element in X denoted by x . A SVNS M in X is characterized by truth (T_M), indeterminacy (I_M) and falsity (F_M) membership functions respectively, where, $T_M(x), I_M(x), F_M(x) \in [0,1]$, for each point x in X , When X is continuous, a SVNS M can be written as:

$$M = \int_X \langle T_M(x), I_M(x), F_M(x) \rangle / x, x \in X \tag{12}$$

When X is discrete, a SVNS M can be written as

$$M = \sum_{i=1}^n \langle T_M(x_i), I_M(x_i), F_M(x_i) \rangle / x, x \in X \tag{13}$$

Definition 7: [30] Let M and P be two SVNS. Then the basics operations of SVNS are

$$M + P = \{ \langle x, (T_M(x) + T_P(x) - T_M(x)T_P(x)), I_M(x)I_P(x), F_M(x)F_P(x)) \rangle : x \in X \} \tag{14}$$

$$M \times P = \{ \langle x, (T_M(x)T_P(x), I_M(x) + I_P(x) - I_M(x)I_P(x), F_M(x) + F_P(x) - F_M(x)F_P(x)) \rangle : x \in X \} \tag{15}$$

$$\gamma M = \{ \langle x, (1 - (1 - T_M(x))^\gamma), (I_M(x))^\gamma, (F_M(x))^\gamma \rangle : x \in X \} \tag{16}$$

$$M^\gamma = \{ \langle (T_M(x))^\gamma, 1 - (1 - I_M(x))^\gamma, 1 - (1 - F_M(x))^\gamma \rangle : x \in X \} \tag{17}$$

where, γ is a constant and $\gamma > 0$.

Application and data analysis usually includes the concept of decision making therefore, neutrosophic-based analysis adheres to the methods and theorems in multi criteria decision making (MCDM). A summary of the steps in MCDM [4, 31] as follows:

Step 1: (Neutrosophication) Determine the linguistic variables using Single Valued Neutrosophic Numbers (SVNN) that are suitable for the assessment objective. Some choices of linguistic variables according to a certain scale and according to linguistic terms such as importance, appropriateness, level, influence and so on, are:

Table 2: The scale and SVNNs

Score/ Scale	SVNNs	Score/ Scale	SVNNs	Score/ Scale	SVNNs
			< 0.10, 0.80, 0.90 >		
			>		
	< 0.10, 0.80, 0.90 >		< 0.20, 0.70, 0.80 >		< 0.00, 1.00, 1.00 >
	>		>		< 0.10, 0.90, 0.90 >
	< 0.35, 0.60, 0.70 >		< 0.35, 0.60, 0.60 >		< 0.20, 0.85, 0.80 >
	>		>		< 0.30, 0.75, 0.70 >
5	< 0.50, 0.40, 0.45 >	7	< 0.50, 0.40, 0.45 >	11	< 0.40, 0.65, 0.60 >
	>		>		< 0.50, 0.50, 0.50 >
	< 0.80, 0.20, 0.15 >		< 0.65, 0.30, 0.25 >		< 0.60, 0.35, 0.40 >
	>		>		< 0.70, 0.25, 0.30 >
	< 0.90, 0.10, 0.10 >		< 0.80, 0.20, 0.15 >		< 0.80, 0.15, 0.20 >
	>		>		< 0.90, 0.10, 0.10 >
			< 0.90, 0.10, 0.10 >		< 1.00, 0.00, 0.00 >
			>		

		< 0.01, 0.99, 0.99
		>
		< 0.10, 0.90, 0.90
		>
	< 0.25, 0.90, 0.75	>
	>	< 0.20, 0.75, 0.80
	< 0.45, 0.75, 0.55	>
	>	< 0.30, 0.65, 0.60
	< 0.60, 0.45, 0.40	>
	>	< 0.50, 0.50, 0.50
6	< 0.70, 0.35, 0.30	9 >
	>	< 0.70, 0.30, 0.30
	< 0.80, 0.25, 0.20	>
	>	< 0.80, 0.20, 0.15
	< 0.90, 0.10, 0.10	>
	>	< 0.90, 0.10, 0.10
		>
		< 0.99, 0.01, 0.01
		>

Step 2: Form SVNS decision matrix based on evaluation or preference.

Definition 8: [31] Let $Q = \{x_1, x_2, x_3, \dots, x_n\}$ be a set of alternatives, $U = \{o_1, o_2, o_3, \dots, o_m\}$ be the set of attributes. The ratings (or evaluations) of alternatives $x_j \in Q (j = 1, 2, \dots, n)$ on attributes $o_i \in U$ are expressed with SVNS $M_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$, called a SVNS decision matrix

$$[M_{ij}]_{m \times n} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \dots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \dots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \dots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{bmatrix}$$

Step 3: Normalize the SVNS decision matrix

Definition 9: [31] Let $[M_{ij}]_{m \times n}$ is SVNS decision matrix, it is necessary to normalize the decision matrix into the new SVNN matrix, $[\bar{M}_{ij}]_{m \times n}$ by:

$$\bar{M}_{ij} = \langle \frac{T_{ij} - \min T_{ij}}{\max T_{ij} - \min T_{ij}}, \frac{I_{ij} - \min I_{ij}}{\max I_{ij} - \min I_{ij}}, \frac{F_{ij} - \min F_{ij}}{\max F_{ij} - \min F_{ij}} \rangle \tag{18}$$

for benefit attributes, and

$$\bar{M}_{ij} = \langle \frac{\max T_{ij} - T_{ij}}{\max T_{ij} - \min T_{ij}}, \frac{\max I_{ij} - I_{ij}}{\max I_{ij} - \min I_{ij}}, \frac{\max F_{ij} - F_{ij}}{\max F_{ij} - \min F_{ij}} \rangle \tag{19}$$

for cost attributes. The normalized SVNN decision matrix will be obtained as follows:

$$[\bar{M}_{ij}]_{m \times n} = \begin{bmatrix} \langle \bar{T}_{11}, \bar{I}_{11}, \bar{F}_{11} \rangle & \langle \bar{T}_{12}, \bar{I}_{12}, \bar{F}_{12} \rangle & \dots & \langle \bar{T}_{1n}, \bar{I}_{1n}, \bar{F}_{1n} \rangle \\ \langle \bar{T}_{21}, \bar{I}_{21}, \bar{F}_{21} \rangle & \langle \bar{T}_{22}, \bar{I}_{22}, \bar{F}_{22} \rangle & \dots & \langle \bar{T}_{2n}, \bar{I}_{2n}, \bar{F}_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \bar{T}_{m1}, \bar{I}_{m1}, \bar{F}_{m1} \rangle & \langle \bar{T}_{m2}, \bar{I}_{m2}, \bar{F}_{m2} \rangle & \dots & \langle \bar{T}_{mn}, \bar{I}_{mn}, \bar{F}_{mn} \rangle \end{bmatrix}$$

Step 4: Before calculating and implementing the aggregating average operator, first determine the weight vector for the attributes.

Definition 10: [32] The weighted vector of attribute set U is as

$$w = \{w_1, w_2, \dots, w_m\} = (\langle r_1, s_1, t_1 \rangle, \langle r_2, s_2, t_2 \rangle, \dots, \langle r_m, s_m, t_m \rangle), \text{ where } \sum_{i=1}^m w_i = 1$$

There are several methods that can be used to obtain attribute weighted vectors such as determining the priority position according to experts (Sun and Sun, 2016) or using SVNS-entropy weights-based technique [33] with the following steps:

Step 4.1: Determine the entropy values

$$e_j = 1 - \frac{1}{n} \sum_{i=1}^m (T_j + F_j) |2(I_j) - 1| \tag{20}$$

Step 4.2: Calculate the degree of divergence d_j against rating based on attributes o_j obtained through the following equation:

$$d_j = (1 - e_j) \quad (21)$$

Step 4.3: Determining attribute weights based on entropy values using the following equation:

$$w_j = \frac{(1-e_j)}{\sum_{j=1}^n (1-e_j)} \quad (22)$$

According to [34, 35], most statistical analyzes use the mean method to determine the weight of the linguistic term of the component, in many cases the weight of the linguistic term is balanced considering that this component is also in a state of ambiguity. Therefore, $w_i = \{w_1, w_2, \dots, w_m\} = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})$, where $\sum_{i=1}^m w_i = 1$

Step 5: Get aggregating average values $[R_{ij}] = w[\bar{M}_{ij}]_{m \times n} = \sum_{j=1}^n w_j \bar{M}_j$

Definition 11: [33] The set of n SVNN is denoted by $R = (\bar{M}_1, \bar{M}_2, \bar{M}_3, \dots, \bar{M}_n)$, where $\bar{M}_j = (T_j, I_j, F_j); j = 1, 2, 3 \dots n$, the neutrosophic weighted arithmetic average values obtained through:

$$R_w = (R_1, R_2, \dots, R_n) = \left(1 - \prod_{j=1}^n (1 - T_j)^w, 1 - \prod_{j=1}^n (1 - I_j)^w, 1 - \prod_{j=1}^n (1 - F_j)^w\right) \quad (23)$$

Or the weighted geometric average values by:

$$R_G = (R_1, R_2, \dots, R_n) = \sum_{j=1}^n w_j \bar{M}_j = \left(\prod_{j=1}^n (T_j)^w, \prod_{j=1}^n (I_j)^w, \prod_{j=1}^n (F_j)^w\right) \quad (24)$$

In order to obtain information from the decision making result, it is necessary to implement aggregating neutrosophic information with steps such as aggregated single value neutrosophic through comprehensive evaluation of each alternative, calculate the score, accuracy and certainty function and then model the alternative either by comparing weighting values, ranking, or can also classify it.

Definition 12: [30] (Deneutrosophication) The comprehensive evaluation of each alternative (result) $x_j \in Q(j = 1, 2, \dots, n)$ denote as R_j , is given by

$$R_j = \langle T_j, I_j, F_j \rangle = \sum_{i=1}^m \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle \quad (25)$$

Step 6: Calculating score, accuracy and certainty function

Definition 13: [26, 31] Let $R_j = \langle T_j, I_j, F_j \rangle$ be a single valued neutrosophic number, a score function, S , accuracy function, α and certainty, c of a single valued neutrosophic value, based on the truth, indeterminacy and falsity membership degrees is defined by:

$$S(R_j) = \frac{2+T_j-I_j-F_j}{3} \quad (26)$$

$$\alpha(R_j) = T_j - F_j \quad (27)$$

$$c(R_j) = T_j \quad (28)$$

Step 7: Modeling alternatives or rank them for evaluation and interpretation purposes.

3.1 Conceptual Framework for Integrating Neutrosophic-based Analysis into Assessment Strategies

The neutrosophic set offers a flexible and robust framework for implementing assessments in an educational context, specifically for assessing student learning outcomes. It effectively handles uncertainty, variability, and ambiguity in assessment data, resulting in assessments that are more relevant and meaningful. By capturing subtle nuances and variations, this approach allows educators to make more accurate decisions and achieve comprehensive assessments. Therefore, certain components are given priority in the evaluation strategy, as summarized in table 3 below.

Table 3: The components of neutrosophic-based analysis assessment strategy

Components	Description
Data Collection	Prioritize accuracy in selecting and using data. Therefore, educational data, including student performance metrics, assessment scores, and qualitative feedback, are collected using a variety of methods, such as tests, surveys, and classroom observations.

Neutrosophication	In order to reduce problems in data orientation, then, the collected data are represented using neutrosophic sets, with each data point characterized by truth-membership, indeterminacy-membership, and falsity-membership values. This representation captures the uncertainty and ambiguity inherent in educational data.
Analysis and Interpretation	Neutrosophic-based analysis techniques are applied to the represented data to extract meaningful insights into educational outcomes. This involves employing algorithms and computational methods to process and interpret the neutrosophic data.
Decision-Making	Based on the analyzed data, educators make informed decisions regarding instructional strategies, curriculum development, and student support interventions. The flexibility and adaptability of neutrosophic sets allow for dynamic decision-making in response to changing educational contexts.

Based on the literature collected by Bakar and Ghani [36] on the application of MCDM, the aspects of assessment used in educational contexts are such as evaluating factors or effects, analyzing opinions or perceptions, choosing and determining the suitability of strategies, methods, models or activities and also predicting student performance and achievement. Based on these aspects, the researchers introduced a more ideal framework to improve the efficiency of assessment, in addition to fulfilling the wishes of digitization in the education system.

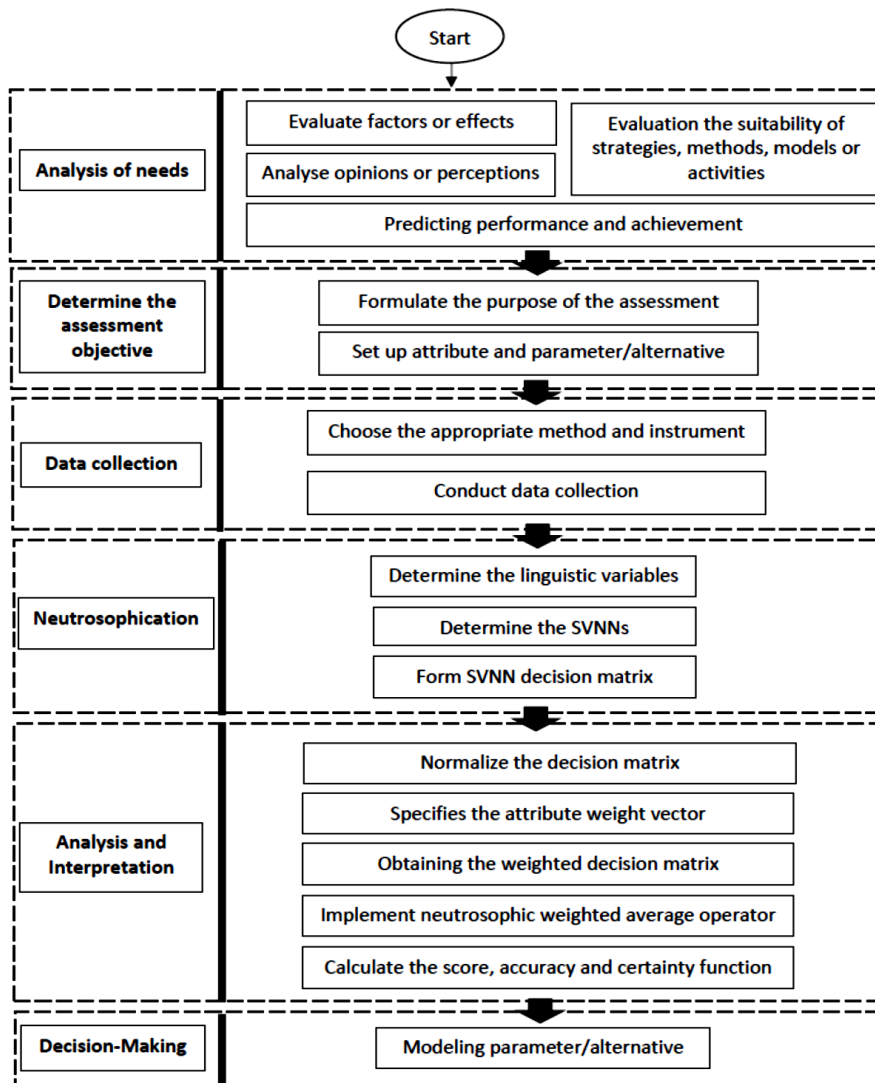


Figure 1. The framework of the proposed Neutrosophic-based Analysis Assessment Model (N-bAAM)

The presented assessment framework describes the potential of neutrosophic-based analysis to revolutionize assessment strategies in the educational context. By accepting uncertainty and ambiguity, a neutrosophic-based assessment approach offers a more comprehensive and nuanced understanding and follow-up of educational outcomes such as student learning, management, and so on. This foundation opens up the space for change towards digitization and ultimately leads to more effective educational practices and contributes to better outcomes for students.

4. Illustrative Example: Factors that affect students' ability to learn mathematics

Next, in this section, the researcher explains and shows how the N-bAAM framework is applied in the context of educational assessment by presenting examples based on formulations with real data. The researcher will present a neutrosophic-based analysis technique to determine what factors affect the learning of mathematics among students. This assessment is made based on the perspective of mathematics teachers. The difference and novelty highlighted is to prove that neutrosophic-based analysis can meet the orientation of data in ambiguity, uncertainty and doubt but still provide significant evaluation results in the situation. The researcher has administered a questionnaire survey to 37 mathematics teachers who have more than ten years of experience teaching mathematics subjects with a purposeful random method. The steps in the study guided by the N-bAAM framework are as follows:

- Step 1: Identification of problems (analysis of needs). In order to start the study, the researcher first carried out a literature review on the problem. In this process, researchers found several factors that affect students' mathematics learning ability, namely the level of motivation, emotional orientation of students, and level of attention, metacognitive tendencies, executive function strength and working memory conditions [37-41]. Previous studies have shown the importance, strength, influence and relationship of these six factors in determining the level of students' mathematics learning ability. However, there is still a need to clarify which factors have priority and strongly influence student ability.
- Step 2: Next, through the need analysis stage, the objective of this study was formed which is to determine factors that affect students' ability to learn mathematics. After setting the six factors as attributes, the researcher formed parameter items as content in the questionnaire to measure the influence of the six factors based on the teacher's point of view. Set attributes, $A (Q_1, Q_2, Q_3, Q_4, Q_5, Q_6)$ is composed of Emotion (Q_1), Motivation (Q_2), Attention (Q_3), Executive function (Q_4), Metacognitive (Q_5) and Working memory (Q_6) masing-masing. While, set alternatives, $T (T_1, T_2, T_3 \dots T_n; n=37)$ is representing the number of teachers.
- Step 3: Acquiring teachers' opinions. The next step is the data collection process. As already informed, purposive sampling method is used. In this method, a questionnaire survey is carried out where the mathematics teacher will rate the level of influence of the six factors using a scale of 1-7, which ranges from no influence at all to very important influence. This rated score will be converted to Single Valued Neutrosophic Numbers (SVNN).
- Step 4: Based on SVNN, the score assigned by the teacher is changed and a neutrosophic decision matrix is formed as shown in the following table.

Table 4: Neutrosophic decision matrix

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
T_1	< 0.65, 0.30, 0.25 >	< 0.90, 0.10, 0.10 >	< 0.90, 0.10, 0.10 >	< 0.90, 0.10, 0.10 >	< 0.90, 0.10, 0.10 >	< 0.65, 0.30, 0.25 >
T_2	< 0.80, 0.20, 0.15 >	< 0.50, 0.40, 0.45 >	< 0.50, 0.40, 0.45 >	< 0.35, 0.60, 0.60 >	< 0.35, 0.60, 0.60 >	< 0.35, 0.60, 0.60 >
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
T_{36}	< 0.35, 0.60, 0.60 >	< 0.50, 0.40, 0.45 >	< 0.65, 0.30, 0.25 >	< 0.90, 0.10, 0.10 >	< 0.65, 0.30, 0.25 >	< 0.65, 0.30, 0.25 >
T_{37}	< 0.65, 0.30, 0.25 >	< 0.80, 0.20, 0.15 >	< 0.80, 0.20, 0.15 >	< 0.90, 0.10, 0.10 >	< 0.80, 0.20, 0.15 >	< 0.80, 0.20, 0.15 >

- Step 5: Normalizing neutrosophic decision matrix using Eq (18).

Table 5: Normalized SVN decision matrix

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
T_1	$\langle 0.6875, 0.2857, 0.1875 \rangle$	$\langle 1.0000, 0.0000, 0.0000 \rangle$	$\langle 1.0000, 0.0000, 0.0000 \rangle$	$\langle 1.0000, 0.0000, 0.0000 \rangle$	$\langle 1.0000, 0.0000, 0.0000 \rangle$	$\langle 0.5455, 0.4000, 0.3000 \rangle$
T_2	$\langle 0.8750, 0.1429, 0.0625 \rangle$	$\langle 0.2727, 0.6000, 0.7000 \rangle$	$\langle 0.0000, 1.0000, 1.0000 \rangle$	$\langle 0.0000, 1.0000, 1.0000 \rangle$	$\langle 0.0000, 1.0000, 1.0000 \rangle$	$\langle 0.0000, 1.0000, 1.0000 \rangle$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
T_{36}	$\langle 0.3125, 0.7143, 0.6250 \rangle$	$\langle 0.2727, 0.6000, 0.7000 \rangle$	$\langle 0.3750, 0.6667, 0.4286 \rangle$	$\langle 1.0000, 0.0000, 0.0000 \rangle$	$\langle 0.5455, 0.4000, 0.3000 \rangle$	$\langle 0.5455, 0.4000, 0.3000 \rangle$
T_{37}	$\langle 0.6875, 0.2857, 0.1875 \rangle$	$\langle 0.8182, 0.2000, 0.1000 \rangle$	$\langle 0.7500, 0.3333, 0.1429 \rangle$	$\langle 1.0000, 0.0000, 0.0000 \rangle$	$\langle 0.8182, 0.2000, 0.1000 \rangle$	$\langle 0.8182, 0.2000, 0.1000 \rangle$

Step 6: Aggregated opinions using neutrosophic weighted averaging operator Eq (24). Next, rank the attributes after getting the score, accuracy and certainty function through Eq (26), Eq (27) and Eq (28).

Table 6: The weighted, score, accuracy, certainty functions and ranking of attributes

Attributes	Weighted Average	Score function	Accuracy function	Certainty function	Ranking
Q_1	(0.7162, 0.2664, 0.2061)	0.7479	0.5101	0.7162	4
Q_2	(0.6462, 0.3189, 0.2784)	0.6830	0.3678	0.6462	5
Q_3	(0.6284, 0.3964, 0.3012)	0.6436	0.3272	0.6284	6
Q_4	(0.8624, 0.1405, 0.1027)	0.8731	0.7597	0.8624	1
Q_5	(0.7371, 0.2432, 0.2108)	0.7610	0.5263	0.7371	2
Q_6	(0.7346, 0.2541, 0.2027)	0.7593	0.5319	0.7346	3

Step 7: Interpretation is carried out based on all results.

The results of the analysis show that the factors that affect students' ability to learn mathematics are in the positions of Executive function (Q_4), Metacognitive (Q_5), Working memory (Q_6), Emotion (Q_1), Motivation (Q_2) and, Attention (Q_3) respectively based on ranking agreed by the teacher. These results show that cognitive function is very influential in building the ability to learn mathematics.

5. Discussion and Future Research

The results of this study describe how soft computer analysis can be used in educational assessment, specifically through the combination of neutrosophic set techniques. The main focus of this framework is to address the inherent uncertainty and complexity in learning assessment, resulting in a more comprehensive and accurate assessment strategy for analyzing student abilities. This finding has significant implications for educational assessment, as it offers a personalized approach to learning assessment and enables targeted interventions. By analyzing real data, such as identifying the dominant factors influencing students' mathematical learning abilities, educators can adjust their teaching methods to cater to the cognitive needs of their students. This indirectly sheds light on the pedagogical requirements for strengthening overall academic success. These results further support the effectiveness of implementing the neutrosophic set approach in education, as demonstrated by previous studies [8, 10, 13, 20, 21, 28]. Also, continuing the continuity and significance of the neutrosophic-based analysis method as implemented in several other field studies [42-45].

Even so, there are limitations in this study's exploration. While the neutrosophic set technique offers efficiency, its application may require extensive knowledge and detailed resources. These constraints potentially limit its widespread use. Therefore, further research is needed to explore the long-term effectiveness and scalability of assessment frameworks in diverse educational contexts. One important aspect to consider is the sampling method. The illustrations presented in this study are based on a small sample size and may not represent the entire population of mathematics teachers. Conducting future research with larger and more focused samples may

improve the generalizability of the results. Additionally, the researcher did not compare this analysis technique to others. As a validity assessment, the effectiveness of this strategy requires more empirical evidence. Limitations in implementation may also stem from limited resource access. Educators need specific resources, training, and guidance to access these strategies. Despite the existence of this framework, further exploration and specialization within the field are still necessary.

As a suggestion, this framework-based assessment strategy requires collective cooperation. Collaborating with stakeholders, such as education departments, universities, educators, students, and education policymakers, can facilitate the development and widespread dissemination of this assessment framework. This network and interweaving will ensure that the evaluation strategy can meet the needs of all parties involved. Additionally, by utilizing technology platforms and tools, engagement sessions can be made more systematic and practical. For instance, the research and development of user-friendly digital platforms or software can greatly increase accessibility and implementation. By addressing these limitations and adopting the recommended approach, future research can further advance the use of neutrosophic set techniques in educational assessment, ultimately promoting more equitable and effective learning outcomes for all students.

6. Conclusion

Overall, this study highlights the potential of neutrosophic-based analysis to improve the quality of educational assessment thus improving learning outcomes for students. Through innovative approaches like these, educators can meet the needs of diverse students and promote academic success. The case studies presented also prove that the application of neutrosophic-based analysis techniques can yield encouraging results in translating students' learning abilities in mathematics. Relying on the value of truth-degree, uncertainty-degree, and false-degree, assessment formulation and analysis can provide a more nuanced understanding of student performance, taking into account the uncertainty and complexity inherent in the actual learning process.

Funding: This research was supported by Ministry of Higher Education (MOHE) through Fundamental Research Grant Scheme (FRGS/1/2022/STG06/UMT/02/4, Grant No. 59722).

Conflicts of Interest: The authors declare no conflict of interest.

References

- [1] Sri Andayani, Sri Hartati, Wardoyo, R. & Mardapi, D. (2017). Decision-making model for student assessment by unifying numerical and linguistic data. *International Journal of Electrical and Computer Engineering*, 7(1), 363-373. doi: 10.11591/ijece.v7i1.pp363-373
- [2] Sato-Ilic, M. & Ilic, P. (2013). Fuzzy dissimilarity based multidimensional scaling and its application to collaborative learning data. *Procedia Computer Science*, 20(2013), 490-495.
- [3] Stojanović, J., Petkovic, D., Alarifi, I. M., Cao, Y., Denic, N., Ilic, J., . . . Milickovic, M. (2021). Application of distance learning in mathematics through adaptive neuro-fuzzy learning method. *Computers & Electrical Engineering*, 93, 107270.
- [4] Deli, İ. (2020). Linear optimization method on single valued neutrosophic set and its sensitivity analysis. *TWMS Journal of Applied and Engineering Mathematics*, 10(1), 128-137.
- [5] Voskoglou, M. G. (2023). Application of neutrosophic sets to assessment of student learning skills. In *Handbook of Research on the Applications of Neutrosophic Sets Theory and Their Extensions in Education* (pp. 89-110). IGI Global.
- [6] Alzyoudi, M., Moussa, N., Almazroui, K., & Alnuaimi, S. (2023). Analyzing Digital Education using Neutrosophic Sets. *International Journal of Neutrosophic Science (IJNS)*, 20(2).
- [7] Martin, N., & Broumi, S. (2023). Neutrosophic cognitive impact study on role transformation of teachers to facilitators. In *Handbook of Research on the Applications of Neutrosophic Sets Theory and Their Extensions in Education* (pp. 215-234). IGI Global.
- [8] Wu, F., & Fang, Y. (2022). Multilevel evaluation of teaching quality in higher education using single-valued neutrosophic set. *Mobile Information Systems*, 2022.
- [9] Al-Quran, A., & Alkhazaleh, S. (2018). Relations between the complex neutrosophic sets with their applications in decision making. *Axioms*, 7(3), 64.
- [10] Feng, Q. (2023). An integrated decision approach with triangular fuzzy neutrosophic sets for higher vocational education quality evaluation in the new era. *Journal of Intelligent & Fuzzy Systems*, (Preprint), 1-14.

- [11] Alnaqbi, N. M., & Fouda, W. (2023). Exploring the role of ChatGPT and social media in enhancing student evaluation of teaching styles in higher education using neutrosophic sets. *International Journal of Neutrosophic Science*, 20(4), 181-190.
- [12] Mishra, A. R., Pamucar, D., Rani, P., Shrivastava, R., & Hezam, I. M. (2024). Assessing the sustainable energy storage technologies using single-valued neutrosophic decision-making framework with divergence measure. *Expert Systems with Applications*, 238, 121791.
- [13] Vasantha, W.B., Kandasamy, I., Smarandache, F., Devvrat, V., & Ghildiyal, S. (2020). Study of imaginative play in children using single-valued refined neutrosophic sets. *Symmetry*, 12(3), 402.
- [14] Kwok, R. C. W., Ma, J., Vogel, D., & Zhou, D. (2001). Collaborative assessment in education: An application of a fuzzy GSS. *Information & Management*, 39(3), 243-253.
- [15] Sarala, N. & Kavitha, R. (2015). Model of mathematics teaching: A fuzzy set approach. *IOSR Journal of Mathematics*, 11(1-1), 19-22. doi: 10.9790/5728-11111922
- [16] Jeong, J.S. & Gonzalez-Gomez, D. (2020). Assessment of sustainability science education criteria in online-learning through fuzzy-operational and multi-decision analysis and professional survey. *Heliyon*, 6(2020), 1-11. <https://doi.org/10.1016/j.heliyon.2020.e04706>
- [17] Sodenkamp, M. A., Tavana, M., & Di Caprio, D. (2018). An aggregation method for solving group multi-criteria decision-making problems with single-valued neutrosophic sets. *Applied Soft Computing*, 71, 715-727.
- [18] Eisa, A., Fattouh, M. & ElShabshery, A. A. (2024). Single-Valued Neutrosophic Sets Based Score Function and WASPAS Method for Plant Location Selection Problem. *Journal of Advanced Research in Applied Sciences and Engineering Technology*, 41(2), 139-151. <https://doi.org/10.37934/araset.41.2.139151>
- [19] Yiğit, F. (2023). A three-stage fuzzy neutrosophic decision support system for human resources decisions in organizations. *Decision Analytics Journal*, 7, 100259.
- [20] Yilmaz, H., Karadayi-Usta, S., & Yanık, S. (2022). A novel neutrosophic AHP-Copeland approach for distance education: towards sustainability. *Interactive Learning Environments*, 1–23. <https://doi.org/10.1080/10494820.2022.2141265>
- [21] Wardat, Y., Alali, R., Jarrah, A. M., & Alzyoudi, M. (2023). Neutrosophic theory framework for building mathematics teachers capacity in assessment of high school students in the United Arab Emirates. *International Journal of Neutrosophic Science*, 21(1).
- [22] Zadeh, L. A. (1994). Fuzzy Logic, Neural Networks, and Soft Computing. *Communications of the ACM*, 37, 77-84. <https://doi.org/10.1145/175247.175255>
- [23] Smarandache, F. (2005). Neutrosophic set-A generalisation of the intuitionistic fuzzy sets. *Int. J. Pure Appl. Math.*, 24, 287-297.
- [24] Abdel-Basset, M., Atef, A., & Smarandache, F. (2019). A hybrid Neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research*, 57, 216-227.
- [25] Atanassov, K. T., & Atanassov, K. T. (1999). Intuitionistic fuzzy sets (pp. 1-137). Physica-Verlag HD.
- [26] Smarandache, F. (2020). The score, accuracy, and certainty functions determine a total order on the set of neutrosophic triplets (T, I, F). *Neutrosophic Sets and Systems*, 38(1), 1-14.
- [27] Smarandache, F. (1999). A unifying field in Logics: Neutrosophic Logic. In *Philosophy* (pp. 1-141). American Research Press.
- [28] Embarak, O. H., Aldarmaki, F. R., & Almesmari, M. J. (2022). Towards Smart Education in IoT and IoB Environment using the Neutrosophic Approach. *International Journal of Neutrosophic Science (IJNS)*, 19(1).
- [29] Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. (2010). *Single Valued Neutrosophic Sets; Infinite Study: Phoenix, AZ, USA*, p. 10.
- [30] Ye, J. (2017). Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method. *Informatica*, 28(2), 387-402.
- [31] Liu, Y., Wu, S., Li, C., & Dong, Y. (2023). Exploring 2-rank strategic weight manipulation in multiple attribute decision making and its applications in project review and university ranking. *Engineering Applications of Artificial Intelligence*, 117, 105525.
- [32] Şahin, R., & Yiğider, M. (2014). A Multi-criteria neutrosophic group decision making method based TOPSIS for supplier selection. *arXiv preprint arXiv:1412.5077*.
- [33] Elshabshery, A., & Fattouh, M. (2021). On some Information Measures of Single-Valued Neutrosophic Sets and their Applications in MCDM Problems. *Int. J. Eng. Res. Technol*, 10(5), 406-415.
- [34] Gou, L., & Wang, M. (2019). Semantic risk analysis based on single-valued neutrosophic sets. *IEEE Access*, 7, 76480-76488.
- [35] Liu, Y., Li, Y., Zhang, Z., Xu, Y., & Dong, Y. (2022). Classification-based strategic weight manipulation in multiple attribute decision making. *Expert Systems with Applications*, 197, 116781.

- [36] Bakar, M.A.A. and Ab Ghani, A.T. (2022). Capturing the Contribution of Fuzzy and Multi-Criteria Decision-Making Analytics: A Review of the Computational Intelligence Approach to Classroom Assessment Sustainability. *International Journal of Industrial Engineering & Production Research*, 33(4), 1-15. doi: 10.22068/ijiepr.33.4.13
- [37] McRae, K. (2016). Cognitive emotion regulation: a review of theory and scientific findings. *Current Opinion in Behavioral Sciences*, 10, 119-124.
- [38] Alpar, G. & Hovee, M.V. (2019). Towards Growth-Mindset Mathematics Teaching in the Netherlands in C.M. Stracke (ed.), *LINQ, EPiC Series in Education Science*, 2, 1-17.
- [39] Otoo, D., Iddrisu, W.A., Kessie, J.A. & Larbi, E. (2018). Structural model of students' interest and self-motivation to learning mathematics. *Education Research International*, 2018, 1-10.
- [40] Molenberghs, P., Trautwein, F.M., Bockler, A., Singer, T. & Kanske, P. (2016). Neural correlates of metacognitive ability and of feeling confident: a large-scale fMRI study. *Social Cognitive and Affective Neuroscience*, 2016, 1942-1951. doi: 10.1093/scan/nsw093.
- [41] Ridderinkhof, K. R., Wildenberg, W. P. M. V. D., Segalowitz, S. J. & Carter, C. S. (2004). Neurocognitive mechanisms of cognitive control: The role of prefrontal cortex in action selection, response inhibition, performance monitoring, and reward-based learning. *Brain and Cognition*, 56(2004), 129-140. doi:10.1016/j.bandc.2004.09.016
- [42] Edalatpanah, S. A. (2018). Neutrosophic perspective on DEA. *Journal of applied research on industrial engineering*, 5(4), 339-345.
- [43] Ismail, J. N., Rodzi, Z., Al-Sharqi, F., Hashim, H., & Sulaiman, N. H. (2023). The integrated novel framework: linguistic variables in pythagorean neutrosophic set with DEMATEL for enhanced decision support. *Int. J. Neutrosophic Sci*, 21(2), 129-141.
- [44] Kara, K., Yalçın, G. C., Çetinkaya, A., Simic, V., & Pamucar, D. (2024). A single-valued neutrosophic CIMAS-CRITIC-RBNAR decision support model for the financial performance analysis: A study of technology companies. *Socio-Economic Planning Sciences*, 101851.
- [45] Sun, H. and Sun, M. (2016). Simplified neutrosophic weighted average operators and their application to e-commerce. *ICIC Express Letters*, 10(1),27-33.