Cubic Spherical Linguistic Neutrosophic Topological Space

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Abstract

In this article, we introduce and establish a novel concept called 'cubic spherical linguistic neutrosophic topological spaces' by employing cubic spherical linguistic neutrosophic sets and topological frameworks. Various foundational definitions, theorems, and properties are provided along with illustrative examples.

Keywords: Cubic Spherical Linguistic Neutrosophic topological Space; Cubic Spherical Linguistic Neutrosophic open set; Cubic Spherical Linguistic Neutrosophic closed set; Cubic Spherical Linguistic Neutrosophic continuous function; Cubic Spherical Linguistic Neutrosophic derived sets

1 Introduction

Many investigators in business, science, economy, and a variety of other fields deal with modeling unknown data on a regular basis. For these ambiguities and uncertainties, traditional techniques are not always successful. Lotfi A. Zadeh[19] introduced the idea of assigning a membership or truth value to the elements of a well-defined collection of objects, known as sets. These systems can handle a variety of inputs, including ambiguous, distorted, or inaccurate data. The idea of fuzzy topology was initially developed by Chang[2] in 1967. Over time, many topological structures and generalizations have been developed using fuzzy sets. In addition to the degree of truth membership, Atanassov[1] introduced a non-membership value which generalized fuzzy sets into intuitionistic fuzzy sets. In 1997, Coker[4] extended these ideas to define intuitionistic fuzzy topology. Later, Smarandache[14] introduced the concept of an indeterminacy membership function in 1999, adding another dimension to the theory. Neutrosophic sets, with their ability to model uncertainty, have found important applications in areas such as decision making and medical diagnosis. Wang and Smarandache[16] further developed this idea by introducing interval-valued neutrosophic sets. Qualitative attributes can be easily expressed in linguistic terms, a concept developed by Zadeh.[20] The idea of linguistic variables was applied in decision making by Herrera *et al.*[12] in 2000 and by Herrera-Viedma and Verdegay[11] in 1996. Su[15] used linguistic preference information in group decision making. Chen, Liu *et al.*[3] introduced linguistic intuitionistic fuzzy numbers (LIFN) in 2015. Since LIFNs lack indeterminacy, Ye[18] proposed the notion of single-valued neutrosophic linguistic numbers (SVNLNs) in 2015 and developed an extended TOPSIS model for multi-attribute group decision-making (MAGDM) utilizing SVNLNs. An extended COPRAS model for MAGDM based on a single-valued neutrosophic 2-tuple linguistic environment was developed by Wei, Wu *et al.*[17] Fang, Zebo *et al.*[6] introduced linguistic neutrosophic numbers in 2017 with a concrete definition. S.Gomathi,S.Krishnaprakash,M.Karpagadevi, and Said Broumi[9] introduced cubic spherical neutrosophic sets in 2023 and S.Gomathi,M.Karpagadevi, and S.Krishnaprakash[10] introduced cubic spherical neutrosophic topological spaces in 2024. N.Gayathri and M.Helen[7] introduced Linguistic Neutrosophic Topology in 2021. In This paper section 2 deals with the basic definitions of LNNs. In Section 3, the concept of cubic spherical linguistic neutrosophic topology is introduced, and some properties are discussed. cubic spherical Linguistic

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neutrosophic derived sets are presented in Section 4. Finally, in Section 5, the notion of cubic spherical linguistic neutrosophic continuity and cubic spherical linguistic neutrosophic dense sets are defined and discussed with suitable examples.

2 Preliminaries

Definition 2.1.[14] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is,

$$T_A : S \to]0^-, 1^+[, \quad I_A : S \to]0^-, 1^+[, \quad F_A : S \to]0^-, 1^+[$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so

$$0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+.$$

Definition 2.2.[14] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by the truth-membership function T_A, the indeterminacy-membership function I_A, and the falsity-membership function F_A. For each point x in S, $T_A(x)$, $I_A(x)$, and $F_A(x) \in [0, 1]$.

When S is continuous, a SVNS A can be written as:

$$A = \int \langle T(x), I(x), F(x)\rangle / x \in S.$$

When S is discrete, a SVNS A can be written as:

$$A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S.$$

Definition 2.3.[6] Let $S = \{s_\theta \mid \theta = 0, 1, 2, \ldots, \tau\}$ be a finite and totally ordered discrete term set, where τ is an even value and s_θ represents a possible value for a linguistic variable. For example, when $\tau = 6$, S can be expressed as:

$$S = \{\text{very bad, bad, fair, very fair, good, very good}\}.$$

Su[15] extended the discrete linguistic term set S into a continuous term set $S = \{s_\theta \mid \theta \in [0, q]\}$, where, if $s_\theta \in S$, then we call s_θ the original term, otherwise it is called a virtual term.

Definition 2.4.[6] Let $Q = \{s_0, s_1, s_2, \ldots, s_t\}$ be a linguistic term set (LTS) with odd cardinality $t + 1$, and $Q = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by:

$$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle \mid x \in S\},$$

where $s_\theta(x), s_\psi(x), s_\sigma(x) \in Q$ represent the linguistic truth, linguistic indeterminacy, and linguistic falsity degrees of S to A, respectively, with the condition:

$$0 \leq \theta + \psi + \sigma \leq 3t.$$

This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5.[6] Let $\alpha = (s_\theta, s_\psi, s_\sigma)$, $\alpha_1 = (s_{\theta_1}, s_{\psi_1}, s_{\sigma_1})$, $\alpha_2 = (s_{\theta_2}, s_{\psi_2}, s_{\sigma_2})$ be three linguistic single valued neutrosophic numbers (LSVNNs), then:

- (1) $\alpha^c = (s_\sigma, s_\psi, s_\theta)$;
- (2) $\alpha_1 \cup \alpha_2 = (\max(\theta_1, \theta_2), \min(\psi_1, \psi_2), \min(\sigma_1, \sigma_2))$;
- (3) $\alpha_1 \cap \alpha_2 = (\min(\theta_1, \theta_2), \max(\psi_1, \psi_2), \max(\sigma_1, \sigma_2))$;

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- (4) $\alpha_1 = \alpha_2$ iff $\theta_1 = \theta_2, \psi_1 = \psi_2, \sigma_1 = \sigma_2$.

Definition 2.6.[6] Let $\alpha = (l_\theta, l_\psi, l_\sigma)$ be a LSVNN. The set of all labels is $L = \{l_0, l_1, l_2, \ldots, l_t\}$. Then the unit linguistic neutrosophic set (1_{LN}) is defined as:

$$1_{LN} = (l_t, l_0, l_0),$$

which represents the truth membership, and the zero linguistic neutrosophic set (0_{LN}) is defined as:

$$0_{LN} = (l_0, l_t, l_t),$$

which represents the falsehood membership.

Example 2.7. For the linguistic neutrosophic set

$$L = \{\text{very bad, bad, fair, very fair, good, very good}\}$$

the set of all labels is $L = \{l_0, l_1, l_2, l_3, l_4, l_5\}$. Then, the unit linguistic neutrosophic set is defined as:

$$1_{LN} = (l_5, l_0, l_0),$$

and the zero linguistic neutrosophic set is defined as:

$$0_{LN} = (l_0, l_5, l_5).$$

Definition 2.8.[7]For a linguistic neutrosophic topology π, the collection of linguistic neutrosophic sets should obey,

1. $0_{LN}, 1_{LN} \in \pi$
2. $K_1 \cap K_2 \in \pi$ for any $K_1, K_2 \in \pi$
3. $\bigcup K_i \in \pi, \forall \{K_i : i \in J\} \subset \pi$

We call the pair (S_{LN}, π_{LN}), a linguistic neutrosophic topological space.

Definition 2.9.[10] Let $\tau_\odot \in CSN(\mathbb{X})$, then $\tau_\odot$ is called a cubic spherical neutrosophic topology on $\mathbb{X}$, if the following hold:

1. $1_\odot, 0_\odot \in \tau_\odot$.
2. $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in \tau_\odot \implies \delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2} \in \tau_\odot$.
3. $\{\delta_{\mathbb{R}_i} \mid i \in \Delta\} \subseteq \tau_\odot \implies \bigcup \delta_{\mathbb{R}_i} \in \tau_\odot$.

Definition 2.10.[9] Let's assume a fixed universe X and its subset csA. The set

$$csA_\rho = \{\langle x, cs\mu(x), csv(x), csn(x); \rho\rangle : x \in X\}$$

where $cs\mu, csv, csn : X \to [0,1]$ are functions such that $cs\mu A + csvA + csnA \leq 3$ and $\rho \in [0,1]$. The radius ρ of the sphere with center $(cs\mu(x), csv(x), csn(x))$ inside the cube or cube inside the sphere is called cubic spherical neutrosophic set (CSNS) csA_ρ. This sphere represents the membership degree, indeterminacy degree and non-membership degree of $x \in X$.

Let

$$\{\langle cs\mu_{i,1}, csv_{i,1}, csn_{i,1}\rangle, \langle cs\mu_{i,2}, csv_{i,2}, csn_{i,2}\rangle, \ldots, \langle cs\mu_{i,k_i}, csv_{i,k_i}, csn_{i,k_i}\rangle\}$$

be a collection of NSs assigned for any $x_i \in X$. We construct the center of the sphere by

$$\langle cs\mu(x_i), csv(x_i), csn(x_i)\rangle = \left\langle \frac{\sum_{j=1}^{k_i} cs\mu_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} csv_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} csn_{i,j}}{k_i} \right\rangle \tag{1}$$

and the radius using

$$\rho_i = \min\left\{ \max_{1\leq j\leq k_i} \sqrt{(cs\mu(x_i) - cs\mu_{i,j})^2 + (csv(x_i) - csv_{i,j})^2 + (csn(x_i) - csn_{i,j})^2}, 1 \right\}. \tag{2}$$

Then the spheres inside the cube or cube inside the sphere by

$$csA_\rho = \{\langle x_i, cs\mu(x_i), csv(x_i), csn(x_i); \rho\rangle : x_i \in X\}. \tag{3}$$

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3 Cubic Spherical Linguistic Neutrosophic Topological Space

In this section, be introduced the concepts of cubic spherical linguistic neutrosophic topological spaces.

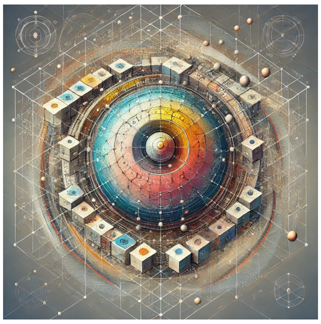
Definition 3.1:

For a *cubic spherical linguistic neutrosophic topological space* τ_{CSLN}, the collection of cubic spherical linguistic neutrosophic sets should obey:

1. $0_{CSLN}, 1_{CSLN} \in \tau_{CSLN}$,
2. $\mathbb{R}_1 \cap \mathbb{R}_2 \in \tau_{CSLN}$ for any $\mathbb{R}_1, \mathbb{R}_2 \in \tau_{CSLN}$,
3. $\bigcup \mathbb{R}_i \in \tau_{CSLN}$ for all $\mathbb{R}_i : i \in J \subseteq \tau_{CSLN}$.

We call the pair $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ a *cubic spherical linguistic neutrosophic topological space (CSLNTS).*
Remark 3.2: Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a *cubic spherical linguistic neutrosophic topological space (CSLNTS).* Then, $(\mathbb{Q}_{CSLN}, \tau_{CSLN})^c$ is the dual CSLN topology, whose elements are $\mathbb{R}^c_{CSLN}$ for $\mathbb{R}_{CSLN} \in (\mathbb{Q}_{CSLN}, \tau_{CSLN})$.

Any open set in τ_{CSLN} is known as a *cubic spherical linguistic neutrosophic open set (CSLNOsS)*. Any closed set in τ_{CSLN} is known as a *cubic spherical linguistic neutrosophic closed set (CSLNCS)* if its complement is a cubic spherical linguistic neutrosophic open set.



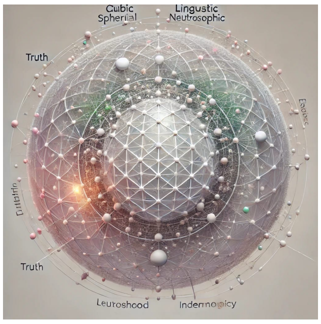


Figure 1: Geometric representation of CSLNTS

Example 3.3:

Let the universe of discourse $U_{CSLN} = \{u, v, w, z\}$ and the set $\mathbb{Q}_{CSLN} = \{u, v\}$, with the linguistic term set $L = \{$very poor, poor, very bad, bad, fair, good, very good$\}$.

For each linguistic term, we define a corresponding cubic spherical linguistic neutrosophic number in terms of truth-membership (T), indeterminacy-membership (I), and falsity-membership (F), satisfying $T^2 + I^2 + F^2 \leq 1$.

Let:

$$\mathbb{R}_{CSLN} = \{(u, (l_5, l_3, l_4)), (v, (l_4, l_2, l_3))\}$$

For the element u: $l_5 =$ good $= [0.7, 0.8]$ $l_3 =$ bad $= [0.3, 0.4]$ $l_4 =$ fair $= [0.5, 0.6]$

Thus, for u:

Truth-membership $T_u = [0.7, 0.8]$ Indeterminacy-membership $I_u = [0.3, 0.4]$ Falsity-membership $F_u = [0.5, 0.6]$

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These values satisfy the spherical neutrosophic condition:

$$T^2 + I^2 + F^2 = (0.7)^2 + (0.3)^2 + (0.5)^2 = 0.49 + 0.09 + 0.25 = 0.83 \leq 1$$

For the element v:

l_4 = fair = $[0.5, 0.6]$ l_2 = very bad = $[0.1, 0.2]$ l_3 = bad = $[0.3, 0.4]$

Thus, for v:

Truth-membership $T_v = [0.5, 0.6]$ Indeterminacy-membership $I_v = [0.1, 0.2]$ Falsity-membership $F_v = [0.3, 0.4]$

These values satisfy:

$$T^2 + I^2 + F^2 = (0.5)^2 + (0.1)^2 + (0.3)^2 = 0.25 + 0.01 + 0.09 = 0.35 \leq 1$$

Let:

$$\mathbb{H}_{CSLN} = \{(u, (l_6, l_2, l_2)), (v, (l_6, l_1, l_0))\}$$

For the element u:

l_6 = very good = $[0.8, 1.0]$ l_2 = very bad = $[0.1, 0.2]$

Thus, for u:

Truth-membership $T_u = [0.8, 1.0]$ Indeterminacy-membership $I_u = [0.1, 0.2]$ Falsity-membership $F_u = [0.1, 0.2]$

These values satisfy:

$$T^2 + I^2 + F^2 = (0.8)^2 + (0.1)^2 + (0.1)^2 = 0.64 + 0.01 + 0.01 = 0.66 \leq 1$$

For the element v:

l_6 = very good = $[0.8, 0.9]$ l_1 = poor = $[0.2, 0.3]$ l_0 = very poor = $[0.0, 0.1]$

Thus, for v:

Truth-membership $T_v = [0.8, 0.9]$ Indeterminacy-membership $I_v = [0.2, 0.3]$ Falsity-membership $F_v = [0.0, 0.1]$

These values satisfy:

$$T^2 + I^2 + F^2 = (0.8)^2 + (0.2)^2 + (0.1)^2 = 0.64 + 0.04 + 0.01 = 0.69 \leq 1$$

Finally, the collection:

$$\tau_{CSLN} = \{0_{CSLN}, \mathbb{R}_{CSLN}, \mathbb{H}_{CSLN}, 1_{CSLN}\}$$

forms a cubic spherical linguistic neutrosophic topology on $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$.

Definition 3.4:

Let $\mathbb{R}_{CSLN}$ represent a set in cubic spherical linguistic neutrosophic topological space (CSLN).

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- **Cubic Spherical Linguistic Neutrosophic Interior:**

$$CSLNint(\mathbb{R}_{CSLN}) = \bigcup\{O_{CSLN} \mid O_{CSLN} \text{ is a } CSLNOSin\mathbb{Q}_{CSLN} \text{ where } O_{CSLN} \subseteq \mathbb{R}_{CSLN}\}$$

  where O_{CSLN} is the cubic spherical linguistic neutrosophic open set and the interior represents the largest cubic spherical linguistic neutrosophic open subset of $\mathbb{R}_{CSLN}$.

- **Cubic Spherical Linguistic Neutrosophic Closure:**

$$CSLNcl(\mathbb{H}_{CSLN}) = \bigcap\{J_{CSLN} \mid J_{CSLN} \text{ is a } CSLCNSin\mathbb{Q}_{CSLN} \text{ where } \mathbb{H}_{CSLN} \subseteq J_{CSLN}\}$$

  where J_{CSLN} is a cubic spherical linguistic neutrosophic closed set and the closure represents the smallest cubic spherical linguistic neutrosophic closed set containing $\mathbb{H}_{CSLN}$.

Example3.5:

$$CSLNint(\mathbb{R}_{CSLN}) = N_{CSLN}, \quad CSLNcl(\mathbb{R}_{CSLN}) = 1_{CSLN}$$

where N_{CSLN} is the cubic spherical linguistic neutrosophic interior and 1_{CSLN} is the total space in CSLN.

Theorem 3.6

Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a *Cubic Spherical Linguistic Neutrosophic Topological Space (CSLNTS)*, and let $\mathbb{R}_{CSLN}, \mathbb{H}_{CSLN} \subseteq \mathbb{Q}_{CSLN}$. Then:

1. $\mathbb{R}_{CSLN} \subseteq$ CSLNcl$(\mathbb{R}_{CSLN})$,
2. $\mathbb{R}_{CSLN}$ is CSLN closed if and only if $\mathbb{R}_{CSLN} =$ CSLNcl$(\mathbb{R}_{CSLN})$,
3. CSLNcl$(0_{CSLN}) = 0_{CSLN}$ and CSLNcl$(\mathbb{Q}_{CSLN}) = \mathbb{Q}_{CSLN}$,
4. $\mathbb{R}_{CSLN} \subseteq \mathbb{H}_{CSLN} \implies$ CSLNcl$(\mathbb{R}_{CSLN}) \subseteq$ CSLNcl(H_{CSLN}),
5. CSLNcl$(\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN}) =$ CSLNcl$(\mathbb{R}_{CSLN}) \cup$ CSLNcl$(\mathbb{H}_{CSLN})$,
6. CSLNcl$(\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{R}_{CSLN}) \cap$ CSLNcl$(\mathbb{H}_{CSLN})$,
7. CSLNcl(CSLNcl$(\mathbb{R}_{CSLN})) =$ CSLNcl$(\mathbb{R}_{CSLN})$.

Proof:

1. From the definition, $\mathbb{R}_{CSLN} \subseteq$ CSLNcl$(\mathbb{R}_{CSLN})$.
2. If $\mathbb{R}_{CSLN}$ is CSLN closed, then $\mathbb{R}_{CSLN}$ is the smallest CSLN closed set containing $\mathbb{R}_{CSLN}$. Hence, $\mathbb{R}_{CSLN} =$ CSLNcl$(\mathbb{R}_{CSLN})$. Conversely, if $\mathbb{R}_{CSLN} =$ CSLNcl$(\mathbb{R}_{CSLN})$, then $\mathbb{R}_{CSLN}$ is the smallest CSLN closed set containing $\mathbb{R}_{CSLN}$, and hence $\mathbb{R}_{CSLN}$ is CSLN closed.
3. If $\mathbb{R}_{CSLN}$ is CSLN closed, then $\mathbb{R}_{CSLN} =$ CSLNcl$(\mathbb{R}_{CSLN})$. Since 0_{CSLN} and $\mathbb{Q}_{CSLN}$ are CSLN closed, we have CSLNcl$(0_{CSLN}) = 0_{CSLN}$ and CSLNcl$(\mathbb{Q}_{CSLN}) = \mathbb{Q}_{CSLN}$.
4. If $\mathbb{R}_{CSLN} \subseteq \mathbb{H}_{CSLN}$, since $\mathbb{H}_{CSLN} \subseteq$ CSLNcl$(\mathbb{H}_{CSLN})$ and $\mathbb{R}_{CSLN} \subseteq$ CSLNcl$(\mathbb{H}_{CSLN})$, then CSLNcl$(\mathbb{R}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{H}_{CSLN})$.
5. As $\mathbb{R}_{CSLN} \subseteq \mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN}$ and $\mathbb{H}_{CSLN} \subseteq \mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN}$, CSLNcl$(\mathbb{R}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN})$ and CSLNcl$(\mathbb{H}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN})$. Thus, CSLNcl$(\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{R}_{CSLN}) \cup$ CSLNcl$(\mathbb{H}_{CSLN})$. Since CSLNcl$(\mathbb{R}_{CSLN}) \cup$ CSLNcl$(\mathbb{H}_{CSLN})$ is CSLN closed and $\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN} \subseteq$ CSLNcl$(\mathbb{R}_{CSLN}) \cup$CSLNcl$(\mathbb{H}_{CSLN})$, we conclude that CSLNcl$(\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN}) =$ CSLNcl$(\mathbb{R}_{CSLN}) \cup$ CSLNcl$(\mathbb{H}_{CSLN})$.

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6. Since $\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN} \subseteq \mathbb{R}_{CSLN}$ and $\mathbb{R}_{CSLN} \cap H_{CSLN} \subseteq \mathbb{H}_{CSLN}$, CSLNcl$(\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{R}_{CSLN}) \cap$ CSLNcl$(\mathbb{H}_{CSLN})$.
7. As CSLNcl$(\mathbb{R}_{CSLN})$ is a CSLN closed set, CSLNcl(CSLNcl$(\mathbb{R}_{CSLN})) =$ CSLNcl$(\mathbb{R}_{CSLN})$.

Remark 3.7

If CSLNint$(\mathbb{R}_{CSLN}) =$ CSLNcl$(\mathbb{R}_{CSLN})$ is a cubic spherical linguistic neutrosophic closed set (CSLNCS), then we have:

1. CSLNint$(\mathbb{R}_{CSLN}) = \mathbb{R}_{CSLN}$ if and only if $\mathbb{R}_{CSLN}$ is CSLNOS (cubic spherical linguistic neutrosophic open set) in $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$.
2. CSLNcl$(\mathbb{R}_{CSLN}) = \mathbb{R}_{CSLN}$ if and only if $\mathbb{R}_{CSLN}$ is CSLNCS (cubic spherical linguistic neutrosophic closed set) in $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$.

Theorem 3.8
Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a cubic spherical linguistic neutrosophic topological space (CSLNTS), and $\mathbb{R}_{CSLN} \in \mathbb{Q}_{CSLN}$. Then:

1. $\mathbb{Q}_{CSLN} -$ CSLNint$(\mathbb{R}_{CSLN}) =$ CSLNint$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$
2. $\mathbb{Q}_{CSLN} -$ CSLNcl$(\mathbb{R}_{CSLN}) =$ CSLNcl$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$

Proof:

(i) Let $S \in \mathbb{Q}_{CSLN} -$ CSLNint$(\mathbb{R}_{CSLN}) \Rightarrow S \notin$ CSLNint$(\mathbb{R}_{CSLN})$. Thus, there is no cubic spherical linguistic neutrosophic open set G containing S, i.e., $C_{CSLN} \cap (S - \mathbb{R}_{CSLN}) \neq \emptyset$ for every CSLN open set G.

Since $\mathbb{Q}_{CSLN} -$CSLNint$(\mathbb{R}_{CSLN}) \subseteq$ CSLNcl$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$, we conclude that $S \in$ CSLNcl$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$.

Conversely, if $S \in$ CSLNcl$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$, then $G_{CSLN} \cap (\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN}) \neq \emptyset$ for every CSLN open set G. Hence, $S \notin$ CSLNint(A), which implies $S \in \mathbb{Q}_{CSLN} -$ CSLNint$(\mathbb{R}_{CSLN})$.

Thus, $\mathbb{Q}_{CSLN} -$ CSLNint$(\mathbb{R}_{CSLN}) =$ CSLNint$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$.

(ii) The proof for this part is similar to (i).

Remark 3.9

On taking complements on both sides of $\mathbb{Q}_{CSLN} -$ CSLNint$(\mathbb{R}_{CSLN}) =$ CSLNint$(S_{CSLN} - \mathbb{R}_{CSLN})$ and $\mathbb{Q}_{CSLN} -$ CSLNcl$(\mathbb{R}_{CSLN}) =$ CSLNcl$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$, we have:

1. CSLNint$(\mathbb{R}_{CSLN}) = \mathbb{Q}_{CSLN} -$ CSLNcl$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$
2. CSLNcl$(\mathbb{R}_{CSLN}) = \mathbb{Q}_{CSLN} -$ CSLNint$(\mathbb{Q}_{CSLN} - \mathbb{R}_{CSLN})$

Theorem 3.10

Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a Cubic Spherical Linguistic Neutrosophic Topological Space (CSLNTS), and let $\mathbb{R}_{CSLN}, \mathbb{H}_{CSLN} \in \mathbb{Q}_{CSLN}$. Then:

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1. CSLNint($\mathbb{R}_{CSLN}$) $= \mathbb{R}_{CSLN}$ if and only if $\mathbb{R}_{CSLN}$ is CSLN open.
2. CSLNint($\varnothing_{CSLN}$) $= \varnothing_{CSLN}$ and CSLNint($\mathbb{Q}_{CSLN}$) $= \mathbb{Q}_{CSLN}$.
3. $\mathbb{R}_{CSLN} \subseteq \mathbb{H}_{CSLN} \Rightarrow$ CSLNint($\mathbb{R}_{CSLN}$) $\subseteq$ CSLNint($\mathbb{H}_{CSLN}$).
4. CSLNint($\mathbb{R}_{CSLN}$) $\cup$ CSLNint($\mathbb{H}_{CSLN}$) $\subseteq$ CSLNint($\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN}$).
5. CSLNint($\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN}$) $=$ CSLNint($\mathbb{R}_{CSLN}$) $\cap$ CSLNint($\mathbb{H}_{CSLN}$).
6. CSLNint(CSLNint($\mathbb{R}_{CSLN}$)) $=$ CSLNint($\mathbb{R}_{CSLN}$).

Proof: Obvious

Definition 3.11

Let $\mathbb{Q}_{CSLN}$ be a non-void set, and let $\mathbb{R}_{CSLN} = \{(S, [T_{\mathbb{R}_{CSLN}}, I_{\mathbb{R}_{CSLN}}, F_{\mathbb{R}_{CSLN}}])\}$ and $H_{CSLN} = \{(S, [T_{\mathbb{H}_{CSLN}}, I_{\mathbb{H}_{CSLN}}, F_{\mathbb{H}_{CSLN}}])\}$ be cubic spherical linguistic neutrosophic sets in CSLNTS.

1. **The union of $\mathbb{R}_{CSLN}$ and $\mathbb{H}_{CSLN}$ is defined as:**
$$\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN} = \{(S, [T_{\mathbb{R}_{CSLN}} \cup T_{\mathbb{H}_{CSLN}}, I_{\mathbb{R}_{CSLN}} \cap I_{\mathbb{H}_{CSLN}}, F_{\mathbb{R}_{CSLN}} \cap F_{\mathbb{H}_{CSLN}}])\}$$
2. **The intersection of $\mathbb{R}_{CSLN}$ and $\mathbb{H}_{CSLN}$ is defined as:**
$$\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN} = \{(S, [T_{\mathbb{R}_{CSLN}} \cap T_{\mathbb{H}_{CSLN}}, I_{\mathbb{R}_{CSLN}} \cup I_{\mathbb{H}_{CSLN}}, F_{\mathbb{R}_{CSLN}} \cup F_{\mathbb{H}_{CSLN}}])\}$$
3. **The complement of $\mathbb{R}_{CSLN}$ is defined as:**
$$\mathbb{R}^C_{CSLN} = \{(S, [1 - T_{\mathbb{R}_{CSLN}}, 1 - I_{\mathbb{R}_{CSLN}}, 1 - F_{\mathbb{R}_{CSLN}}])\}$$
4. **Additional properties:**
   (a) $(\mathbb{R}^C_{CSLN})^C = \mathbb{R}_{CSLN}$
   (b) $(\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN})^C = \mathbb{R}^C_{CSLN} \cup \mathbb{H}^C_{CSLN}$
   (c) $(\mathbb{R}_{CSLN} \cup \mathbb{H}_{CSLN})^C = \mathbb{R}^C_{CSLN} \cap \mathbb{H}^C_{CSLN}$

Theorem 3.12
Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a Cubic Spherical Linguistic Neutrosophic Topological Space (CSLNTS). A point $S \in$ CSLNcl($\mathbb{R}_{CSLN}$) if and only if $U_{CSLN} \cap \mathbb{R}_{CSLN} \neq \varnothing_{CSLN}$ for every cubic spherical linguistic neutrosophic open set U_{CSLN} containing S, where $\mathbb{R}_{CSLN} \subseteq \mathbb{Q}_{CSLN}$.

Proof:

(a) If U_{CSLN} is a CSLN open set and if $S \in$ CSLNcl($\mathbb{R}_{CSLN}$), then $\mathbb{Q}_{CSLN} - U_{CSLN}$ is CSLN closed. If $\mathbb{R}_{CSLN} \cap U_{CSLN} = \varnothing_{CSLN}$, then $\mathbb{R}_{CSLN} \subseteq \mathbb{Q}_{CSLN} - U_{CSLN}$. That is, $\mathbb{Q}_{CSLN} - U_{CSLN}$ is a CSLN closed set containing $\mathbb{R}_{CSLN}$. Therefore, CSLNcl($\mathbb{R}_{CSLN}$) $\subseteq \mathbb{Q}_{CSLN} - U_{CSLN}$, which is a contradiction, since $S \in$ CSLNcl($\mathbb{R}_{CSLN}$) but $S \notin \mathbb{Q}_{CSLN} - U_{CSLN}$. Hence, $\mathbb{R}_{CSLN} \cap U_{CSLN} \neq \varnothing_{CSLN}$, for every CSLN open set U_{CSLN} containing S.

(b) Conversely, if $\mathbb{R}_{CSLN} \cap U_{CSLN} \neq \varnothing_{CSLN}$ for every CSLN open set U_{CSLN} containing S, and if $S \notin$ CSLNcl($\mathbb{R}_{CSLN}$), then $S \in \mathbb{Q}_{CSLN} -$ CSLNcl($\mathbb{R}_{CSLN}$), which is CSLN open. Hence, ($\mathbb{Q}_{CSLN} -$ CSLNcl($\mathbb{R}_{CSLN}$)) $\cap \mathbb{R}_{CSLN} = \varnothing_{CSLN}$. But $\mathbb{R}_{CSLN} \subseteq$ CSLNcl($\mathbb{R}_{CSLN}$), and hence $\mathbb{Q}_{CSLN} -$ CSLNcl($\mathbb{R}_{CSLN}$) $\cap \mathbb{R}_{CSLN} = \varnothing_{CSLN}$, a contradiction. Therefore, $S \in$ CSLNcl($\mathbb{R}_{CSLN}$).

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Definition 3.13

Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a Cubic Spherical Linguistic Neutrosophic Topological Space (CSLNTS) and $\tau_{CSLN} = \{0_{CSLN}, \mathbb{Q}_{CSLN}\}$. Then, τ_{CSLN} is called the CSLN indiscrete topology over $\mathbb{Q}_{CSLN}$.

Definition 3.14

Let τ_{CSLN} be the collection of all CSLN sets that can be defined over $\mathbb{Q}_{CSLN}$. Then, $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ is called the CSLN discrete topology over $\mathbb{Q}_{CSLN}$.

Theorem 3.15

Let $(\mathbb{Q}_{CSLN}, \tau^1_{CSLN})$ and $(\mathbb{Q}_{CSLN}, \tau^2_{CSLN})$ be two Cubic Spherical Linguistic Neutrosophic Topological Spaces (CSLNTSs), then $(\mathbb{Q}_{CSLN}, \tau^1_{CSLN} \cap \tau^2_{CSLN})$ is a CSLNTS on $\mathbb{Q}_{CSLN}$.

Proof:

(a) Clearly, $\varnothing_{CSLN}$ and $\mathbb{Q}_{CSLN} \in \tau^1_{CSLN} \cap \tau^2_{CSLN}$.

(b) Let $F_i \in \tau^1_{CSLN} \cap \tau^2_{CSLN}$. Then, $F_i \in \tau^1_{CSLN}$ and $F_i \in \tau^2_{CSLN}$ $\forall i \in I$. Therefore, $\bigcup_{i\in I} F_i \in \tau^1_{CSLN}$ and $\bigcup_{i\in I} F_i \in \tau^2_{CSLN}$. Thus, $\bigcup_{i\in I} F_i \in \tau^1_{CSLN} \cap \tau^2_{CSLN}$.

(c) Let $\mathbb{R}_{CSLN}$ and $\mathbb{Q}_{CSLN} \in \tau^1_{CSLN} \cap \tau^2_{CSLN}$, which implies $\mathbb{R}_{CSLN}, \mathbb{H}_{CSLN} \in \tau^1_{CSLN}$ and $\mathbb{H}_{CSLN}, \mathbb{H}_{CSLN} \in \tau^2_{CSLN}$. Since $\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN} \in \tau^1_{CSLN}$ and $\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN} \in \tau^2_{CSLN}$, we have $\mathbb{R}_{CSLN} \cap \mathbb{Q}_{CSLN} \in \tau^1_{CSLN} \cap \tau^2_{CSLN}$.

Thus, $(\mathbb{Q}_{CSLN}, \tau^1_{CSLN} \cap \tau^2_{CSLN})$ is a CSLNTS on $\mathbb{Q}_{CSLN}$.

Remark 3.16

The union of two Cubic Spherical Linguistic Neutrosophic Topological Spaces (CSLNTSs) may not be a CSLN topology over $\mathbb{Q}_{CSLN}$.

Example 3.17.

Let the universe of discourse be U = {a, b, c} and S = {a}. The set of all linguistic terms is

$$L = \{\text{very salt}(l_0), \text{salt}(l_1), \text{very sour}(l_2), \text{sour}(l_3), \text{bitter}(l_4), \text{sweet}(l_5), \text{very sweet}(l_6)\}$$

and

$$\tau_1^{\text{CSLN}} = \{0_{\text{CSLN}}, 1_{\text{CSLN}}, \mathbb{R}_{\text{CSLN}}\}$$

where $\mathbb{R}_{\text{CSLN}} = \{(a, (l_6, l_3, l_4))\}$, where the element a's degree of appurtenance to the set $\mathbb{R}_{\text{CSLN}}$ is very sweet (l_6), the element a's degree of indeterminate appurtenance to the set $\mathbb{R}_{\text{CSLN}}$ is sour (l_3), and the element a's degree of non-appurtenance to the set $\mathbb{R}_{\text{CSLN}}$ is bitter (l_4).

Let

$$\tau_2^{CSLN} = \{0_{CSLN}, 1_{CSLN}, H_{CSLN}\}$$

where $\mathbb{H}_{\text{CSLN}} = \{(a, (l_4, l_5, l_2))\}$, where the element a's degree of appurtenance to the set $\mathbb{H}_{\text{CSLN}}$ is bitter (l_4), the element a's degree of indeterminate appurtenance to the set $\mathbb{H}_{\text{CSLN}}$ is sweet (l_5), and the element a's degree of non-appurtenance to the set $\mathbb{H}_{\text{CSLN}}$ is very sour (l_2). Let τ_1^{CSLN} and τ_2^{CSLN} be two CSLN topologies on $\mathbb{Q}_{\text{CSLN}}$.

Then,

$$\tau_1^{CSLN} \cup \tau_2^{CSLN} = \{0_{CSLN}, 1_{CSLN}, \mathbb{R}_{CSLN}, \mathbb{Q}_{CSLN}\} = \{0_{CSLN}, 1_{CSLN}, \{(a, (l_6, l_3, l_3))\}, \{(a, (l_4, l_5, l_2))\}\}.$$

Now,

$$\mathbb{R}_{CSLN} \cup \mathbb{Q}_{CSLN} = \{(a, (l_6, l_5, l_2))\} \notin \tau_1^{CLN} \cup \tau_2^{CSLN}.$$

$$\mathbb{R}_{CSLN} \cap \mathbb{H}_{CSLN} = \{(a, (l_4, l_3, l_3))\} \notin \tau_1^{CSLN} \cup \tau_2^{CSLN}.$$

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Therefore, union of any two linguistic neutrosophic topologies need not be a linguistic neutrosophic topology.

Definition 3.18
Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a **Cubic Spherical Linguistic Neutrosophic Topological Space (CSLNTS)** and U_{CSLN} be a **Cubic Spherical Linguistic Neutrosophic Set (CSLN)** over $\mathbb{Q}_{CSLN}$. Then any point S is a **CSLN interior point** of U_{CSLN}, if there exists a **CSLN open set** V_{CSLN} such that $S \in U_{CSLN} \subseteq V_{CSLN}$.

Definition 3.19
Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a **Cubic Spherical Linguistic Neutrosophic Topological Space (CSLNTS)** and U_{CSLN} be a **CSLN set** over $\mathbb{Q}_{CSLN}$. Then, V_{CSLN} is called a **CSLN neighborhood** if there exists a **CSLN open set** V_{CSLN} such that $S \in U_{CSLN} \subseteq V_{CSLN}$.

Theorem 3.20

Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a **CSLNTS**, then:

1. Each $s \in S$ has a neighborhood.
2. If U_{CSLN} and V_{CSLN} are **CSLN neighborhoods** of some $x \in \mathbb{Q}_{CSLN}$, then $U_{CSLN} \cap V_{CSLN}$ is also a **CSLN neighborhood** of s.
3. If U_{CSLN} is a **CSLN neighborhood** of S and $U_{CSLN} \cap V_{CSLN}$, then V_{CSLN} is also a **CSLN neighborhood** of $s \in \mathbb{Q}_{CSLN}$.

Proof:

(1) : (2): Let U_{CSLN} and V_{CSLN} be **CSLN neighborhoods** of some $s \in S$, then there exists U^1_{CSLN} and $V^1_{CSLN} \in \tau$ such that $S \in U^1_{CSLN} \subseteq U_{CSLN}$ and $S \in V^1_{CSLN} \subseteq V_{CSLN}$.

Now, $S \in U_{CSLN}$ and $S \in V_{CSLN}$ implies that $S \in U^1_{CSLN} \cap V^1_{CSLN}$ and $U^1_{CSLN} \cap V^1_{CSLN} \in \tau$. So we have $S \in U^1_{CSLN} \cap V^1_{CSLN} \subseteq U_{CSLN} \cap V_{CSLN}$. Thus, $U_{CSLN} \cap V_{CSLN}$ is a **CSLN neighborhood** of s.

(3): Let U_{CSLN} be a **CSLN neighborhood** of s and $U_{CSLN} \cap V_{CSLN}$. By definition, there exists a **CSLN open set** U^1_{CSLN} such that $s \in U^1_{CSLN} \subseteq U_{CSLN} \cap V_{CSLN}$. Then, $s \in U_{CSLN} \subseteq V_{CSLN}$.

Therefore, V_{CSLN} is also a **CSLN neighborhood** of $s \in S$.

Theorem 3.21

Let $(\mathbb{Q}_{\text{CSLN}}, \tau_{\text{CSLN}})$ be a CSLNTS. For any CSLN open set $\mathbb{R}_{\text{CSLN}}$ over S, $\mathbb{R}_{\text{CSLN}}$ is a CSLN neighborhood of each point in $\bigcap_{i \in I} A_i$.

Proof:

Let $\mathbb{R}_{\text{CSLN}} \in \pi_{\text{CSLN}}$. For any $S \in \bigcap_{i \in I} K_{\text{CSLNi}}$, we have $S \in A_i$ for all $i \in I$. Thus, $S \in \mathbb{R}_{\text{CSLN}}$, and hence $\mathbb{R}_{\text{CSLN}}$ is a CSLN neighborhood of S.

4. Cubic Spherical Linguistic Neutrosophic Derived Sets

Definition 4.1

Let $(\mathbb{Q}_{CSLN}, \tau_{CSLN})$ be a CSLNS and $\mathbb{R}_{CSLN} \subseteq \mathbb{Q}_{CSLN}$. Let $s \in \mathbb{Q}_{CSLN}$. s is called a CSLN limit point of $\mathbb{R}_{CSLN}$ if

$$E_{CSLN} \cap (\mathbb{R}_{CSLN} - \{s\}) \neq \emptyset$$

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for every CSLN open set E_{CSLN} containing s.

The collection of all CSLN limit points of $\mathbb{R}_{CSLN}$ is the CSLN derived set $LND(\mathbb{R}_{CSLN})$ of $\mathbb{R}_{CSLN}$

Theorem 4.2

$$CSLNcl(\mathbb{R}_{CSLN}) = \mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN}) \quad \text{where} \quad \mathbb{R}_{CSLN} \subseteq \mathbb{Q}_{CSLN}$$

Proof:

If $s \in \mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN})$, then $s \in \mathbb{R}_{CSLN}$ or $s \in LND(\mathbb{R}_{CSLN})$. If $s \in \mathbb{R}_{CSLN}$, then $s \in CSLNcl(\mathbb{R}_{CSLN})$. Therefore, let $s \notin \mathbb{R}_{CSLN}$. That is, $s \in LND(\mathbb{R}_{CSLN})$. Then, for every CSLN open set E_{CSLN} containing s,

$$E_{CSLN} \cap (\mathbb{R}_{CSLN} - s) \neq \emptyset.$$

Since $s \notin \mathbb{R}_{CSLN}, E_{CSLN} \cap \mathbb{R}_{CSLN} \neq \emptyset$. Thus, $s \in CSLNcl(\mathbb{R}_{CSLN})$.

Hence,

$$\mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN}) \subseteq CSLNcl(\mathbb{R}_{CSLN}).$$

If $s \in CSLNcl(\mathbb{R}_{CSLN})$ and $s \in \mathbb{R}_{CSLN}$, then $s \in \mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN})$. If $s \in CSLNcl(\mathbb{R}_{CSLN})$ but $s \notin \mathbb{R}_{CSLN}$, then $E_{CSLN} \cap \mathbb{R}_{CSLN} \neq \emptyset$ for every CSLN open set E_{CSLN} containing s, and hence,

$$E_{CSLN} \cap (\mathbb{R}_{CSLN} - s) \neq \emptyset.$$

Therefore, $s \in LND(\mathbb{R}_{CSLN})$, i.e., $s \in \mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN})$.

Thus,

$$CSLNcl(\mathbb{R}_{CSLN}) \subseteq \mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN}).$$

Therefore,

$$CSLNcl(\mathbb{R}_{CSLN}) = \mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN}).$$

Theorem 4.3

If the derived set of $\mathbb{R}_{CSLN}$ is a subset of $\mathbb{R}_{CSLN}$, then $\mathbb{R}_{CSLN}$ is CSLN closed.

Proof:

$\mathbb{R}_{CSLN}$ is CSLN closed if and only if

$$CSLNcl(\mathbb{R}_{CSLN}) = \mathbb{R}_{CSLN},$$

iff

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$$\mathbb{R}_{CSLN} \cup LND(\mathbb{R}_{CSLN}) = \mathbb{R}_{CSLN},$$

iff

$$LND(\mathbb{R}_{CSLN}) \subseteq \mathbb{R}_{CSLN}.$$

Theorem 4.4.
If $\mathbb{R}_{CSLN}$ is a singleton subset of $\mathbb{Q}_{CSLN}$, then $LND(\mathbb{R}_{CSLN}) = CSLNcl(\mathbb{R}_{CSLN}) - \mathbb{R}_{CSLN}$.

Proof:

If $s \in LND(\mathbb{R}_{CSLN})$, then for every CSLN open set E_{CSLN} containing s,

$$E_{CSLN} \cap (\mathbb{R}_{CSLN} - s) \neq \emptyset.$$

Then $s \notin \mathbb{R}_{CSLN}$. Suppose if $s \in \mathbb{R}_{CSLN}$, then $\mathbb{R}_{CSLN} = \{s\}$, and

$$E_{CSLN} \cap (\mathbb{R}_{CSLN} - s) = \emptyset.$$

It is true that

$$LND(\mathbb{R}_{CSLN}) \subseteq CSLNcl(\mathbb{R}_{CSLN}).$$

Then, $s \in CSLNcl(\mathbb{R}_{CSLN})$ but $s \notin \mathbb{R}_{CSLN}$, when $s \in LND(\mathbb{R}_{CSLN})$. Thus,

$$LND(\mathbb{R}_{CSLN}) \subseteq CSLNcl(\mathbb{R}_{CSLN}) - \mathbb{R}_{CSLN}.$$

Thus, $s \in CSLNcl(\mathbb{R}_{CSLN}) - \mathbb{R}_{CSLN}$, $s \in CSLNcl(\mathbb{R}_{CSLN})$ but $s \notin \mathbb{R}_{CSLN}$.

But $s \notin \mathbb{R}_{CSLN}$. Thus, $E_{CSLN} \cap \mathbb{R}_{CSLN} \neq \emptyset$ for every CSLN open set E_{CSLN} containing s. Thus, $s \in LND(\mathbb{R}_{CSLN})$.

Therefore,

$$LND(\mathbb{R}_{CSLN}) = CSLNcl(\mathbb{R}_{CSLN}) - \mathbb{R}_{CSLN},$$

if $\mathbb{R}_{CSLN}$ is a singleton set.

Definition 4.5

1. **Cubic Spherical Linguistic Neutrosophic semi-closed set** if

$$CSLNint(CSLNcl(\mathbb{R}_{CSLN})) \subseteq \mathbb{R}_{CSLN}.$$

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2. **Cubic Spherical Linguistic Neutrosophic semi-open set** if

$$\mathbb{R}_{CSLN} \subseteq CSLNcl(CSLNint(\mathbb{R}_{CSLN})).$$

3. **Cubic Spherical Linguistic Neutrosophic semi-pre closed** if

$$CSLNint(CSLNcl(CSLNint(\mathbb{R}_{CSLN}))) \subseteq \mathbb{R}_{CSLN}.$$

4. **Cubic Spherical Linguistic Neutrosophic semi-pre open** if

$$\mathbb{R}_{CSLN} \subseteq CSLNcl(CSLNint(CSLNcl(\mathbb{R}CSLN))).$$

5. **Cubic Spherical Linguistic Neutrosophic pre-closed** if

$$CSLNcl(CSLNint(\mathbb{R}_{CSLN})) \subseteq \mathbb{R}_{CSLN}.$$

6. **Cubic Spherical Linguistic Neutrosophic pre-open** if

$$\mathbb{R}_{CSLN} \subseteq CSLNint(CSLNcl(\mathbb{R}_{CSLN})).$$

7. **Cubic Spherical Linguistic Neutrosophic regular closed** if

$$\mathbb{R}_{CSLN} = CSLNint(CSLNcl(\mathbb{R}_{CSLN})).$$

8. **Cubic Spherical Linguistic Neutrosophic regular open** if

$$\mathbb{R}_{CSLN} = CSLNcl(CSLNint(\mathbb{R}_{CSLN})).$$

4 Conclusion

”We have developed a new type of topology called cubic spherical linguistic neutrosophic topological space, accompanied by illustrative examples. The foundational properties of this topology were explored, as well as the concepts of cubic spherical linguistic neutrosophic continuity and cubic spherical linguistic neutrosophic neighborhoods. Additionally, we investigated cubic spherical linguistic neutrosophic derived sets and cubic spherical linguistic neutrosophic dense sets.”

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