



Solving of First Order Initial Value Problem Using Fuzzy Kamal Transform in Neutrosophic Environment

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Abstract

This manuscript presents a novel approach for solving first-order initial value problems by leveraging the Fuzzy Kamal Transform within a Neutrosophic framework. By integrating fuzzy logic with Neutrosophic set theory, the method adeptly addresses uncertainties inherent in differential equations. The efficacy of this method is demonstrated through the exposition of various illustrative examples.

Keywords: Fuzzy transform; Neutrosophic logic; Kamal transform; Fuzzy Kamal transform

1. Introduction

The initial value problem (IVP) holds significant importance in modeling real-world phenomena, in the field of differential equations. Traditional techniques for solving these problems typically rely on precise and deterministic data. However, real-world data often presents challenges due to its imprecision, uncertainty, or incompleteness. To address these issues, fuzzy set theory and Neutrosophic logic have emerged as powerful tools for effectively managing uncertainty and vagueness.

Differential equation models typically feature well-defined initial conditions, while the associated variables commonly involve uncertainties. Hence, fuzzy notions are used in the field of differential equations. Zadeh [1] initially presented the core ideas of fuzzy sets and set operations. Fuzzy differential equations were solved using the Hukuhara differentiability notion [13, 14]. Abdelilah Kamal invented the Kamal transform, which is essentially derived from the Fourier transform. Kamal transforms, is exploited from Abdelilah and Hassan to solve an ordinary differential equation 2016 [16]. Fuzzy Kamal transform was developed by Le Yu and Yumei Chen [9]. Finally, the senses of some types of neutrosophic topological groups with respect to neutrosophic alpha open sets, new types of weakly neutrosophic crisp continuity, and new concepts of neutrosophic crisp open sets were informed by Imran et al. [24-26].

The Fuzzy Kamal Transform (FKT), a revolutionary mathematical method that expands the standard Kamal Transform to the fuzzy domain, offers a strong foundation for resolving differential equations with fuzzy beginning conditions. We investigate the use of FKT in a Neutrosophic setting in this publication, which extends the idea by including degrees of truth, indeterminacy, and falsehood.

Definition 1.1 [16] The definition of the Kamal transforms for the function $u(s)$ is

$$\mathcal{K}[u(s)] = \int_0^{\infty} u(s)e^{-\frac{s}{v}} ds = \mathcal{G}(v), \quad s \geq 0, \quad k_1 \leq v \leq k_2.$$

We say Kamal transform of $u(s)$ exists, if $\forall s \geq 0$, and $u(s)$ is piecewise continuous and of exponential order. These are the necessary and sufficient requirements for the Kamal transform to exist.

2. Neutrosophic Kamal Transform (\mathcal{N}_k – Transform)

This part illustrates how a fuzzy differential equation may be reduced to an algebraic system using the Fuzzy Integral Transform and the Neutrosophic Kamal Transform, represented by \mathcal{N}_k .

Definition 2.1: Suppose $u(s)$ represent a continuous function with Neutrosophic values. Assuming that $u(s)e^{-\frac{s}{v}}$ is an improper Neutrosophic Riemann integrable function over the interval $[0, \infty)$, the function satisfies the specified conditions. Then Neutrosophic Kamal transform is

$$\int_0^{\infty} u(s)e^{-\frac{s}{v}} ds,$$

And it is defined as:

$$\mathcal{G}(v) = \mathcal{K}[u(s)] = \int_0^{\infty} u(s)e^{-\frac{s}{v}} ds,$$

we have

$$u(s, \alpha) = \{u_T(s, \alpha), u_I(s, \alpha), u_F(s, \alpha)\},$$

These items are represented in a Neutrosophic triangular form

$$\bar{u}_T(s, \alpha) = [\underline{u}_T(s, \alpha), \bar{u}_T(s, \alpha)]$$

$$u_I(s, \alpha) = [\underline{u}_I(s, \alpha), \bar{u}_I(s, \alpha)]$$

$$u_F(s, \alpha) = [\underline{u}_F(s, \alpha), \bar{u}_F(s, \alpha)]$$

So:

$$\int_0^{\infty} u_T(s)e^{-\frac{s}{v}} ds = \left(\int_0^{\infty} \underline{u}_T(s)e^{-\frac{s}{v}} ds, \int_0^{\infty} \bar{u}_T(s)e^{-\frac{s}{v}} ds \right)$$

$$\int_0^{\infty} u_I(s)e^{-\frac{s}{v}} ds = \left(\int_0^{\infty} \underline{u}_I(s)e^{-\frac{s}{v}} ds, \int_0^{\infty} \bar{u}_I(s)e^{-\frac{s}{v}} ds \right)$$

$$\int_0^{\infty} u_F(s)e^{-\frac{s}{v}} ds = \left(\int_0^{\infty} \underline{u}_F(s)e^{-\frac{s}{v}} ds, \int_0^{\infty} \bar{u}_F(s)e^{-\frac{s}{v}} ds \right)$$

By using the classical Kamal Transform:

$$\mathcal{K}[\underline{u}_T(s, \alpha)] = \int_0^{\infty} \underline{u}_T(s)e^{-\frac{s}{v}} ds$$

$$\mathcal{K}[\bar{u}_T(s, \alpha)] = \int_0^{\infty} \bar{u}_T(s)e^{-\frac{s}{v}} ds$$

$$\mathcal{K}[\underline{u}_I(s, \alpha)] = \int_0^{\infty} \underline{u}_I(s)e^{-\frac{s}{v}} ds$$

$$\mathcal{K}[\bar{u}_I(s, \alpha)] = \int_0^{\infty} \bar{u}_I(s)e^{-\frac{s}{v}} ds$$

$$\mathcal{K}[\underline{u}_F(s, \alpha)] = \int_0^{\infty} \underline{u}_F(s)e^{-\frac{s}{v}} ds$$

$$\mathcal{K}[\bar{u}_F(s, \alpha)] = \int_0^{\infty} \bar{u}_F(s)e^{-\frac{s}{v}} ds$$

Then it follows:

$$\mathcal{N}_k [u(s)] = (\mathcal{K}[\underline{u}(s, \alpha)], \mathcal{K}[\bar{u}(s, \alpha)]) .$$

Theorem 2.1: suppose $u(s)$ be a continuous neutrosophic valued function, represent, $u_T(s) = [\underline{u}_{T\alpha}(s), \bar{u}_{T\alpha}(s)]$, $u_I(s) = [\underline{u}_{I\beta}(s), \bar{u}_{I\beta}(s)]$, $u_F(s) = [\underline{u}_{F\gamma}(s), \bar{u}_{F\gamma}(s)] \quad \forall \alpha, \beta, \gamma \in [0, 1]$, then

a. If u_T is first form- differentiable, then the differentiable functions $\underline{u}_{T\alpha}(s)$ and $\bar{u}_{T\alpha}(s)$ are represented by $u'(s) = [\underline{u}'_{T\alpha}(s), \bar{u}'_{T\alpha}(s)]$,

b. If u_T is second form-differentiable, then the differentiable functions $\underline{u}_{T\alpha}(s)$ and $\bar{u}_{T\alpha}(s)$ are represented by $u'(s) = [\underline{u}'_{T\alpha}(s), \bar{u}'_{T\alpha}(s)]$,

c. If u_I is differentiable in first form, then the differentiable functions $\underline{u}_{I\beta}(s)$ and $\bar{u}_{I\beta}(s)$ are represented by $u'(s) = [\underline{u}'_{I\beta}(s), \bar{u}'_{I\beta}(s)]$

d. If u_I is differentiable in second form, then the differentiable functions $\underline{u}_{I\beta}(s)$ and $\bar{u}_{I\beta}(s)$ are represented by $u'(s) = [\underline{u}'_{I\beta}(s), \bar{u}'_{I\beta}(s)]$

e. If u_F is differentiable in first form, then the differentiable functions $\underline{u}_{F\gamma}(s)$ and $\bar{u}_{F\gamma}(s)$ are represented by $u'(s) = [\underline{u}'_{F\gamma}(s), \bar{u}'_{F\gamma}(s)]$.

f. If u_F is differentiable in second form, then the differentiable functions $\underline{u}_{F\gamma}(s)$ and $\bar{u}_{F\gamma}(s)$ are represented by $u'(s) = [\underline{u}'_{F\gamma}(s), \bar{u}'_{F\gamma}(s)]$

Theorem2.2: Given an integrable neutrosophic v -valued function $u(s)$, and a Primitive $u'(s)$ of $u(s)$ on $[0, \infty]$, then

a- Where u is first form- differentiable, $\mathcal{N}_{\mathcal{X}} [u'(s)] = \frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)] \ominus u(0)$.

b- Where u is second form-differentiable, $\mathcal{N}_{\mathcal{X}} [u'(s)] = -u(0) \ominus -\frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)]$.

Proof: **a-** for any arbitrary fixed $\alpha \in [0,1]$, since $u'(s)$ is continuous fuzzy v -valued function then

$$\mathcal{N}_{\mathcal{X}} [u'(s)] = (\mathcal{K}[\underline{u}'(s, \alpha)], \mathcal{K}[\bar{u}'(s, \alpha)]) \quad (1.1)$$

By using the first-order Kamal derivative definition, we obtain:

$$\left. \begin{aligned} \mathcal{N}_{\mathcal{X}} [\underline{u}'(s)] &= \frac{1}{v} \mathcal{K} [\underline{u}(s, \alpha)] - \underline{u}(0, \alpha), \\ \mathcal{N}_{\mathcal{X}} [\bar{u}'(s)] &= \frac{1}{v} \mathcal{K} [\bar{u}(s, \alpha)] - \bar{u}(0, \alpha). \end{aligned} \right\} \quad (1.2)$$

If we substitute (1.2) in (1.1), we get :

$$\mathcal{N}_{\mathcal{X}} [u'(s)] = \left(\frac{1}{v} \mathcal{K} [\underline{u}(s, \alpha)] - \underline{u}(0, \alpha), \frac{1}{v} \mathcal{K} [\bar{u}(s, \alpha)] - \bar{u}(0, \alpha) \right)$$

$$\mathcal{N}_{\mathcal{X}} [u'(s)] = \frac{1}{v} \mathcal{K} [\underline{u}(s, \alpha), \bar{u}(s, \alpha)] - [\underline{u}(0, \alpha) - \bar{u}(0, \alpha)]$$

We obtain:

$$\mathcal{N}_{\mathcal{X}} [u'(s)] = \frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)] \ominus u(0)$$

b- Let us now assume that u is second form -differentiable, for any arbitrary $\alpha \in [0,1]$.

$$-u(0) \ominus -\frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)] = \left(-\bar{u}(0, \alpha) + \frac{1}{v} \mathcal{K} [\bar{u}(s, \alpha)], \underline{u}(0, \alpha) + \frac{1}{v} \mathcal{K} [\underline{u}(s, \alpha)] \right)$$

Since

$$(-\bar{u}(0, \alpha) + \frac{1}{v} \mathcal{K} [\bar{u}(s, \alpha)]) = \mathcal{K} [\bar{u}'(s, \alpha)]$$

$$\text{, and } (-\underline{u}(0, \alpha) + \frac{1}{v} \mathcal{K} [\underline{u}(s, \alpha)]) = \mathcal{K} [\underline{u}'(s, \alpha)]$$

$$-u(0) \ominus -\frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)] = (\mathcal{K}[\bar{u}'(s, \alpha)], \mathcal{K}[\underline{u}'(s, \alpha)])$$

$$-u(0) \ominus -\frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)] = (\mathcal{K} [\bar{u}'(s, \alpha)], [\underline{u}'(s, \alpha)])$$

Since u is second form -differentiable, it follows that :

$$-u(0) \ominus -\frac{1}{v} \mathcal{N}_{\mathcal{X}} [u(s)] = \mathcal{K} [\bar{u}'(s)].$$

3. Solution of Neutrosophic Initial value Problem of First order

In a Neutrosophic context, the differential equation is solved. This proves that the Neutrosophic environment is a differential equation with initial conditions that involve belongingness, non-belongingness, and indeterminacy parameters.

Example 3.1

Regard the problem of the Neutrosophic initial value

$$\begin{aligned} u'(s) &= u(s), & u_{T\alpha}(0, \alpha) &= [-\alpha, \alpha] \\ u_{T\beta}(0, \beta) &= [\beta - 1, 1 - \beta] \\ u_{T\gamma}(0, \gamma) &= [\gamma, 2 - \gamma] \end{aligned}$$

Solution By utilizing the Neutrosophic Kamal transform, we see how truth, indeterminacy, and falsity materialize as:

$$\mathcal{N}_{\mathcal{K}} [u'(s)] = \mathcal{N}_{\mathcal{K}} [u(s)]$$

Case 1: if $u(s)$ be first form -differentiable,

$$\begin{aligned} \frac{1}{v} \mathcal{N}_{\mathcal{K}} [u(s)] \ominus u(0) &= \mathcal{N}_{\mathcal{K}} [u(s)] \\ \frac{1}{v} \mathcal{K}[\underline{u}_T(s, \alpha) - \underline{u}_T(0, \alpha)] &= \mathcal{K}[\underline{u}_T(s, \alpha)] \\ \text{Or } \frac{1}{v} \mathcal{K}[\underline{u}_{T\alpha}(s) - \underline{u}_{T\alpha}(0)] &= \mathcal{K}[\underline{u}_{T\alpha}(s)] \\ \left(\frac{1}{v} - 1\right) \mathcal{K}[\underline{u}_{T\alpha}(s)] &= -\alpha \\ \mathcal{K}[\underline{u}_{T\alpha}(s)] &= \frac{-\alpha v}{1-v} \text{ also } \mathcal{K}[\bar{u}_{T\alpha}(s)] = \frac{-\alpha v}{1-v} \end{aligned}$$

Now we use inverse Kamal Transform, obtain

$$\underline{u}_{T\alpha}(s) = -\alpha e^s, \quad \bar{u}_{T\alpha}(s) = \alpha e^s$$

In similar way, we get

$$\begin{aligned} \underline{u}_{T\beta}(s) &= (\beta - 1)e^s, & \bar{u}_{T\beta}(s) &= (1 - \beta)e^s \\ \underline{u}_{T\gamma}(s) &= \gamma e^s, & \bar{u}_{T\gamma}(s) &= (2 - \gamma)e^s \end{aligned}$$

Case 2: if $u(s)$ be second form -differentiable,

$$\begin{aligned} -u(0) \ominus -\frac{1}{v} \mathcal{N}_{\mathcal{K}} [u(s)] &= \mathcal{N}_{\mathcal{K}} [u(s)] \\ -\bar{u}_{T\alpha}(0) + \frac{1}{v} \mathcal{K}[\bar{u}_{T\alpha}(s)] &= \mathcal{K}[\underline{u}_{T\alpha}(s)] \\ -\underline{u}_{T\alpha}(0) + \frac{1}{v} \mathcal{K}[\underline{u}_{T\alpha}(s)] &= \mathcal{K}[\bar{u}_{T\alpha}(s)] \\ \frac{1}{v} \mathcal{K}[\bar{u}_{T\alpha}(s)] - \mathcal{K}[\underline{u}_{T\alpha}(s)] &= \bar{u}_{T\alpha}(0) \tag{3.1} \end{aligned}$$

$$\frac{1}{v} \mathcal{K}[\underline{u}_{T\alpha}(s)] - \mathcal{K}[\bar{u}_{T\alpha}(s)] = \underline{u}_{T\alpha}(0) \tag{3.2}$$

Solving equation 3.1 and 3.2 we have:

$$\begin{aligned} \underline{u}_{T\alpha}(s) &= \alpha \sinh s - \alpha \cosh s \\ \bar{u}_{T\alpha}(s) &= -\alpha \sinh s + \alpha \cosh s \end{aligned}$$

In similar way, we can obtain

$$\begin{aligned} \underline{u}_{T\beta}(s) &= (1 - \beta) \sinh s + (\beta - 1) \cosh s \\ \bar{u}_{T\beta}(s) &= (\beta - 1) \sinh s + (1 - \beta) \cosh s \end{aligned}$$

and

$$\begin{aligned}\underline{u}_{T\gamma}(s) &= (2 - \gamma) \sinh s + \gamma \cosh s, \\ \bar{u}_{T\gamma}(s) &= \gamma \sinh s + (2 - \gamma) \cosh s\end{aligned}$$

Example 3.2

Regard the problem of the Neutrosophic initial value

$$\begin{aligned}u'(s) - u(s) &= s^2, & u_{T\alpha}(0, \alpha) &= [\alpha - 0.5, 0.5 - \alpha] \\ u_{T\beta}(0, \beta) &= [\beta - 0.2, 0.2 - \beta] \\ u_{T\gamma}(0, \gamma) &= [\gamma - 0.1, 0.1 - \gamma]\end{aligned}$$

Solution In the similar way of example 3.1, we observe the manifestations of truth, indeterminacy, and falsity as:

$$\mathcal{N}_{\mathcal{K}}[u'(s)] - \mathcal{K}[u(s)] = \mathcal{K}[s^2]$$

Case 1: if $u(s)$ be first form- differentiable,

$$\begin{aligned}\frac{1}{\nu} \mathcal{N}_{\mathcal{K}}[u(s)] \ominus u(0) - \mathcal{N}_{\mathcal{K}}[u(s)] &= 2\nu^2 \\ \left(\frac{1}{\nu} - 1\right) \mathcal{K}[u_T(s, \alpha) - \underline{u}_T(0, \alpha)] &= \mathcal{K}[u_T(s, \alpha)] \\ \left(\frac{1}{\nu} - 1\right) \mathcal{K}[u_{T\alpha}(s)] &= (\alpha - 0.5) + 2\nu^2 \\ \left(\frac{1}{\nu} - 1\right) \mathcal{K}[\bar{u}_{T\alpha}(s)] &= (\alpha - 0.5) + 2\nu^3 \\ \mathcal{K}[u_{T\alpha}(s)] &= \frac{(\alpha - 0.5)\nu}{1 - \nu} + \frac{2\nu^4}{1 - \nu}\end{aligned}$$

Now we use inverse Kamal Transform, we have

$$\begin{aligned}\underline{u}_{T\alpha}(s) &= (\alpha - 0.5)e^s + 2\left(\frac{-s^2}{2} - s - 1 + e^s\right) \\ \bar{u}_{T\alpha}(s) &= (0.5 - \alpha)e^s + 2\left(\frac{-s^2}{2} - s - 1 + e^s\right)\end{aligned}$$

In similar way, obtain

$$\begin{aligned}\underline{u}_{T\beta}(s) &= (\beta - 0.2)e^s + 2\left(\frac{-s^2}{2} - s - 1 + e^s\right) \\ \bar{u}_{T\beta}(s) &= (0.2 - \beta)e^s + 2\left(\frac{-s^2}{2} - s - 1 + e^s\right) \\ \underline{u}_{T\gamma}(s) &= (\gamma - 0.1)e^s + 2\left(\frac{-s^2}{2} - s - 1 + e^s\right) \\ \bar{u}_{T\gamma}(s) &= (0.1 - \gamma)e^s + 2\left(\frac{-s^2}{2} - s - 1 + e^s\right)\end{aligned}$$

Case 2: if $u(s)$ be second form-differentiable,

$$\begin{aligned}-u(0) \ominus -\frac{1}{\nu} \mathcal{N}_{\mathcal{K}}[u(s)] - \mathcal{N}_{\mathcal{K}}[u(s)] &= 2\nu^3 \\ -\bar{u}_{T\alpha}(0) + \frac{1}{\nu} \mathcal{K}[\bar{u}_{T\alpha}(s)] - \mathcal{K}[u_{T\alpha}(s)] &= 2\nu^3 \\ -\underline{u}_{T\alpha}(0) + \frac{1}{\nu} \mathcal{K}[u_{T\alpha}(s)] - \mathcal{K}[\bar{u}_{T\alpha}(s)] &= 2\nu^3 \\ \frac{1}{\nu} \mathcal{K}[\bar{u}_{T\alpha}(s)] - \mathcal{K}[u_{T\alpha}(s)] &= (0.5 - \alpha) + 2\nu^3 \quad (3.3)\end{aligned}$$

$$\frac{1}{\nu} \mathcal{K}[u_{T\alpha}(s)] - \mathcal{K}[\bar{u}_{T\alpha}(s)] = (\alpha - 0.5) + 2\nu^3 \quad (3.4)$$

Solving equation 3.3 and 3.4 then we have :

$$\underline{u}_{T\alpha}(s) = (\alpha - 0.5) \cosh s - (0.5 - \alpha) \sinh s + 2e^s - s$$

$$\bar{u}_{T\alpha}(s) = (0.5 - \alpha) \cosh s - (\alpha - 0.5) \sinh s + 2e^s - s$$

In similar way, we can get

$$\underline{u}_{T\beta}(s) = (\beta - 0.2) \cosh s - (0.2 - \beta) \sinh s + 2e^s - s$$

$$\bar{u}_{T\beta}(s) = (0.2 - \beta) \cosh s - (\beta - 0.2) \sinh s + 2e^s - s$$

and

$$\underline{u}_{T\gamma}(s) = (\gamma - 0.1) \cosh s - (0.1 - \gamma) \sinh s + 2e^s - s$$

$$\bar{u}_{T\gamma}(s) = (0.1 - \gamma) \cosh s - (\gamma - 0.1) \sinh s + 2e^s - s$$

Conclusion

In this manuscript, we have presented a novel approach for solving first-order initial value problems (IVPs) by integrating the Fuzzy Kamal Transform (FKT) within a Neutrosophic environment. Our method effectively addresses the inherent uncertainties and indeterminacies present in real-world data by leveraging the flexibility of fuzzy set theory and Neutrosophic logic. The results obtained from our methodology highlight the potential of FKT in handling complex differential equations with fuzzy initial conditions. This hybrid approach not only enhances the precision of the solutions but also offers deeper insights into the dynamics of the modeled phenomena. Future research can explore the extension of this method to higher-order differential equations and other types of boundary value problems. Additionally, the integration of FKT with other advanced mathematical techniques could further expand its applicability and effectiveness in various scientific and engineering domains. In summary, the Fuzzy Kamal Transform in a Neutrosophic environment offers a promising framework for solving differential equations under uncertainty, paving the way for more accurate and insightful analyses of complex systems.

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References

- [1] Zadeh, L.A.(1965)"Fuzzy sets", Information and Control ,Vol.8,No.3,pp.338-353.
- [2] Atanassov, K.T.(1986)"Intuitionistic fuzzy sets", Fuzzy Sets and Systems,Vol.20,No.1,pp.87-96.
- [3] Ejegwa, P.A.; Akowe, S.O.; Otene, P.M.; Ikyule, J.M.(2014)"An Overview on intuitionistic Fuzzy sets ,Int. J. of Scientific and Technology Research ,Vol.3,No.3,pp.142-144.
- [4] Smarandache, F.(2006)"Neutrosophic set-A generalization of the intuitionistic Fuzzy set", Conference:2006 IEEE Int.Con. On Granular Computing, GrC, 2006, Atlanta,Georgia,USA.
- [5] Wang,H.;Smarandache,F.;Zhang,Y.Q.;Sunderraman,R.(2010) "Single Valued Neutrosophic sets", Multispace a Multistructure,Vol.4,pp.410- 413.
- [6] Chakraborty,A.;Mondal,S.P.;Ahmadian,A.;Senu,N.;Alam,S.; Salahshour ,S.(2018) "Different forms of triangular Neutrosophic numbers,de-Neutrosophication techniques,And their applications",Symmetry,Vol.10,pp.327.
- [7] Jabbar, S.M., Alkiffai, A.N., Mera, A.(2023)" Solving Thermal System Problem Via Fuzzy Mohand Transform Under Neutrosophic Initial Conditions", Int. j. of Neutrosophic Science 21(1), pp. 8–13
- [8] Jabbar, S.M., Alkiffai, A.N., Mera, A.(2024)" Fuzzy Logic Used to Solve ODEs of Second Order Under Neutrosophic Initial Conditions", Int. j. of Neutrosophic Science 2024, 23(1), pp. 51–58
- [9] Le Yu, Yumei Chen. (2022). Solving Fuzzy Volterra Integro-Differential Equations By Using Fuzzy Kamal Transform. International Journal of Research in Advent Technology, 10(4), 1-5.
- [10] D. Dubois, H. Prade, (1982) "Towards fuzzy differential calculus part 1: Integration of fuzzy mappings". Fuzzy Sets and Systems, vol. 8, pp. 1-17.
- [11] D. Dubois, H. Prade, (1982) "Towards fuzzy differential calculus part 2: Integration on fuzzy intervals".

- Fuzzy Sets and Systems. vol. 8, pp. 105-116.
- [12] D. Dubois, H. Prade.(1982) "Towards fuzzy differential calculus part 3: Differentiation". Fuzzy Sets and Systems. vol. 8, pp. 225-233.
- [13] M.L. Puri, D.A. Ralescu.(1983) "Differentials of fuzzy functions". Journal of Mathematical Analysis and Applications. vol. 91, pp. 552-558.
- [14] J.J. Buckley, T. Feuring.(2000) "Fuzzy differential equations". Fuzzy Sets Systems. vol. 110, pp. 43-54.
- [15] R. Goetschel, W. Voxman,(1986) "Elementary fuzzy calculus". Fuzzy Sets Systems. vol. 18, pp. 31-43.
- [16] K. Abdelilah, S. Hassan,(2016) "The new integral transform ``Kamal Transform''". Advances in Theoretical and Applied Mathematics, vol. 11, pp. 451-458.
- [17] B. Bede, S.G. Gal,(2005) "Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations". Fuzzy Sets and Systems, vol. 151, pp.581-599.
- [18] Y. Chalco-Cano, H. Roman-Flores, (2008) "On new solutions of fuzzy differential equations". Chaos, Solitons Fractals, vol. 38, pp.112-119.
- [19] R.A. Fadhil.(2017) "Convolution for Kamal and Mahgoub transforms". Bulletin of Mathematics and Statistics Research, vol. 5, pp.11-16.
- [20] S. Aggarwal, A.R. Gupta, N. Asthana, D.P. Singh,(2018) "Application of Kamal transform for solving population growth and decay problems". Global Journal of Engineering Science and Researches, vol. 5, pp.254-260.
- [21] S. Aggarwal,(2018) "Kamal transform of Bessel's functions". International Journal of Research and Innovation in Applied Science, vol. 3, pp.1-4.
- [22] Saini,R.K.;Sangal,A.;Manisha(2020)"Application of single valued Trapezoidal Neutrosophic numbers in transportation problem",Neutrosophic Sets and Systems",Vol.35,pp.563-583.
- [23] Awolola,J.(2020)"A Note on the concept of α -level sets of Neutrosophic set", Neutrosophic Sets and Systems,Vol.31,pp.120-126.
- [24] Q. H. Imran, A. H. M. Al-Obaidi, & Smarandache, F., On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets. Neutrosophic Sets and Systems, 32, (2020), 425-434.
- [25] Q. H. Imran, R. K. Al-Hamido and A. H. M. Al-Obaidi, on new types of weakly neutrosophic crisp continuity. Neutrosophic Sets and Systems, 38(2020), 179-187.
- [26] Q. H. Imran, K. S. Tanak and A. H. M. Al-Obaidi, on new concepts of neutrosophic crisp open sets. Journal of Interdisciplinary Mathematics, 25(2)(2022), 563-572.