A new generalization of interval-valued Q-neutrosophic soft matrix and its applications

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Abstract

Decision-making theory is an effective way to help the decision-maker take the right path to solve a problem. Among the applications of this theory is the medical field, i.e. allowing the decision maker (doctor) to analyze patient data and judge the result of this analysis as to whether the patient is infected or not. In this path and to enrich this theory with more flexible mathematical methods, we present in this work a more flexible expanded method for a previous concept called Interval-valued Q-neutrosophic soft matrix (IV-Q-NSM) as a new generalization of previous mathematical tools. These tools deal with the two-dimensional uncertainty issues that exist in many areas of life. Next, some ordinary algebraic properties and matrix operations have also been studied. After that, we present a new methodology for the decision-making (DM) selection problems in medical diagnoses.

Keywords: Neutrosophic set; Q- neutrosophic set; Q-neutrosophic soft matrix; interval-valued neutrosophic set; soft set; Interval-valued neutrosophic soft set

1. Introduction

In this contemporary world, researchers and practitioners work together in different scientific areas to find solutions using efficient and effective decision-making. Individuals sometimes make quick decisions, but some decisions require a well-designed method that fits the problem environment. Some risks and uncertainties may arise during the decision-making process. Factors such as vagueness, uncertainty, and ambiguity can make the decision-making process more complex in real-life problems. In this context, experts incorporate special theories, new approaches, and methodologies to understand circumstances and evaluate alternatives before selecting the most appropriate one. From macro to micro levels, the decision-making process is not only used in scientific research areas but has also been applied to different fields such as finance, economics, games, and marketing. The first researcher to deal with this environment was Zadeh [1] when he presented the fuzzy sets (FS) theory as an extension of a traditional set (crisp set). With FS, Zadeh gives every component in fuzziness environment one membership degree belonged [0,1]. This concept has inspired researchers and prompted them to present a large number of studies in various fields [2-5]. Day by day life situations are becoming more complicated, so it is necessary to create new mathematical tools or develop existing ones and so Smarandache [6] in 1998 presented a new theory called neutrosophic set (NS) theory that is considered an advanced glimpse of Zadeh's previous theory. With NS, Smarandache gives every component in fuzziness environment three membership degrees all of them belonged [0,1]. Deli [7] introduced the concept of an interval-valued neutrosophic soft set (IV-NSS) as an

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extension of both NSs and soft sets (SSs). Al-Sharqi et al [8] put up IV-NSS with some mathematical tools like algebra [9,10], graph [11,12], complex [13,14], and possibility [15,16]. Follow these works Al-Quran et al [17] combin' NSs with some fuzziness extensions [18,19]. Ali and Mohammed [20,21] introduce new topological tools for fuzzy neutrosophic environments. Palanikumar et al. [22,23] combine NSs with many mathematical tools to increase the accuracy of MCDM methods. Al-Qudah et al. [24,25] handled complex uncertainty sets in DM problems when they presented some complex uncertainty works [26,27]. Furthermore, when information is converted from human knowledge to mathematical formulas and vice versa, its representation retains its entire meaning. Accordingly, these tools have been applied in many life fields such as science [28,29], engineering[30,31], and administrative sciences[32,33]. Recently Ghazi and Al-Salihi [34] introduced a new hyper model known as IV-Q-NSSs as both SS and NSs under interval setting Q-two-dimensional universal information [35,36]. In this work we reintroduce the concept of matrix but in the form of a matrix in order to benefit as much as possible from the properties and advantages of the techniques present in the matrix when we proposed a new model called IV-QNS-Matrix.

2. Basic definitions and background

Definition 2.1 [1]**:** A following structure

$$\mathrm{N} = \left\{\mathrm{v}_j, \hat{P}^t(\mathrm{v}_j), \hat{P}^i(\mathrm{v}_j), \hat{P}^f(\mathrm{v}_j) \middle| \mathrm{v}_j \in \mathrm{V}\right\}$$

called NS on universal set V. Such that $\mathrm{N} = \hat{P}^t(\mathrm{v}_j), \hat{P}^i(\mathrm{v}_j), \hat{P}^f(\mathrm{v}_j): \mathrm{V} \to [0,1]$ and $\hat{P}^t(\mathrm{v}_j), \hat{P}^i(\mathrm{v}_j), \hat{P}^f(\mathrm{v}_j) \in [0,1]$.

Definition 2.2 [1]: A following structure

$$\mathrm{N}_{\mathfrak{Q}} = \left\{\mathrm{v}_j, \hat{P}^t_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v, \mathrm{q}) \middle| (v, \mathrm{q}) \in \mathrm{V} \times \mathfrak{Q}\right\}$$

called Q-NS on Q× V. Such that $\mathrm{N}_{\mathfrak{Q}} = \hat{P}^t_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v, \mathrm{q}): \mathrm{V} \to [0,1]$ and $\hat{P}^t_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v, \mathrm{q}) \in [0,1]$.

Definition 2.3 [1]: A following structure

$$IVN_Q = \{e \in \bar{\mathrm{A}}, < \hat{P}^t_{\mathfrak{Q}}(v, \mathrm{q})(e), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v, \mathrm{q})(e), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v, \mathrm{q})(e) > | (v, \mathrm{q}) \in V \times \mathfrak{Q}\}$$

Where

$$\hat{P}^t_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e}) = [\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(e), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e})]$$

$$\hat{P}^i_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e}) = [\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(e), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e})]$$

$$\hat{P}^f_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e}) = [\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e}), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}}}(v, \mathrm{q})(\mathrm{e})]$$

called IV-Q-NS on Q× V. Such that $\mathrm{IVN}_{\mathfrak{Q}} = \hat{P}^t_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v, \mathrm{q}): \mathrm{V} \to [0,1]$ and $\hat{P}^t_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(v, \mathrm{q}), \hat{P}^{\mathrm{f}}_{\mathfrak{Q}}(v, \mathrm{q}) \in [0,1]$.

Example 2.4 Let $\mathrm{V} = \{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ present three houses we are interested to analyzing their attractiveness that one person (user) is wishes to buying one of them and $Q = \{q_1, q_2\}$ be a set constituting two cities and $\mathcal{E} = \{e_1, e_2, e_3\}$ be a collection of attribute of these three houses. Now, let us evaluate this attractiveness according to our model $(IV - Q - NSS)$ as following:

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$\widehat{\mathfrak{P}}_{\mathcal{Q}_{\bar{A}}} =$

$\left\{\left(e_1, \frac{\langle [0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,q_1)}, \frac{\langle [0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1,q_2)}\right.\right.$

$\frac{\langle [0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_2,q_1)}, \frac{\langle [0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_2,q_2)}$

$\left.\frac{\langle [0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(v_3,q_1)}, \frac{\langle [0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(v_3,q)}\right)$

$\left(e_2, \frac{\langle [0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(v_1,q_1)}, \frac{\langle [0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(v_1,q_2)}\right.$

$\frac{\langle [0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(v_2,q_1)}, \frac{\langle [0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(v_2,q_2)}$

$\left.\frac{\langle [0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(v,q_1)}, \frac{\langle [0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(v_3,q_2)}\right)$

$\left(e_3, \frac{\langle [0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(v_1,q_1)}, \frac{\langle [0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(v_1,q_2)}\right.$

$\frac{\langle [0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(v_2,q_1)}, \frac{\langle [0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(v_2,q_2)}$

$\left.\left.\frac{\langle [0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(v_3,q_1)}, \frac{\langle [0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(v_3,q_2)}\right)\right\}$

3. An IV-Q-NSs in Matrix Form

In this section, we will demonstrate the algebraic ability with our proposed concept by presenting our proposed concept in this thesis with a matrix system.

Definition 3.1. Let $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, < \hat{P}^t_{\mathfrak{Q}}(\hat{v},\breve{q})(e) > |(\hat{v},\breve{q}) \in V \times \mathfrak{Q}\} \in IV-Q-NSS(V)$.Then, the IV-Q-NSs in matrix form given as a following matrix :

$$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{m\times n} = \begin{bmatrix} \backslash & e_1 & e_1 & \cdots & e_m \\ (\hat{v}_1,\breve{q}_1) & \hat{P}_{\mathfrak{Q}_{\bar{A}_{1,1}}} & \hat{P}_{\mathfrak{Q}_{\bar{A}_{1,2}}} & \cdots & \hat{P}_{\mathfrak{Q}_{\bar{A}_{1,j}}} \\ (\hat{v}_2,\breve{q}_2) & \hat{P}_{\mathfrak{Q}_{\bar{A}_{2,1}}} & \hat{P}_{\mathfrak{Q}_{\bar{A}_{2,2}}} & \cdots & \hat{P}_{\mathfrak{Q}_{\bar{A}_{2,j}}} \\ \vdots & \vdots & \vdots & \vdots & \\ (\hat{v}_n,\breve{q}_j) & \hat{P}_{\mathfrak{Q}_{\bar{A}_{i,1}}} & \hat{P}_{\mathfrak{Q}_{\bar{A}_{i,2}}} & \cdots & \hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \end{bmatrix}_{m\times n}$$

Where $\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} = \left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle$

, $i = 1,2,3,\ldots,n$ number of Rows and $j = 1,2,3,\ldots,m$ number of Columns.

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Example 3.2. Assume that the values given in the **example 2.4** are as follows:

$\widehat{\mathfrak{P}}_{\mathcal{Q}_{\bar{A}}} =$

$$\left\{\left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,q_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1,q_2)}\right.\right.$$

$$\frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_2,q_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_2,q_2)}$$

$$\left.\frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(v_3,q_1)}, \frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(v_3,q)}\right)$$

$$\left(e_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(v_1,q_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(v_1,q_2)}\right.$$

$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(v_2,q_1)}, \frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(v_2,q_2)}$$

$$\left.\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(v,q_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(v_3,q_2)}\right)$$

$$\left(e_3, \frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(v_1,q_1)}, \frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(v_1,q_2)}\right.$$

$$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(v_2,q_1)}, \frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(v_2,q_2)}$$

$$\left.\left.\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(v_3,q_1)}, \frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(v_3,q_2)}\right)\right\}$$

Now, through the **definition 3.1**, these values can be represented as a matrix as follows:

$$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{6\times 3} = \begin{bmatrix} \backslash & e_1 & e_2 & e_3 \\ (u_1,q_1) & \langle[0.2,0.5]\rangle & \langle[0.2,0.2]\rangle & \langle[0.5,0.8]\rangle \\ (u_1,q_2) & \langle[0.1,0.4]\rangle & \langle[0.1,0.8]\rangle & \langle[0.4,0.7]\rangle \\ (u_2,q_1) & \langle[0.3,0.4]\rangle & \langle[0.5,0.8]\rangle & \langle[0.3,0.6]\rangle \\ (u_2,q_2) & \langle[0.6,0.8]\rangle & \langle[0.1,0.2]\rangle & \langle[0.5,0.6]\rangle \\ (u_3,q_1) & \langle[0.1,0.5]\rangle & \langle[0.2,0.5]\rangle & \langle[0.3,0.6]\rangle \\ (u_3,q_2) & \langle[0.4,0.8]\rangle & \langle[0.4,0.5]\rangle & \langle[0.4,0.8]\rangle \end{bmatrix}_{6\times 3}$$

In the above matrix, the row represents a set of $\mathcal{E} = \{e_1, e_2, e_3\}$, while the column represents a set of both $\mathfrak{U} = \{u_1, u_2, u_3\}$ and $\mathfrak{Q} = \{q_1, q_2\}$.

Definition 3.3 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then we have the following point:

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1. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is called zero-IV-Q-NSM if $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = [\langle [0,0],[1,1],[1,1\rangle]_{m\times n}$.
2. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is absolute-IV-Q-NSM if $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = [\langle [1,1],[0,0],[0,0\rangle]_{m\times n}$.
3. An IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is IV-Q-NS-submatrix $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ and denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \subseteq \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ if the degrees: $\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \leq \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \leq \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$ and $\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.
4. A square IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ has a transpose by exchange of rows and columns of $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and it denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^t_{m\times n}$ such that

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^t_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]^t_{m\times n}$$
$$= \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right]\right\rangle\right]^t_{m\times n}.$$

5. A square IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ is denoted by symmetric square IV-Q-NSM if $\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} = \hat{P}_{\mathfrak{Q}_{\bar{A}_{j,i}}}$ i.e $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^t_{m\times n}$.

Definition 3.4 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then, the fundamental algebraic operations on IV-Q-NSM are defined as follows:

1. **Addition:** The addition operation of two IV-Q-NSM is defined as follow:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} + \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}} - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$,

$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$

$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \times \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.

2. **Subtraction:** The operation of subtraction of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} - \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|$,

$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = max\left(\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = max\left(\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right)$

$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \left|\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} - \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right|$.

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3. Multiplication: The operation of multiplication for two IV-Q-NSMs are defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right]_{m\times n} \times \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} \times \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} \times \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}$$

$$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}} - \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} \times \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}} - \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} \times \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}$$

$$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}} - \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} \times \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}} - \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} \times \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}$$

4. Scalar Multiplication: The operation of scalar multiplication for two IV-Q-NSMs are defined as follows:

$$K\left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[K\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}, K\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right], \left[K\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}, \mathrm{K}\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right], \left[K\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}, K\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where $K \in R$.

5. Arithmetic mean: The arithmetic mean of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right]_{m\times n} \circledast \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}$$

$$\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}$$

6. Weighted arithmetic mean: The weighted arithmetic mean of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right]_{m\times n} \circledast^{w} \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{w\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + w\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\widehat{wP}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + w\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{w\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + w\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}$$

$$, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\widehat{wP}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + \widehat{wP}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{w\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + w\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \frac{\widehat{wP}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} + w\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}{2}, \text{ where } w \in R.$$

7. Geometric mean: The geometric of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}}\right]_{m\times n} \odot \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Where

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \sqrt{\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} . \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \sqrt{\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} . \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \sqrt{\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} . \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \sqrt{\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} . \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}$$

$$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \sqrt{\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} . \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{\mathrm{C}}_{i,j}}} = \sqrt{\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{A}}_{i,j}}} . \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{\mathrm{B}}_{i,j}}}}$$

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8. Weighted Geometric mean: The weighted geometric of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \circledcirc^{w} \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$Where$

$$\widehat{wP}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \sqrt{w\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . w\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \widehat{wP}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \sqrt{\widehat{wP}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . w\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}},$$
$$\widehat{wP}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \sqrt{w\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . w\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \widehat{wP}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \sqrt{\widehat{wP}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . w\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}$$
$$\widehat{wP}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \sqrt{\widehat{wP}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \widehat{wP}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \widehat{wP}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \sqrt{w\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . w\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}.$$

9. Harmonic mean: The Harmonic mean of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \boxdot \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$Where$

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \frac{2\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \frac{2\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \frac{2\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \frac{2\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}$$
$$\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \frac{2\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = \frac{2\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}$$

10. Weighted Harmonic mean: The Weighted Harmonic mean of two IV-Q-NSMs is defined as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \boxdot^{W} \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$Where$

$$\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = W\left(\frac{2W\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}\right), \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = W\left(\frac{2\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}\right), \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = W\left(\frac{2\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}\right), \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} =$$
$$W\left(\frac{2\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}\right), \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = W\left(\frac{2\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}\right), \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}} = W\left(\frac{2\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} . \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}{\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}}\right), W \in R.$$

Proposition 3.5 : Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$, $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2, \dots, r$, $j = 1,2, \dots, s$. Then, the following point have been satsfied:

1. For both $\alpha, \beta \in R$ then $\alpha\left(\beta\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right) = \alpha.\beta\left(\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)$.

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2. For $\alpha \leq \beta$ then $\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \leq \beta\left[\hat{P}_{\mathfrak{Q}_{B_{i,j}}}\right]_{m\times n}$.

3. If $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \leq \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ then $\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \leq \beta\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$.

4. $\left(\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^t = \alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^t$.

5. $\left(\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^t\right)^t = \left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$.

Prof (1). Take

$$\alpha\left(\beta\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right) = \alpha\left[\left\langle\left[\widehat{\beta P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \beta\,\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\beta\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \beta\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\beta\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \beta\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$$= \left[\left\langle\left[\alpha\widehat{\beta P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \alpha\beta\,\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\alpha\beta\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \alpha\beta\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\alpha\beta\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \alpha\beta\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

$$= \alpha\beta\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n} = \alpha\beta\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}.$$

Prof (3). Suppose that $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \leq \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ then $\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \leq \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \leq \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$ and $\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.

Now for $\alpha \in R$, then
$\alpha\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \leq \widehat{\alpha P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \alpha\,\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \leq \alpha\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \alpha\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \alpha\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \alpha\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \alpha\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$ and
$\alpha\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \alpha\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \alpha\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} \geq \alpha\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}$.

Then $\alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \leq \alpha\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$.

Prof (4). We have $\left(\left[\widehat{\alpha P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^t \in IV-Q-NSM$, then

$$\left(\left[\widehat{\alpha P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^t = \left[\left\langle\left[\widehat{\alpha P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}}, \alpha\,\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right],\left[\alpha\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}}, \alpha\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right],\left[\alpha\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{j,i}}}, \alpha\hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{j,i}}}\right]\right\rangle\right]_{m\times n}$$

$$= \alpha\left(\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]\right)^t$$

$$= \alpha\left(\left[\widehat{\alpha P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}\right)^t.$$

Definition 3.6 The unionof two IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \cup \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ where

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}, \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} =$
$\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} =$
$\left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right],\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then,

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$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{cases} \left(max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], max\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \\ \left(min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \\ \left(min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \end{cases}$$

Definition 3.7 The intersectionof two IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \cap \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ where

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}, \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{C}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{C_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then,

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{cases} \left(min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], max\left[\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \\ \left(max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \\ \left(max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], min\left[\hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \end{cases}$$

Definition 3.8 The complement of two IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{c}$ where

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{c} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^c, \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^c, \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]^c\right\rangle\right]_{m\times n}, \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ =for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then,

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{c} = \begin{cases} \left(\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right) \\ \left(\left[1 - \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, 1 - \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right) \\ \left(\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]\right) \end{cases}$$

Definition 3.9 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then the OR-operation ($\vee$) of IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \otimes \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ and defined as following form:

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \vee \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}$ such that

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \max min\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \min max\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \min max\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

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Definition 3.10 Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ be two IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{B}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{B_{i,j}}}\right]\right\rangle\right]_{m\times n}$ for all $i = 1,2,\dots,r, j = 1,2,\dots,s$. Then the AND-operation ($\wedge$) of IV-Q-NSMs denoted by $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \odot \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ and defined as following form:

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \wedge \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n}$ such that

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \left[\left\langle \min max\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \max min\left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \max min\left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$$

Example 3.11. Take two IV-Q-NSSs:

$$\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}} = \left\{\left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,q_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1,q_2)}\right)\right\} \text{ and}$$

$$\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}} = \left\{\left(e_1, \frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_1,q_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_1,q_2)}\right)\right\}$$ Then:both $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ and $\left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n}$ it is presented as follows:

$$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{2\times 1} = \begin{bmatrix}[0.2,0.8],[0.1,0.7],[0.4,0.8]\\ [0.1,0.4],[0.5,0.8],[0.7,0.8]\end{bmatrix}_{2\times 1}, \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{2\times 1} = \begin{bmatrix}[0.3,0.6],[0.2,0.7],[0.5,0.8]\\ [0.4,0.6],[0.2,0.9],[0.5,0.7]\end{bmatrix}_{2\times 1}$$

Then:

1. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} + \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.3,0.9],[0.4,0.9],[0.5,0.9]\\ [0.4,0.5],[0.7,0.6],[0.8,0.9]\end{bmatrix}_{2\times 1}$
2. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} - \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.8],[0.1,0.7],[0.4,0.8]\\ [0.1,0.4],[0.5,0.8],[0.7,0.8]\end{bmatrix}_{2\times 1}$
3. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \times \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.8],[0.1,0.7],[0.4,0.8]\\ [0.1,0.4],[0.5,0.8],[0.7,0.8]\end{bmatrix}_{2\times 1}$
4. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \cup \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.3,0.8],[0.1,0.7],[0.4,0.8]\\ [0.4,0.6],[0.2,0.8],[0.5,0.7]\end{bmatrix}_{2\times 1}$
5. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \cap \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \left[\hat{P}_{\mathfrak{Q}_{\bar{C}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.6],[0.2,0.7],[0.5,0.8]\\ [0.1,0.4],[0.5,0.9],[0.7,0.8]\end{bmatrix}_{2\times 1}$
6. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}^{c} = \begin{bmatrix}[0.4,0.8],[0.3,0.9],[0.2,0.8]\\ [0.7,0.8],[0.2,0.5],[0.1,0.4]\end{bmatrix}_{2\times 1}$
7. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \vee \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.3,0.8],[0.1,0.7],[0.4,0.8]\\ [0.4,0.6],[0.2,0.8],[0.5,0.7]\end{bmatrix}_{2\times 1}$
8. $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} \wedge \left[\hat{P}_{\mathfrak{Q}_{\bar{B}_{i,j}}}\right]_{m\times n} = \begin{bmatrix}[0.2,0.6],[0.2,0.7],[0.5,0.8]\\ [0.1,0.4],[0.5,0.9],[0.7,0.8]\end{bmatrix}_{2\times 1}$

4. Practical application of IV-Q-NSM in Medical Field

In this section, we will show the apparatus for appealing to our put-forward model in dealing with daily life situations. By narrating an issue in the medical field and showing the mechanism for representing its data proposed by our proposed model. After that, we will work on creating an algorithm consisting of a number of sequential steps that analyze the algebraic structure of our proposed model and the data it represents. Now we will provide some definitions that will be useful to us in building the above algorithm.

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Definition 4.1. Let $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n}$ be IV-Q-NSMs, where $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]_{m\times n} = \left[\left\langle\left[\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right], \left[\hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}, \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right]\right\rangle\right]_{m\times n}$ then IV-Q-NSMs reduce to IV-Q-FSMs $\Psi_{\bar{A}_{i,j}}$ where the lower and upper given as following formalhs $\Psi^{l}_{\bar{A}_{i,j}} = \frac{1}{3}\left(\hat{P}^{t,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,l}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right)$, $\Psi^{u}_{\bar{A}_{i,j}} = \frac{1}{3}\left(\hat{P}^{t,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{i,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}} + \hat{P}^{f,u}_{\mathfrak{Q}_{\bar{A}_{i,j}}}\right)$.

Definition 4.2. (Comparison Matrix) Comparison matrix is define as a matrix that provides a comprehensive comparison between the membership function values of each object with other objects according to a rule that gives a value of 1 if it is larger and a value of 0 if the value is smaller.

Algorithm

Step 1. Build the IV-Q-NSM based on the IV-Q-NSS.

Step 2. Convert IV-Q-NSM to IV-Q-FSM using definition.

Step 3. Convert IV-Q-FSM to SV-Q-FSM using the following formal $\Psi_{\bar{A}_{i,j}} = \frac{\Psi^{l,t}_{\bar{A}_{i,j}} + \Psi^{u,t}_{\bar{A}_{i,j}}}{2}$

Step 4. Find **c**omparison matrix based on given definition above.

Step 5. Find the score values M_t for all $(v,q) \in V \times Q$, where $t = 1,2,\dots n$.

Step 6. The decision taken is to pick the highest score of M_t,i.e. Decision taken= $max\{M_t\}$.

Step 7. End Algorithm.

Case Study

Now, we provide a case study related to the medical field for IV-Q-NSS strategic decision-making method.

On a cold winter day, numerous patients visited the office of a respiratory doctor to diagnose their health conditions (whether COVID-positive or not) based on the symptoms they were experiencing. To assist the doctor in organizing and analyzing patient data according to our proposed model, we requested that he select a value between 0 and 1 that reflects the severity of symptoms and their association with the disease (COVID). In this context, a ratio closer to 1 indicates more severe symptoms and a greater impact on the disease. Therefore:

Suppose that $V = \{v_1, v_2, v_3, v_4\}$ be a patient set contains four patients, $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ where $\mathfrak{q}_1$ =infected and $\mathfrak{q}_2$ =uninfected,while $\bar{A} \subseteq \mathcal{E} = \{e_1, \bar{e}_2, \bar{e}_3\}$ be a set of symptoms contains four symptoms such that $\bar{a}_1$ =Headache, $\bar{a}_2$ =Sore throat, $\bar{a}_3$ =Muscle pain

Now, after the doctor has examined each patient and set a numerical value between 0 and 1 for each of the symptoms above, our proposed model can be built in a way that is consistent with the examining doctor's report. The examining physician's opinion will be analyzed and help him make the appropriate decision about which of the examined patients are infected or not .

$\hat{P}_{\mathfrak{Q}_{\bar{A}}} =$

$$\left\{\left(\bar{e}_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(v_1,\mathfrak{q}_2)}\right.\right.$$

$$\frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(v_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(v_2,\mathfrak{q}_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(v_3,\mathfrak{q}_1)}, \frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(v_3,\mathfrak{q}_2)}$$

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$$\frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(v_4,q_1)},\frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(v_4,q_2)}\Bigg)$$

$$\Bigg(e_2,\frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(v_1,q_1)},\frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(v_1,q_2)}$$

$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(v_2,q_1)},\frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(v_2,q_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(v_3,q_1)},\frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(v_3,q_2)}$$

$$\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(v_4,q_1)},\frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(v_4,q_2)}\Bigg)$$

$$\Bigg(e_3,\frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(v_1,q_1)},\frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(v_1,q_2)}$$

$$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(v_2,q_1)},\frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(v_2,q_2)}$$

$$\frac{\langle[0.6,0.8],[0.4,0.5],[0.3,0.5]\rangle}{(v_3,q_1)},\frac{\langle[0.3,0.7],[0.2,0.4],[0.1,0.8]\rangle}{(v_3,q_2)}$$

$$\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(v_4,q_1)},\frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(v_4,q_2)}\Bigg)\Bigg\}$$

To approximate the working mechanism, we take the part

$\left(e_1,\frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(v_1,q_1)}\right)$ Here, this part indicates that the strength of the symptom $\bar{a}_1$ of the first patient suspected of having the disease is strongly estimated between $[0.2,0.8]$, the degree of non-injury rating $[0.4,0.8]$ and the degree of neutrality in whether or not an injury is present is estimated $[0.1,0.7]$.

Now we use the above algorithm to solve this problem by applying its steps sequentially and clearly.

Step 1. Build the IV-Q-NSM $\left[\hat{P}_{\mathfrak{Q}_{\bar{A}}}\right]_{m\times n}$ based on the IV-Q-NSS $\hat{P}_{\mathfrak{Q}_{\bar{A}}}$.

$\left[\hat{P}_{\mathfrak{Q}_{\bar{A}}}\right]_{3\times 8}$

= (matrix, 3×8)

\	e_1	e_2	e_3
(v_1,q_1)	⟨[0.2,0.8], [0.1,0.7], [0.4,0.8]⟩	⟨[0.1,0.8], [0.5,0.7], [0.3,0.4]⟩	⟨[0.7,0.9], [0.2,0.8], [0.3,0.6]⟩
(v_1,q_2)	[0.1,0.4], [0.5,0.8], [0.7,0.8]	⟨[0.1,0.8], [0.4,0.7], [0.2,0.6]⟩	⟨[0.4,0.7], [0.2,0.5], [0.1,0.7]⟩
(v_2,q_1)	⟨[0.3,0.6], [0.2,0.7], [0.5,0.8]⟩	⟨[0.5,0.8], [0.4,0.9], [0.2,0.7]⟩	⟨[0.1,0.8], [0.1,0.4], [0.3,0.6]⟩
(v_2,q_2)	⟨[0.4,0.6], [0.2,0.9], [0.5,0.7]⟩	⟨[0.1,0.2], [0.2,0.5], [0.4,0.7]⟩	⟨[0.5,0.6], [0.3,0.6], [0.2,0.7]⟩
(v_3,q_1)	⟨[0.6,0.8], [0.4,0.5], [0.3,0.5]⟩	⟨[0.6,0.8], [0.4,0.5], [0.3,0.5]⟩	⟨[0.6,0.8], [0.4,0.5], [0.3,0.5]⟩
(v_3,q_2)	⟨[0.3,0.7], [0.2,0.4], [0.1,0.8]⟩	⟨[0.3,0.7], [0.2,0.4], [0.1,0.8]⟩	⟨[0.3,0.7], [0.2,0.4], [0.1,0.8]⟩
(v_4,q_1)	⟨[0.1,0.5], [0.3,0.7], [0.2,0.8]⟩	⟨[0.1,0.4], [0.2,0.5], [0.3,0.7]⟩	⟨[0.4,0.6], [0.2,0.7], [0.3,0.6]⟩
(v_4,q_2)	⟨[0.4,0.8], [0.4,0.6], [0.2,0.8]⟩	⟨[0.1,0.6], [0.4,0.5], [0.5,0.7]⟩	⟨[0.4,0.8], [0.8,0.9], [0.3,0.7]⟩

Step 2. Convert IV-Q-NSM to IV-Q-FSM using definition.

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$$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8} = \begin{bmatrix} \backslash & e_1 & e_2 & e_3 \\ (v_1,q_1) & \langle[0.21,0.69]\rangle & \langle[0.54,0.76]\rangle & \langle[0.43,0.65]\rangle \\ (v_1,q_2) & \langle[0.43,0.64]\rangle & \langle[0.24,0.79]\rangle & \langle[0.24,0.74]\rangle \\ (v_2,q_1) & \langle[0.21,0.78]\rangle & \langle[0.57,0.86]\rangle & \langle[0.35,0.76]\rangle \\ (v_2,q_2) & \langle[0.34,0.89]\rangle & \langle[0.34,0.78]\rangle & \langle[0.25,0.68]\rangle \\ (v_3,q_1) & \langle[0.56,0.80]\rangle & \langle[0.53,0.86]\rangle & \langle[0.45,0.79]\rangle \\ (v_3,q_2) & \langle[0.46,0.84]\rangle & \langle[0.32,0.69]\rangle & \langle[0.24,0.78]\rangle \\ (v_4,q_1) & \langle[0.58,0.94]\rangle & \langle[048,0.72]\rangle & \langle[0.21,0.74]\rangle \\ (v_4,q_2) & \langle[0.36,0.78]\rangle & \langle[0.43,0.79]\rangle & \langle[0.68,0.91]\rangle \end{bmatrix}_{3\times 8}$$

Step 3. Convert IV-Q-FSM to SV-Q-FSM using the following formal $\Psi_{\bar{A}_{i,j}} = \frac{\Psi^{l,t}_{\bar{A}_{i,j}} + \Psi^{u,t}_{\bar{A}_{i,j}}}{2}$

$$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8} = \begin{bmatrix} \backslash & e_1 & e_2 & e_3 \\ (v_1,q_1) & \langle 0.45\rangle & \langle 0.65\rangle & \langle 0.54\rangle \\ (v_1,q_2) & \langle 0.53\rangle & \langle 0.52\rangle & \langle 0.49\rangle \\ (v_2,q_1) & \langle 0.50\rangle & \langle 0.72\rangle & \langle 0.60\rangle \\ (v_2,q_2) & \langle 0.61\rangle & \langle 0.56\rangle & \langle 0.47\rangle \\ (v_3,q_1) & \langle 0.68\rangle & \langle 0.70\rangle & \langle 0.62\rangle \\ (v_3,q_2) & \langle 0.65\rangle & \langle 0.51\rangle & \langle 0.51\rangle \\ (v_4,q_1) & \langle 0.76\rangle & \langle 0.6\rangle & \langle 0.48\rangle \\ (v_4,q_2) & \langle 0.57\rangle & \langle 0.61\rangle & \langle 0.80\rangle \end{bmatrix}_{3\times 8}$$

Step 4. Find comparison matrix based on given definition above.

$$[\hat{P}_{\mathfrak{Q}_{\bar{A}}}]_{3\times 8} = \begin{bmatrix} \backslash & e_1 & e_2 & e_3 \\ (v_1,q_1) & 1 & 5 & 4 \\ (v_1,q_2) & 2 & 1 & 2 \\ (v_2,q_1) & 1 & 7 & 5 \\ (v_2,q_2) & 4 & 2 & 0 \\ (v_3,q_1) & 6 & 6 & 6 \\ (v_3,q_2) & 5 & 0 & 2 \\ (v_4,q_1) & 7 & 3 & 1 \\ (v_4,q_2) & 3 & 4 & 7 \end{bmatrix}_{3\times 8}$$

Step 5. Find the score values M_t for all $(v,q) \in V \times Q$, where $t = 1,2, \dots n$.

$$\begin{bmatrix} (v_i,q_j) & M_i \\ (v_1,q_1) & M_1 = 10 \\ (v_1,q_2) & M_2 = 5 \\ (v_2,q_1) & M_3 = 13 \\ (v_2,q_2) & M_4 = 6 \\ (v_3,q_1) & M_5 = 18 \\ (v_3,q_2) & M_6 = 7 \\ (v_4,q_1) & M_7 = 11 \\ (v_4,q_2) & M_8 = 14 \end{bmatrix}$$

Step 6. The decision taken is to pick the highest score of M_t,i.e. Decision taken= $max\{M_t\}$. Here the Decision taken showing in the following table:

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Table 1: Comparison of the results obtained from the above algorithm

Patients	Degree of (v, q_1)	Degree of (v, q_2)	Comparison	Result
v_1	10	5	$q_1 > q_2$	Yes
v_2	13	6	$q_1 > q_2$	Yes
v_3	18	7	$q_1 > q_2$	Yes
v_4	11	14	$q_1 < q_2$	No

By looking at Table 1. above, which contains a comparison between the results obtained, it is clear that all patients $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are infected except the patient $\mathbf{v_4}$.

Based on the results obtained from Algorithm 1, we can construct the following statistical chart, which illustrates the variation in the severity of symptoms among the patients examined in the doctor's clinic.

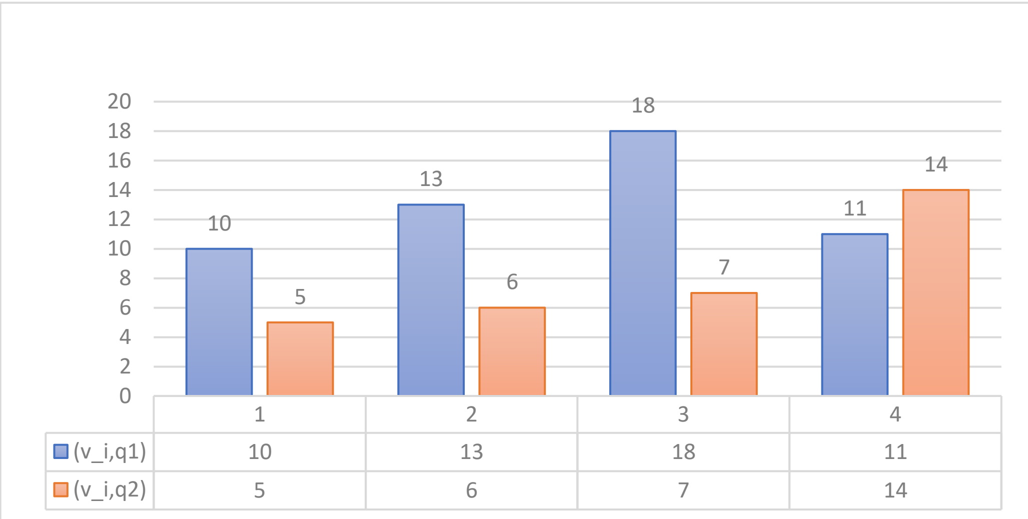

Figure 1. Statistical chart, which shows the disparity in the severity of the injury among those examined in the doctor's clinic.

5. Conclusion

In this work, a new framework was presented for a previous concept, where the concept of IV-Q-NSS was presented in a matrix system. In this work, first, the concept of IV-Q-NSM was presented, then the properties of this concept were clarified and shown. Based on this concept, some theories were given that show the mechanism of the presented tools. In addition, work was done to show the importance of this concept in solving daily life problems by creating a multi-step algorithm that helps in solving one of the decision-making problems, specifically in the medical field. Finally, as recommendations for future studies, these tools can be developed by integrating them with mathematical tools explained in the following works see [37-45].

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