Image and Inverse Image of Neutrosophic Cubic Sets in UP-Algebras under UP-Homomorphisms

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Abstract

The concept of a neutrosophic cubic set in a UP-algebra was introduced by Songsaeng and Iampan [Neutrosophic cubic set theory applied to UP-algebras, 2019]. In this paper, we define the image and inverse image of a neutrosophic cubic set in a non-empty set under any function and study the image and inverse image of a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of a UP-algebra under some UP-homomorphisms.
Keywords: UP-algebra, UP-homomorphism, neutrosophic cubic UP-subalgebra, neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal

1 Introduction

The type of the logical algebra, a UP-algebra was introduced by Iampan.[7] Later Somjanta et al.[27] studied a fuzzy UP-subalgebra, a fuzzy UP-ideal and a fuzzy UP-filter of a UP-algebra. Guntasow et al.[5] studied a fuzzy translation of a fuzzy set in a UP-algebra. Kesorn et al.[17] studied an intuitionistic fuzzy set in a UP-algebra. Kaijae et al.[16] studied anti-fuzzy UP-ideals and anti-fuzzy UP-subalgebras. Tanamoon et al.[36] studied a Q-fuzzy set in a UP-algebra. Sripaeng et al.[34] studied an anti Q-fuzzy UP-ideal and an anti Q-fuzzy UP-subalgebra of a UP-algebra. Dokkhamdang et al.[4] studied a generalized fuzzy set in a UP-algebra. Songsaeng and Iampan[28,29] studied an $\mathcal{N}$-fuzzy UP-algebra and a fuzzy proper UP-filter of a UP-algebra. Senapati et al.[24,25] studies a cubic set and an interval-valued intuitionistic fuzzy structure in a UP-algebra.

A fuzzy set f in a non-empty set A is a function from A to the closed interval $[0,1]$. The concept of a fuzzy set in a non-empty set was first introduced by Zadeh.[38] The fuzzy set theory developed by Zadeh and others have found many applications in the domain of mathematics and other domains. Zadeh[39] introduced an interval-value fuzzy sets. The concept of a neutrosophic set was introduced by Smarandache[26] in 1999. Wang et al.[37] introduced the concept of an interval-valued neutrosophic set in 2005. Jun et al.[13] introduced the concept of an interval-valued neutrosophic set in a BCK/BCI-algebra. The concept of a neutrosophic $\mathcal{N}$-structure in a semigroup was introduced by Khan et al.[19] in 2017. Jun et al.[14] applied the concept of a neutrosophic $\mathcal{N}$-structure to a BCK/BCI-algebra in 2017. Songsaeng and Iampan[31–33] applied the concept of a neutrosophic set to a UP-algebra. Ibrahim et. al.[10] introduced the concept of a neutrosophic subtraction algebra and a neutrosophic subtraction semigroup, and Al-Tahan and Davvaz[1] introduced the concept of a neutrosophic $\aleph$-ideal of a subtraction algebra in 2020.

A neutrosophic cubic set which is the generalized form of fuzzy sets, cubic sets and neutrosophic sets was introduced by Jun et al.[15] in 2017. Iqbal et al.[11] introduced the concept of a neutrosophic cubic subalgebra and a neutrosophic cubic closed ideal of a B-algebra in 2016. Songsaeng and Iampan[30] introduced the concept of a neutrosophic cubic set in a UP-algebra in 2020. Khalid et. al.[18] applied the concept of a multiplicative interpretation of a neutrosophic cubic set to a B-algebra in 2020.

From literature review, we will study the image and inverse image of neutrosophic cubic UP-subalgebras (resp., neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, neutrosophic cubic strong UP-ideals) under some UP-homomorphisms.

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2 Basic concepts and preliminary notes on a UP-algebra

Before the study, we will review the definition of a UP-algebra.

Definition 2.1. [7] An algebra $X = (X, \circ, 0)$ of type $(2, 0)$ is said to be a *UP-algebra*, where X is a non-empty set, $\circ$ is a binary operation on X, and 0 is a fixed element of X if it holds the followings:

(UP-1) (for all $x, y, z \in X)((y \circ z) \circ ((x \circ y) \circ (x \circ z)) = 0)$,

(UP-2) (for all $x \in X)(0 \circ x = x)$,

(UP-3) (for all $x \in X)(x \circ 0 = 0)$, and

(UP-4) (for all $x, y \in X)(x \circ y = 0, y \circ x = 0 \Rightarrow x = y)$.

From,[7] we already know that the concept of a UP-algebra is a generalization of a KU-algebra (see[21]).

Example 2.2. [23] Let Y be a universal set and let $\Omega \in \mathcal{P}(Y)$, where $\mathcal{P}(Y)$ means the power set of Y. Let $\mathcal{P}_\Omega(Y) = \{A \in \mathcal{P}(Y) \mid \Omega \subseteq A\}$. Define a binary operation $\circ$ on $\mathcal{P}_\Omega(Y)$ by putting $A \circ B = B \cap (A^C \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(Y)$, where A^C means the complement of a subset A. Then $(\mathcal{P}_\Omega(Y), \circ, \Omega)$ is a UP-algebra. Let $\mathcal{P}^\Omega(Y) = \{A \in \mathcal{P}(Y) \mid A \subseteq \Omega\}$. Define a binary operation $\bullet$ on $\mathcal{P}^\Omega(Y)$ by putting $A \bullet B = B \cup (A^C \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(Y)$. Then $(\mathcal{P}^\Omega(Y), \bullet, \Omega)$ is a UP-algebra. In particular, $(\mathcal{P}(Y), \circ, \emptyset)$ and $(\mathcal{P}(Y), \bullet, X)$ are UP-algebras.

Example 2.3. [4] Let $\mathbb{N}_0$ be the set of all natural numbers with zero. Define two binary operations $\cdot$ and $*$ on $\mathbb{N}_0$ by

$$\text{(for all } m, n \in \mathbb{N}_0) \left(m \cdot n = \begin{cases} n & \text{if } m < n, \\ 0 & \text{otherwise} \end{cases} \right)$$

and

$$\text{(for all } m, n \in \mathbb{N}_0) \left(m * n = \begin{cases} n & \text{if } m > n \text{ or } m = 0, \\ 0 & \text{otherwise} \end{cases} \right).$$

Then $(\mathbb{N}_0, \cdot, 0)$ and $(\mathbb{N}_0, *, 0)$ are UP-algebras.

For more examples of a UP-algebra, see.[2,3,8,22–25]

In a UP-algebra $X = (X, \circ, 0)$, the followings are valid (see[7,8]).

$$\text{(for all } x \in X)(x \circ x = 0), \tag{2.1}$$
$$\text{(for all } x, y, z \in X)(x \circ y = 0, y \circ z = 0 \Rightarrow x \circ z = 0), \tag{2.2}$$
$$\text{(for all } x, y, z \in X)(x \circ y = 0 \Rightarrow (z \circ x) \circ (z \circ y) = 0), \tag{2.3}$$
$$\text{(for all } x, y, z \in X)(x \circ y = 0 \Rightarrow (y \circ z) \circ (x \circ z) = 0), \tag{2.4}$$
$$\text{(for all } x, y \in X)(x \circ (y \circ x) = 0), \tag{2.5}$$
$$\text{(for all } x, y \in X)((y \circ x) \circ x = 0 \Leftrightarrow x = y \circ x), \tag{2.6}$$
$$\text{(for all } x, y \in X)(x \circ (y \circ y) = 0), \tag{2.7}$$
$$\text{(for all } a, x, y, z \in X)((x \circ (y \circ z)) \circ (x \circ ((a \circ y) \circ (a \circ z))) = 0), \tag{2.8}$$
$$\text{(for all } a, x, y, z \in X)((((a \circ x) \circ (a \circ y)) \circ z) \circ ((x \circ y) \circ z) = 0), \tag{2.9}$$
$$\text{(for all } x, y, z \in X)(((x \circ y) \circ z) \circ (y \circ z) = 0), \tag{2.10}$$
$$\text{(for all } x, y, z \in X)(x \circ y = 0 \Rightarrow x \circ (z \circ y) = 0), \tag{2.11}$$
$$\text{(for all } x, y, z \in X)(((x \circ y) \circ z) \circ (x \circ (y \circ z)) = 0), \text{ and} \tag{2.12}$$
$$\text{(for all } a, x, y, z \in X)(((x \circ y) \circ z) \circ (y \circ (a \circ z)) = 0). \tag{2.13}$$

From,[7] the binary relation $\leq$ on a UP-algebra $X = (X, \circ, 0)$ is defined as follows:

$$\text{(for all } x, y \in X)(x \leq y \Leftrightarrow x \circ y = 0).$$

In a UP-algebra, 5 types of special subsets are defined as follows.

Definition 2.4. [5–7,27] A non-empty subset A of a UP-algebra $X = (X, \circ, 0)$ is said to be

(1) a *UP-subalgebra* of X if (for all $x, y \in A)(x \circ y \in A)$.

(2) a *near UP-filter* of X if

   (i) the constant 0 of X is in A, and

   (ii) (for all $x, y \in X)(y \in A \Rightarrow x \circ y \in A)$.

(3) a *UP-filter* of X if

   (i) the constant 0 of X is in A, and

   (ii) (for all $x, y \in X)(x \circ y \in A, x \in A \Rightarrow y \in A)$.

(4) a *UP-ideal* of X if

   (i) the constant 0 of X is in A, and

   (ii) (for all $x, y, z \in X)(x \circ (y \circ z) \in A, y \in A \Rightarrow x \circ z \in A)$.

(5) a *strong UP-ideal* of X if

   (i) the constant 0 of X is in A, and

   (ii) (for all $x, y, z \in X)((z \circ y) \circ (z \circ x) \in A, y \in A \Rightarrow x \in A)$.

Guntasow et al.[5] and Iampan[6] proved that the concept of a UP-subalgebra is a generalization of a near UP-filter, a near UP-filter is a generalization of a UP-filter, a UP-filter is a generalization of a UP-ideal, and a UP-ideal is a generalization of a strong UP-ideal. Moreover, they proved that the only strong UP-ideal of a UP-algebra X is X.

Definition 2.5. [7] Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras. A function f from X to Y is said to be a *UP-homomorphism* if

$$\text{(for all } x, y \in X)(f(x \circ y) = f(x) \bullet f(y)).$$

A UP-homomorphism $f\colon X \to Y$ is said to be a *UP-epimorphism* if f is surjective, a *UP-monomorphism* if f is injective, and a *UP-isomorphism* if f is bijective.

Theorem 2.6. [9] *Let X and Y be two UP-algebras with fixed elements of 0_X and 0_Y, respectively, and let $f\colon X \to Y$ be a UP-homomorphism. Then the followings hold:*

(1) $f(0_X) = 0_Y$, *and*

(2) (*for all* $x_1, x_2 \in X)(x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2))$.

In 1965, the concept of a fuzzy set in a non-empty set was introduced by Zadeh[38] with the following definition.

Definition 2.7. A *fuzzy set* (briefly, FS) in a non-empty set X (or a fuzzy subset of X) is defined to be a function $\lambda : X \to [0, 1]$, where $[0, 1]$ is the unit segment of the real line. Denote by $[0, 1]^X$ the collection of all FSs in X. Define a binary relation $\leq$ on $[0, 1]^X$ as follows:

$$\text{(for all } \lambda, \mu \in [0, 1]^X)(\lambda \leq \mu \Leftrightarrow \text{(for all } x \in X)(\lambda(x) \leq \mu(x))). \tag{2.14}$$

Definition 2.8. [27] Let λ be a FS in a non-empty set X. The *complement of* λ, denoted by λ^C, is defined by

$$\text{(for all } x \in X)(\lambda^C(x) = 1 - \lambda(x)). \tag{2.15}$$

Definition 2.9. [20] Let $\{\lambda_j \mid j \in J\}$ be a family of FSs in a non-empty set X. We define the *join* and the *meet* of $\{\lambda_j \mid j \in J\}$, denoted by $\vee_{j\in J}\lambda_j$ and $\wedge_{j\in J}\lambda_j$, respectively, as follows:

$$\text{(for all } x \in X)((\vee_{j\in J}\lambda_j)(x) = \sup_{j\in J}\{\lambda_j(x)\}), \tag{2.16}$$

$$\text{(for all } x \in X)((\wedge_{j\in J}\lambda_j)(x) = \inf_{j\in J}\{\lambda_j(x)\}). \tag{2.17}$$

In particular, if λ and μ be FSs in X, we have the join and meet of λ and μ as follows:

$$\text{(for all } x \in X)((\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\}), \tag{2.18}$$

$$\text{(for all } x \in X)((\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\}), \tag{2.19}$$

respectively.

An *interval number* we mean a close subinterval $\widehat{a} = [a^-, a^+]$ of $[0, 1]$, where $0 \leq a^- \leq a^+ \leq 1$. The interval number $\widehat{a} = [a^-, a^+]$ with $a^- = a^+$ is denoted by $\mathbf{a}$. Denote by $\text{int}[0, 1]$ the set of all interval numbers.

Definition 2.10. [15] Let $\{\widehat{a}_j \mid j \in J\}$ be a family of interval numbers. We define the *refined infimum* and the *refined supremum* of $\{\widehat{a}_j \mid j \in J\}$, denoted by $\text{rinf}_{j\in J}\widehat{a}_j$ and $\text{rsup}_{j\in J}\widehat{a}_j$, respectively, as follows:

$$\text{rinf}_{j\in J}\{\widehat{a}_j\} = [\inf_{j\in J}\{a_j^-\}, \inf_{j\in J}\{a_j^+\}], \tag{2.20}$$

$$\text{rsup}_{j\in J}\{\widehat{a}_j\} = [\sup_{j\in J}\{a_j^-\}, \sup_{j\in J}\{a_j^+\}]. \tag{2.21}$$

In particular, if $\widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1]$, we define the *refined minimum* and the *refined maximum* of $\widehat{a}_1$ and $\widehat{a}_2$, denoted by $\text{rmin}\{\widehat{a}_1, \widehat{a}_2\}$ and $\text{rmax}\{\widehat{a}_1, \widehat{a}_2\}$, respectively, as follows:

$$\text{rmin}\{\widehat{a}_1, \widehat{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}], \tag{2.22}$$

$$\text{rmax}\{\widehat{a}_1, \widehat{a}_2\} = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}]. \tag{2.23}$$

Definition 2.11. [15] Let $\widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1]$. We define the symbols "$\succeq$", "$\preceq$", "$=$" in case of $\widehat{a}_1$ and $\widehat{a}_2$ as follows:

$$\widehat{a}_1 \succeq \widehat{a}_2 \Leftrightarrow a_1^- \geq a_2^- \text{ and } a_1^+ \geq a_2^+, \tag{2.24}$$

and similarly we may have $\widehat{a}_1 \preceq \widehat{a}_2$ and $\widehat{a}_1 = \widehat{a}_2$. To say $\widehat{a}_1 \succ \widehat{a}_2$ (resp., $\widehat{a}_1 \prec \widehat{a}_2$) we mean $\widehat{a}_1 \succeq \widehat{a}_2$ and $\widehat{a}_1 \neq \widehat{a}_2$ (resp., $\widehat{a}_1 \preceq \widehat{a}_2$ and $\widehat{a}_1 \neq \widehat{a}_2$).

Definition 2.12. [39] Let $\widehat{a} \in \text{int}[0, 1]$. The *complement of* $\widehat{a}$, denoted by $\widehat{a}^C$, is defined by the interval number

$$\widehat{a}^C = [1 - a^+, 1 - a^-]. \tag{2.25}$$

In the $\text{int}[0, 1]$, the followings are valid (see[35]).

$$(\text{for all } \widehat{a} \in \text{int}[0, 1])(\widehat{a} \succeq \widehat{a}), \tag{2.26}$$

$$(\text{for all } \widehat{a} \in \text{int}[0, 1])((\widehat{a}^C)^C = \widehat{a}), \tag{2.27}$$

$$(\text{for all } \widehat{a} \in \text{int}[0, 1])(\text{rmax}\{\widehat{a}, \widehat{a}\} = \widehat{a} \text{ and } \text{rmin}\{\widehat{a}, \widehat{a}\} = \widehat{a}), \tag{2.28}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\text{rmax}\{\widehat{a}_1, \widehat{a}_2\} = \text{rmax}\{\widehat{a}_2, \widehat{a}_1\} \text{ and } \text{rmin}\{\widehat{a}_1, \widehat{a}_2\} = \text{rmin}\{\widehat{a}_2, \widehat{a}_1\}), \tag{2.29}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\text{rmax}\{\widehat{a}_1, \widehat{a}_2\} \succeq \widehat{a}_1 \text{ and } \widehat{a}_2 \succeq \text{rmin}\{\widehat{a}_1, \widehat{a}_2\}), \tag{2.30}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \widehat{a}_2 \Leftrightarrow \widehat{a}_1^C \preceq \widehat{a}_2^C), \tag{2.31}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3, \widehat{a}_4 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \widehat{a}_2, \widehat{a}_3 \succeq \widehat{a}_4 \Rightarrow \text{rmin}\{\widehat{a}_1, \widehat{a}_3\} \succeq \text{rmin}\{\widehat{a}_2, \widehat{a}_4\}), \tag{2.32}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \widehat{a}_2, \widehat{a}_3 \succeq \widehat{a}_2 \Leftrightarrow \text{rmin}\{\widehat{a}_1, \widehat{a}_3\} \succeq \widehat{a}_2), \tag{2.33}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3, \widehat{a}_4 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \widehat{a}_2, \widehat{a}_3 \succeq \widehat{a}_4 \Rightarrow \text{rmax}\{\widehat{a}_1, \widehat{a}_3\} \succeq \text{rmax}\{\widehat{a}_2, \widehat{a}_4\}), \tag{2.34}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3 \in \text{int}[0, 1])(\widehat{a}_2 \succeq \widehat{a}_1, \widehat{a}_2 \succeq \widehat{a}_3 \Leftrightarrow \widehat{a}_2 \succeq \text{rmax}\{\widehat{a}_1, \widehat{a}_3\}), \tag{2.35}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \widehat{a}_2 \Leftrightarrow \text{rmin}\{\widehat{a}_1, \widehat{a}_2\} = \widehat{a}_2), \tag{2.36}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \widehat{a}_2 \Leftrightarrow \text{rmax}\{\widehat{a}_1, \widehat{a}_2\} = \widehat{a}_1), \tag{2.37}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\text{rmin}\{\widehat{a}_1^C, \widehat{a}_2^C\} = \text{rmax}\{\widehat{a}_1, \widehat{a}_2\}^C), \tag{2.38}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2 \in \text{int}[0, 1])(\text{rmax}\{\widehat{a}_1^C, \widehat{a}_2^C\} = \text{rmin}\{\widehat{a}_1, \widehat{a}_2\}^C), \tag{2.39}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3 \in \text{int}[0, 1])(\widehat{a}_1 \preceq \text{rmax}\{\widehat{a}_2, \widehat{a}_3\} \Leftrightarrow \widehat{a}_1^C \succeq \text{rmin}\{\widehat{a}_2^C, \widehat{a}_3^C\}), \tag{2.40}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \text{rmax}\{\widehat{a}_2, \widehat{a}_3\} \Leftrightarrow \widehat{a}_1^C \preceq \text{rmin}\{\widehat{a}_2^C, \widehat{a}_3^C\}), \tag{2.41}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3 \in \text{int}[0, 1])(\widehat{a}_1 \preceq \text{rmin}\{\widehat{a}_2, \widehat{a}_3\} \Leftrightarrow \widehat{a}_1^C \succeq \text{rmax}\{\widehat{a}_2^C, \widehat{a}_3^C\}), \text{ and} \tag{2.42}$$

$$(\text{for all } \widehat{a}_1, \widehat{a}_2, \widehat{a}_3 \in \text{int}[0, 1])(\widehat{a}_1 \succeq \text{rmin}\{\widehat{a}_2, \widehat{a}_3\} \Leftrightarrow \widehat{a}_1^C \preceq \text{rmax}\{\widehat{a}_2^C, \widehat{a}_3^C\}). \tag{2.43}$$

In 1975, the concept of an interval-valued fuzzy set in a non-empty set was first introduced by Zadeh[38] with the following definition.

Definition 2.13. An *interval-valued fuzzy set* (briefly, IVFS) in a non-empty set X is an arbitrary function $A : X \to \text{int}[0, 1]$. Let $IVFS(X)$ stands for the set of all IVFS in X. For every $A \in IVFS(X)$ and $x \in X, A(x) = [A^-(x), A^+(x)]$ is said to be the *degree of membership* of an element x to A, where A^-, A^+ are FSs in X which are called a *lower fuzzy set* and an *upper fuzzy set* in X, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Definition 2.14. [15] Let A and B be IVFSs in a non-empty set X. We define the symbols "$\subseteq$", "$\supseteq$", "$=$" in case of A and B as follows:

$$A \subseteq B \Leftrightarrow (\text{for all } x \in X)(A(x) \preceq B(x)), \tag{2.44}$$

and similarly we may have $A \supseteq B$ and $A = B$.

Definition 2.15. [39] Let A be an IVFS in a non-empty set X. The *complement of* A, denoted by A^C, is defined as follows: $A^C(x) = A(x)^C$ for all $x \in X$, that is,

$$(\text{for all } x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x)]). \tag{2.45}$$

We note that $A^{C^-}(x) = 1 - A^+(x)$ and $A^{C^+}(x) = 1 - A^-(x)$ for all $x \in X$.

Definition 2.16. [39] Let $\{A_j \mid j \in J\}$ be a family of IVFSs in a non-empty set X. We define the *intersection* and the *union* of $\{A_j \mid j \in J\}$, denoted by $\cap_{j\in J} A_j$ and $\cup_{j\in J} A_j$, respectively, as follows:

$$(\text{for all } x \in X)((\cap_{j\in J} A_j)(x) = \mathrm{rinf}_{j\in J}\{A_j(x)\}), \tag{2.46}$$

$$(\text{for all } x \in X)((\cup_{j\in J} A_j)(x) = \mathrm{rsup}_{j\in J}\{A_j(x)\}). \tag{2.47}$$

We note that

$$(\text{for all } x \in X)((\cap_{j\in J} A_j)^-(x) = (\wedge_{j\in J} A_j^-)(x) = \inf_{j\in J}\{A_j^-(x)\})$$

and

$$(\text{for all } x \in X)((\cap_{j\in J} A_j)^+(x) = (\wedge_{j\in J} A_j^+)(x) = \inf_{j\in J}\{A_j^+(x)\}).$$

Similarly,

$$(\text{for all } x \in X)((\cup_{j\in J} A_j)^-(x) = (\vee_{j\in J} A_j^-)(x) = \sup_{j\in J}\{A_j^-(x)\})$$

and

$$(\text{for all } x \in X)((\cup_{j\in J} A_j)^+(x) = (\vee_{j\in J} A_j^+)(x) = \sup_{j\in J}\{A_j^+(x)\}).$$

In particular, if A_1 and A_2 are IVFSs in X, we have the intersection and the union of A_1 and A_2 as follows:

$$(\text{for all } x \in X)((A_1 \cap A_2)(x) = \mathrm{rmin}\{A_1(x), A_2(x)\}), \tag{2.48}$$

$$(\text{for all } x \in X)((A_1 \cup A_2)(x) = \mathrm{rmax}\{A_1(x), A_2(x)\}). \tag{2.49}$$

In 1999, the concept of a neutrosophic set in a non-empty set was introduced by Smarandache[26] with the following definition.

Definition 2.17. A *neutrosophic set* (briefly, NS) in a non-empty set X is a structure of the form:

$$\Lambda = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}, \tag{2.50}$$

where $\lambda_T : X \to [0,1]$ is a *truth membership function*, $\lambda_I : X \to [0,1]$ is an *indeterminate membership function*, and $\lambda_F : X \to [0,1]$ is a *false membership function*. For our convenience, we will denote a NS as $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F) = (X, \lambda_{T,I,F}) = \{(x, \lambda_T(x), \lambda_I(x), \lambda_F(x)) \mid x \in X\}$.

Definition 2.18. [26] Let Λ be a NS in a non-empty set X. The NS $\Lambda^C = (X, \lambda_T^C, \lambda_I^C, \lambda_F^C)$ in X is said to be the *complement* of Λ in X.

In 2005, the concept of an interval neutrosophic set in a non-empty set was introduced by Wang et al.[37] with the following definition.

Definition 2.19. An *interval-valued neutrosophic set* (briefly, IVNS) in a non-empty set X is a structure of the form:

$$\mathbf{A} := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}, \tag{2.51}$$

where A_T, A_I and A_F are IVFSs in X, which are called an *interval truth membership function*, an *interval indeterminacy membership function* and an *interval falsity membership function*, respectively. For our convenience, we will denote a IVNS as $\mathbf{A} = (X, A_T, A_I, A_F) = (X, A_{T,I,F}) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}$.

Definition 2.20. [37] Let $\mathbf{A} = (X, A_T, A_I, A_F)$ be an IVNS in a non-empty set X. The IVNS $\mathbf{A}^C = (X, A_T^C, A_I^C, A_F^C)$ in X is said to be the *complement* of $\mathbf{A}$ in X.

In 2012, the concept of a cubic set in a non-empty set was introduced by Jun et al.[12] with the following definition.

Definition 2.21. A *cubic set* (briefly, CS) in a non-empty set X is a structure of the form:

$$\mathbf{C} = \{(x, A(x), \lambda(x)) \mid x \in X\}, \tag{2.52}$$

where A is an IVFS in X and λ is a FS in X. For our convenience, we will denote a CS as $\mathbf{C} = (X, A, \lambda) = \{(x, A(x), \lambda(x)) \mid x \in X\}$.

In 2017, Jun et al.[15] introduced the concept of a neutrosophic cubic set with the following definition.

Definition 2.22. A neutrosophic cubic set (briefly, NCS) in a non-empty set X is a pair $\mathscr{A} = (\mathbf{A}, \Lambda)$, where $\mathbf{A} = (X, A_T, A_I, A_F)$ is an IVNS in X and $\Lambda = (X, \lambda_T, \lambda_I, \lambda_F)$ is a neutrosophic set in X. For simplicity, we denote $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in a non-empty set X is said to be *constant* if $A_T, A_I, A_F, \lambda_T, \lambda_I$, and λ_F are constant functions. The complement of a NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ is defined to be the NCS $\mathscr{A}^C = (\mathbf{A}^C, \Lambda^C)$.

In 2020, the concepts of a neutrosophic cubic UP-subalgebra, a neutrosophic cubic near UP-filter, a neutrosophic cubic UP-filter, a neutrosophic cubic UP-ideal, and a neutrosophic cubic strong UP-ideal of a UP-algebra were introduced by Songsaeng and Iampan[30] with the following definition.

Definition 2.23. A NCS $\mathscr{A} = (\mathbf{A}, \Lambda)$ in a UP-algebra $X = (X, \circ, 0)$ is said to be

(1) a *neutrosophic cubic UP-subalgebra* of X if

$$(\text{for all } x, y \in X) \begin{pmatrix} A_T(x \circ y) \succeq \operatorname{rmin}\{A_T(x), A_T(y)\} \\ A_I(x \circ y) \preceq \operatorname{rmax}\{A_I(x), A_I(y)\} \\ A_F(x \circ y) \succeq \operatorname{rmin}\{A_F(x), A_F(y)\} \end{pmatrix}, \tag{2.53}$$

$$(\text{for all } x, y \in X) \begin{pmatrix} \lambda_T(x \circ y) \leq \max\{\lambda_T(x), \lambda_T(y)\} \\ \lambda_I(x \circ y) \geq \min\{\lambda_I(x), \lambda_I(y)\} \\ \lambda_F(x \circ y) \leq \max\{\lambda_F(x), \lambda_F(y)\} \end{pmatrix}. \tag{2.54}$$

(2) a *neutrosophic cubic near UP-filter* of X if

$$(\text{for all } x \in X) \begin{pmatrix} A_T(0) \succeq A_T(x) \\ A_I(0) \preceq A_I(x) \\ A_F(0) \succeq A_F(x) \end{pmatrix}, \tag{2.55}$$

$$(\text{for all } x \in X) \begin{pmatrix} \lambda_T(0) \leq \lambda_T(x) \\ \lambda_I(0) \geq \lambda_I(x) \\ \lambda_F(0) \leq \lambda_F(x) \end{pmatrix}, \tag{2.56}$$

$$(\text{for all } x, y \in X) \begin{pmatrix} A_T(x \circ y) \succeq A_T(y) \\ A_I(x \circ y) \preceq A_I(y) \\ A_F(x \circ y) \succeq A_F(y) \end{pmatrix}, \tag{2.57}$$

$$(\text{for all } x, y \in X) \begin{pmatrix} \lambda_T(x \circ y) \leq \lambda_T(y) \\ \lambda_I(x \circ y) \geq \lambda_I(y) \\ \lambda_F(x \circ y) \leq \lambda_F(y) \end{pmatrix}. \tag{2.58}$$

(3) a *neutrosophic cubic UP-filter* of X if it holds the followings: (2.55), (2.56), and

$$(\text{for all } x, y \in X) \begin{pmatrix} A_T(y) \succeq \operatorname{rmin}\{A_T(x \circ y), A_T(x)\} \\ A_I(y) \preceq \operatorname{rmax}\{A_I(x \circ y), A_I(x)\} \\ A_F(y) \succeq \operatorname{rmin}\{A_F(x \circ y), A_F(x)\} \end{pmatrix}, \tag{2.59}$$

$$(\text{for all } x, y \in X) \begin{pmatrix} \lambda_T(y) \leq \max\{\lambda_T(x \circ y), \lambda_T(x)\} \\ \lambda_I(y) \geq \min\{\lambda_I(x \circ y), \lambda_I(x)\} \\ \lambda_F(y) \leq \max\{\lambda_F(x \circ y), \lambda_F(x)\} \end{pmatrix}. \tag{2.60}$$

(4) a *neutrosophic cubic UP-ideal* of X if it holds the followings: (2.55), (2.56), and

$$\text{(for all } x,y,z\in X) \begin{pmatrix} A_T(x\circ z)\succeq \text{rmin}\{A_T(x\circ(y\circ z)),A_T(y)\} \\ A_I(x\circ z)\preceq \text{rmax}\{A_I(x\circ(y\circ z)),A_I(y)\} \\ A_F(x\circ z)\succeq \text{rmin}\{A_F(x\circ(y\circ z)),A_F(y)\}\end{pmatrix}, \tag{2.61}$$

$$\text{(for all } x,y,z\in X) \begin{pmatrix} \lambda_T(x\circ z)\leq \max\{\lambda_T(x\circ(y\circ z)),\lambda_T(y)\} \\ \lambda_I(x\circ z)\geq \min\{\lambda_I(x\circ(y\circ z)),\lambda_I(y)\} \\ \lambda_F(x\circ z)\leq \max\{\lambda_F(x\circ(y\circ z)),\lambda_F(y)\}\end{pmatrix}. \tag{2.62}$$

(5) a *neutrosophic cubic strong UP-ideal* of X if it holds the followings: (2.55), (2.56), and

$$\text{(for all } x,y,z\in X) \begin{pmatrix} A_T(x)\succeq \text{rmin}\{A_T((z\circ y)\circ(z\circ x)),A_T(y)\} \\ A_I(x)\preceq \text{rmax}\{A_I((z\circ y)\circ(z\circ x)),A_I(y)\} \\ A_F(x)\succeq \text{rmin}\{A_F((z\circ y)\circ(z\circ x)),A_F(y)\}\end{pmatrix}, \tag{2.63}$$

$$\text{(for all } x,y,z\in X) \begin{pmatrix} \lambda_T(x)\leq \max\{\lambda_T((z\circ y)\circ(z\circ x)),\lambda_T(y)\} \\ \lambda_I(x)\geq \min\{\lambda_I((z\circ y)\circ(z\circ x)),\lambda_I(y)\} \\ \lambda_F(x)\leq \max\{\lambda_F((z\circ y)\circ(z\circ x)),\lambda_F(y)\}\end{pmatrix}. \tag{2.64}$$

Songsaeng and Iampan[30] proved that the concept of a neutrosophic cubic UP-subalgebra is a generalization of a neutrosophic cubic near UP-filter, a neutrosophic cubic near UP-filter is a generalization of a neutrosophic cubic UP-filter, a neutrosophic cubic UP-filter is a generalization of a neutrosophic cubic UP-ideal, and a neutrosophic cubic UP-ideal is a generalization of a neutrosophic cubic strong UP-ideal. Moreover, they proved that a neutrosophic cubic strong UP-ideal and a constant NCS coincide.

3 Homomorphic properties of a NCSs in a UP-algebra

In this section, the image and inverse image of a NCS are defined and some results are studied.

Definition 3.1. Let f be a function from a non-empty set X into a non-empty set Y and $\mathscr{A}=(A_{T,I,F},\lambda_{T,I,F})$ be a NCS in X. Then the image of $\mathscr{A}$ under f is defined as a NCS $f(\mathscr{A})=(f(A)_{T,I,F},f(\lambda)_{T,I,F})$ in Y, where

$$f(A)_T(y)=\begin{cases}\text{rsup}_{x\in f^{-1}(y)}\{A_T(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,}\\ [0,0] & \text{otherwise,}\end{cases}$$

$$f(A)_I(y)=\begin{cases}\text{rinf}_{x\in f^{-1}(y)}\{A_I(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,}\\ [1,1] & \text{otherwise,}\end{cases}$$

$$f(A)_F(y)=\begin{cases}\text{rsup}_{x\in f^{-1}(y)}\{A_F(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,}\\ [0,0] & \text{otherwise,}\end{cases}$$

$$f(\lambda)_T(y)=\begin{cases}\inf_{x\in f^{-1}(y)}\{\lambda_T(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,}\\ 1 & \text{otherwise,}\end{cases}$$

$$f(\lambda)_I(y)=\begin{cases}\sup_{x\in f^{-1}(y)}\{\lambda_I(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,}\\ 0 & \text{otherwise,}\end{cases}$$

$$f(\lambda)_F(y)=\begin{cases}\inf_{x\in f^{-1}(y)}\{\lambda_F(x)\} & \text{if } f^{-1}(y) \text{ is non-empty,}\\ 1 & \text{otherwise.}\end{cases}$$

Example 3.2. Let $X=\{0_X,1_X,2_X\}$ be a UP-algebra with a fixed element 0_X and a binary operation $\circ$ defined by the following Cayley table:

$\circ$	0_X	1_X	2_X
0_X	0_X	1_X	2_X
1_X	0_X	0_X	1_X
2_X	0_X	0_X	0_X

and let $Y = \{0_Y, 1_Y, 2_Y\}$ be a UP-algebra with a fixed element 0_Y and a binary operation $\bullet$ defined by the following Cayley table:

$\bullet$	0_Y	1_Y	2_Y
0_Y	0_Y	1_Y	2_Y
1_Y	0_Y	0_Y	2_Y
2_Y	0_Y	0_Y	0_Y

We define a function $f : X \to Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$

We define a NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0_X	$([0.4, 0.7], [0.5, 0.7], [0.2, 0.4])$	$(0.1, 0.3, 0.4)$
1_X	$([0.1, 0.2], [0.1, 0.5], [0.4, 0.5])$	$(0.3, 0.8, 0.4)$
2_X	$([0.8, 0.9], [0.7, 0.8], [0.1, 0.6])$	$(0.1, 0.5, 0.7)$

Then $f(\mathscr{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ in Y with the tabular representation as follows:

Y	$\mathbf{A}(x)$	$\Lambda(x)$
0_Y	$([0.4, 0.7], [0.5, 0.7], [0.2, 0.4])$	$(0.1, 0.3, 0.4)$
1_Y	$([0.8, 0.9], [0.1, 0.5], [0.4, 0.6])$	$(0.1, 0.8, 0.4)$
2_Y	$([0, 0], [1, 1], [0, 0])$	$(1, 0, 1)$

Hence, $f(\mathscr{A}) = (f(A)_{T,I,F}, f(\lambda)_{T,I,F})$ is a NCS in Y.

Definition 3.3. Let f be a function from a non-empty set X into a non-empty set Y and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y. Then the inverse image of $\mathscr{A}$ is defined as a NCS $f^{-1}(\mathscr{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in X, where

$$(\text{for all } x \in X)(f^{-1}(A)_{T,I,F}(x) = A_{T,I,F}(f(x))), \tag{3.1}$$

$$(\text{for all } x \in X)(f^{-1}(\lambda)_{T,I,F}(x) = \lambda_{T,I,F}(f(x))). \tag{3.2}$$

Example 3.4. In Example 3.2, we have $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ are two UP-algebras. We define a function $f : X \to Y$ as follows:

$$f(0_X) = 0_Y, f(1_X) = 1_Y, \text{ and } f(2_X) = 1_Y.$$

We define a NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in Y with the tabular representation as follows:

Y	$\mathbf{A}(x)$	$\Lambda(x)$
0_Y	$([0.3, 0.7], [0.3, 0.5], [0.1, 0.4])$	$(0.5, 0.4, 0.7)$
1_Y	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$
2_Y	$([0.5, 0.9], [0.3, 0.5], [0.5, 0.8])$	$(0.3, 0.5, 0.4)$

Then $f^{-1}(\mathscr{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ in X with the tabular representation as follows:

X	$\mathbf{A}(x)$	$\Lambda(x)$
0_X	$([0.3, 0.7], [0.3, 0.5], [0.1, 0.4])$	$(0.5, 0.4, 0.7)$
1_X	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$
2_X	$([0.6, 0.7], [0.1, 0.3], [0.4, 0.5])$	$(0.2, 0.7, 0.8)$

Hence, $f^{-1}(\mathscr{A}) = (f^{-1}(A)_{T,I,F}, f^{-1}(\lambda)_{T,I,F})$ is a NCS in X.

Definition 3.5. A NCS
$\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X is said to be *order preserving* if

$$(\text{for all } x, y \in X)\left(x \leq y \Rightarrow \left\{\begin{array}{l} A_T(x) \preceq A_T(y), A_I(x) \succeq A_I(y), A_F(x) \preceq A_F(y), \\ \lambda_T(x) \geq \lambda_T(y), \lambda_I(x) \leq \lambda_I(y), \lambda_F(x) \geq \lambda_F(y) \end{array}\right.\right). \tag{3.3}$$

Lemma 3.6. *Every neutrosophic cubic UP-filter (resp., neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of X is order preserving.*

Proof. Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is a neutrosophic cubic UP-filter of X. Let $x, y \in X$ be such that $x \leq y$ in X. Then $x \circ y = 0$. Thus

$$A_T(y) \succeq \text{rmin}\{A_T(x \circ y), A_T(x)\} = \text{rmin}\{A_T(0), A_T(x)\} = A_T(x), \quad ((2.59),(2.55),(2.36))$$
$$A_I(y) \preceq \text{rmax}\{A_I(x \circ y), A_I(x)\} = \text{rmin}\{A_I(0), A_I(x)\} = A_I(x), \quad ((2.59),(2.55),(2.37))$$
$$A_F(y) \succeq \text{rmin}\{A_F(x \circ y), A_F(x)\} = \text{rmin}\{A_F(0), A_F(x)\} = A_F(x), \quad ((2.59),(2.55),(2.36))$$
$$\lambda_T(y) \leq \max\{\lambda_T(x \circ y), \lambda_T(x)\} = \max\{\lambda_T(0), \lambda_T(x)\} = \lambda_T(x), \quad ((2.60),(2.56))$$
$$\lambda_I(y) \geq \min\{\lambda_I(x \circ y), \lambda_I(x)\} = \min\{\lambda_I(0), \lambda_I(x)\} = \lambda_I(x), \quad ((2.60),(2.56))$$
$$\lambda_F(y) \leq \max\{\lambda_F(x \circ y), \lambda_F(x)\} = \max\{\lambda_F(0), \lambda_F(x)\} = \lambda_F(x). \quad ((2.60),(2.56))$$

Hence, $\mathscr{A}$ is order preserving. □

Theorem 3.7. *Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras, $f\colon X \to Y$ be a UP-homomorphism, and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in Y. Then the followings hold:*

(1) *If $\mathscr{A}$ is a neutrosophic cubic UP-subalgebra of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-subalgebra of X.*

(2) *If $\mathscr{A}$ is a neutrosophic cubic near UP-filter of Y which is order preserving, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic near UP-filter of X.*

(3) *If $\mathscr{A}$ is a neutrosophic cubic UP-filter of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-filter of X.*

(4) *If $\mathscr{A}$ is a neutrosophic cubic UP-ideal of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-ideal of X.*

(5) *If $\mathscr{A}$ is a neutrosophic cubic strong UP-ideal of Y, then the inverse image $f^{-1}(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic strong UP-ideal of X.*

Proof. (1) Assume that $\mathscr{A}$ is a neutrosophic cubic UP-subalgebra of Y. Then for all $x, y \in X$,

$$\begin{aligned}
f^{-1}(A)_T(x \circ y) &= A_T(f(x \circ y)) && ((3.1))\\
&= A_T(f(x) \bullet f(y))\\
&\succeq \text{rmin}\{A_T(f(x)), A_T(f(y))\} && ((2.53))\\
&= \text{rmin}\{f^{-1}(A)_T(x), f^{-1}(A)_T(y)\}, && ((3.1))\\
f^{-1}(A)_I(x \circ y) &= A_I(f(x \circ y)) && ((3.1))\\
&= A_I(f(x) \bullet f(y))\\
&\preceq \text{rmax}\{A_I(f(x)), A_I(f(y))\} && ((2.53))\\
&= \text{rmax}\{f^{-1}(A)_I(x), f^{-1}(A)_I(y)\}, && ((3.1))\\
f^{-1}(A)_F(x \circ y) &= A_F(f(x \circ y)) && ((3.1))\\
&= A_F(f(x) \bullet f(y))\\
&\succeq \text{rmin}\{A_F(f(x)), A_F(f(y))\} && ((2.53))\\
&= \text{rmin}\{f^{-1}(A)_F(x), f^{-1}(A)_F(y)\}, && ((3.1))\\
f^{-1}(\lambda)_T(x \circ y) &= \lambda_T(f(x \circ y)) && ((3.2))\\
&= \lambda_T(f(x) \bullet f(y))\\
&\leq \max\{\lambda_T(f(x)), \lambda_T(f(y))\} && ((2.54))\\
&= \max\{f^{-1}(\lambda)_T(x), f^{-1}(\lambda)_T(y)\}, && ((3.2))\\
f^{-1}(\lambda)_I(x \circ y) &= \lambda_I(f(x \circ y)) && ((3.2))\\
&= \lambda_I(f(x) \bullet f(y))\\
&\geq \min\{\lambda_I(f(x)), \lambda_I(f(y))\} && ((2.54))\\
&= \min\{f^{-1}(\lambda)_I(x), f^{-1}(\lambda)_I(y)\}, && ((3.2))
\end{aligned}$$

$$\begin{aligned} f^{-1}(\lambda)_F(x \circ y) &= \lambda_F(f(x \circ y)) && ((3.2)) \\ &= \lambda_F(f(x) \bullet f(y)) \\ &\leq \max\{\lambda_F(f(x)), \lambda_F(f(y))\} && ((2.54)) \\ &= \max\{f^{-1}(\lambda)_F(x), f^{-1}(\lambda)_F(y)\}. && ((3.2)) \end{aligned}$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic UP-subalgebra of X.

(2) Assume that $\mathscr{A}$ is a neutrosophic cubic near UP-filter of Y which is order preserving. By Theorem 2.6 (2) and (UP-3), we have for all $x \in X$,

$$\begin{aligned} f^{-1}(A)_T(0_X) = A_T(f(0_X)) \succeq A_T(f(x)) = f^{-1}(A)_T(x), \\ f^{-1}(A)_I(0_X) = A_I(f(0_X)) \preceq A_I(f(x)) = f^{-1}(A)_I(x), \\ f^{-1}(A)_F(0_X) = A_F(f(0_X)) \succeq A_F(f(x)) = f^{-1}(A)_F(x), \\ f^{-1}(\lambda)_T(0_X) = \lambda_T(f(0_X)) \leq \lambda_T(f(x)) = f^{-1}(\lambda)_T(x), \\ f^{-1}(\lambda)_I(0_X) = \lambda_I(f(0_X)) \geq \lambda_I(f(x)) = f^{-1}(\lambda)_I(x), \\ f^{-1}(\lambda)_F(0_X) = \lambda_F(f(0_X)) \leq \lambda_F(f(x)) = f^{-1}(\lambda)_F(x). \end{aligned}$$

Let $x, y \in X$. Then

$$\begin{aligned} f^{-1}(A)_T(x \circ y) = A_T(f(x \circ y)) = A_T(f(x) \bullet f(y)) \succeq A_T(f(y)) = f^{-1}(A)_T(y), && ((2.57),(3.1)) \\ f^{-1}(A)_I(x \circ y) = A_I(f(x \circ y)) = A_I(f(x) \bullet f(y)) \preceq A_I(f(y)) = f^{-1}(A)_I(y), && ((2.57),(3.1)) \\ f^{-1}(A)_F(x \circ y) = A_F(f(x \circ y)) = A_F(f(x) \bullet f(y)) \succeq A_F(f(y)) = f^{-1}(A)_F(y), && ((2.57),(3.1)) \\ f^{-1}(\lambda)_T(x \circ y) = \lambda_T(f(x \circ y)) = \lambda_T(f(x) \bullet f(y)) \leq \lambda_T(f(y)) = f^{-1}(\lambda)_T(y), && ((2.58),(3.2)) \\ f^{-1}(\lambda)_I(x \circ y) = \lambda_I(f(x \circ y)) = \lambda_I(f(x) \bullet f(y)) \geq \lambda_I(f(y)) = f^{-1}(\lambda)_I(y), && ((2.58),(3.2)) \\ f^{-1}(\lambda)_F(x \circ y) = \lambda_F(f(x \circ y)) = \lambda_F(f(x) \bullet f(y)) \leq \lambda_F(f(y)) = f^{-1}(\lambda)_F(y). && ((2.58),(3.2)) \end{aligned}$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic near UP-filter of X.

(3) Assume that $\mathscr{A}$ is a neutrosophic cubic UP-filter of Y. Then $\mathscr{A}$ is a neutrosophic cubic near UP-filter of Y. By Lemma 3.6 and the proof of (2), we have $f^{-1}(\mathscr{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y \in X$. Then

$$\begin{aligned} f^{-1}(A)_T(y) &= A_T(f(y)) && ((3.1)) \\ &\succeq \text{rmin}\{A_T(f(x) \bullet f(y)), A_T(f(x))\} && ((2.59)) \\ &= \text{rmin}\{A_T(f(x \circ y)), A_T(f(x))\} \\ &= \text{rmin}\{f^{-1}(A)_T(x \circ y), f^{-1}(A)_T(x)\}, && ((3.1)) \\ f^{-1}(A)_I(y) &= A_I(f(y)) && ((3.1)) \\ &\preceq \text{rmax}\{A_I(f(x) \bullet f(y)), A_I(f(x))\} && ((2.59)) \\ &= \text{rmax}\{A_I(f(x \circ y)), A_I(f(x))\} \\ &= \text{rmax}\{f^{-1}(A)_I(x \circ y), f^{-1}(A)_I(x)\}, && ((3.1)) \\ f^{-1}(A)_F(y) &= A_F(f(y)) && ((3.1)) \\ &\succeq \text{rmin}\{A_F(f(x) \bullet f(y)), A_F(f(x))\} && ((2.59)) \\ &= \text{rmin}\{A_F(f(x \circ y)), A_F(f(x))\} \\ &= \text{rmin}\{f^{-1}(A)_F(x \circ y), f^{-1}(A)_F(x)\}, && ((3.1)) \\ f^{-1}(\lambda)_T(y) &= \lambda_T(f(y)) && ((3.2)) \\ &\leq \max\{\lambda_T(f(x) \bullet f(y)), \lambda_T(f(x))\} && ((2.60)) \\ &= \max\{\lambda_T(f(x \circ y)), \lambda_T(f(x))\} \\ &= \max\{f^{-1}(\lambda)_T(x \circ y), f^{-1}(\lambda)_T(x)\}, && ((3.2)) \\ f^{-1}(\lambda)_I(y) &= \lambda_I(f(y)) && ((3.2)) \\ &\geq \min\{\lambda_I(f(x) \bullet f(y)), \lambda_I(f(x))\} && ((2.60)) \\ &= \min\{\lambda_I(f(x \circ y)), \lambda_I(f(x))\} \\ &= \min\{f^{-1}(\lambda)_I(x \circ y), f^{-1}(\lambda)_I(x)\}, && ((3.2)) \end{aligned}$$

$$\begin{aligned}
f^{-1}(\lambda)_F(y) &= \lambda_F(f(y)) && ((3.2))\\
&\leq \max\{\lambda_F(f(x)\bullet f(y)), \lambda_F(f(x))\} && ((2.60))\\
&= \max\{\lambda_F(f(x\circ y)), \lambda_F(f(x))\}\\
&= \max\{f^{-1}(\lambda)_F(x\circ y), f^{-1}(\lambda)_F(x)\}. && ((3.2))
\end{aligned}$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic UP-filter of X.

(4) Assume that $\mathscr{A}$ is a neutrosophic cubic UP-ideal of Y. Then $\mathscr{A}$ is a neutrosophic cubic UP-filter of Y. By the proof of (3), we have $f^{-1}(\mathscr{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in X$. Then

$$\begin{aligned}
f^{-1}(A)_T(x\circ z) &= A_T(f(x\circ z)) && ((3.1))\\
&= A_T(f(x)\bullet f(z))\\
&\succeq \text{rmin}\{A_T(f(x)\bullet(f(y)\bullet f(z))), A_T(f(y))\} && ((2.61))\\
&= \text{rmin}\{A_T(f(x)\bullet(f(y\circ z))), A_T(f(y))\}\\
&= \text{rmin}\{A_T(f(x\circ(y\circ z))), A_T(f(y))\}\\
&= \text{rmin}\{f^{-1}(A)_T(x\circ(y\circ z)), f^{-1}(A)_T(y)\}, && ((3.1))\\
f^{-1}(A)_I(x\circ z) &= A_I(f(x\circ z)) && ((3.1))\\
&= A_I(f(x)\bullet f(z))\\
&\preceq \text{rmax}\{A_I(f(x)\bullet(f(y)\bullet f(z))), A_I(f(y))\} && ((2.61))\\
&= \text{rmax}\{A_I(f(x)\bullet(f(y\circ z))), A_I(f(y))\}\\
&= \text{rmax}\{A_I(f(x\circ(y\circ z))), A_I(f(y))\}\\
&= \text{rmax}\{f^{-1}(A)_I(x\circ(y\circ z)), f^{-1}(A)_I(y)\}, && ((3.1))\\
f^{-1}(A)_F(x\circ z) &= A_F(f(x\circ z)) && ((3.1))\\
&= A_F(f(x)\bullet f(z))\\
&\succeq \text{rmin}\{A_F(f(x)\bullet(f(y)\bullet f(z))), A_F(f(y))\} && ((2.61))\\
&= \text{rmin}\{A_F(f(x)\bullet(f(y\circ z))), A_F(f(y))\}\\
&= \text{rmin}\{A_F(f(x\circ(y\circ z))), A_F(f(y))\}\\
&= \text{rmin}\{f^{-1}(A)_F(x\circ(y\circ z)), f^{-1}(A)_F(y)\}, && ((3.1))\\
f^{-1}(\lambda)_T(x\circ z) &= \lambda_T(f(x\circ z)) && ((3.2))\\
&= \lambda_T(f(x)\bullet f(z))\\
&\leq \max\{\lambda_T(f(x)\bullet(f(y)\bullet f(z))), \lambda_T(f(y))\} && ((2.62))\\
&= \max\{\lambda_T(f(x)\bullet(f(y\circ z))), \lambda_T(f(y))\}\\
&= \max\{\lambda_T(f(x\circ(y\circ z))), \lambda_T(f(y))\}\\
&= \max\{f^{-1}(\lambda)_T(x\circ(y\circ z)), f^{-1}(\lambda)_T(y)\}, && ((3.2))\\
f^{-1}(\lambda)_I(x\circ z) &= \lambda_I(f(x\circ z)) && ((3.2))\\
&= \lambda_I(f(x)\bullet f(z))\\
&\geq \min\{\lambda_I(f(x)\bullet(f(y)\bullet f(z))), \lambda_I(f(y))\} && ((2.62))\\
&= \min\{\lambda_I(f(x)\bullet(f(y\circ z))), \lambda_I(f(y))\}\\
&= \min\{\lambda_I(f(x\circ(y\circ z))), \lambda_I(f(y))\}\\
&= \min\{f^{-1}(\lambda)_I(x\circ(y\circ z)), f^{-1}(\lambda)_I(y)\}, && ((3.2))\\
f^{-1}(\lambda)_F(x\circ z) &= \lambda_F(f(x\circ z)) && ((3.2))\\
&= \lambda_F(f(x)\bullet f(z))\\
&\leq \max\{\lambda_F(f(x)\bullet(f(y)\bullet f(z))), \lambda_F(f(y))\} && ((2.62))\\
&= \max\{\lambda_F(f(x)\bullet(f(y\circ z))), \lambda_F(f(y))\}\\
&= \max\{\lambda_F(f(x\circ(y\circ z))), \lambda_F(f(y))\}\\
&= \max\{f^{-1}(\lambda)_F(x\circ(y\circ z)), f^{-1}(\lambda)_F(y)\}. && ((3.2))
\end{aligned}$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic UP-ideal of X.

(5) Assume that $\mathscr{A}$ is a neutrosophic cubic strong UP-ideal of Y. Then $\mathscr{A}$ is a neutrosophic cubic UP-ideal of Y. By the proof of (4), we have $f^{-1}(\mathscr{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in X$. Then

$$\begin{aligned}
f^{-1}(A)_T(x) &= A_T(f(x)) && ((3.1))\\
&\succeq \text{rmin}\{A_T((f(z)\bullet f(y))\bullet(f(z)\bullet f(x))), A_T(f(y))\} && ((2.63))\\
&= \text{rmin}\{A_T(f(z\circ y)\bullet f(z\circ x)), A_T(f(y))\}\\
&= \text{rmin}\{A_T(f((z\circ y)\circ(z\circ x))), A_T(f(y))\}\\
&= \text{rmin}\{f^{-1}(A)_T((z\circ y)\circ(z\circ x)), f^{-1}(A)_T(y)\}, && ((3.1))\\
f^{-1}(A)_I(x) &= A_I(f(x)) && ((3.1))\\
&\preceq \text{rmax}\{A_I((f(z)\bullet f(y))\bullet(f(z)\bullet f(x))), A_I(f(y))\} && ((2.63))\\
&= \text{rmax}\{A_I(f(z\circ y)\bullet f(z\circ x)), A_I(f(y))\}\\
&= \text{rmax}\{A_I(f((z\circ y)\circ(z\circ x))), A_I(f(y))\}\\
&= \text{rmax}\{f^{-1}(A)_I((z\circ y)\circ(z\circ x)), f^{-1}(A)_I(y)\}, && ((3.1))\\
f^{-1}(A)_F(x) &= A_F(f(x)) && ((3.1))\\
&\succeq \text{rmin}\{A_F((f(z)\bullet f(y))\bullet(f(z)\bullet f(x))), A_F(f(y))\} && ((2.63))\\
&= \text{rmin}\{A_F(f(z\circ y)\bullet f(z\circ x)), A_F(f(y))\}\\
&= \text{rmin}\{A_F(f((z\circ y)\circ(z\circ x))), A_F(f(y))\}\\
&= \text{rmin}\{f^{-1}(A)_F((z\circ y)\circ(z\circ x)), f^{-1}(A)_F(y)\}, && ((3.1))\\
f^{-1}(\lambda)_T(x) &= \lambda_T(f(x)) && ((3.2))\\
&\leq \max\{\lambda_T((f(z)\bullet f(y))\bullet(f(z)\bullet f(x))), \lambda_T(f(y))\} && ((2.64))\\
&= \max\{\lambda_T(f(z\circ y)\bullet f(z\circ x)), \lambda_T(f(y))\}\\
&= \max\{\lambda_T(f((z\circ y)\circ(z\circ x))), \lambda_T(f(y))\}\\
&= \max\{f^{-1}(\lambda)_T((z\circ y)\circ(z\circ x)), f^{-1}(\lambda)_T(y)\}, && ((3.2))\\
f^{-1}(\lambda)_I(x) &= \lambda_I(f(x)) && ((3.2))\\
&\geq \min\{\lambda_I((f(z)\bullet f(y))\bullet(f(z)\bullet f(x))), \lambda_I(f(y))\} && ((2.64))\\
&= \min\{\lambda_I(f(z\circ y)\bullet f(z\circ x)), \lambda_I(f(y))\}\\
&= \min\{\lambda_I(f((z\circ y)\circ(z\circ x))), \lambda_I(f(y))\}\\
&= \min\{f^{-1}(\lambda)_I((z\circ y)\circ(z\circ x)), f^{-1}(\lambda)_I(y)\}, && ((3.2))\\
f^{-1}(\lambda)_F(x) &= \lambda_F(f(x)) && ((3.2))\\
&\leq \max\{\lambda_F((f(z)\bullet f(y))\bullet(f(z)\bullet f(x))), \lambda_F(f(y))\} && ((2.64))\\
&= \max\{\lambda_F(f(z\circ y)\bullet f(z\circ x)), \lambda_F(f(y))\}\\
&= \max\{\lambda_F(f((z\circ y)\circ(z\circ x))), \lambda_F(f(y))\}\\
&= \max\{f^{-1}(\lambda)_F((z\circ y)\circ(z\circ x)), f^{-1}(\lambda)_F(y)\}. && ((3.2))
\end{aligned}$$

Hence, $f^{-1}(\mathscr{A})$ is a neutrosophic cubic strong UP-ideal of X. □

Definition 3.8. A NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X has *NCS-property* if for any non-empty subset A of X, there exist elements $\alpha_{T,I,F}, \beta_{T,I,F} \in A$ (instead of $\alpha_T, \alpha_I, \alpha_F, \beta_T, \beta_I, \beta_F \in A$) such that

$$A_T(\alpha_T) = \text{rsup}_{s\in A}\{A_T(s)\}, A_I(\alpha_I) = \text{rinf}_{s\in A}\{A_I(s)\}, A_F(\alpha_F) = \text{rsup}_{s\in A}\{A_F(s)\},$$
$$\lambda_T(\beta_T) = \inf_{s\in A}\{\lambda_T(s)\}, \lambda_I(\beta_I) = \sup_{s\in A}\{\lambda_I(s)\}, \lambda_F(\beta_F) = \inf_{s\in A}\{\lambda_F(s)\}.$$

Definition 3.9. Let X and Y be any two non-empty sets and let $f : X \to Y$ be any function. A NCS $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ in X is said to be *f-invariant* if

$$(\text{for all } x, y \in X)(f(x) = f(y) \Rightarrow A_{T,I,F}(x) = A_{T,I,F}(y), \lambda_{T,I,F}(x) = \lambda_{T,I,F}(y)). \tag{3.4}$$

Lemma 3.10. *Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras and let $f\colon X \to Y$ be a UP-epimorphism. Let $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be an f-invariant NCS in X with NCS-property. For any $x, y \in Y$, there exist* *elements* $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ *and* $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ *such that*

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$
$$f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F),$$
$$f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F).$$

Proof. Let $x, y \in Y$. Since f is surjective, we have $f^{-1}(x), f^{-1}(y)$, and $f^{-1}(x \circ y)$ are non-empty subsets of X. Since $\mathscr{A}$ has NCS-property, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x), \beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$, and $a_{T,I,F}, b_{T,I,F} \in f^{-1}(x \bullet y)$ such that

$$f(A)_T(x) = \text{rsup}_{s\in f^{-1}(x)}\{A_T(s)\} = A_T(\alpha_T),$$
$$f(A)_I(x) = \text{rinf}_{s\in f^{-1}(x)}\{A_I(s)\} = A_I(\alpha_I),$$
$$f(A)_F(x) = \text{rsup}_{s\in f^{-1}(x)}\{A_F(s)\} = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \inf_{s\in f^{-1}(x)}\{\lambda_T(s)\} = \lambda_T(\gamma_T),$$
$$f(\lambda)_I(x) = \sup_{s\in f^{-1}(x)}\{\lambda_I(s)\} = \lambda_I(\gamma_I),$$
$$f(\lambda)_F(x) = \inf_{s\in f^{-1}(x)}\{\lambda_F(s)\} = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = \text{rsup}_{s\in f^{-1}(y)}\{A_T(s)\} = A_T(\beta_T),$$
$$f(A)_I(y) = \text{rinf}_{s\in f^{-1}(y)}\{A_I(s)\} = A_I(\beta_I),$$
$$f(A)_F(y) = \text{rsup}_{s\in f^{-1}(y)}\{A_F(s)\} = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \inf_{s\in f^{-1}(y)}\{\lambda_T(s)\} = \lambda_T(\phi_T),$$
$$f(\lambda)_I(y) = \sup_{s\in f^{-1}(y)}\{\lambda_I(s)\} = \lambda_I(\phi_I),$$
$$f(\lambda)_F(y) = \inf_{s\in f^{-1}(y)}\{\lambda_F(s)\} = \lambda_F(\phi_F),$$

and

$$f(A)_T(x \bullet y) = \text{rsup}_{s\in f^{-1}(x\bullet y)}\{A_T(s)\} = A_T(a_T),$$
$$f(A)_I(x \bullet y) = \text{rinf}_{s\in f^{-1}(x\bullet y)}\{A_I(s)\} = A_I(a_I),$$
$$f(A)_F(x \bullet y) = \text{rsup}_{s\in f^{-1}(x\bullet y)}\{A_F(s)\} = A_F(a_F),$$
$$f(\lambda)_T(x \bullet y) = \inf_{s\in f^{-1}(x\bullet y)}\{\lambda_T(s)\} = \lambda_T(b_T),$$
$$f(\lambda)_I(x \bullet y) = \sup_{s\in f^{-1}(x\bullet y)}\{\lambda_I(s)\} = \lambda_I(b_I),$$
$$f(\lambda)_F(x \bullet y) = \inf_{s\in f^{-1}(x\bullet y)}\{\lambda_F(s)\} = \lambda_F(b_F).$$

Since

$$f(a_T) = x \bullet y = f(\alpha_T) \bullet f(\beta_T) = f(\alpha_T \circ \beta_T),$$
$$f(a_I) = x \bullet y = f(\alpha_I) \bullet f(\beta_I) = f(\alpha_I \circ \beta_I),$$
$$f(a_F) = x \bullet y = f(\alpha_F) \bullet f(\beta_F) = f(\alpha_F \circ \beta_F),$$
$$f(b_T) = x \bullet y = f(\gamma_T) \bullet f(\phi_T) = f(\gamma_T \circ \phi_T),$$
$$f(b_I) = x \bullet y = f(\gamma_I) \bullet f(\phi_I) = f(\gamma_I \circ \phi_I),$$
$$f(b_F) = x \bullet y = f(\gamma_F) \bullet f(\phi_F) = f(\gamma_F \circ \phi_F),$$

and $\mathscr{A}$ is f-invariant, it follows that

$$f(A)_T(x \bullet y) = A_T(a_T) = A_T(\alpha_T \circ \beta_T),$$
$$f(A)_I(x \bullet y) = A_I(a_I) = A_I(\alpha_I \circ \beta_I),$$
$$f(A)_F(x \bullet y) = A_F(a_F) = A_F(\alpha_F \circ \beta_F),$$
$$f(\lambda)_T(x \bullet y) = \lambda_T(b_T) = \lambda_T(\gamma_T \circ \phi_T),$$
$$f(\lambda)_I(x \bullet y) = \lambda_I(b_I) = \lambda_I(\gamma_I \circ \phi_I),$$
$$f(\lambda)_F(x \bullet y) = \lambda_F(b_{TF}) = \lambda_F(\gamma_F \circ \phi_F).$$

The proof is completed. □

Theorem 3.11. *Let $(X, \circ, 0_X)$ and $(Y, \bullet, 0_Y)$ be two UP-algebras, $f\colon X \to Y$ be a UP-epimorphism, and $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ be a NCS in X. Then the followings hold:*

(1) *If $\mathscr{A}$ is an f-invariant neutrosophic cubic UP-subalgebra of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-subalgebra of Y.*

(2) *If $\mathscr{A}$ is an f-invariant neutrosophic cubic near UP-filter of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic near UP-filter of Y.*

(3) *If $\mathscr{A}$ is an f-invariant neutrosophic cubic UP-filter of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-filter of Y.*

(4) *If $\mathscr{A}$ is an f-invariant neutrosophic cubic UP-ideal of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic UP-ideal of Y.*

(5) *If $\mathscr{A}$ is an f-invariant neutrosophic cubic strong UP-ideal of X with NCS-property, then the image $f(\mathscr{A})$ of $\mathscr{A}$ under f is a neutrosophic cubic strong UP-ideal of Y.*

Proof. (1) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic UP-subalgebra of X with NCS-property. Let $x, y \in Y$. Since f is surjective, we have $f^{-1}(x), f^{-1}(y)$, and $f^{-1}(x \bullet y)$ are non-empty. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$\begin{gathered}
f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),\\
f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),\\
f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),\\
f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),\\
f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F),\\
f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F).
\end{gathered}$$

Then

$$\begin{aligned}
f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T) &\succeq \text{rmin}\{A_T(\alpha_T), A_T(\beta_T)\} = \text{rmin}\{f(A)_T(x), f(A)_T(y)\}, && ((2.53))\\
f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I) &\preceq \text{rmax}\{A_I(\alpha_I), A_I(\beta_I)\} = \text{rmax}\{f(A)_I(x), f(A)_I(y)\}, && ((2.53))\\
f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F) &\succeq \text{rmin}\{A_F(\alpha_F), A_F(\beta_F)\} = \text{rmin}\{f(A)_F(x), f(A)_F(y)\}, && ((2.53))\\
f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T) &\leq \max\{\lambda_T(\gamma_T), \lambda_T(\phi_T)\} = \max\{f(\lambda)_T(x), f(\lambda)_T(y)\}, && ((2.54))\\
f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I) &\geq \min\{\lambda_I(\gamma_I), \lambda_I(\phi_I)\} = \min\{f(\lambda)_I(x), f(\lambda)_I(y)\}, && ((2.54))\\
f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F) &\leq \max\{\lambda_F(\gamma_F), \lambda_F(\phi_F)\} = \max\{f(\lambda)_F(x), f(\lambda)_F(y)\}. && ((2.54))
\end{aligned}$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic UP-subalgebra of Y.

(2) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic near UP-filter of X with NCS-property. By Theorem 2.6 (1), we have $0_X \in f^{-1}(0_Y)$ and so $f^{-1}(0_Y)$ is non-empty. Thus

$$\begin{pmatrix}
f(A)_T(0_Y) = \text{rsup}_{s \in f^{-1}(0_Y)}\{A_T(s)\} \succeq A_T(0_X)\\
f(A)_I(0_Y) = \text{rinf}_{s \in f^{-1}(0_Y)}\{A_I(s)\} \preceq A_I(0_X)\\
f(A)_F(0_Y) = \text{rsup}_{s \in f^{-1}(0_Y)}\{A_F(s)\} \succeq A_F(0_X)\\
f(\lambda)_T(0_Y) = \inf_{s \in f^{-1}(0_Y)}\{\lambda_T(s)\} \leq \lambda_T(0_X)\\
f(\lambda)_I(0_Y) = \sup_{s \in f^{-1}(0_Y)}\{\lambda_I(s)\} \geq \lambda_I(0_X)\\
f(\lambda)_F(0_Y) = \inf_{s \in f^{-1}(0_Y)}\{\lambda_F(s)\} \leq \lambda_F(0_X)
\end{pmatrix}. \tag{3.5}$$

Let $y \in Y$. Since f is surjective, we have $f^{-1}(y)$ is non-empty. By (2.55) and (2.56), we have $A_T(0_X) \succeq A_T(s), A_I(0_X) \preceq A_I(s), A_F(0_X) \succeq A_F(s), \lambda_T(0_X) \leq \lambda_T(s), \lambda_I(0_X) \geq \lambda_I(s), \lambda_F(0_X) \leq \lambda_F(s)$ for all $s \in f^{-1}(y)$. Then $A_T(0_X)$ is an upper bound of $\{A_T(s)\}_{s \in f^{-1}(y)}$, $A_I(0_X)$ is a lower bound of $\{A_I(s)\}_{s \in f^{-1}(y)}$, $A_F(0_X)$ is an upper bound of $\{A_F(s)\}_{s \in f^{-1}(y)}$, $\lambda_T(0_X)$ is a lower bound of $\{\lambda_T(s)\}_{s \in f^{-1}(y)}$, $\lambda_I(0_X)$ is an upper bound of $\{\lambda_I(s)\}_{s\in f^{-1}(y)}$, and $\lambda_F(0_X)$ is a lower bound of $\{\lambda_F(s)\}_{s\in f^{-1}(y)}$. By (3.5), we have

$$f(A)_T(0_Y) \succeq A_T(0_X) \succeq \text{rsup}_{s\in f^{-1}(y)}\{A_T(s)\} = f(A)_T(y),$$
$$f(A)_I(0_Y) \preceq A_I(0_X) \preceq \text{rinf}_{s\in f^{-1}(y)}\{A_I(s)\} = f(A)_I(y),$$
$$f(A)_F(0_Y) \succeq A_F(0_X) \succeq \text{rsup}_{s\in f^{-1}(y)}\{A_F(s)\} = f(A)_F(y),$$
$$f(\lambda)_T(0_Y) \leq \lambda_T(0_X) \leq \inf_{s\in f^{-1}(y)}\{\lambda_T(s)\} = f(\lambda)_T(y),$$
$$f(\lambda)_I(0_Y) \geq \lambda_I(0_X) \geq \sup_{s\in f^{-1}(y)}\{\lambda_I(s)\} = f(\lambda)_I(y),$$
$$f(\lambda)_F(0_Y) \leq \lambda_F(0_X) \leq \inf_{s\in f^{-1}(y)}\{\lambda_F(s)\} = f(\lambda)_F(y).$$

Let $x, y \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$
$$f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F),$$
$$f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F).$$

Then

$$f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T) \succeq A_T(\beta_T) = f(A)_T(y), \quad ((2.57))$$
$$f(A)_I(x \bullet y) = A_T(\alpha_I \circ \beta_I) \preceq A_I(\beta_I) = f(A)_I(y), \quad ((2.57))$$
$$f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F) \succeq A_F(\beta_F) = f(A)_F(y), \quad ((2.57))$$
$$f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T) \leq \lambda_T(\phi_T) = f(\lambda)_T(y), \quad ((2.58))$$
$$f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I) \geq \lambda_I(\phi_I) = f(\lambda)_I(y), \quad ((2.58))$$
$$f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F) \leq \lambda_F(\phi_F) = f(\lambda)_F(y). \quad ((2.58))$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic near UP-filter of Y.

(3) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic UP-filter of X with NCS-property. Then $\mathscr{A}$ is a neutrosophic cubic near UP-filter of X. By the proof of (2), we have $f(\mathscr{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$ and $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ such that

$$f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F),$$
$$f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F),$$
$$f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F),$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F),$$
$$f(A)_T(x \bullet y) = A_T(\alpha_T \circ \beta_T), f(A)_I(x \bullet y) = A_I(\alpha_I \circ \beta_I), f(A)_F(x \bullet y) = A_F(\alpha_F \circ \beta_F),$$
$$f(\lambda)_T(x \bullet y) = \lambda_T(\gamma_T \circ \phi_T), f(\lambda)_I(x \bullet y) = \lambda_I(\gamma_I \circ \phi_I), f(\lambda)_F(x \bullet y) = \lambda_F(\gamma_F \circ \phi_F).$$

Then

$$f(A)_T(y) = A_T(\beta_T) \succeq \text{rmin}\{A_T(\alpha_T \circ \beta_T), A_T(\alpha_T)\} = \text{rmin}\{f(A)_T(x \bullet y), f(A)_T(x)\}, \quad ((2.59))$$
$$f(A)_I(y) = A_I(\beta_I) \preceq \text{rmax}\{A_I(\alpha_I \circ \beta_I), A_I(\alpha_I)\} = \text{rmax}\{f(A)_I(x \bullet y), f(A)_I(x)\}, \quad ((2.59))$$
$$f(A)_F(y) = A_F(\beta_F) \succeq \text{rmin}\{A_F(\alpha_F \circ \beta_F), A_F(\alpha_F)\} = \text{rmin}\{f(A)_F(x \bullet y), f(A)_F(x)\}, \quad ((2.59))$$
$$f(\lambda)_T(y) = \lambda_T(\phi_T) \leq \max\{\lambda_T(\gamma_T \circ \phi_T), \lambda_T(\gamma_T)\} = \max\{f(\lambda)_T(x \bullet y), f(\lambda)_T(x)\}, \quad ((2.60))$$
$$f(\lambda)_I(y) = \lambda_I(\phi_I) \geq \min\{\lambda_I(\gamma_I \circ \phi_I), \lambda_I(\gamma_I)\} = \min\{f(\lambda)_I(x \bullet y), f(\lambda)_I(x)\}, \quad ((2.60))$$
$$f(\lambda)_F(y) = \lambda_F(\phi_F) \leq \max\{\lambda_F(\gamma_F \circ \phi_F), \lambda_F(\gamma_F)\} = \max\{f(\lambda)_F(x \bullet y), f(\lambda)_F(x)\}. \quad ((2.60))$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic UP-filter of Y.

(4) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic UP-ideal of X with NCS-property. Then $\mathscr{A}$ is a neutrosophic cubic UP-filter of X. By the proof of (3), we have $f(\mathscr{A})$ satisfies the

assertions (2.55) and (2.56). Let $x, y, z \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that

$$\begin{gathered}
f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\
f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\
f(A)_T(x \bullet z) = A_T(\alpha_T \circ \psi_T), f(A)_I(x \bullet z) = A_I(\alpha_I \circ \psi_I), f(A)_F(x \bullet z) = A_F(\alpha_F \circ \psi_F), \\
f(\lambda)_T(x \bullet z) = \lambda_T(\gamma_T \circ \omega_T), f(\lambda)_I(x \bullet z) = \lambda_I(\gamma_I \circ \omega_I), f(\lambda)_F(x \bullet z) = \lambda_F(\gamma_F \circ \omega_F), \\
f(A)_T(x \bullet (y \bullet z)) = A_T(\alpha_T \circ (\beta_T \circ \psi_T)), \\
f(A)_I(x \bullet (y \bullet z)) = A_I(\alpha_I \circ (\beta_I \circ \psi_I)), \\
f(A)_F(x \bullet (y \bullet z)) = A_F(\alpha_F \circ (\beta_F \circ \psi_F)), \\
f(\lambda)_T(x \bullet (y \bullet z)) = \lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \\
f(\lambda)_I(x \bullet (y \bullet z)) = \lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \\
f(\lambda)_F(x \bullet (y \bullet z)) = \lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)).
\end{gathered}$$

Then

$$\begin{aligned}
f(A)_T(x \bullet z) &= A_T(\alpha_T \circ \psi_T) \\
&\succeq \text{rmin}\{A_T(\alpha_T \circ (\beta_T \circ \psi_T)), A_T(\beta_T)\} && ((2.61)) \\
&= \text{rmin}\{f(A)_T(x \bullet (y \bullet z)), f(A)_T(y)\}, \\
f(A)_I(x \bullet z) &= A_I(\alpha_I \circ \psi_I) \\
&\preceq \text{rmax}\{A_I(\alpha_I \circ (\beta_I \circ \psi_I)), A_I(\beta_I)\} && ((2.61)) \\
&= \text{rmax}\{f(A)_I(x \bullet (y \bullet z)), f(A)_I(y)\}, \\
f(A)_F(x \bullet z) &= A_F(\alpha_F \circ \psi_F) \\
&\succeq \text{rmin}\{A_F(\alpha_F \circ (\beta_F \circ \psi_F)), A_F(\beta_F)\} && ((2.61)) \\
&= \text{rmin}\{f(A)_F(x \bullet (y \bullet z)), f(A)_F(y)\}, \\
f(\lambda)_T(x \bullet z) &= \lambda_T(\gamma_T \circ \omega_T) \\
&\leq \max\{\lambda_T(\gamma_T \circ (\phi_T \circ \omega_T)), \lambda_T(\phi_T)\} && ((2.62)) \\
&= \max\{f(\lambda)_T(x \bullet (y \bullet z)), f(\lambda)_T(y)\}, \\
f(\lambda)_I(x \bullet z) &= \lambda_I(\gamma_I \circ \omega_I) \\
&\geq \min\{\lambda_I(\gamma_I \circ (\phi_I \circ \omega_I)), \lambda_I(\phi_I)\} && ((2.62)) \\
&= \min\{f(\lambda)_I(x \bullet (y \bullet z)), f(\lambda)_I(y)\}, \\
f(\lambda)_F(x \bullet z) &= \lambda_F(\gamma_F \circ \omega_F) \\
&\leq \max\{\lambda_F(\gamma_F \circ (\phi_F \circ \omega_F)), \lambda_F(\phi_F)\} && ((2.62)) \\
&= \max\{f(\lambda)_F(x \bullet (y \bullet z)), f(\lambda)_F(y)\}.
\end{aligned}$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic UP-ideal of Y.

(5) Assume that $\mathscr{A} = (A_{T,I,F}, \lambda_{T,I,F})$ is an f-invariant neutrosophic cubic strong UP-ideal of X with NCS-property. Then $\mathscr{A}$ is a neutrosophic cubic UP-ideal of X. By the proof of (4), we have $f(\mathscr{A})$ satisfies the assertions (2.55) and (2.56). Let $x, y, z \in Y$. By Lemma 3.10, there exist elements $\alpha_{T,I,F}, \gamma_{T,I,F} \in f^{-1}(x)$, $\beta_{T,I,F}, \phi_{T,I,F} \in f^{-1}(y)$ and $\psi_{T,I,F}, \omega_{T,I,F} \in f^{-1}(z)$ such that

$$\begin{gathered}
f(A)_T(x) = A_T(\alpha_T), f(A)_I(x) = A_I(\alpha_I), f(A)_F(x) = A_F(\alpha_F), \\
f(\lambda)_T(x) = \lambda_T(\gamma_T), f(\lambda)_I(x) = \lambda_I(\gamma_I), f(\lambda)_F(x) = \lambda_F(\gamma_F), \\
f(A)_T(y) = A_T(\beta_T), f(A)_I(y) = A_I(\beta_I), f(A)_F(y) = A_F(\beta_F), \\
f(\lambda)_T(y) = \lambda_T(\phi_T), f(\lambda)_I(y) = \lambda_I(\phi_I), f(\lambda)_F(y) = \lambda_F(\phi_F), \\
f(A)_T((z \bullet y) \bullet (z \bullet x)) = A_T((\psi_T \circ \beta_T) \circ (\psi_T \circ \alpha_T)), \\
f(A)_I((z \bullet y) \bullet (z \bullet x)) = A_I((\psi_I \circ \beta_I) \circ (\psi_I \circ \alpha_I)), \\
f(A)_F((z \bullet y) \bullet (z \bullet x)) = A_F((\psi_F \circ \beta_F) \circ (\psi_F \circ \alpha_F)), \\
f(\lambda)_T((z \bullet y) \bullet (z \bullet x)) = \lambda_T((\omega_T \circ \phi_T) \circ (\omega_T \circ \gamma_T)), \\
f(\lambda)_I((z \bullet y) \bullet (z \bullet x)) = \lambda_I((\omega_I \circ \phi_I) \circ (\omega_I \circ \gamma_I)), \\
f(\lambda)_F((z \bullet y) \bullet (z \bullet x)) = \lambda_F((\omega_F \circ \phi_F) \circ (\omega_F \circ \gamma_F)).
\end{gathered}$$

Then

$$\begin{aligned}
f(A)_T(x) = A_T(\alpha_T) &\succeq \text{rmin}\{A_T((\psi_T \circ \beta_T) \circ (\psi_T \circ \alpha_T)), A_T(\beta_T)\} && ((2.63))\\
&= \text{rmin}\{f(A)_T((z \bullet y) \bullet (z \bullet x)), f(A)_T(y)\},\\
f(A)_I(x) = A_I(\alpha_I) &\preceq \text{rmax}\{A_I((\psi_I \circ \beta_I) \circ (\psi_I \circ \alpha_I)), A_I(\beta_I)\} && ((2.63))\\
&= \text{rmax}\{f(A)_I((z \bullet y) \bullet (z \bullet x)), f(A)_I(y)\},\\
f(A)_F(x) = A_F(\alpha_F) &\succeq \text{rmin}\{A_F((\psi_F \circ \beta_F) \circ (\psi_F \circ \alpha_F)), A_F(\beta_F)\} && ((2.63))\\
&= \text{rmin}\{f(A)_F((z \bullet y) \bullet (z \bullet x)), f(A)_F(y)\},\\
f(\lambda)_T(x) = \lambda_T(\gamma_T) &\le \max\{\lambda_T((\omega_T \circ \phi_T) \circ (\omega_T \circ \gamma_T)), \lambda_T(\phi_T)\} && ((2.64))\\
&= \max\{f(\lambda)_T((z \bullet y) \bullet (z \bullet x)), f(\lambda)_T(y)\},\\
f(\lambda)_I(x) = \lambda_I(\gamma_I) &\ge \min\{\lambda_I((\omega_I \circ \phi_I) \circ (\omega_I \circ \gamma_I)), \lambda_I(\phi_I)\} && ((2.64))\\
&= \min\{f(\lambda)_I((z \bullet y) \bullet (z \bullet x)), f(\lambda)_I(y)\},\\
f(\lambda)_F(x) = \lambda_F(\gamma_F) &\le \max\{\lambda_F((\omega_F \circ \phi_F) \circ (\omega_F \circ \gamma_F)), \lambda_F(\phi_F)\} && ((2.64))\\
&= \max\{f(\lambda)_F((z \bullet y) \bullet (z \bullet x)), f(\lambda)_F(y)\}.
\end{aligned}$$

Hence, $f(\mathscr{A})$ is a neutrosophic cubic strong UP-ideal of Y. □

4 Conclusions and future work

In this paper, we have studied the image and inverse image of a neutrosophic cubic UP-subalgebra (resp., neutrosophic cubic near UP-filter, neutrosophic cubic UP-filter, neutrosophic cubic UP-ideal, neutrosophic cubic strong UP-ideal) of a UP-algebra under some UP-homomorphisms. The results of the study, in the case of inverse image, we noticed that only a neutrosophic cubic near UP-filter required order preserving condition. In the case of image, we noticed that all concepts of NCSs required f-invariant and NCS-property assertions and UP-epimorphism.

In our future study, we will apply this concept/results to other types of NCSs in a UP-algebra. Also, we will study the P-intersection, P-union, R-intersection, R-union of neutrosophic cubic UP-subalgebras, neutrosophic cubic near UP-filters, neutrosophic cubic UP-filters, neutrosophic cubic UP-ideals, and neutrosophic cubic strong UP-ideals of a UP-algebra.

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