Implementation of the Neutrosophic Sets in Measurable Space with Respect to Neutrosophic Ring

Ibrahim S. Ahmed[1,*], Ali Al-Fayadh[2], Hassan H. Ebrahim[3], Luma S. Abdalbaqi[3]

[1]Department of Mathematics. College of Education-Tuzkhurmatu. Tikrit University. Iraq
[2] Department of Mathematics and Computer Applications. College of Science. Al – Nahrain University. Iraq
[3]Department of Mathematics. College of Computer Science and Mathematics. Tikrit University. Iraq

Emails: ibrahim1992@tu.edu.iq; aalfayadh@yahoo.com; hassan1962pl@tu.edu.iq; lumahhany1977@tu.edu.iq

Abstract

The generalization for interval fuzzy set name as neutrosophic set employed to construct a measurable space in this work. The measurable space with respect to a ring of sets that is closed under difference and union, is studied. The objective of this study is to extend the notion of a ring of sets by using neutrosophic sets. Neutrosophic set concept has gained popularity in various fields of mathematics, probability, and other sciences due to its many uses, especially when dealing with uncertainties. Several different properties of neutrosophic ring are studied. Examples and characterizations to the proposed extension are given.

Keywords: σ-algebra; Ring; Neutrosophic sets; Measurable spaces; Fuzzy sets

1. Introduction

Generalized measure theory is arise from the well-established conventional measure theory by the process of generalization, which done by replacing some conditions with a considerably weak conditions [13]. The majority of our conventional tools for formal modeling, computation, and reasoning are clear and deterministic. However, a lot of complex issues in the fields of medicine, engineering, economics, etc., require evidence that isn't always obvious. Due to the various types of uncertainty that these situations offer, we are not always able to employ the classical methodologies. Mathematical tools for handling uncertainties include the theory of fuzzy set [15, 16], the probability theory [17], the theory of intuitionistic fuzzy sets [18] which is an extension of fuzzy sets, and the theory of interval mathematics [19,20]. But as pointed in [21], all these theories have their possess challenges. These challenges could result from the theories' inadequate parametrization tool, which is why Smarandache [12] developed the idea of neutrosophic set by introducing an independent component, indeterminacy degree, for handling uncertainty that is unaffected by the aforementioned issues. The neutrosophic set concept represent a significant generalization of fuzzy sets and intuitionistic fuzzy sets, serving as an effective framework for addressing incomplete, indeterminate, and inconsistent information that is prevalent in real-world scenarios. The truth membership function, indeterminacy membership function, and falsity membership function are characteristics of neutrosophic sets. Since indeterminacy is explicitly quantified and the truth, indeterminacy, and falsity membership functions are independent, this theory is crucial in a wide range of application domains. The notion of a single-valued neutrosophic set was presented by Wang et al. [22]. The single-valued neutrosophic set deals with inconsistent, indeterminate, and incomplete information and can independently express truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Given the limitations of the knowledge that humans acquire or observe from the outside world, all of the elements covered by the single-valued neutrosophic set are highly appropriate for human thought. The broad range of application areas and theoretical sophistication of single valued neutrosophic sets led to their rapid development, for examples, see [23-31]. The interval neutrosophic set was introduced by Wang as extension of the neutrosophic set [32]. Uncertain, incomplete, imprecise, and inconsistent information that exists in the real world can be represented by the interval neutrosophic set. A special instance to the single valued neutrosophic sets, the single valued fuzzy number is

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024

significant for decision-making issues. In decision making issues, several researchers investigated the idea of trapezoidal neutrosophic fuzzy numbers to generalize the triangular fuzzy numbers, trapezoidal intuitionistic fuzzy numbers, and triangular intuitionistic fuzzy numbers [33, 34]. The ranking for neutrosophic trapezoidal numbers was examined by Deli [35] and Biswas [29], who then used the idea to resolve decision making difficulties. The rough neutrosophic set was defined and its fundamental characteristics were demonstrated by Broumi et al. [36]. The literature has documented some theoretical developments and applications [37-40]. Many scholars have examined a neutrosophic refined sets, which are generalization of neutrosophic sets, and their applications [41-44].

The σ-algebra concept is studied by Ahmed et al. [5, 10] which is stronger form of ring. In [2, 4] Ahmed et al. were introduced an idea of fuzzy σ-algebra as an extension for each of σ-algebra and ring concepts. It is shown that the fuzzy σ-algebra is closed under the countable union of fuzzy sets and it is closed under complementation of fuzzy sets, while the ring is closed under a finite union of ordinary sets and a finite union of the differences of these sets. The neutrosophic σ-algebra has been introduced in 2022 by Sahin [11].

In recent decades, the matter of uncertainty model has received many concern from authors in the field of theory of probability, various ideas, inclusive of fuzzy sets theory and possibility measures were proposed [8]. The study of the concept of neutrosophic branched into several fields of mathematics such as neutrosophis topological spaces, see [45, 46]

The theory of neutrosophic sets were advanced to treat uncertainty modelling best than conventional styles such as fuzzy sets theory, so the neutrosophic sets theory is outperform by the theory of fuzzy sets for uncertainty modelling.

In this article, a novel type of measurable spaces are introduced and studied by using neutrosophic sets; namely neutrosophic ring. The objective of employing neutrosophic sets in measurable spaces provides various benefits, particularly when dealing with uncertainty which could be of mighty significance in the field of applied mathematics.

2. Related Work

Definition 2.1 [2]. The fuzzy set Ą on universal U define by Ą= {(v, $ţ_{Ą}$(v)) : v∈U} where $ţ_{Ą}$: U →[0,1] is function with for all v ϵ U, $ţ_{Ą}$(v) represent a degree for membership of v ϵ Ą.

Definition 2.2 [7]. The intuitionistic fuzzy set Ą in universal U defining by Ą= {(v, $ţ_{Ą}$(v), $ʄ_{Ą}$(v)) : v ϵ U} where $ţ_{Ą}$ and $ʄ_{Ą}$: U →[0,1] are function having for each v ϵ U, $ţ_{Ą}$(v) is a degree for membership of v ϵ Ą and $ʄ_{Ą}$(v) represent a degree for non-membership of v ϵ Ą with $ţ_{Ą}$(v) + $ʄ_{Ą}$(v)≤1.

Definition 2.3 [12]. The neutrosophic set Ą on universal U is represent as Ą= {(v, $ţ_{Ą}$(v), $ɩ_{Ą}$(v), $ʄ_{Ą}$(v)) :v ϵ U} where $ţ_{Ą}$, $ɩ_{Ą}$ and $ʄ_{Ą}$: U →[0,1] are function such that for any v ϵ U, $ţ_{Ą}$(v) is called a degree for trust-membership of v ϵ Ą , $ɩ_{Ą}$(v) is named a degree for indeterminacy-membership of v ϵ Ą and $ʄ_{Ą}$(v) represent to a degree for falsity-membership of v ϵ Ą .

Definition 2.4 [9]. The nonempty-collection Ψ of subsets in universal U is called ring if Ą-Ɓ ϵ Ψ and Ą ∪ Ɓ ϵ Ψ whenever Ą, Ɓ ϵ Ψ.

Definition 2.5 [1,14].

1) The neutrosophic power set is denoted by NP(U), where U is universal set.

2) A union for neutrosophic set Ą & Ɓ defined in following forms:

a) Ą $\cup_N$ Ɓ = {(v, Max{$ţ_{Ą}$(v), $ţ_{B}$(v)}, Min{$ɩ_{Ą}$(v),$ɩ_{B}$(v)}, Min{$ʄ_{Ą}$(v), $ʄ_{B}$(v) }) : v ϵ U}.

b) Ą $\cup_N$ Ɓ = {(v, Max{$ţ_{Ą}$(v), $ţ_{B}$(v)}, Max{$ɩ_{Ą}$(v),$ɩ_{B}$(v)}, Min{$ʄ_{Ą}$(v), $ʄ_{B}$(v) }) : v ϵ U}.

3) An intersection for neutrosophic set Ą & Ɓ possible have forms in below:

a) Ą $\cap_N$ Ɓ = {(v, Min{$ţ_{Ą}$(v), $ţ_{B}$(v)}, Max{$ɩ_{Ą}$(v),$ɩ_{B}$(v)}, Max{$ʄ_{Ą}$(v), $ʄ_{B}$(v) }) : v ϵ U}.

b) Ą $\cap_N$ Ɓ = {(v, Min{$ţ_{Ą}$(v), $ţ_{B}$(v)}, Min{$ɩ_{Ą}$(v),$ɩ_{B}$(v)}, Max{$ʄ_{Ą}$(v), $ʄ_{B}$(v) }) : v ϵ U}.

Definition 2.6 [1].

1) The empty neutrosophic set can be define by

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024

a) N(Φ) = {(v,0,0,1): v ϵ U}.

b) N(Φ) = {(v,0,1,1): v ϵ U}.

c) N(Φ) = {(v,0,1,0): v ϵ U}.

d) N(Φ) = {(v,0,0,0): v ϵ U}.

2) The universal neutrosophic set may be define by:

a) N(U) = {(v,1,0,0): v ϵ U}.

b) N(U) = {(v,1,01): v ϵ U}.

c) N(U) = {(v,1,1,0): v ϵ U}.

d) N(U) = {(v,1,1,1): v ϵ U}.

3) A complement for neutrosophic set Ą can be writing in the following forms:

a) $Ą^{C_N}$= {(r, $ƒ_Ą$(r), $l_Ą$(r), $ƫ_Ą$(r)) : r ϵ U}.

b) $Ą^{C_N}$= {(r, $ƒ_Ą$(r), 1- $l_Ą$(r), $ƫ_Ą$(r)) : r ϵ U}

c) $Ą^{C_N}$= {(r, 1- $ƫ_Ą$(r), 1- $l_Ą$(r), 1- $ƒ_Ą$(r)) : r ϵ U}.

4) The neutrosophic sets Ą contained in Ɓ is possible having the forms:

a) Ą $\subseteq_N$ Ɓ iff $ƫ_Ą(\varkappa) \le ƫ_Ɓ(\varkappa)$ & $l_Ą(\varkappa) \ge l_Ɓ(\varkappa)$ and $ƒ_Ą(\varkappa) \ge ƒ_Ɓ(\varkappa)$ for all $\varkappa$ ϵ U.

b) Ą $\subseteq_N$ Ɓ iff $ƫ_Ą(\varkappa) \le ƫ_Ɓ(\varkappa)$ & $l_Ą(\varkappa) \le l_Ɓ(\varkappa)$ and $ƒ_Ą(\varkappa) \ge ƒ_Ɓ(\varkappa)$ for all $\varkappa$ ϵ U.

3. Results

We start this section by introducing the neutrosophic ring concept and we studying its basic properties.

Definition 3.1. A difference among two neutrosophic set Ą & Ɓ may be define by the following forms:

a) Ą $-_N$ Ɓ = Ą $\cap_N$ $Ɓ^{C_N}$

= {(v, Min{$ƫ_Ą$(v), $ƒ_Ɓ$(v)}, Max{$l_Ą$(v),$l_Ɓ$(v)}, Max{$ƒ_Ą$(v), $ƫ_Ɓ$(v) }) : v ϵ U}.

b) Ą $-_N$ Ɓ = Ą $\cap_N$ $Ɓ^{C_N}$

= {(v, Min{$ƫ_Ą$(v), $ƒ_Ɓ$(v)}, Min{$l_Ą$(v),$l_Ɓ$(v)}, Max{$ƒ_Ą$(v), $ƫ_Ɓ$(v) }) : v ϵ U}.

c) Ą $-_N$ Ɓ = Ą $\cap_N$ $Ɓ^{C_N}$

= {(v, Min{$ƫ_Ą$(v), $ƒ_Ɓ$(v)}, Max{1-$l_Ą$(v),1-$l_Ɓ$(v)}, Max{$ƒ_Ą$(v), $ƫ_Ɓ$(v) }) : v ϵ U}.

d) Ą $-_N$ Ɓ = Ą $\cap_N$ $Ɓ^{C_N}$

= {(v, Min{$ƫ_Ą$(v), $ƒ_Ɓ$(v)}, Min{1-$l_Ą$(v),1-$l_Ɓ$(v)}, Max{$ƒ_Ą$(v), $ƫ_Ɓ$(v) }) : v ϵ U}.

e) Ą $-_N$ Ɓ = Ą $\cap_N$ $Ɓ^{C_N}$

= {(v, Min{1-$ƫ_Ą$(v), 1-$ƫ_Ɓ$(v)}, Max{1-$l_Ą$(v),1-$l_Ɓ$(v)}, Max{1-$ƒ_Ą$(v), 1-$ƒ_Ɓ$(v) }): v ϵ U}.

f) Ą $-_N$ Ɓ = Ą $\cap_N$ $Ɓ^{C_N}$

= {(v, Min{1-$ƫ_Ą$(v), 1-$ƫ_Ɓ$(v)}, Min{1-$l_Ą$(v),1-$l_Ɓ$(v)}, Max{1-$ƒ_Ą$(v), 1-$ƒ_Ɓ$(v) }): v ϵ U}.

Definition 3.2. The family Ƒ of neutrosophic subsets of a universal neutrosophic set N(U) is called neutrosophic ring on N(U) if :

1) N(Φ) ϵ Ƒ.

2) (Ą $-_N$ Ɓ) ϵ Ƒ, for any Ą & Ɓ ϵ Ƒ.

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024

3) (Ą $\cup_N$Ɓ) ϵ Ƒ whenever Ą & Ɓ ϵ Ƒ .

An ordered pair (N(U), Ƒ) is named neutrosophic measurable space with respect to neutrosophic ring and its members is called neutrosophic measurable sets with respect to neutrosophic ring.

Example 3.3. Let us U = {s, k} and Ƒ = { Ą = {(s,0,1,1),(k,0,1,1)}, Ɓ ={(s,1,1,0),(k,1,1,0)}}. then Ƒ is neutrosophic ring on N(U) where N(U) ={(s,1,0,0),(k,1,0,0)}, because if we consider Ą = {(s,0,1,1),(k,0,1,1)} & Ɓ ={(s,1,1,0),(k,1,1,0)}, then Ą & Ɓ $\subseteq_N$ N(U) and by using form (b) in definition 2.6 (1), form (a) in definition 2.5 (2), form (a) in definition 2.6 (3) and form (a) in definition 3.1, then we note that (Ą $\cup_N$Ɓ) = Ɓ, (Ɓ $-_N$ Ą) = Ɓ, (Ą $-_N$ Ɓ) = Ą. Thus the all condition in Definition 3.2 are holds.

Theorem 3.4. If $\{Ƒ_\alpha\}_{\alpha\epsilon\Lambda}$ is a family of neutrosophic ring on N(U), then so is $\bigcap_{\alpha\epsilon\Lambda} Ƒ_\alpha$.

Proof. Assume that for all αϵΛ, $Ƒ_\alpha$ is a neutrosophic ring, then N(Φ) ϵ $Ƒ_\alpha$ ∀αϵΛ, hence N(Φ) ϵ $\bigcap_{\alpha\epsilon\Lambda} Ƒ_\alpha$.

Let us Ą & Ɓ ϵ $\bigcap_{\alpha\epsilon\Lambda} Ƒ_\alpha$. Then Ą & Ɓ ϵ $Ƒ_\alpha$ ∀αϵΛ.

Since ∀αϵΛ, $Ƒ_\alpha$ is a neutrosophic ring, then (Ą $-_N$ Ɓ) ϵ $Ƒ_\alpha$ ∀αϵΛ and (Ą $\cup_N$Ɓ) ϵ $Ƒ_\alpha$ ∀αϵΛ, thus

(Ą $-_N$ Ɓ) ϵ $\bigcap_{\alpha\epsilon\Lambda} Ƒ_\alpha$ and (Ą $\cup_N$Ɓ) ϵ$\bigcap_{\alpha\epsilon\Lambda} Ƒ_\alpha$. Therefore, $\bigcap_{\alpha\epsilon\Lambda} Ƒ_\alpha$ is a neutrosophic ring on N(U).

Definition 3.5. If Γ is a collection of neutrosophic subsets of a universal neutrosophic set N(U). Then the intersection of all neutrosophic ring on N(U) which includes Γ is called neutrosophic ring on N(U) that generated by Γ and its denoted by NR(Γ).

Proposition 3.6. NR(Γ) is a smallest neutrosophic ring on N(U) which includes Γ.

Proof. Since NR(Γ) is intersection of all neutrosophic ring on N(U) which includes Γ. Then by

Theorem 3.4, we get NR(Γ) is neutrosophic ring on N(U). By definition of NR(Γ), then NR(Γ) is includes Γ.

Now, if Ƒ is neutrosophic ring on N(U) which includes Γ, so NR(Γ) is contained in Ƒ, hence NR(Γ) is smallest neutrosophic ring on N(U) which includes Γ.

Proposition 3.7. Γ is a neutrosophic ring on N(U) if and only if Γ = NR(Γ).

Proof. The result follows by the definition of NR(Γ) and by **Proposition 3.6**.

Theorem 3.8. If Ψ &Γ are families of neutrosophic subsets of a universal neutrosophic set N(U)such that Ψ contained in Γ, then NR(Ψ) contained in NR(Γ).

Proof. To prove NR(Ψ) contained in NR(Γ). We must prove that NR(Γ) is a neutrosophic ring on N(U) that contains Ψ.

Let Ƒ ϵ NR(Γ). Then Ƒ is a neutrosophic ring on N(U) that contain Γ, but Ψ contained in Γ by hypothesis, thus Ƒ is a neutrosophic ring on N(U) which include Ψ which implies that NR(Γ) is a neutrosophic ring on N(U) that contain Ψ.

But NR(Ψ) is a the smallest neutrosophic ring on N(U) that contain Ψ. Therefore, NR(Ψ) contained in NR(Γ).

Theorem 3.9. If Ψ & Γ are families of neutrosophic subsets of a universal neutrosophic set N(U) such that Ψ contained in Γ. If Γ contained in NR(Ψ), then NR(Ψ) = NR(Γ).

Proof. From **Theorem 3.8**, we have NR(Ψ) contained in NR(Γ).

So, its only remains to prove NR(Γ) contained in NR(Ψ).

By hypothesis we have Γ contained in NR(Ψ) and by definition of NR(Ψ) we known that NR(Ψ) is neutrosophic ring on N(U), so NR(Ψ) is neutrosophic ring on N(U) which includes Γ, but NR(Γ) is a the smallest neutrosophic ring on N(U) that contain Γ, hence NR(Γ) contained in NR(Ψ). Therefore NR(Ψ) = NR(Γ).

Theorem 3.10. If Ψ is class of neutrosophic subsets of universal neutrosophic set N(U), then

NR(Γ) $\cap_N$ Ѵ = NR(Γ $\cap_N$ Ѵ) for every Ѵ $\subseteq_N$ N(U).

Proof. To prove NR(Γ $\cap_N$ Ѵ) is contained in NR(Γ) $\cap_N$ Ѵ, we claim NR(Γ) $\cap_N$ Ѵ is neutrosophic ring on N(U) which include Γ $\cap_N$ Ѵ. Since NR(Γ) $\cap_N$ Ѵ = { S : S = T $\cap_N$ Ѵ, where T ϵ NR(Γ) } and NR(Γ) is neutrosophic ring on N(U), then N(Φ) ϵ NR(Γ), but N(Φ) = N(Φ) $\cap_N$ Ѵ, so N(Φ) ϵ NR(Γ) $\cap_N$ Ѵ.

Let us S & F ϵ NR(Γ) $\cap_N$ Ѵ, then S = T $\cap_N$ Ѵ, where T ϵ NR(Γ) and F = L $\cap_N$ Ѵ, where L ϵ NR(Γ)

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024

Thus, we get S $\cup_N$ F = (T $\cap_N$ Ѵ) $\cup_N$ (L $\cap_N$ Ѵ) = (T $\cup_N$ L) $\cap_N$ Ѵ, but T & L ϵ NR(Γ) and NR(Γ) is neutrosophic ring on N(U), then T $\cup_N$ L ϵ NR(Γ), hence by definition of NR(Γ) $\cap_N$ Ѵ we get S $\cup_N$ F ϵ NR(Γ) $\cap_N$ Ѵ.

Now, S $-_N$ F = (T $\cap_N$ Ѵ) $-_N$ (L $\cap_N$ Ѵ) = (T $-_N$ L) $\cap_N$ Ѵ, but T & L ϵ NR(Γ) and NR(Γ) is neutrosophic ring on N(U), then T $-_N$ L ϵ NR(Γ), hence by definition of NR(Γ) $\cap_N$ Ѵ we get S $-_N$ F ϵ NR(Γ) $\cap_N$ Ѵ.

Therefore, NR(Γ) $\cap_N$ Ѵ is neutrosophic ring on N(U).

Now, to prove Γ $\cap_N$ Ѵ is contained in NR(Γ) $\cap_N$ Ѵ, suppose Ɓ ϵ Γ $\cap_N$ Ѵ, then Ɓ = T $\cap_N$ Ѵ when T ϵ Γ, since NR(Γ) includes Γ, then T ϵ NR(Γ), hence Ɓ = T $\cap_N$ Ѵ ϵ NR(Γ) $\cap_N$ Ѵ.

Therefore, NR(Γ) $\cap_N$ Ѵ is neutrosophic ring on N(U) which include Γ $\cap_N$ Ѵ.

Since NR(Γ $\cap_N$ Ѵ) is smallest neutrosophic ring on N(U) which includes Γ $\cap_N$ Ѵ, then NR(Γ $\cap_N$ Ѵ) is contained in NR(Γ) $\cap_N$ Ѵ.

Now, to complete the proof it is only remains to prove NR(Γ) $\cap_N$ Ѵ is contained in NR(Γ $\cap_N$ Ѵ).

So, we define a family Ƒ as follows

Ƒ = { T : (T $\cap_N$ Ѵ) ϵ NR(Γ $\cap_N$ Ѵ) & T ϵ NR(Γ) } and we claims that Ƒ is neutrosophic ring on N(U) includes Γ and NR(Γ).

Now, since NR(Γ $\cap_N$ Ѵ) & NR(Γ) are neutrosophic ring on N(U), then N(Φ) ϵ NR(Γ $\cap_N$ Ѵ) & NR(Γ), but N(Φ) = N(Φ) $\cap_N$ Ѵ, then N(Φ) ϵ Ƒ.

Assume that T & L ϵ Ƒ, then (T $\cap_N$ Ѵ) ϵ NR(Γ $\cap_N$ Ѵ) & T ϵ NR(Γ) and (L $\cap_N$ Ѵ) ϵ NR(Γ $\cap_N$ Ѵ) & L ϵ NR(Γ). Now, since NR(Γ $\cap_N$ Ѵ) & NR(Γ) are neutrosophic ring on N(U), then we get

(T $\cap_N$ Ѵ) $\cup_N$ (L $\cap_N$ Ѵ) ϵ NR(Γ $\cap_N$ Ѵ) , (T $\cap_N$ Ѵ) $-_N$ (L $\cap_N$ Ѵ) ϵ NR(Γ $\cap_N$ Ѵ), (T $\cup_N$ L) ϵ NR(Γ) and (T $-_N$ L) ϵ NR(Γ), but (T $\cap_N$ Ѵ) $\cup_N$ (L $\cap_N$ Ѵ) = (T $\cup_N$ L) $\cap_N$ Ѵ, so by definition of Ƒ we get (T $\cup_N$ L) ϵ Ƒ and (T $-_N$ L) ϵ Ƒ. Hence Ƒ is neutrosophic ring on N(U).

Now, to prove Ƒ includes Γ we assume that S ϵ Γ, then S $\cap_N$Ѵ ϵ (Γ $\cap_N$ Ѵ), hence S ϵ NR(Γ) because NR(Γ include Γ and S $\cap_N$Ѵ ϵ NR(Γ $\cap_N$ Ѵ) because NR(Γ $\cap_N$ Ѵ) includes (Γ $\cap_N$ Ѵ), so by definition of Ƒ we get S ϵ Ƒ.

Therefore, Ƒ is neutrosophic ring on N(U) includes Γ.

Now, from **Proposition 3.6** we have, NR(Γ) is a smallest neutrosophic ring on N(U) which includes Γ, so Ƒ include NR(Γ).

Now, let us Y ϵ NR(Γ) $\cap_N$ Ѵ, then Y = T $\cap_N$ Ѵ where T ϵ NR(Γ), so T ϵ Ƒ since Ƒ include NR(Γ), hence (T $\cap_N$ Ѵ) ϵ NR(Γ $\cap_N$ Ѵ) by definition of Ƒ, that is Y ϵ NR(Γ $\cap_N$ Ѵ), this fact means that

NR(Γ) $\cap_N$ Ѵ contained in NR(Γ $\cap_N$ Ѵ).

Therefore, NR(Γ) $\cap_N$ Ѵ = NR(Γ $\cap_N$ Ѵ).

4. Conclusion

The present paper investigates the use of neutrosophic sets to extend the concept of ring. Several properties of neutrosophic rings are studied. It is shown that any intersection of neutrosophic rings is also neutrosophic ring as well as the smallest neutrosophic ring is determined. In addition, we prove that if we have two classes of neutrosophic sets such that the first class is contained in the second class, then the smallest neutrosophic ring that generated by the first class is also contained in the smallest neutrosophic ring that generated by the second class.

References

[1] Albowi, S.A. ; Salama, A.A. (2012). Neutrosophic Set and Neutrosophic Topological Spaces. IOSR Journal of Mathematics, 3(4), 31–35. https://doi.org/10.9790/5728-0343135.

[2] Ahmed, I. S. ; Al-Fayadh, A.; Ebrahim, H. H. (2023). Fuzzy σ-algebra and some related concepts. AIP Conference Proceedings, 2834(1). https://doi.org/10.1063/5.0162037

[3] Ahmed, I. S. ; Ebrahim, H. H. (2022). On measure relatively to fuzzy σ-algebra. Journal of Physics: Conference Series, 2322(1). https://doi.org/10.1088/1742-6596/2322/1/012020.

[4] Ahmed, I. S.; Ebrahim, H. H., ; Al-Fayadh, A. (2022). γ– algebra of Sets and Some of its Properties. Iraqi Journal of Science, 63(11), 4918–4927. https://doi.org/10.24996/ijs.2022.63.11.28.

[5] Asaad, S. H.; Ahmed, I. S. ; Ebrahim, H. H. (2022). On Finitely Null-additive and Finitely Weakly Null-additive Relative to the σ–ring. Baghdad Science Journal, 19(5), 1148–1154.

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024

[6] Asaad, S. H. ; Ahmed, I. S. ; Mohammed, A. S. (2020). Generalization Fuzzy Completely Intra G-ideals in G-Semi Groups. Journal of Physics: Conference Series, 1664(1). https://doi.org/10.1088/1742-6596/1664/1/012028.

[7] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

[8] Dubois, D. ; Prade, H. (1989). Fuzzy sets, probability and measurement. European Journal of Operational Research, 40(January), 135–154.

[9] Halmos, P. R. (1974). Measure Theory (P. R. ; C. C. M. Halmos & University (eds.); 1st ed). Springer-VerlagNewYork-Heidelberg-Berlin.

[10] Khalaf, K. H. ; Salih, A. M.; Abdullah, O. A.; Asaad, S. H. ; Ahmed, I. S. (2023). Study of some properties for pre measure. AIP Conference Proceedings, 2834(1). https://doi.org/10.1063/5.0162036

[11] Şahin, M. (2022). Neutro-Sigma Algebras and Anti-Sigma Algebras. Neutrosophic Sets and Systems, 51, 909–922.

[12] Smarandache, F. (2000). neutrosophic probability, set, and logic (first. collected papers, III, 59–72. https://doi.org/10.5281/zenodo.57726.

[13] Wang, Z. ; Klir, G. (2009). Generalized Measure Theory. In G. J. Klir (Ed.), Springer Science and Business Media, LLC (1st ed). Springer Science and Business Media, LLC, New York. https://doi.org/10.1007/978-0-387-76852-6.

[14] Xuan Thao, N. ; Smarandache, F. (2018). Divergence Measure of Neutrosophic Sets and Applications. Neutrosophic Sets and Systems, 21, 142–152.

[15] Zadeh, L. . (1965). Fuzzy Sets. In Information and Control (Vol. 8, pp. 338–353).

[16] H. J. Zimmermann (1996). Fuzzy Set Theory and Its Applications, Kluwer Academic, Boston, MA, (1996).

[17] H. Prade and D. Dubois (1980). Fuzzy Sets and Systems Theory and Applications, Academic Press, London.

[18] K. Atanassov (1986). Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20, 87-96.

[19] K. Atanassov (1994). Operators Over Interval Valued Intuitionstic Fuzzy sets, Fuzzy Sets and Systems, 64, 159-174.

[20] K. Atanassov (1999). Intuitionistic Fuzzy Sets, Physica-Verlag, Heidelberg, N.Y.

[21] D. Molodtsov (1999), Soft Set Theory-first Results, Computers Math. Applic., 37 (4/5), 19-31.

[22] Wang, H., Smarandache F., Zhang, Y.Q., & Sunderraman, R. (2010). Single Valued Neutrosophic Sets. Multispace and Multistructure, 4, 410-413.

[23] Sodenkamp, M. (2013). Models, methods and applications of group multiple-criteria decision analysis in complex and uncertain systems. Dissertation, University of Paderborn, Germany.

[24] Kharal, A (2014). A neutrosophic multi-criteria decision making method. New Mathematics and Natural Computation, 10(2), 143–162.

[25] Broumi, S., Smarandache, F. (2014). Single valued neutrosophic trapezoid linguistic aggregation operators based multiattribute decision making. Bulletin of Pure & Applied Sciences- Mathematics and Statistics,135-155. doi: 10.5958/2320-3226.2014.00006.X

[26] Hai-Long, Y., Zhi-Lian, G., Yanhong, S., & Xiuwu, L. (2016). On single valued neutrosophic relations. Journal of Intelligent and Fuzzy Systems, 30(2), 1045-1056. doi: 10.3233/IFS-151827.

[27] Broumi, S.; Smarandache, F. (2013). Several similarity measures of neutrosophic sets. Neutrosophic Sets and Systems, 1, 54–62.

[28] Broumi, S.; Smarandache, F. (2014). Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets and Systems, 6, 42–48.

[29] Biswas, P., Pramanik, S., & Giri, B. C. (2016). Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems 12, 127-138.

[30] Biswas, P., Pramanik, S., & Giri, B. C. (2016). Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems 12, 20-40.

[31] Ye, J., Smarandache, F. (2016). Similarity measure of refined single-valued neutrosophic sets and its multicriteria decision making method. Neutrosophic Sets and Systems, 12, 41–44.

[32] Wang, H., Smarandache, F., Zhang, Y.Q., & Sunderraman, R. (2005). Interval neutrosophic sets and logic: theory and applications in computing. Arizona, Hexis.

[33] Ye, J. (2015). Trapezoidal neutrosophic set and its application to multiple attribute decision-making, Neural Computing and Applications, 26, 1157-1166.

[34] Biswas, P., Pramanik, S., & Giri, B.C. (2014). Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets and Systems, 8, 46-56.

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024

[35] Deli, I. & Y. Subas, Y. (2017). A ranking method of single valued neutrosophic numbers and its application to multiattribute decision making problems. International Journal of Machine Learning and Cybernetics, 8 (4), 1309- 1322.

[36] Broumi, S., Smarandache, F., & Dhar, M. (2014). Rough neutrosophic sets. Neutrosophic Sets and Systems, 3, 60-66.

[37] Mondal, K., & Pramanik, S. (2015). Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7, 62-68.

[38] Mondal, K., & Pramanik, S. (2015). Rough neutrosophic multi-attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 7, 8-17.

[39] Mondal, K., Pramanik, S., & Smarandache, F. (2016). Multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. Neutrosophic Sets and Systems, 13, 3-17.

[40] Mondal, K., Pramanik, S., & Smarandache, F. (2016). Rough neutrosophic hyper-complex set and its application to multiattribute decision making. Critical Review, 13, 111-126.

[41] Chen, J., Ye, J., & Du, S. (2107). Vector similarity measures between refined simplified neutrosophic sets and their multiple attribute decision-making method. Symmetry, 9(8), 153; doi:10.3390/sym9080153.

[42] Ye, J., Smarandache, F. (2016). Similarity measure of refined single-valued neutrosophic sets and its multicriteria decision making method. Neutrosophic Sets and Systems, 12, 41–44.

[43] Uluçay, V, Deli, I, & Şahin, M. (2016). Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. Neural Computing and Applications. https://doi.org/10.1007/s00521-016-2479-1

[44] Pramanik, S., Dalapati, S, Alam, S., Roy, T. K., & Smarandache, F. (2017). Neutrosophic cubic MCGDM method based on similarity measure. Neutrosophic Sets and Systems, 16, 44-56.

[45] Huda, F. Khudair and Fatimah, M. Mohammed, (2022). " Generalized of A-Closed Set and C̈-Closed Set in Fuzzy Neutrosophic Topological Spaces, International Journal of Neutrosophic Science, vol. 19(2), 8–18.

[46] Zahraa, A. Khalaf and Fatimah, M. Mohammed, (2023). " Weakly Generalized M-Closed and Strongly M-Generalized Closed Sets in Fuzzy Neutrosophic Topological Spaces", International Journal of Neutrosophic Science, vol. 21(2), pp. 8-19.

DOI: https://doi.org/10.54216/IJNS.250317
Received: February 25, 2024 Revised: May 27, 2024 Accepted: October 02, 2024