Modern Free-Derivative Numerical Optimization of Approximate Algorithms Convergence and Neutrosophic Convergence

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Abstract

The aim of this study is to compare common and previously used numerical algorithms for nonlinear problems under different conditions. This study proposes a parallel implementation of two free derivative optimization methods, Powell's method and Nelder-Mead's method, combined with two restart strategies to achieve a global search. In terms of total time, the Powell method converges faster than the Nelder-Mead method. The final function value obtained by the Powell method is slightly lower. Both are optimization techniques used to find the minimum of an objective function in multidimensional space, without requiring derivatives. Also, we extend our results to apply to some neutrosophic non-linear problems under different neutrosophic-based conditions with many examples that explain the validity of our approach.

Keywords: Numerical Optimization, Nonlinear programming, Approximate methods, Neutrosophic based condition, Neutrosophic non-linear problem

1. Introduction

Optimization is a vital tool in decision science and mathematical model analysis. This tool can only be used once we have determined a goal (a quantitative measure of the performance of a system). This can be profit, time, potential energy, or any quantity or combination of quantities represented by a single number. Certain properties of the system (called variables or unknowns) determine the goal. In order to optimize the goal, we need to find the values of the variables. It is common for the variables to be constrained or restricted in some way. Using an optimization algorithm (usually a computer), the solution to the model can be found after it has been formulated.[3]

There is no universal optimization algorithm, but rather a collection of algorithms tailored for different types of optimization problems. In many cases, it is up to the user to select an algorithm that is appropriate for a particular application. One of the most common mathematical problems, which is also widely used in real-world problems, is the optimization of a function. We would like to be able to do this for functions that are not described in a way that allows us to compute derivatives, although for well-defined functions this is as simple as computing the gradient and finding the zero. Powell's method is a direction theorem method that does not require computing the gradient of the function. It works by performing successive one-dimensional minimizations along a sequence of directions.[4]

An example of this is the optimization of the number of iterations required for a numerical algorithm according to some tuning factor. To achieve this goal, various numerical minimization and optimization methods are studied. While Powell's method (the second approach) uses one-dimensional optimization techniques, the Downhill Simplex method is completely independent. In this study, we introduced the most important numerical optimization tools: the Nelder-Mead method and the Powell method. Our main goal is to use these optimization techniques to find robust algorithm optimization strategies [5, 10, 11, 12].

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Also, we use neutrosophic real variables defined in [13-15] to extend our results to be applicable to some neutrosophic non-linear problems under different neutrosophic-based conditions.

2. Convex sets and Convex Functions

Definition 2.1 Convex set [1]

Suppose that S is a set such that $S \subseteq R^n, S$ is **Convex set** if condition is hold $\delta x + (1-\delta)y \in S, 0 \le \delta \le 1, \forall x, y \in S$

i.e, if any two points x and y lie on a line segment lies in S. The set S is **Non-convex** if this condition is false.

Definition 2.2 Convex Function [1]

A set B is a nonempty convex set, where $v \in R^n, G: B \to R$, G is said a **Convex function** on B if satisfy;

$$G(\alpha y_1 + (1-\alpha)y_2 \le \alpha G(y_1) + (1-\alpha)G(y_2), \forall y_1, y_2 \in v, \forall \alpha \in [0,1]$$

$y_1 \neq y_2, \forall \alpha \in (0,1)$ As a rigorous inequality

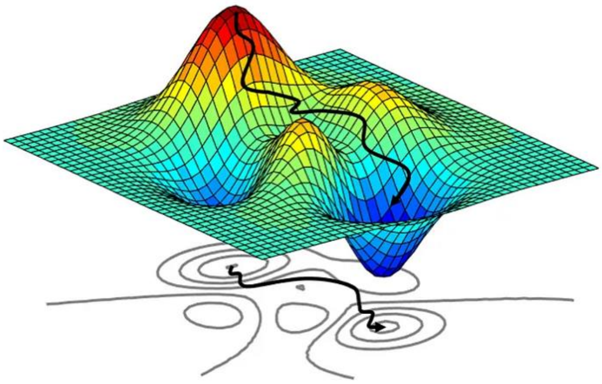

Figure 1. Optimizing a convex and non-convex function

Definition 2.2 the Global Minimum [2]

Assume that $m^* \in q, m^*$it is known a **Global minimize** of G over q if satisfy $G(m^*) \le G(m), \forall m \in q, m^*$ is known **Strict global minimize** if $G(m^*)G(m), \forall m \in R^n$ with $m^* \neq m$.

Definition 2.2.3 the Feasible Region [3]

A feasible region is one that satisfies all constraints or a practical option that yields the best possible value of the objective function.

3. Mathematical Optimizations Model

We introduce the functions of several variables, assume that $g(x_1, x_2, \dots, x_k)$ which is defined in the region

$$Z = \left\{ x_1, x_2, \dots, x_k : \sum_{n=1}^{k} (x_n - p_n)^2 < r^2 \right\}$$

The function $g(x_1, x_2, \dots, x_k)$ has a local minimum at the point $(p_1, p_2, \dots, p_n)$ provided that

$g(p_1, p_2, \dots, p_n) \le g(x_1, x_2, \dots, x_k)$

For each point $(x_1, x_2, \dots, x_k) \in R$, the function $g(x_1, x_2, \dots, x_k)$ has a local maximum at the point $(p_1, p_2, \dots, p_n)$ provided that

$g(p_1, p_2, \dots, p_n) \ge g(x_1, x_2, \dots, x_k)$,

for each point $(x_1, x_2, \dots, x_k) \in R$.

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4. The Approximate Methods

Approximate methods are regarded as analytical techniques used to generate solutions that are near, in some way, to the precise solution of the nonlinear problem [6]. In this section we introduced the modified of comparison between Nelder Mead method and Powell's method [7, 8]

Table 1: The Nelder Mead method

The Nelder Mead method	
Also referred to as the simplex method or the downhill simplex method. Utilized for unconstrained optimization of multi-dimensional functions. Does not rely on gradient information. Performs effectively with non-smooth functions. Utilizes a simplex, which is a geometric figure consisting of n+1 vertices in n dimensions. Iteratively adjusts the simplex to approach the optimal solution. Implements reflection, expansion, contraction, and shrinkage techniques.	
Advantages:	**Disadvantages:**
Easy to implement Efficient for issues with discontinuities Performs effectively in the presence of noisy objective functions	It may converge slowly, particularly in high-dimensional spaces. There is a risk of becoming trapped in local optima. Finding the global optimum is not guaranteed.

Table 2: The Powell's method

The Powell's method	
A method for unconstrained optimization that utilizes direction sets Does not require gradient information Appropriate for smooth, continuous functions Iteratively updates the simplex to approach the optimum Utilizes reflection, expansion, contraction, and shrinkage operations. Advantages are its simplicity and ease of implementation 6. Effective for problems with discontinuities 7. Performs well in cases where the objective function is noisy	
Advantages:	**Disadvantages:**
Typically demonstrates quicker convergence compared to Nelder-Mead for smooth functions. More efficient at handling higher-dimensional problems. Generally considered more robust than Nelder-Mead in practical applications.	May have difficulties with functions that are highly non-smooth or discontinuous Can be influenced by the selection of initial directions May underperform when encountering functions with narrow valleys

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5. The Modified Approximate Algorithms

The approximation an algorithm that computes a solution for an optimization with an objective value that is probably within a bounded factor of the optimal objective value.

Modified Nelder-Mead method algorithm
Order: Evaluate the objective function at each vertex and order them from best to worst: $x_1, x_2, \ldots, x_{k+1}$. Compute the reflected point x_j: $x_j = \bar{x} + \beta(\bar{x} - x_{k+1})$, where x̄ is the centroid of all points except $x_{k+1}$and $\beta > 0$ is the reflection coefficient. Expand: If $f(x_j) < f(x_1)$, compute the expanded point x_s: $x_s = \bar{x} + \beta(x_j - \bar{x})$. Contract: If $f(x_k) \leq f(x_j) < f(x_{k+1})$, compute the contracted point $$x_v: x_v = \bar{x} + \rho(x_j - \bar{x}),$$ where $0 < \rho \leq 0.5$ is the contraction coefficient. Compute new vertices: $w_i = x_1 + \sigma(x_i - x_1)$ for $i = 2, \ldots, n+1$, where σ is the shrink coefficient, if none of the above improved the simplex

Modified Powell's Method algorithm
Choose initial point u_0 and a set of search directions $\{u_1, u_2, \ldots, u_k\}$, typically the unit vectors. For $n = 1, 2, \ldots, k$: Find z_n that minimizes $f(u_{n-1} + z_n x_n)$ Set $u_n = u_{n-1} + z_n x_n$ Define a new direction: $u_{k+1} = u_k - u_0$. Find α that minimizes $f(u_k - \alpha x_{k+1})$, set. $u_0 = u_k + \alpha x_{k+1}$. Update the direction set by discarding the direction along which the function made its largest decrease, and add x_{k+1}. Repeat steps b-e until convergence. The mathematical form for updating the directions in Powell's method can be expressed as: $$u_i = u_{i+1} \, for \; i = 1, \ldots, K-1, \qquad u_k = u_k - u_0$$

6. Numerical Computational

Computational methods and optimization are widely used in applied mathematics. These problems are complex and highly nonlinear and difficult to predict. In this section, we present the implementation results of the approximate methods. This comparison demonstrates that both Nelder-Mead and Powell's methods are effective for this optimization problem. The choice between them might depend on the specific characteristics of your problem and any constraints on computation time or function evaluations. Powell's method requires fewer iterations but more function evaluations than Nelder-Mead with high accuracy. In this case, Powell's method achieves a slightly lower final function value, but the difference is negligible for practical purposes. The comparison demonstrates that both Nelder-Mead and Powell's methods are effective for this optimization problem. The choice between them might depend on the specific characteristics of your problem and any constraints on computation time or function evaluations. In this work, we implemented the highly nonlinear function to find the global minimum at (1,1).

$$Min \;\; f(x,y) = -10e^{\sqrt{x^2+y^2}} - e^{0.5(\cos(2\pi x)+\cos(2\pi y))} + Ln(x+y)$$

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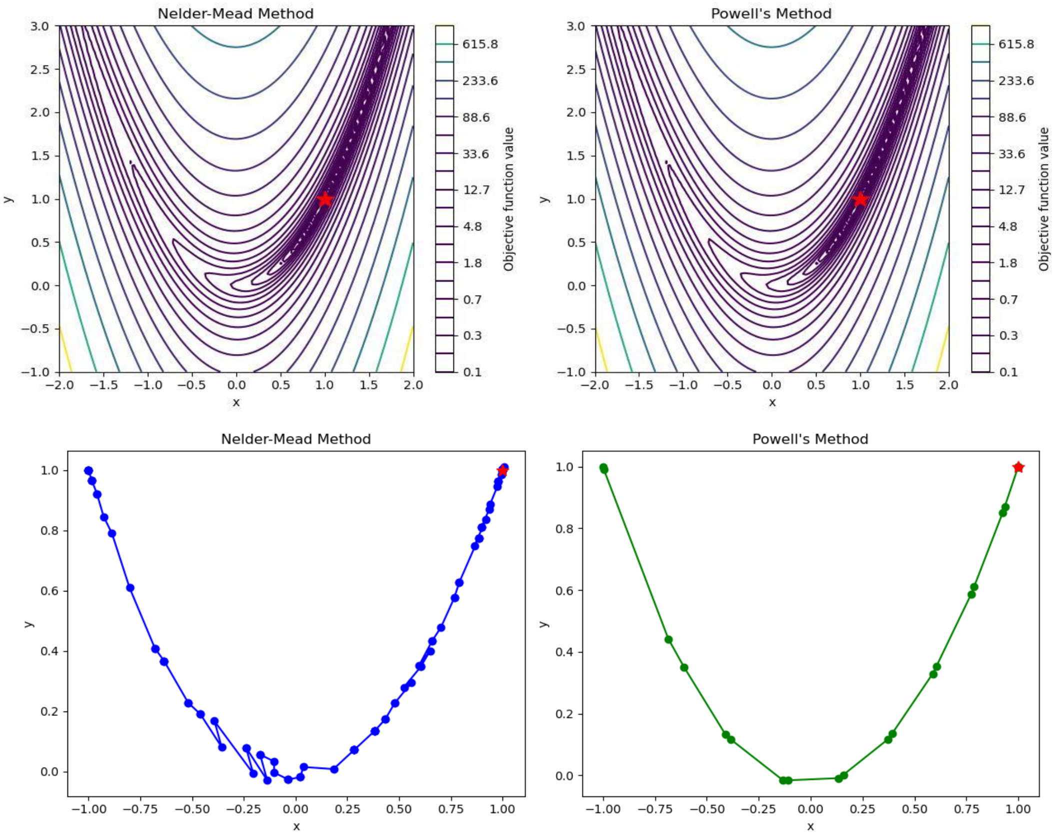

Figure 2. Comparison results, Nelder-Mead Method via Powell Method

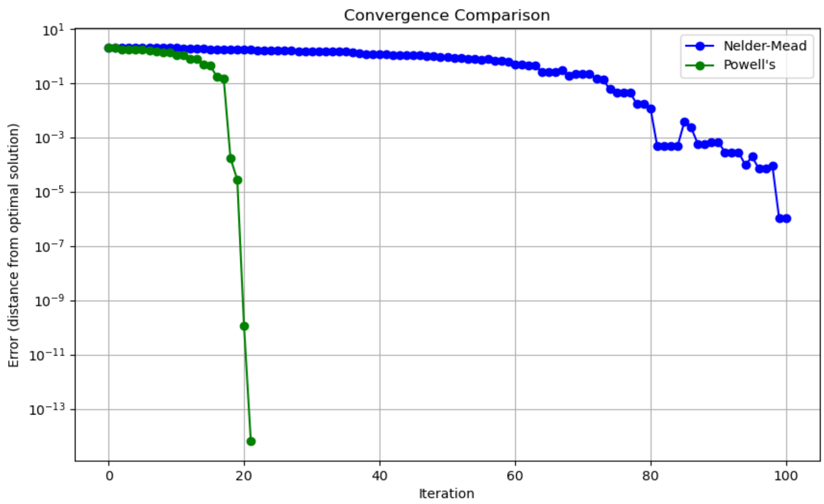

Figure 3. Improving Convergence Properties

The convergence properties of Powell's method typically converges faster for smooth functions, while Nelder-Mead may be better for non-smooth functions. However, Powell's method generally handles higher-dimensional problems better than Nelder-Mead. Also, Function characteristics: Nelder-Mead is more suitable for noisy or discontinuous functions, while Powell's method works best on smooth, continuous functions. The implementation: Nelder-Mead is generally simpler to implement than Powell's method. Finally the robustness: Powell's method is often considered more robust in practice, but this can depend on the specific problem. In figure (1.2) the first set of iterations plots shows the path taken by each method in the 2D space. The red star indicates the global minimum (1,1). In figure (1.3) shows the convergence plot of both methods over iterations. It plots the error distance from the optimal solution on a logarithmic scale against the iteration number.

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7. Neutrosophic Mathematical Optimizations Model

We introduce the functions of several neutrosophic real variables, assume that $g(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI)$ which is defined in the region

$$Z=\left\{x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI : \sum_{n=1}^{k}(x_n+y_nI-p_n)^2<r^2\right\}$$

The function $g(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI)$ has a local minimum at the point $(p_1,p_2,\dots,p_n)$ provided that

$g(p_1,p_2,\dots,p_n) \le g(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI)$

For each point $(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI) \in R(I)$, the function $g(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI)$ has a local maximum at the point $(p_1,p_2,\dots,p_n)$ provided that

$g(p_1,p_2,\dots,p_n) \ge g(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI)$,

for each point $(x_1+y_1I, x_2+y_2I, \dots, x_k+y_kI) \in R(I)$.

8. The Modified Neutrosophic Approximate Algorithms

The approximation algorithm that computes a solution for an optimization with an objective value that is probably within a bounded factor of the optimal objective value.

Modified neutrosophic Nelder-Mead method algorithm
Order: Evaluate the objective function at each vertex and order them from best to worst: $x_1+y_1I, x_2+y_2I, \dots, x_{k+1}+y_{k+1}I$. Compute the reflected point x_j+y_jI: $x_j=\bar{x}+\beta(\bar{x}-x_{k+1}), y_j=\hat{y}+\beta(\hat{y}-y_{k+1})$, Expand: If $f(x_j+y_jI) < f(x_1+y_1I)$, compute the expanded point $x_s: x_s=\bar{x}+\beta(x_j-\bar{x}), y_s: y_s=\hat{y}+\beta(y_j-\hat{y})$. Contract: If $f(x_k+y_kI) \le f(x_j+y_jI) < f(x_{k+1}+y_{k+1}I)$, compute the contracted point $x_v: x_v=\bar{x}+\rho(x_j-\bar{x}), y_v: y_v=\hat{y}+\rho(y_j-\hat{y})$ where $0<\rho\le 0.5$ is the contraction coefficient. Compute new vertices: $w_i=x_1+\sigma(x_i-x_1), u_i=y_1+\sigma(y_i-y_1)$ for $i=2,\dots,n+1$, where σ is the shrink coefficient, if none of the above improved the simplex

Modified Powell's Method algorithm
Choose initial point u_0 and a set of search directions $\{u_1,u_2,\dots,u_k\}$, typically the unit vectors. For $n=1,2,\dots,k$: Find z_n that minimizes $f(u_{n-1}+z_nx_n)$ (the same for y coordinate) Set $u_n=u_{n-1}+z_nx_n$ (the same for y coordinate) Define a new direction: $u_{k+1}=u_k-u_0$, (the same for y coordinate). Find α that minimizes $f(u_k-\alpha x_{k+1})$, set. $u_0=u_k+\alpha x_{k+1}$ ((the same for y coordinate)). Update the direction set by discarding the direction along which the function made its largest decrease, and add x_{k+1}, ((the same for y coordinate)). Repeat steps b-e until convergence. The mathematical form for updating the directions in Powell's method can be expressed as: $u_i=u_{i+1}\ for\ i=1,\dots,K-1, \quad u_k=u_k-u_0$

9. Numerical Neutrosophic Computational

$$Min\ \ f(x,y)=-10e^{\sqrt{x^2+y^2}}-e^{0.5(\cos(2\pi x)+\cos(2\pi y))}+Ln(x+y); x=a+bI, y=c+dI$$

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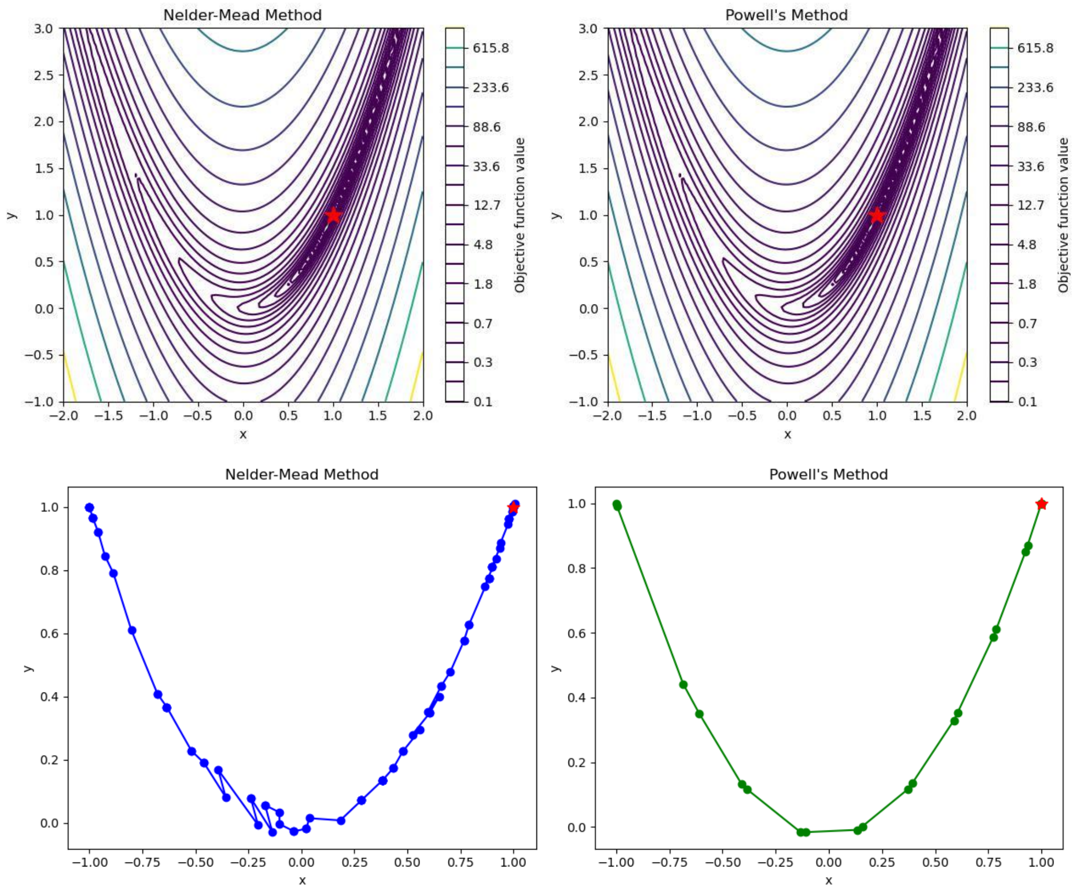

Figure 4. Comparison results, Nelder-Mead Method via Powell Method for neutrosophic case

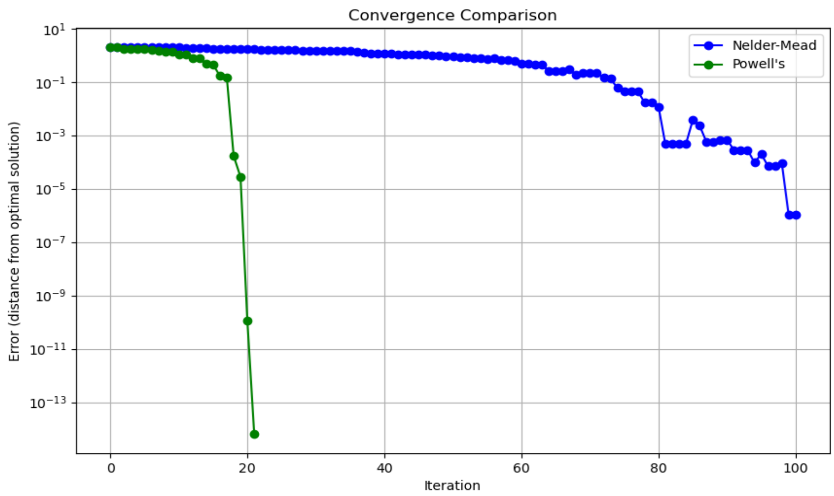

Figure 5. Improving neutrosophic Convergence Properties

10. Conclusion

In this work, we introduced novel approximation techniques for algorithms grounded in non-linear optimization problems. We evaluated several derivative-free optimization algorithms by comparing them to one another [9]. These optimization techniques aim to find the minimum of an objective function across multidimensional spaces. Our study focuses on comparing Nelder-Mead and Powell's methods, depending on the specific characteristics of the optimization problem being addressed. This comparison provides valuable insights into the performance of these methods across various optimization scenarios. Generally, Powell's method is more effective for handling

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higher-dimensional problems compared to Nelder-Mead. However, Powell's method tends to converge more quickly for smooth functions, while Nelder-Mead may be more effective for non-smooth functions. Additionally, Nelder-Mead is typically simpler to implement than Powell's method, making it more suitable for noisy or discontinuous functions. Conversely, Powell's method excels with smooth, continuous functions and is often considered more robust in practical applications. Also, we extend our results to be applicable to some neutrosophic non-linear problems under different neutrosophic based conditions with many examples that explain the validity of our approach.

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