# RETRACTED ARTICLE: Different algebraic structures and their properties setting logarithm operator applied to extended neutrosophic interval-valued set

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**Abstract**

We present the neutrosophic interval-valued set applied to the q-rung logarithmic operator (q-RLOANIVS). One might develop a q-rung neutrosophic interval-valued set by extending the Pythagorean interval-valued fuzzy set (PIVFS) and neutrosophic set (NS). We discuss the q-Rlogarithimic operator applied neutrosophic interval-valued weighted averaging (q-RLOANIVWA), q-Rlogarithimic operator applied neutrosophic interval-valued weighted geometric (q-RLOANIVWG), extended q-Rlogarithimic operator applied neutrosophic interval-valued weighted averaging (q-RELOANIVWA) and extended q-Rlogarithimic operator applied neutrosophic interval-valued weighted geometric (q-RELOANIVWG). Several algebraic attributes have been established, including distributivity, idempotency, and associativity of q-RLOANIVSs.

**Keywords:** q-RLOANIVWA; q-RLOANIVWG; q-RELOANIVWA; q-RELOANIVWG

## 1 Introduction

Many authors have used different techniques to contribute to this field of study. As a result of the uncertainties, fuzzy set (FS),[1] intuitionistic FS (IFS),[2] interval-valued FS (IVFS),[3] vague set (VS),[4] Pythagorean FS (PFS),[5] IVPFS,[6] spherical FS (SFS)[7] neutrosophic set (NS).[8] Numerous theories pertaining to uncertainty have been put forth, such as fuzzy set (FS),[1] wherein membership grade (MG) varies between 0 and 1. Atanassov[2] developed an intuitionistic fuzzy set (IFS), in which $0 \leq \gamma + \phi \leq 1$, for $\gamma, \phi \in [0, 1]$, is satisfied by each object's two MGs, positive $\gamma$ and negative $\phi$. Pythagorean fuzzy sets (PFS) were introduced by Yager.[5] They are characterized by their MG and non-membership grade (NMG) with the hypothesis that $\gamma + \phi \geq 1$ to $\gamma^2 + \phi^2 \leq 1$. A great deal of studies have been done on the application of PFSs and IFSs in various disciplines. They are still not very good at communicating ideas. As a result, the experts still had trouble explaining the data in these sets and the data that went along with them. To get around this knowledge, Cuong et al.[9] created the idea of a picture fuzzy set. Consequently, it has been observed that the picture fuzzy set is an enlarged form of IFS that can handle more ambiguities. It was observed that in the picture fuzzy set, MG $\gamma$, NMG $\wp$, and neutral grade $\alpha$ with $0 \leq \gamma + \alpha + \wp \leq 1$; for $\gamma, \alpha, \wp \in [0, 1]$. By keeping to the PFS definition, it will be possible to guarantee that expert opinions such as ”yes,” ”abstain,” ”no,” and ”refusal” are communicated. Additionally, the outcome information and the real decision environment will be consistent, and no evaluation detail will be overlooked. While picture fuzzy sets have been applied and explored extensively, their notion

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has not received as much attention. The novel aggregating operator was discussed by Palanikumar et al.[10–15] Liu et al.[16] described how aggregation operators (AO) were added along with extended PFS. challenges with multiple truth membership values (TMD), false membership values (FMD), and indeterminacy membership values (IMD) that are addressed by the DM approach. Palanikumar and associates have recently explored the novel aggregating operators.[17,18] Smarandache et al.[19] introduced the idea of neutrosophic sets. One of the main distinctions between FS and IFS is the ability to think neutrally. The knowledge of neutral thought is known as neutrosophy. This reasoning uses TMD, IMD, and FD to determine a value for each proposition. A universe where all elements have values between 0 and 1, with a total value of 1.[20] provided a method for MCDM under interval NS based on AOs. Palanikumar et al.[21] explain AOs and their algebraic structures in numerous applications.

Xu et al.[22] recommended employing multiple IFS averaging operators to manage IFS data. Furthermore, Xu et al.[23] created weighted, ordered weighted, and hybrid geometric operators based on IFSs. The generalized ordered weighted averaging (GOWA) operators were proposed by Li et al.[24] to build the IF-ordered weighted distance (IFOWD) operators that are covered in[25] by Zeng et al. Peng et al.[26] investigated the fundamental features of PFS using AOs. Ashraf et al.[27] have lately proposed the idea of spherical fuzzy Dombi AOs. Temel et al.[28] established this new power Muirhead mean in the SFS in 2022, which also applies to MADM. Recently,[29–31] developed new aggregating operators. There is an introduction in Section 1. Section 2 for details on PNS and FS. We define and explain some of the functions of q-RLOANIVNs in Section 3. The paper has two primary conclusions: 1) The algebraic properties of the q-rungs of LOANIVS have been demonstrated; they are distributive, idempotent, and associative. The q-RLOANIVWA, q-RLOANIVWG, q-RELOANIVWA, and q-RELOANIVWA are subsequently addressed.

## 2 Preliminaries

We are going to talk through PFS and PIVFS ideas in this part.

**Definition 2.1.** [5] Let $\mathbb{U}$ be the universal set. The PFS $A = \Big\{u, \langle \tau_A^{\mathcal{T}}(u), \tau_A^{\mathcal{F}}(u) \rangle \big| u \in \mathbb{U}\Big\}$, $\tau_A^{\mathcal{T}} : \mathbb{U} \to (0,1)$ and $\tau_A^{\mathcal{F}} : \mathbb{U} \to (0,1)$ denotes MD and NMD of $u \in \mathbb{U}$ to $A$, respectively and $0 \leq (\tau_A^{\mathcal{T}}(u))^2 + (\tau_A^{\mathcal{F}}(u))^2 \leq 1$. For convenience, $A = \langle \tau_A^{\mathcal{T}}, \tau_A^{\mathcal{F}} \rangle$ is called the Pythagorean fuzzy number (PFN).

**Definition 2.2.** [6] The Pythagorean IVFS (PIVFS) $A = \Big\{u, \Big\langle \widetilde{\tau_A^{\mathcal{T}}}(u), \widetilde{\tau_A^{\mathcal{F}}}(u) \Big\rangle \Big| u \in \mathbb{U}\Big\}$, where $\widetilde{\tau_A^{\mathcal{T}}} : \mathbb{U} \to Int((0,1))$ and $\widetilde{\tau_A^{\mathcal{F}}} : \mathbb{U} \to Int((0,1))$ denotes MD and NMD of $u \in \mathbb{U}$ to $A$, respectively, and $0 \leq (\tau_A^{\mathcal{T}+}(u))^2 + (\tau_A^{\mathcal{F}+}(u))^2 \leq 1$. For convenience, $A = \Big\langle \Big(\tau_A^{\mathcal{T}-}, \tau_A^{\mathcal{T}+}\Big), \Big(\tau_A^{\mathcal{F}-}, \tau_A^{\mathcal{F}+}\Big) \Big\rangle$ is called the PIVFN.

**Definition 2.3.** The Pythagorean NS $A = \Big\{u, \langle \tau_A^{\mathcal{T}}(u), \tau_A^{\mathcal{I}}(u), \tau_A^{\mathcal{F}}(u) \rangle \big| u \in \mathbb{U}\Big\}$, where $\tau_A^{\mathcal{T}} : \mathbb{U} \to (0,1)$, $\tau_A^{\mathcal{I}} : \mathbb{U} \to (0,1)$ and $\tau_A^{\mathcal{F}} : \mathbb{U} \to (0,1)$ denotes TMD, IMD and FMD of $u \in \mathbb{U}$ to $A$, respectively and $0 \leq (\tau_A^{\mathcal{T}}(u))^2 + (\tau_A^{\mathcal{I}}(u))^2 + (\tau_A^{\mathcal{F}}(u))^2 \leq 2$. For convenience, $A = \langle \tau_A^{\mathcal{T}}, \tau_A^{\mathcal{I}}, \tau_A^{\mathcal{F}} \rangle$ is called the Pythagorean neutrosophic number.

## 3 q-RLOANIVN and its fundamental operations

The q-RLOANIVN is related to numerous interesting fundamental operations.

**Definition 3.1.** The q-RLOANIVS $A = \Big\{u, \Big\langle \Big[log\mathcal{T}_A^l[u], log[\mathcal{T}_A^u[u]]\Big], \Big[log\mathcal{I}_A^l[u], log\mathcal{I}_A^u[u]\Big], \Big[log\mathcal{F}_A^l[u], log[\mathcal{F}_A^u[u]]\Big] \Big\rangle \Big| u \in \mathbb{U}\Big\}$, $\widetilde{\tau_A^{\mathcal{T}}} : \mathbb{U} \to Int[[0,1]]$, $\widetilde{\tau_A^{\mathcal{I}}} : \mathbb{U} \to Int[[0,1]]$ and $\widetilde{\tau_A^{\mathcal{F}}} : \mathbb{U} \to Int[[0,1]]$ denotes TMD, IMD and FMD of $u \in \mathbb{U}$ to $A$, respectively and $0 \leq [log_{Z_i}\mathcal{T}_A^u[u]]^q + [log_{Z_i}\mathcal{I}_A^l[u]]^q + [log_{Z_i}\mathcal{F}_A^u[u]]^q \leq 2$, where $q, q, q$ are positive integers and $Z = \circledast[\mathcal{T}_A^l, \mathcal{T}_A^u], [\mathcal{I}_A^u, \mathcal{I}_A^l], [\mathcal{F}_A^l, \mathcal{F}_A^u]$. For convenience, $A = \Big\langle \Big[log\mathcal{T}_A^l, log[\mathcal{T}_A^u]\Big], \Big[log\mathcal{I}_A^l, log\mathcal{I}_A^u\Big], \Big[log\mathcal{F}_A^l, log[\mathcal{F}_A^u]\Big] \Big\rangle$ is called the q-RLOANIVN, where $q \geq 1$.

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**Definition 3.2.** Let $A = \left\langle \left[log\mathcal{T}_A^l, log[\mathcal{T}_A^u]\right], \left[log\mathcal{I}_A^l, log\mathcal{I}_A^u\right], \left[log\mathcal{F}_A^l, log[\mathcal{F}_A^u]\right] \right\rangle$ be the q-RLOANIVN, the score function of $A$ is defined as $\mathbb{S}[A] = \frac{\mathbb{S}_1[A]+\mathbb{S}_2[A]}{2}$, $-1 \leq \mathbb{S}[A] \leq 1$, where

$$\mathbb{S}_1[A] = \left[\frac{M_1}{2} + 1 - \frac{M_2}{2} + 1 - \frac{M_3}{2}\right], \mathbb{S}_2[A] = \left[\frac{M_1}{2} + 1 - \frac{M_2}{2} + 1 - \frac{M_3}{2}\right],$$

The accuracy function of $A$ is $\mathbb{A}[A] = \frac{\mathbb{A}_1[A]+\mathbb{A}_2[A]}{2}$, where $0 \leq \mathbb{A}[A] \leq 1$.

$$\mathbb{A}_1[A] = \left[\frac{M_1}{2} + 1 + \frac{M_2}{2} + 1 + \frac{M_3}{2}\right], \mathbb{A}_2[A] = \left[\frac{M_1}{2} + 1 + \frac{M_2}{2} + 1 + \frac{M_3}{2}\right],$$

where $M_1 = [log_{Z_i}\mathcal{T}_A^l]^2 + [log_{Z_i}[\mathcal{T}_A^u]]^2, M_2 = [log_{Z_i}\mathcal{I}_A^l]^2 + [log_{Z_i}\mathcal{I}_A^u]^2, M_3 = [log_{Z_i}\mathcal{F}_A^l]^2 + [log_{Z_i}[\mathcal{F}_A^u]]^2$

**Definition 3.3.** Let $A = \left\langle [log\mathcal{T}_A^l, log[\mathcal{T}_A^u]], [log\mathcal{I}_A^l, log\mathcal{I}_A^u], [log\mathcal{F}_A^l, log[\mathcal{F}_A^u]] \right\rangle$,
$B = \left\langle [log\mathcal{T}_B^l, log[\mathcal{T}_B^u]], [log\mathcal{I}_B^l, log\mathcal{I}_B^u], [log\mathcal{F}_B^l, log[\mathcal{F}_B^u]] \right\rangle$
and $C = \left\langle [log\mathcal{T}_C^l, log[\mathcal{T}_C^u]], [log\mathcal{I}_C^l, log\mathcal{I}_C^u], [log\mathcal{F}_C^l, log[\mathcal{F}_C^u]] \right\rangle$ be any three q-RLOANIVNs and
$Z = \circledast[T_{A_i}, \mathcal{T}_{A_i}^u], [I_{A_i}, I_{A_i}], [F_{A_i}, \mathcal{F}_{A_i}^u]$. Their following operations are defined as follows:

1. $B \bigvee C =$

$$\left[\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{[log_{Z_i}\mathcal{T}_B^l]^q + [log_{Z_i}\mathcal{T}_C^l]^q - [log_{Z_i}\mathcal{T}_B^l]^q \cdot [log_{Z_i}\mathcal{T}_C^l]^q}, \\ \sqrt[q]{[log_{Z_i}[\mathcal{T}_B^u]]^q + [log_{Z_i}[\mathcal{T}_C^u]]^q - [log_{Z_i}[\mathcal{T}_B^u]]^q \cdot [log_{Z_i}[\mathcal{T}_C^u]]^q} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{[log_{Z_i}\mathcal{I}_B^l]^q + [log_{Z_i}\mathcal{I}_C^u]^q - [log_{Z_i}\mathcal{I}_B^l]^q \cdot [log_{Z_i}\mathcal{I}_C^u]^q}, \\ \sqrt[q]{[log_{Z_i}\mathcal{I}_B^l]^q + [log_{Z_i}\mathcal{I}_C^u]^q - [log_{Z_i}\mathcal{I}_B^l]^q \cdot [log_{Z_i}\mathcal{I}_C^u]^q} \end{array}\right], \\ \left[log_{Z_i}[\mathcal{F}_B^l]^q \cdot log_{Z_i}[\mathcal{F}_C^l]^q, log_{Z_i}[\mathcal{F}_B^u]^q \cdot log_{Z_i}[\mathcal{F}_C^u]^q\right] \end{array}\right],$$

2. $B \bigwedge C =$

$$\left[\begin{array}{c} \left[log_{Z_i}[\mathcal{T}_B^l]^q \cdot log_{Z_i}[\mathcal{T}_C^l]^q, log_{Z_i}[\mathcal{T}_B^u]^q \cdot log_{Z_i}[\mathcal{T}_C^u]^q\right], \\ \left[\begin{array}{c} \sqrt[q]{[log_{Z_i}\mathcal{I}_B^l]^q + [log_{Z_i}\mathcal{I}_C^u]^q - [log_{Z_i}\mathcal{I}_B^l]^q \cdot [log_{Z_i}\mathcal{I}_C^u]^q}, \\ \sqrt[q]{[log_{Z_i}\mathcal{I}_B^l]^q + [log_{Z_i}\mathcal{I}_C^u]^q - [log_{Z_i}\mathcal{I}_B^l]^q \cdot [log_{Z_i}\mathcal{I}_C^u]^q} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{[log_{Z_i}\mathcal{F}_B^l]^q + [log_{Z_i}\mathcal{F}_C^l]^q - [log_{Z_i}\mathcal{F}_B^l]^q \cdot [log_{Z_i}\mathcal{F}_C^l]^q}, \\ \sqrt[q]{[log_{Z_i}[\mathcal{F}_B^u]]^q + [log_{Z_i}[\mathcal{F}_C^u]]^q - [log_{Z_i}[\mathcal{F}_B^u]]^q \cdot [log_{Z_i}[\mathcal{F}_C^u]]^q} \end{array}\right] \end{array}\right],$$

3. $$\Xi \cdot A = \left[\begin{array}{c} \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{T}_A^l]^q\right]^{\Xi}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}[\mathcal{T}_A^u]]^q\right]^{\Xi}}\right], \\ \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}_A^l]^q\right]^{\Xi}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}_A^u]^q\right]^{\Xi}}\right], \\ \left[[log_{Z_i}\mathcal{F}_A^l]^{q\Xi}, [log_{Z_i}[\mathcal{F}_A^u]]^{q\Xi}\right] \end{array}\right],$$

4. $$A^{\Xi} = \left[\begin{array}{c} \left[[log_{Z_i}\mathcal{T}_A^l]^{q\Xi}, [log_{Z_i}[\mathcal{T}_A^u]]^{q\Xi}\right], \\ \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}_A^l]^q\right]^{\Xi}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}_A^u]^q\right]^{\Xi}}\right], \\ \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{F}_A^l]^q\right]^{\Xi}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}[\mathcal{F}_A^u]]^q\right]^{\Xi}}\right] \end{array}\right].$$

## 4 q-RLOANIVS concept

The weighted averaging operators for q-RLOANIVN are given based on the operational rules of q-RLOANIVNs.

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### 4.1 q-RLOANIV weighted averaging [q-RLOANIVWA] operator

**Definition 4.1.** Let $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}], [log\mathcal{F}^l_{Ai}, log\mathcal{F}^u_{Ai}] \right\rangle$ be the collection of q-RLOANIVNs, $\partial = [\partial_1, \partial_2, ..., \partial_n]$ be the weight of $A_i$, $\partial_i \geq 0$ and $\biguplus_{i=1}^n \partial_i = 1$ and $Z = \circledast[\mathcal{T}^l_{A_i}, \mathcal{T}^u_{A_i}], [I_{A_i}, I_{A_i}], [F_{A_i}, \mathcal{F}^u_{A_i}]$. Then q-RLOANIVWA operator is q-RLOANIVWA $[A_1, A_2, ..., A_n] = \biguplus_{i=1}^n \partial_i A_i$ for $i = 1, 2, ..., n$.

**Theorem 4.2.** *Let $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^{-+}_{A\ i}, log\mathcal{I}^{-+}_{A\ i}], [log\mathcal{F}^l_{Ai}, log\mathcal{F}^u_{Ai}] \right\rangle$ be the collection of q-RLOANIVNs. Then q-RLOANIVWA $[A_1, A_2, ..., A_n]$ = [Associativity property].*

$$\begin{bmatrix} \left[\sqrt[q]{1 - \circledast_{i=1}^n \left[1 - [log_{Z_i}\mathcal{T}^l_{Ai}]^q\right]^{\partial_i}}, \sqrt[q]{1 - \circledast_{i=1}^n \left[1 - [log_{Z_i}[\mathcal{T}^u_{A\,i}]]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1 - \circledast_{i=1}^n \left[1 - [log_{Z_i}\mathcal{I}^l_{Ai}]^q\right]^{\partial_i}}, \sqrt[q]{1 - \circledast_{i=1}^n \left[1 - [log_{Z_i}\mathcal{I}^u_{Ai}]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^n [log_{Z_i}\mathcal{F}^l_{Ai}]^{q\partial_i}, \circledast_{i=1}^n [log_{Z_i}[\mathcal{F}^u_{Ai}]]^{q\partial_i}\right] \end{bmatrix}.$$

**Proof.** If $n = 2$, then q-RLOANIVWA $[A_1 = B, A_2 = C] = \partial_1 B \bigvee \partial_2 C$, where

$$\partial_1 B = \begin{bmatrix} \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{T}^l_B]^q\right]^{\partial_1}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}[\mathcal{T}^u_B]]^q\right]^{\partial_1}}\right], \\ \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}^l_B]^q\right]^{\partial_1}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}^u_B]^q\right]^{\partial_1}}\right], \\ \left[[log_{Z_i}\mathcal{F}^l_B]^{q\partial_1}, [log_{Z_i}[\mathcal{F}^u_B]]^{q\partial_1}\right] \end{bmatrix}$$

and

$$\partial_2 C = \begin{bmatrix} \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{T}^l_C]^q\right]^{\partial_2}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}[\mathcal{T}^u_C]]^q\right]^{\partial_2}}\right], \\ \left[\sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}^l_C]^q\right]^{\partial_2}}, \sqrt[q]{1 - \left[1 - [log_{Z_i}\mathcal{I}^u_C]^q\right]^{\partial_2}}\right], \\ \left[[log_{Z_i}\mathcal{F}^l_C]^{q\partial_2}, [log_{Z_i}[\mathcal{F}^u_C]]^{q\partial_2}\right] \end{bmatrix}.$$

Hence,

$$\partial_1 B \bigvee \partial_2 C = \begin{bmatrix} \left[\begin{matrix} \sqrt[q]{\begin{matrix}\left[1 - \left[1 - [log_{Z_i}\mathcal{T}^l_B]^q\right]^{\partial_1}\right] + \left[1 - \left[1 - [log_{Z_i}\mathcal{T}^l_C]^q\right]^{\partial_2}\right] \\ -\left[1 - \left[1 - [log_{Z_i}\mathcal{T}^l_B]^q\right]^{\partial_1}\right] \cdot \left[1 - \left[1 - [log_{Z_i}\mathcal{T}^l_C]^q\right]^{\partial_2}\right]\end{matrix}}, \\ \sqrt[q]{\begin{matrix}\left[1 - \left[1 - [log_{Z_i}[\mathcal{T}^u_B]]^q\right]^{\partial_1}\right] + \left[1 - \left[1 - [log_{Z_i}[\mathcal{T}^u_C]]^q\right]^{\partial_2}\right] \\ -\left[1 - \left[1 - [log_{Z_i}[\mathcal{T}^u_B]]^q\right]^{\partial_1}\right] \cdot \left[1 - \left[1 - [log_{Z_i}[\mathcal{T}^u_C]]^q\right]^{\partial_2}\right]\end{matrix}} \end{matrix}\right], \\ \left[\begin{matrix} \sqrt[q]{\begin{matrix}\left[1 - \left[1 - [log_{Z_i}\mathcal{I}^l_B]^q\right]^{\partial_1}\right] + \left[1 - \left[1 - [log_{Z_i}\mathcal{I}^u_C]^q\right]^{\partial_2}\right] \\ -\left[1 - \left[1 - [log_{Z_i}\mathcal{I}^l_B]^q\right]^{\partial_1}\right] \cdot \left[1 - \left[1 - [log_{Z_i}\mathcal{I}^u_C]^q\right]^{\partial_2}\right]\end{matrix}}, \\ \sqrt[q]{\begin{matrix}\left[1 - \left[1 - [log_{Z_i}\mathcal{I}^l_B]^q\right]^{\partial_1}\right] + \left[1 - \left[1 - [log_{Z_i}\mathcal{I}^u_C]^q\right]^{\partial_2}\right] \\ -\left[1 - \left[1 - [log_{Z_i}\mathcal{I}^l_B]^q\right]^{\partial_1}\right] \cdot \left[1 - \left[1 - [log_{Z_i}\mathcal{I}^u_C]^q\right]^{\partial_2}\right]\end{matrix}} \end{matrix}\right], \\ \left[[log_{Z_i}\mathcal{F}^l_B]^{q\partial_1} \cdot [log_{Z_i}\mathcal{F}^l_C]^{q\partial_2}, [log_{Z_i}[\mathcal{F}^u_B]]^{q\partial_1} \cdot [log_{Z_i}[\mathcal{F}^u_C]]^{q\partial_2}\right] \end{bmatrix}$$

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$$= \left[ \begin{array}{c} \left[ \begin{array}{c} \sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{T}_B^l]^q\right]^{\partial_1}\cdot\left[1-[log_{Z_i}\mathcal{T}_C^l]^q\right]^{\partial_2}}, \\ \sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{T}_B^u]]^q\right]^{\partial_1}\cdot\left[1-[log_{Z_i}[\mathcal{T}_C^u]]^q\right]^{\partial_2}} \end{array} \right], \\ \left[ \begin{array}{c} \sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{I}_B^l]^q\right]^{\partial_1}\cdot\left[1-[log_{Z_i}\mathcal{I}_C^u]^q\right]^{\partial_2}}, \\ \sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{I}_B^l]^q\right]^{\partial_1}\cdot\left[1-[log_{Z_i}\mathcal{I}_C^u]^q\right]^{\partial_2}} \end{array} \right], \\ \left[[log_{Z_i}\mathcal{F}_B^l]^{q\partial_1}\cdot[log_{Z_i}\mathcal{F}_C^l]^{q\partial_2}, [log_{Z_i}[\mathcal{F}_B^u]]^{q\partial_1}\cdot[log_{Z_i}[\mathcal{F}_C^u]]^{q\partial_2}\right] \end{array} \right].$$

Thus, q-RLOANIVWA $[A_1 = B, A_2 = C]$

$$= \left[ \begin{array}{c} \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{T}_{Ai}^l]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}[\mathcal{T}_{A\,i}^u]]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}_{Ai}^u]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^n [log_{Z_i}\mathcal{F}_{Ai}^l]^{q\partial_i}, \circledast_{i=1}^n[log_{Z_i}[\mathcal{F}_{Ai}^u]]^{q\partial_i}\right] \end{array} \right].$$

It is valid for $n = m$ and $m \geq 3$.
Hence, q-RLOANIVWA $[A_1, A_2, ..., A_m]$

$$= \left[ \begin{array}{c} \left[\sqrt[q]{1-\circledast_{i=1}^m\left[1-[log_{Z_i}\mathcal{T}_{Ai}^l]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^m\left[1-[log_{Z_i}[\mathcal{T}_{A\,i}^u]]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^m\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^m\left[1-[log_{Z_i}\mathcal{I}_{Ai}^u]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^m [log_{Z_i}\mathcal{F}_{Ai}^l]^{q\partial_i}, \circledast_{i=1}^m[log_{Z_i}[\mathcal{F}_{Ai}^u]]^{q\partial_i}\right] \end{array} \right].$$

If $n = l + 1$ and we apply, q-RLOANIVWA $[A_1, A_2, ..., A_m, A_{m+1}]$

$$= \left[ \begin{array}{c} \left[ \begin{array}{c} \sqrt[q]{\begin{array}{c}\biguplus_{i=1}^m\left[1-\left[1-[log_{Z_i}\mathcal{T}_{Ai}^l]^q\right]^{\partial_i}\right]+\left[1-\left[1-[log_{Z_i}\mathcal{T}_{Am+1}^l]^q\right]^{\partial_{m+1}}\right] \\ -\circledast_{i=1}^m\left[1-\left[1-[log_{Z_i}\mathcal{T}_{Ai}^l]^q\right]^{\partial_i}\right]\cdot\left[1-\left[1-[log_{Z_i}\mathcal{T}_{Am+1}^l]^q\right]^{\partial_{m+1}}\right]\end{array}}, \\ \sqrt[q]{\begin{array}{c}\biguplus_{i=1}^m\left[1-\left[1-[log_{Z_i}[\mathcal{T}_{A\,i}^u]]^q\right]^{\partial_i}\right]+\left[1-\left[1-[log_{Z_i}[\mathcal{T}_{A\,m+1}^u]]^q\right]^{\partial_{m+1}}\right] \\ -\circledast_{i=1}^m\left[1-\left[1-[log_{Z_i}[\mathcal{T}_{A\,i}^u]]^q\right]^{\partial_i}\right]\cdot\left[1-\left[1-[log_{Z_i}[\mathcal{T}_{A\,m+1}^u]]^q\right]^{\partial_{m+1}}\right]\end{array}} \end{array} \right], \\ \left[ \begin{array}{c} \sqrt[q]{\begin{array}{c}\biguplus_{i=1}^m\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}\right]+\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Am+1}^u]^q\right]^{\partial_{m+1}}\right] \\ -\circledast_{i=1}^m\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}\right]\cdot\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Am+1}^u]^q\right]^{\partial_{m+1}}\right]\end{array}}, \\ \sqrt[q]{\begin{array}{c}\biguplus_{i=1}^m\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}\right]+\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Am+1}^u]^q\right]^{\partial_{m+1}}\right] \\ -\circledast_{i=1}^m\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}\right]\cdot\left[1-\left[1-[log_{Z_i}\mathcal{I}_{Am+1}^u]^q\right]^{\partial_{m+1}}\right]\end{array}} \end{array} \right], \\ \left[\circledast_{i=1}^m [log_{Z_i}\mathcal{F}_{Ai}^l]^{q\partial_i}\cdot[log_{Z_i}\mathcal{F}_{Am+1}^l]^{q\partial_{m+1}}, \circledast_{i=1}^m[log_{Z_i}[\mathcal{F}_{Ai}^u]]^{q\partial_i}\cdot[log_{Z_i}[\mathcal{F}_{Am+1}^u]]^{q\partial_{m+1}}\right] \end{array} \right]$$

$$= \left[ \begin{array}{c} \left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-[log_{Z_i}\mathcal{T}_{Ai}^l]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-[log_{Z_i}[\mathcal{T}_{A\,i}^u]]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-[log_{Z_i}\mathcal{I}_{Ai}^l]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-[log_{Z_i}\mathcal{I}_{Ai}^u]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^{m+1} [log_{Z_i}\mathcal{F}_{Ai}^l]^{q\partial_i}, \circledast_{i=1}^{m+1}[log_{Z_i}[\mathcal{F}_{Ai}^u]]^{q\partial_i}\right] \end{array} \right].$$

**Theorem 4.3.** *[Idempotency property] If all* $A_i = \left\langle [log\mathcal{T}_{Ai}^l, log\mathcal{T}_{A\,i}^u], [log\mathcal{I}_{Ai}^l, log\mathcal{I}_{Ai}^u][log\mathcal{F}_{Ai}^l, log\mathcal{F}_{Ai}^u] \right\rangle$ $[i = 1, 2, ..., n]$ *are equal and* $A_i = A$*. Then q-RLOANIVWA* $[A_1, A_2, ..., A_n] = A$*.*


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**Proof.** Given that $[log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}] = [log\mathcal{T}^l_A, log[\mathcal{T}^u_A]]$ , $[log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}] = [log\mathcal{I}^l_A, log\mathcal{I}^u_A]$ and $[log\mathcal{F}^l_{Ai}, log\mathcal{F}^u_{Ai}] = [log\mathcal{F}^l_A, log[\mathcal{F}^u_A]]$, for $i = 1, 2, ..., n$ and $\biguplus^n_{i=1} \partial_i = 1$. Now, q-RLOANIVWA $[A_1, A_2, ..., A_n]$

$$
\begin{aligned}
&= \left[\begin{array}{c} \left[\sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [log_{Z_i}\mathcal{T}^l_{Ai}]^q\Big]^{\partial_i}}, \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [log_{Z_i}[\mathcal{T}^u_{A\,i}]]^q\Big]^{\partial_i}}\ \right], \\ \left[\sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [log_{Z_i}\mathcal{I}^l_{Ai}]^q\Big]^{\partial_i}}, \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [log_{Z_i}\mathcal{I}^u_{Ai}]^q\Big]^{\partial_i}}\right], \\ \Big[\circledast^n_{i=1}\ [log_{Z_i}\mathcal{F}^l_{Ai}]^{q\partial_i}, \circledast^n_{i=1}[log_{Z_i}[\mathcal{F}^u_{Ai}]]^{q\partial_i}\Big] \end{array}\right] \\
&= \left[\begin{array}{c} \left[\sqrt[q]{1 - \Big[1 - [log_{Z_i}\mathcal{T}^l_A]^q\Big]^{\biguplus^n_{i=1}\partial_i}}, \sqrt[q]{1\Big[1 - [log_{Z_i}[\mathcal{T}^u_A]]^q\Big]^{\biguplus^n_{i=1}\partial_i}}\ \right], \\ \left[\sqrt[q]{1 - \Big[1 - [log_{Z_i}\mathcal{I}^l_A]^q\Big]^{\biguplus^n_{i=1}\partial_i}}, \sqrt[q]{1\Big[1 - [log_{Z_i}\mathcal{I}^u_A]^q\Big]^{\biguplus^n_{i=1}\partial_i}}\right], \\ \big[[log_{Z_i}\mathcal{F}^l_A]^{\biguplus^n_{i=1}q\partial_i}, [log_{Z_i}[\mathcal{F}^u_A]]^{\biguplus^n_{i=1}q\partial_i}\big] \end{array}\right] \\
&= \left[\begin{array}{c} \left[\sqrt[q]{1 - \Big[1 - [log_{Z_i}\mathcal{T}^l_A]^q\Big]}, \sqrt[q]{1 - \Big[1 - [log_{Z_i}[\mathcal{T}^u_A]]^q\Big]}\ \right], \\ \left[\sqrt[q]{1 - \Big[1 - [log_{Z_i}\mathcal{I}^l_A]^q\Big]}, \sqrt[q]{1 - \Big[1 - [log_{Z_i}\mathcal{I}^u_A]^q\Big]}\right], \\ \Big[[log_{Z_i}\mathcal{F}^l_A]^q, [log_{Z_i}[\mathcal{F}^u_A]]^q\Big] \end{array}\right] \\
&= A.
\end{aligned}
$$

**Theorem 4.4.** *Let* $A_i = \Big\langle [log\mathcal{T}^l_{Aij}, log[\mathcal{T}^u_{A\,ij}]], [log\mathcal{I}^l_{Aij}, log\mathcal{I}^u_{Aij}][log\mathcal{F}^l_{Aij}, log[\mathcal{F}^u_{Aij}]]\Big\rangle[i = 1, 2, ..., n]; [j = 1, 2, ..., i_j]$ *be the collection of q-RLOANIVWA, where* $\overleftarrow{log_{Z_i}\mathcal{T}^l_A} = \min log_{Z_i}\mathcal{T}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{T}^l_A} = \max log_{Z_i}\mathcal{T}^l_{Aij}$, $\overleftarrow{log_{Z_i}[\mathcal{T}^u_A]} = \min log_{Z_i}[\mathcal{T}^u_{A\,ij}]$, $\overrightarrow{log_{Z_i}[\mathcal{T}^u_A]} = \max log_{Z_i}[\mathcal{T}^u_{A\,ij}]$, $\overleftarrow{log_{Z_i}\mathcal{I}^l_A} = \min log_{Z_i}\mathcal{I}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{I}^u_A} = \max log_{Z_i}\mathcal{I}^u_{Aij}$, $\overleftarrow{log_{Z_i}\mathcal{I}^l_A} = \min log_{Z_i}\mathcal{I}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{I}^u_A} = \max log_{Z_i}\mathcal{I}^u_{Aij}$, $\overleftarrow{log_{Z_i}\mathcal{F}^l_A} = \min log_{Z_i}\mathcal{F}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{F}^l_A} = \max log_{Z_i}\mathcal{F}^l_{Aij}$, $\overleftarrow{log_{Z_i}[\mathcal{F}^u_A]} = \min log_{Z_i}[\mathcal{F}^u_{Aij}]$, $\overrightarrow{log_{Z_i}[\mathcal{F}^u_A]} = \max log_{Z_i}[\mathcal{F}^u_{Aij}]$.
*Then,* $\Big\langle [\overleftarrow{log_{Z_i}\mathcal{T}^l_A}, \overleftarrow{log_{Z_i}[\mathcal{T}^u_A]}], [\overleftarrow{log_{Z_i}\mathcal{I}^l_A}, \overleftarrow{log_{Z_i}\mathcal{I}^u_A}], [\overrightarrow{log_{Z_i}\mathcal{F}^l_A}, \overrightarrow{log_{Z_i}[\mathcal{F}^u_A]}]\Big\rangle$

$$
\begin{aligned}
&\leq\ new\ type\ LOANIVWA[A_1, A_2, ..., A_n] \\
&\leq\ \Big\langle [\overrightarrow{log_{Z_i}\mathcal{T}^l_A}, \overrightarrow{log_{Z_i}[\mathcal{T}^u_A]}], [\overrightarrow{log_{Z_i}\mathcal{I}^l_A}, \overrightarrow{log_{Z_i}\mathcal{I}^u_A}], [\overleftarrow{log_{Z_i}\mathcal{F}^l_A}, \overleftarrow{log_{Z_i}[\mathcal{F}^u_A]}]\Big\rangle.
\end{aligned}
$$

*where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$*, [Boundedness property].*

**Proof.** Since, $\overleftarrow{log_{Z_i}\mathcal{T}^l_A} = \min log_{Z_i}\mathcal{T}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{T}^l_A} = \max log_{Z_i}\mathcal{T}^l_{Aij}$ $\overleftarrow{log_{Z_i}[\mathcal{T}^u_A]} = \min log_{Z_i}[\mathcal{T}^u_{A\,ij}]$, $\overrightarrow{log_{Z_i}[\mathcal{T}^u_A]} = \max log_{Z_i}[\mathcal{T}^u_{A\,ij}]$ and $\overleftarrow{log_{Z_i}\mathcal{T}^l_A} \leq log_{Z_i}\mathcal{T}^l_{Aij} \leq \overrightarrow{log_{Z_i}\mathcal{T}^l_A}$ and $\overleftarrow{log_{Z_i}[\mathcal{T}^u_A]} \leq log_{Z_i}[\mathcal{T}^u_A]_{ij} \leq \overrightarrow{log_{Z_i}[\mathcal{T}^u_A]}$.
Now, $\overleftarrow{log_{Z_i}\mathcal{T}^l_A} + \overleftarrow{log_{Z_i}[\mathcal{T}^u_A]}$

$$
\begin{aligned}
&= \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [\overleftarrow{log_{Z_i}\mathcal{T}^l_A}]^q\Big]^{\partial_i}} + \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [\overleftarrow{log_{Z_i}[\mathcal{T}^u_A]}]^q\Big]^{\partial_i}} \\
&\leq \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [log_{Z_i}\mathcal{T}^l_{Aij}]^q\Big]^{\partial_i}} + \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [log_{Z_i}[\mathcal{T}^u_{A\,ij}]]^q\Big]^{\partial_i}} \\
&\leq \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [\overrightarrow{log_{Z_i}\mathcal{T}^l_A}]^q\Big]^{\partial_i}} + \sqrt[q]{1 - \circledast^n_{i=1}\Big[1 - [\overrightarrow{log_{Z_i}[\mathcal{T}^u_A]}]^q\Big]^{\partial_i}} \\
&= \overrightarrow{log_{Z_i}\mathcal{T}^l_A} + \overrightarrow{log_{Z_i}[\mathcal{T}^u_A]}.
\end{aligned}
$$

Since, $\overleftarrow{log_{Z_i}\mathcal{I}^l_A} = \min log_{Z_i}\mathcal{I}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{I}^u_A} = \max log_{Z_i}\mathcal{I}^u_{Aij}$, $\overleftarrow{log_{Z_i}\mathcal{I}^l_A} = \min log_{Z_i}\mathcal{I}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{I}^u_A} =$

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$\max log_{Z_i}\mathcal{I}^u_{Aij}$ and $\overleftarrow{log_{Z_i}\mathcal{I}^l_A} \leq log_{Z_i}\mathcal{I}^l_{Aij} \leq \overrightarrow{log_{Z_i}\mathcal{I}^l_A}$ and $\overleftarrow{log_{Z_i}\mathcal{I}^u_A} \leq log_{Z_i}\mathcal{I}^u_{Aij} \leq \overrightarrow{log_{Z_i}\mathcal{I}^u_A}$. Now,

$$\begin{aligned}
\overleftarrow{log_{Z_i}\mathcal{I}^l_A} + \overleftarrow{log_{Z_i}\mathcal{I}^u_A} &= \sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overleftarrow{log_{Z_i}\mathcal{I}^l_A}]^q\right]^{\partial_i}} + \sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overleftarrow{log_{Z_i}\mathcal{I}^u_A}]^q\right]^{\partial_i}} \\
&\leq \sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[log_{Z_i}\mathcal{I}^l_{Aij}]^q\right]^{\partial_i}} + \sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[log_{Z_i}\mathcal{I}^u_{Aij}]^q\right]^{\partial_i}} \\
&\leq \sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overrightarrow{log_{Z_i}\mathcal{I}^l_A}]^q\right]^{\partial_i}} + \sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overrightarrow{log_{Z_i}\mathcal{I}^u_A}]^q\right]^{\partial_i}} \\
&= \overrightarrow{log_{Z_i}\mathcal{I}^l_A} + \overrightarrow{log_{Z_i}\mathcal{I}^u_A}.
\end{aligned}$$

Since, $\overleftarrow{log_{Z_i}\mathcal{F}^l_A} = \min log_{Z_i}\mathcal{F}^l_{Aij}$, $\overrightarrow{log_{Z_i}\mathcal{F}^l_A} = \max log_{Z_i}\mathcal{F}^l_{Aij}$ $\overleftarrow{log_{Z_i}[\mathcal{F}^u_A]} = \min log_{Z_i}[\mathcal{F}^u_{Aij}]$, $\overrightarrow{log_{Z_i}[\mathcal{F}^u_A]} = \max log_{Z_i}[\mathcal{F}^u_{Aij}]$ and $\overleftarrow{log_{Z_i}\mathcal{F}^l_A} \leq log_{Z_i}\mathcal{F}^l_{Aij} \leq \overrightarrow{log_{Z_i}\mathcal{F}^l_A}$ and $\overleftarrow{log_{Z_i}[\mathcal{F}^u_A]} \leq log_{Z_i}[\mathcal{F}^u_{Aij}] \leq \overrightarrow{log_{Z_i}[\mathcal{F}^u_A]}$. Now,

$$\begin{aligned}
\overleftarrow{log_{Z_i}\mathcal{F}^l_A} + \overleftarrow{log_{Z_i}[\mathcal{F}^u_A]} &= \circledast_{i=1}^{n}[\overleftarrow{log_{Z_i}\mathcal{F}^l_A}]^{q\partial_i} + \circledast_{i=1}^{n}[\overleftarrow{log_{Z_i}[\mathcal{F}^u_A]}]^{q\partial_i} \\
&\leq \circledast_{i=1}^{n}[log_{Z_i}\mathcal{F}^l_{Aij}]^{q\partial_i} + \circledast_{i=1}^{n}[log_{Z_i}[\mathcal{F}^u_{Aij}]]^{q\partial_i} \\
&\leq \circledast_{i=1}^{n}[\overrightarrow{log_{Z_i}\mathcal{F}^l_A}]^{q\partial_i} + \circledast_{i=1}^{n}[\overrightarrow{log_{Z_i}[\mathcal{F}^u_A]}]^{q\partial_i} \\
&= \overrightarrow{log_{Z_i}\mathcal{F}^l_A} + \overrightarrow{log_{Z_i}[\mathcal{F}^u_A]}.
\end{aligned}$$

Therefore,

$$\left[\cfrac{\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overleftarrow{log_{Z_i}\mathcal{T}^l_A}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overleftarrow{log_{Z_i}[\mathcal{T}^u_A]}]^q\right]^{\partial_i}}\right]^2}{2+1-\cfrac{\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overleftarrow{log_{Z_i}\mathcal{I}^l_A}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overleftarrow{log_{Z_i}\mathcal{I}^u_A}]^q\right]^{\partial_i}}\right]^2}{2+1-\cfrac{\left[\circledast_{i=1}^{n}[\overrightarrow{log_{Z_i}\mathcal{F}^l_A}]^{q\partial_i}\right]^2+\left[\circledast_{i=1}^{n}[\overrightarrow{log_{Z_i}[\mathcal{F}^u_A]}]^{q\partial_i}\right]^2}{2}}}\right]$$

$$=\left[\cfrac{\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[log_{Z_i}\mathcal{T}^l_{Aij}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[log_{Z_i}[\mathcal{T}^u_{A\,ij}]]^q\right]^{\partial_i}}\right]^2}{2+1-\cfrac{\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[log_{Z_i}\mathcal{I}^l_{Aij}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[log_{Z_i}\mathcal{I}^u_{Aij}]^q\right]^{\partial_i}}\right]^2}{2+1-\cfrac{\left[\circledast_{i=1}^{n}[log_{Z_i}\mathcal{F}^l_{Aij}]^{q\partial_i}\right]^2+\left[\circledast_{i=1}^{n}[log_{Z_i}[\mathcal{F}^u_{Aij}]]^{q\partial_i}\right]^2}{2}}}\right]$$

$$=\left[\cfrac{\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overrightarrow{log_{Z_i}\mathcal{T}^l_A}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overrightarrow{log_{Z_i}[\mathcal{T}^u_A]}]^q\right]^{\partial_i}}\right]^2}{2+1-\cfrac{\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overrightarrow{log_{Z_i}\mathcal{I}^l_A}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^{n}\left[1-[\overrightarrow{log_{Z_i}\mathcal{I}^u_A}]^q\right]^{\partial_i}}\right]^2}{2+1-\cfrac{\left[\circledast_{i=1}^{n}[\overleftarrow{log_{Z_i}\mathcal{F}^l_A}]^{q\partial_i}\right]^2+\left[\circledast_{i=1}^{n}[\overleftarrow{log_{Z_i}[\mathcal{F}^u_A]}]^{q\partial_i}\right]^2}{2}}}\right].$$

Hence, $\left\langle[\overleftarrow{log\mathcal{T}^l_A}, \overleftarrow{log[\mathcal{T}^u_A]}], [\overleftarrow{log\mathcal{I}^l_A}, \overleftarrow{log\mathcal{I}^u_A}], [\overrightarrow{log\mathcal{F}^l_A}, \overrightarrow{log[\mathcal{F}^u_A]}]\right\rangle$

$$\begin{aligned}
&\leq q - RLOANIVWA[A_1, A_2, ..., A_n] \\
&\leq \left\langle[\overrightarrow{log\mathcal{T}^l_A}, \overrightarrow{log[\mathcal{T}^u_A]}], [\overrightarrow{log\mathcal{I}^l_A}, \overrightarrow{log\mathcal{I}^u_A}], [\overleftarrow{log\mathcal{F}^l_A}, \overrightarrow{log[\mathcal{F}^u_A]}]\right\rangle.
\end{aligned}$$

**Theorem 4.5.** *[Monotonicity property] Let* $A_i = \left\langle[log\mathcal{T}^l_{At_{ij}}, log[\mathcal{T}^u_{A\,t_{ij}}]], [log\mathcal{I}^l_{At_{ij}}, log\mathcal{I}^u_{At_{ij}}], [log\mathcal{F}^l_{At_{ij}}, log[\mathcal{F}^u_{At_{ij}}]]\right\rangle$ *and* $\partial_i = \left\langle[log\mathcal{T}^l_{Ah_{ij}}, log[\mathcal{T}^u_{A\,h_{ij}}]], [log\mathcal{I}^l_{Ah_{ij}}, log\mathcal{I}^u_{Ah_{ij}}], [log\mathcal{F}^l_{Ah_{ij}}, log[\mathcal{F}^u_{Ah_{ij}}]]\right\rangle$


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*$[i=1,2,...,n];[j=1,2,...,i_j]$ be the families of q-RLOANIVWAs. For any i, if there is $\left[log_{Z_i}\mathcal{T}^l_{A t_{ij}}\right]^2+\left[log_{Z_i}[\mathcal{T}^u_{A\,t_{ij}}]\right]^2\leq\left[log_{Z_i}\mathcal{T}^l_{A h_{ij}}\right]^2+\left[log_{Z_i}[\mathcal{T}^u_{A\,h_{ij}}]\right]^2$ and $\left[log_{Z_i}\mathcal{I}^l_{A t_{ij}}\right]^2+\left[log_{Z_i}\mathcal{I}^u_{A t_{ij}}\right]^2\geq\left[log_{Z_i}\mathcal{I}^l_{A h_{ij}}\right]^2+\left[log_{Z_i}\mathcal{I}^u_{A h_{ij}}\right]^2$ and $\left[log_{Z_i}\mathcal{F}^l_{A t_{ij}}\right]^2+\left[log_{Z_i}[\mathcal{F}^u_{A}]_{t_{ij}}\right]^2\geq\left[log_{Z_i}\mathcal{F}^l_{A h_{ij}}\right]^2+\left[log_{Z_i}[\mathcal{F}^u_{A h_{ij}]}\right]^2$ or $A_i\leq\mathscr{W}_i$. Then q-RLOANIVWA $[A_1,A_2,...,A_n]\leq$ q-RLOANIVWA $[\mathscr{W}_1,\mathscr{W}_2,...,\mathscr{W}_n]$.*

**Proof.** For any $i$, $\left[log_{Z_i}\mathcal{T}^l_{A t_{ij}}\right]^q+\left[log_{Z_i}[\mathcal{T}^u_{A\,t_{ij}}]\right]^q\leq\left[log_{Z_i}\mathcal{T}^l_{A h_{ij}}\right]^q+\left[log_{Z_i}[\mathcal{T}^u_{A\,h_{ij}}]\right]^q$.
Therefore, $1-\left[log_{Z_i}\mathcal{T}^l_{A t_i}\right]^q+1-\left[log_{Z_i}[\mathcal{T}^u_{A\,t_i}]\right]^q\geq1-\left[log_{Z_i}\mathcal{T}^l_{A h_i}\right]^q+1-\left[log_{Z_i}[\mathcal{T}^u_{A\,h_i}]\right]^q$.
Hence, $\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{T}^l_{A t_i}\right]^q\right]^{\partial_i}+\circledast_{i=1}^n\left[1-\left[log_{Z_i}[\mathcal{T}^u_{A\,t_i}]\right]^q\right]^{\partial_i}\geq$
$\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{T}^l_{A h_i}\right]^q\right]^{\partial_i}+\circledast_{i=1}^n\left[1-\left[log_{Z_i}[\mathcal{T}^u_{A\,h_i}]\right]^q\right]^{\partial_i}$
and $\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{T}^l_{A t_i}\right]^q\right]^{\partial_i}}+\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}[\mathcal{T}^u_{A\,t_i]}\right]^q\right]^{\partial_i}}$
$\leq\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{T}^l_{A h_i}\right]^q\right]^{\partial_i}}+\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}[\mathcal{T}^u_{A\,h_i]}\right]^q\right]^{\partial_i}}$.

For any $i$, $\left[log_{Z_i}\mathcal{I}^l_{A t_{ij}}\right]^q+\left[log_{Z_i}\mathcal{I}^u_{A t_{ij}}\right]^q\geq\left[log_{Z_i}\mathcal{I}^l_{A h_{ij}}\right]^q+\left[log_{Z_i}\mathcal{I}^u_{A h_{ij}}\right]^q$.
Therefore, $1-\left[log_{Z_i}\mathcal{I}^l_{A t_i}\right]^q+1-\left[log_{Z_i}\mathcal{I}^u_{A t_i}\right]^q\leq1-\left[log_{Z_i}\mathcal{I}^l_{A h_i}\right]^q+1-\left[log_{Z_i}\mathcal{I}^u_{A h_i}\right]^q$.
Hence, $\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^l_{A t_i}\right]^q\right]^{\partial_i}+\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^u_{A t_i}\right]^q\right]^{\partial_i}\leq$
$\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^l_{A h_i}\right]^q\right]^{\partial_i}+\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^u_{A h_i}\right]^q\right]^{\partial_i}$
implies that $\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^l_{A t_i}\right]^q\right]^{\partial_i}}+\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^u_{A t_i}\right]^q\right]^{\partial_i}}$
$\geq\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^l_{A h_i}\right]^q\right]^{\partial_i}}+\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^u_{A h_i}\right]^q\right]^{\partial_i}}$.
Hence, $1-\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^l_{A t_i}\right]^q\right]^{\partial_i}}+\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^u_{A t_i}\right]^q\right]^{\partial_i}}$
$\leq1-\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^l_{A h_i}\right]^q\right]^{\partial_i}}+\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[log_{Z_i}\mathcal{I}^u_{A h_i}\right]^q\right]^{\partial_i}}$.

For any $i$, $\left[log_{Z_i}\mathcal{F}^l_{A t_{ij}}\right]^q+\left[log_{Z_i}[\mathcal{F}^u_{A t_{ij}]}\right]^q\geq\left[log_{Z_i}\mathcal{F}^l_{A h_{ij}}\right]^q+\left[log_{Z_i}[\mathcal{F}^u_{A h_{ij}]}\right]^q$.
Therefore, $1-\frac{\left[\circledast_{i=1}^n log_{Z_i}\mathcal{F}^l_{A t_{ij}}\right]^q+\left[\circledast_{i=1}^n log_{Z_i}[\mathcal{F}^u_{A t_{ij}]}\right]^q}{2}\leq1-\frac{\left[\circledast_{i=1}^n log_{Z_i}\mathcal{F}^l_{A h_{ij}}\right]^q+\left[\circledast_{i=1}^n log_{Z_i}[\mathcal{F}^u_{A h_{ij}]}\right]^q}{2}$.

$$=\left[\frac{\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{T}^l_{Ati}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}[\mathcal{T}^u_{A\,ti}]]^q\right]^{\partial_i}}\right]^2}{2}\atop +1-\frac{\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}^l_{Ati}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}^u_{Ati}]^q\right]^{\partial_i}}\right]^2}{2}\atop +1-\frac{\left[\circledast_{i=1}^n[log_{Z_i}\mathcal{F}^l_{A\,tij}]\right]^2+\left[\circledast_{i=1}^n[log_{Z_i}[\mathcal{F}^u_{A\,tij}]]\right]^2}{2}\right]$$

$$=\left[\frac{\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{T}^l_{Ahi}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}[\mathcal{T}^u_{A\,hi}]]^q\right]^{\partial_i}}\right]^2}{2}\atop +1-\frac{\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}^l_{Ahi}]^q\right]^{\partial_i}}\right]^2+\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}^u_{Ahi}]^q\right]^{\partial_i}}\right]^2}{2}\atop +1-\frac{\left[\circledast_{i=1}^n[log_{Z_i}\mathcal{F}^l_{A\,hij}]\right]^2+\left[\circledast_{i=1}^n[log_{Z_i}[\mathcal{F}^u_{A\,hij}]]\right]^2}{2}\right].$$

Hence, q-RLOANIVWA $[A_1,A_2,...,A_n]\leq$ q-RLOANIVWA $[\mathscr{W}_1,\mathscr{W}_2,...,W_n]$.

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### 4.2 q-RLOANIV weighted geometric [q-RLOANIVWG] operator

**Definition 4.6.** Let $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}], [log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ be the collection of q-RLOANIVNs. Then q-RLOANIVWG operator is q-RLOANIVWG $[A_1, A_2, ..., A_n] = \circledast_{i=1}^n A_i^{\partial_i}\ [i = 1, 2, ..., n]$.

**Theorem 4.7.** *Let* $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{F}^l_{Ai}, log\mathcal{F}^u_{Ai}] \right\rangle$ *be the collection of q-RLOANIVNs. Then q-RLOANIVWG* $[A_1, A_2, ..., A_n]$

$$= \begin{bmatrix} \left[\circledast_{i=1}^n [log_{Z_i}\mathcal{T}^l_{Ai}]^{q\partial_i}, \circledast_{i=1}^n [log_{Z_i}[\mathcal{T}^u_{A\,i}]]^{q\partial_i}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}^l_{Ai}]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{I}^u_{Ai}]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}\mathcal{F}^l_{Ai}]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^n\left[1-[log_{Z_i}[\mathcal{F}^u_{Ai}]]^q\right]^{\partial_i}}\right] \end{bmatrix}.$$

**Theorem 4.8.** *If all* $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}][log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ *are equal and* $A_i = A$, *for* $i = 1, 2, ..., n$. *Then q-RLOANIVWG* $[A_1, A_2, ..., A_n] = A$.

**Corollary 4.9.** *The q-RLOANIVWG operator is used to satisfy the boundedness and monotonicity properties.*

### 4.3 Extended q-RLOANIVWA [q-RELOANIVWA] operator

**Definition 4.10.** Let $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}], [log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ be the collection of q-RLOANIVN. Then q-RELOANIVWA $[A_1, A_2, ..., A_n] = \left[\biguplus_{i=1}^n \partial_i A_i^{\Xi}\right]^{1/\Xi}$ is called the q-RELOANIVWA operator.

**Theorem 4.11.** *Let* $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}], [log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ *be the collection of q-RLOANIVNs. Then q-RELOANIVWA* $[A_1, A_2, ..., A_n]$

$$= \begin{bmatrix} \left[\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}\mathcal{T}^l_{Ai}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}, \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}[\mathcal{T}^u_{A\,i}]]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}\right], \\ \left[\left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}\mathcal{I}^l_{Ai}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}, \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}\mathcal{I}^u_{Ai}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}\right], \\ \left[\begin{array}{c} \sqrt[q]{1-\left[1-\left[\circledast_{i=1}^n\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}^l_{Ai}]^q\right]^q}\right]^{\partial_i}\right]\right]^{q}}^{\,1/\Xi}, \\ \sqrt[q]{1-\left[1-\left[\circledast_{i=1}^n\left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}^u_{Ai}]]^q\right]^q}\right]^{\partial_i}\right]\right]^{q}}^{\,1/\Xi} \end{array}\right] \end{bmatrix}.$$

**Proof.** We have, $\biguplus_{i=1}^n \partial_i A_i^{\Xi}$

$$= \begin{bmatrix} \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}\mathcal{T}^l_{Ai}]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}[\mathcal{T}^u_{A\,i}]]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}\mathcal{I}^l_{Ai}]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^n\left[1-\left[[log_{Z_i}\mathcal{I}^u_{Ai}]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^n\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}^l_{Ai}]^q\right]^q}\right]^{\partial_i}, \circledast_{i=1}^n\left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}^u_{Ai}]]^q\right]^q}\right]^{\partial_i}\right] \end{bmatrix}.$$

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If $n = 2$, then $\partial_1 B \bigvee \partial_2 C$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{T}_B^l]^q\right]^q\right]^{\partial_1}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{T}_C^l]^q\right]^q\right]^{\partial_1}}\right]^q, \\ -\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{T}_B^l]^q\right]^q\right]^{\partial_1}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{T}_C^l]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}}, \\ \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}[\mathcal{T}_B^u]]^q\right]^q\right]^{\partial_1}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}[\mathcal{T}_C^u]]^q\right]^q\right]^{\partial_1}}\right]^q \\ -\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}[\mathcal{T}_B^u]]^q\right]^q\right]^{\partial_1}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}[\mathcal{T}_C^u]]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_B^l]^q\right]^q\right]^{\partial_1}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_C^l]^q\right]^q\right]^{\partial_1}}\right]^q, \\ -\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_B^l]^q\right]^q\right]^{\partial_1}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_C^l]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}}, \\ \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_B^u]^q\right]^q\right]^{\partial_1}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_C^u]^q\right]^q\right]^{\partial_1}}\right]^q \\ -\left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_B^u]^q\right]^q\right]^{\partial_1}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_C^u]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_B^l]^q\right]^q}\right]^{\partial_1} \cdot \left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_C^l]^q\right]^q}\right]^{\partial_1}, \\ \left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}_B^u]]^q\right]^q}\right]^{\partial_1} \cdot \left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}_C^u]]^q\right]^q}\right]^{\partial_1} \end{array}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\sqrt[q]{1-\circledast_{i=1}^{q}\left[1-\left[[log_{Z_i}\mathcal{T}_B^l]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{q}\left[1-\left[[log_{Z_i}[\mathcal{T}_B^u]]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^{q}\left[1-\left[[log_{Z_i}\mathcal{I}_B^l]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{q}\left[1-\left[[log_{Z_i}\mathcal{I}_B^u]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^{q}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^l]^q\right]^q}\right]^{\partial_i}, \circledast_{i=1}^{q}\left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}_{Ai}^u]]^q\right]^q}\right]^{\partial_i}\right] \end{array}\right].$$

It is valid for $n = m$ and $m \geq 3$.
Hence, $\biguplus_{i=1}^{m} \partial_i A_i^{\Xi} =$

$$\left[\begin{array}{c} \left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{T}_B^l]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}[\mathcal{T}_B^u]]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{I}_B^l]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{I}_B^u]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^{m}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^l]^q\right]^q}\right]^{\partial_i}, \circledast_{i=1}^{m}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^l]^q\right]^q}\right]^{\partial_i}\right] \end{array}\right].$$

If $n = m + 1$ and we apply, then $\biguplus_{i=1}^{m} \partial_i A_i^{\Xi} + \partial_{m+1} A_{m+1}^{\Xi} = \biguplus_{i=1}^{m+1} \partial_i A_i^{\Xi}$.

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Now, $\biguplus_{i=1}^{m} \partial_i A_i^{\Xi} + \partial_{m+1} A_{m+1}^{\Xi} = \partial_1 A_1^{\Xi} \bigvee \partial_2 A_2^{\Xi} \bigvee ... \bigvee \partial_m A_m^{\Xi} \bigvee \partial_{m+1} A_{m+1}^{\Xi}$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{T}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{T}_{Am+1}^{l}]^q\right]^q\right]^{\partial_1}}\right]^q, \\ -\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{T}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{T}_{Am+1}^{l}]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}} \\ \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}[\mathcal{T}_{A\,i}^{u}]]^q\right]^q\right]^{\partial_i}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}[\mathcal{T}_{A\,m+1}^{u}]]^q\right]^q\right]^{\partial_1}}\right]^q \\ -\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}[\mathcal{T}_{A\,i}^{u}]]^q\right]^q\right]^{\partial_i}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}[\mathcal{T}_{A\,m+1}^{u}]]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_{Am+1}^{l}]^q\right]^q\right]^{\partial_1}}\right]^q, \\ -\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_{Am+1}^{l}]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}} \\ \sqrt[q]{\begin{array}{c}\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{u}]^q\right]^q\right]^{\partial_i}}\right]^q + \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_{Am+1}^{u}]^q\right]^q\right]^{\partial_1}}\right]^q \\ -\left[\sqrt[q]{1-\circledast_{i=1}^{m}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{u}]^q\right]^q\right]^{\partial_i}}\right]^q \cdot \left[\sqrt[q]{1-\left[1-\left[[log_{Z_i}\mathcal{I}_{Am+1}^{u}]^q\right]^q\right]^{\partial_1}}\right]^q\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \circledast_{i=1}^{m}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^{l}]^q\right]^q}\right]^{\partial_i} \cdot \left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Am+1}^{l}]^q\right]^q}\right]^{\partial_1}, \\ \circledast_{i=1}^{m}\left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}_{Ai}^{u}]]^q\right]^q}\right]^{\partial_i} \cdot \left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}_{Am+1}^{u}]]^q\right]^q}\right]^{\partial_1} \end{array}\right] \end{array}\right].$$

Thus,

$$\biguplus_{i=1}^{m+1} \partial_i A_i^{\Xi} = \left[\begin{array}{c} \left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}\mathcal{T}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}[\mathcal{T}_{Ai}^{u}]]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}, \sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{u}]^q\right]^q\right]^{\partial_i}}\right], \\ \left[\circledast_{i=1}^{m+1}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^{l}]^q\right]^q}\right]^{\partial_i}, \circledast_{i=1}^{m+1}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^{l}]^q\right]^q}\right]^{\partial_i}\right] \end{array}\right].$$

Hence,

$$\left[\biguplus_{i=1}^{m+1} \partial_i A_i^{\Xi}\right]^{1/\Xi} = \left[\begin{array}{c} \left[\left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}\mathcal{T}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}, \left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}[\mathcal{T}_{A\,i}^{u}]]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}\right], \\ \left[\left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{l}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}, \left[\sqrt[q]{1-\circledast_{i=1}^{m+1}\left[1-\left[[log_{Z_i}\mathcal{I}_{Ai}^{u}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}\right], \\ \left[\begin{array}{c} \sqrt[q]{1-\left[1-\left[\circledast_{i=1}^{m+1}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{F}_{Ai}^{l}]^q\right]^q}\right]^{\partial_i}\right]^q\right]^{1/\Xi}}, \\ \sqrt[q]{1-\left[1-\left[\circledast_{i=1}^{m+1}\left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{F}_{Ai}^{u}]]^q\right]^q}\right]^{\partial_i}\right]^q\right]^{1/\Xi}} \end{array}\right] \end{array}\right].$$

It is valid for $m \geq 1$.

**Remark 4.12.** If $\partial_i = 1$, then q-RELOANIVWA operator is modified to the q-RLOANIVWA operator.

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**Theorem 4.13.** *If all* $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}][log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ *are equal and* $A_i = A$. *Then q-RELOANIVWA* $[A_1, A_2, ..., A_n] = A$.

**Remark 4.14.** We use the q-RELOANIVWA operator to satisfy boundedness and monotonicity conditions.

### 4.4 Extended q-RLOANIVWG [q-RELOANIVWG] operator

**Definition 4.15.** Let $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}], [log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ be the collection of q-RLOANIVNs. Then q-RELOANIVWG $[A_1, A_2, ..., A_n] = \frac{1}{\Xi}\left[\circledast^n_{i=1}\left[\Xi A_i\right]^{\partial_i}\right]$ is called the q-RELOANIVWG operator.

**Theorem 4.16.** *Let* $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}], [log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ *be the collection of q-RLOANIVNs. Then q-RELOANIVWG*$[A_1, A_2, ..., A_n]$

$$= \left[\begin{array}{c} \left[\begin{array}{c} \sqrt[q]{1-\left[1-\left[\circledast^n_{i=1}\left[\sqrt[q]{1-\left[1-[log_{Z_i}\mathcal{T}^l_{Ai}]^q\right]^q}\right]^{\partial_i}\right]\right]^q}^{\,1/\Xi}, \\ \sqrt[q]{1-\left[1-\left[\circledast^n_{i=1}\left[\sqrt[q]{1-\left[1-[log_{Z_i}[\mathcal{T}^u_{A\,i}]]^q\right]^q}\right]^{\partial_i}\right]\right]^q}^{\,1/\Xi} \end{array}\right], \\ \left[\left[\sqrt[q]{1-\circledast^n_{i=1}\left[1-\left[[log_{Z_i}\mathcal{I}^l_{Ai}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}, \left[\sqrt[q]{1-\circledast^n_{i=1}\left[1-\left[[log_{Z_i}\mathcal{I}^u_{Ai}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}\right], \\ \left[\left[\sqrt[q]{1-\circledast^n_{i=1}\left[1-\left[[log_{Z_i}\mathcal{F}^l_{Ai}]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}, \left[\sqrt[q]{1-\circledast^n_{i=1}\left[1-\left[[log_{Z_i}[\mathcal{F}^u_{Ai}]]^q\right]^q\right]^{\partial_i}}\right]^{1/\Xi}\right] \end{array}\right].$$

**Remark 4.17.** If $\partial_i = 1$, then q-RELOANIVWG operator is converted to the q-RLOANIVWG operator.

**Remark 4.18.** q-RELOANIVWG operators satisfy boundedness and monotonicity properties.

**Corollary 4.19.** *If all* $A_i = \left\langle [log\mathcal{T}^l_{Ai}, log\mathcal{T}^u_{A\,i}], [log\mathcal{I}^l_{Ai}, log\mathcal{I}^u_{Ai}][log\mathcal{F}^l_{Ai},\ log\mathcal{F}^u_{Ai}] \right\rangle$ *are equal and* $A_i = A$, *for* $i = 1, 2, ..., n$. *Then q-RELOANIVWG*$[A_1, A_2, ..., A_n] = A$.

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