



On ψ_{NC} – Operator in Neutrosophic Crisp Topological Spaces

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Abstract

The research started from Salama's generalization to both ideal and local function through NCSs. We presented some results and properties to reinforce the concept of the generalized local function, which though its properties was used to deduce the properties of the ψ_{NC} - operator that we generalized through NCSS.

Keyword: NCSs; NCTS; NCI; NCLF; Neutrosophic crisp ψ_{NC} - operator

1. Introduction

In 2013 [1], Salama was introduced to the idea of ideal through neutrosophic crisp sets. But the mathematical basis for both the neutrosophic and neutrosophic crisp sets (NCS) was first laid down by Smarandache [2,3]. For any non-empty set $\mathcal{Y}$.

The NCS $\mathcal{A}_N = \langle \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3 \rangle$ with $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$, $\mathcal{A}_1 \cap \mathcal{A}_3 = \emptyset$, and $\mathcal{A}_2 \cap \mathcal{A}_3 = \emptyset$. And the NC empty set $\emptyset_N = \langle \emptyset, \emptyset, \mathcal{Y} \rangle$ and $\mathcal{Y}_N = \langle \mathcal{Y}, \mathcal{Y}, \emptyset \rangle$ and $\mathcal{A}_N \cap \mathcal{B}_N = \langle \mathcal{A}_1 \cap \mathcal{B}_1, \mathcal{A}_2 \cap \mathcal{B}_2, \mathcal{A}_3 \cup \mathcal{B}_3 \rangle$, $\mathcal{A}_N \cup \mathcal{B}_N = \langle \mathcal{A}_1 \cup \mathcal{B}_1, \mathcal{A}_2 \cup \mathcal{B}_2, \mathcal{A}_3 \cap \mathcal{B}_3 \rangle$, $\mathcal{A}_N \subseteq \mathcal{B}_N$ iff $\mathcal{B}_3 \subseteq \mathcal{A}_3$, $\mathcal{A}_i \subseteq \mathcal{B}_i$, $i = 1, 2$ and the complement $\mathcal{B}_N^c = \langle \mathcal{Y} \setminus \mathcal{B}_1, \mathcal{Y} \setminus \mathcal{B}_2, \mathcal{Y} \setminus \mathcal{B}_3 \rangle$. We take the neutrosophic crisp point (NCP)

$$P_{N_1} = \langle \{p\}, \emptyset, \{p\}^c \rangle, P_{N_1} \in \mathcal{A}_N \quad \text{iff} \quad P \in \mathcal{A}_1,$$

$$P_{N_2} = \langle \{p\}, \emptyset, \emptyset^c \rangle, P_{N_2} \in \mathcal{A}_N \quad \text{iff} \quad P \in \mathcal{A}_1,$$

$$P_{N_3} = \langle \emptyset, \{p\}, \{p\}^c \rangle, P_{N_3} \in \mathcal{A}_N \quad \text{iff} \quad P \in \mathcal{A}_2,$$

$P_{N_4} = \langle \emptyset, \{p\}, \emptyset \rangle, P_{N_4} \in \mathcal{A}_N$ iff $P \in \mathcal{A}_3$. The symbol $P \in \mathcal{A}$ means the classic affiliation. The idea of ideal $\mathbb{I}$ is defined by Kuratowski [4,5], which represents a non - empty family of subsets and is closed w.r.t. finite unions and hereditary. Also Kuratowski defined the local function as $\mathcal{A}^* = \{P \in \mathcal{Y}; \forall \mathcal{U} \in \mathcal{T}, P \in \mathcal{U} \text{ and } \mathcal{U} \cap \mathcal{A} \in \mathbb{I}\}$. Al-Obaidi et al. [18,19] gave the view of new types of weakly neutrosophic crisp open mappings and new types of weakly neutrosophic crisp closed functions. Finally, the senses of some types of neutrosophic topological groups with respect to neutrosophic alpha open sets, new types of weakly neutrosophic crisp continuity, new concepts of neutrosophic crisp open sets, new concepts of weakly neutrosophic crisp separation axioms, and neutrosophic crisp generalized sg -closed sets and their continuity were informed by Imran et al. [20-24]. Abdulkadhim et al. [25] presented the view of neutrosophic crisp generalized alpha generalized closed sets.

2. Preliminaries

2.1 Definition [1]: Let $\mathbb{I}_{NC}$ a non-empty family of NCSs on a non-empty set $\mathcal{Y}$ is called a Neutrosophic crisp ideal (NCI), if it is closed w.r.t. finite unions and hereditary.

- The families $\{\emptyset_N\}$ and NCSs are trivial NCIs.

- For any NCP P , $\mathbb{I}_{NCP} = \{\mathcal{A}_N; P \notin \mathcal{A}_N\}$ is the principle NCI. Through own study, when we say "the NCI" we mean the proper NCI (i.e. $\mathcal{Y}_N \notin \mathbb{I}_{NC}$).

2.2 Definition [9]: A collection $\mathcal{T}_{NC}$ of NCSs in $\mathcal{Y}$, is called NCTS if $\mathcal{T}_{NC}$ containing $\emptyset_N, \mathcal{Y}_N$ and closed w.r.t. finite intersection, Also closed w.r.t. unions. If $\mathcal{A}_N \in \mathcal{T}_{NC}$, then $\mathcal{A}_N$ is called NCOS and $\mathcal{A}_N^c$ is called NCCS. Let $(\mathcal{Y}_N, \mathcal{T}_{NC}, \mathbb{I}_{NC})$ is called Neutrosophic crisp topological ideal space (NCTIS). In a simplified way, we symbolize it $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$.

2.3 Definition [9]: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTIS. Let $\mathcal{A}_N$ be any NCS. Then the Neutrosophic crisp local function (NCLF) $NC \mathcal{A}^*(\mathcal{T}_{NC}, \mathbb{I}_{NC}) = NC \mathcal{A}^* = \cup \{P \in \mathcal{Y}_{NC}; \forall \mathcal{U}_N \in \mathcal{T}_{NC} \exists P \in \mathcal{U}_N \text{ and } \mathcal{U}_N \cap \mathcal{A}_N \notin \mathbb{I}_{NC}\}$, for any type on NCPs P . If $\mathbb{I}_{NC} = \{\emptyset_N\}$, then $NC \mathcal{A}^* = NCCI(\mathcal{A})$ and for $\mathbb{I}_{NC} = \text{NCSs}$, then $NC \mathcal{A}^* = \emptyset_N$, for any NCS $\mathcal{A}_N$.

2.4 Theorem [9]: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTIS and for any NCSs $\mathcal{A}_N, \mathcal{B}_N$, the following properties are verified;

- 1- If $\mathcal{A}_N \subseteq \mathcal{B}_N$, then $NC \mathcal{A}_N^* \subseteq NC \mathcal{B}_N^*$.
- 2- $NC \mathcal{A}_N^* = NCCI(\mathcal{A}_N^*) \subseteq NCCI(\mathcal{A}_N)$, where $NCCI(\mathcal{A}_N)$ is Neutrosophic crisp closure of $\mathcal{A}_N$
- 3- $NC \mathcal{A}_N^{**} \subseteq NC \mathcal{A}_N^*$.
- 4- $NC(\mathcal{A}_N \cup \mathcal{B}_N)^* = NC \mathcal{A}_N^* \cup NC \mathcal{B}_N^*$.
- 5- If $\mathcal{I}_N \in \mathbb{I}_{NC}$, then $NC \mathcal{I}_N^* = \emptyset_N$.
- 6- If $\mathcal{I}_N \in \mathbb{I}_{NC}$, then $NC(\mathcal{A}_N \cup \mathcal{I}_N)^* = NC(\mathcal{A}_N \cap \mathcal{I}_N^c)^* = NC \mathcal{A}_N^*$.

2.5 Remark : Through (Definition 2.3) and (Theorem 2.4), we can conclude:

- i. $NC \emptyset_N^* = \emptyset_N$,
- ii. For any NCS $\mathcal{A}_N$, then
- iii. $\mathcal{A}_N \cap (NC \mathcal{A}^*)^c \cap NC(\mathcal{A}_N \cap (NC \mathcal{A}^*)^c)^* = \emptyset_N$
- iv. $NC \mathcal{A}_N^* \cap (NC \mathcal{B}_N^c)^c = [NC(\mathcal{A}_N \cap \mathcal{B}_N^*)^* \cap (NC \mathcal{B}_N^c)^c] \subseteq NC(\mathcal{A}_N \cap \mathcal{B}_N^c)^*$
- v. For any NCOS $\mathcal{U}_N, \mathcal{U}_N \cap NC \mathcal{A}_N^* \subseteq NC(\mathcal{U}_N \cap \mathcal{A}_N)^*$.

2.6 Theorem : For any NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$, the following statements are equivalent;

- 1- $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \{\emptyset_N\}$.
- 2- If $\mathcal{I}_N \in \mathbb{I}_{NC}$, then $NC \text{int}(\mathcal{I}_N) = \emptyset_N$, where $NC \text{int}(\mathcal{I}_N)$ is a Neutrosophic crisp interior of $\mathcal{I}_N$
- 3- For any NCOS G_N , then $G_N \subseteq NC G_N^*$.
- 4- $\mathcal{Y}_{NC} \subseteq NC \mathcal{Y}_{NC}^*$.

Proof 1 $\Rightarrow$ 2. Let $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \{\emptyset_N\}$ and $\mathcal{I}_N \in \mathbb{I}_{NC}$. If possible, $p \in NC \text{int}(\mathcal{I}_N) \Rightarrow \exists \mathcal{U}_N \in \mathcal{T}_{NC}$, $p \in \mathcal{U}_N \subseteq \mathcal{I}_N \Rightarrow$ by (Definition 2.1) $\mathcal{U}_N \in \mathbb{I}_{NC}$, which contradiction part(1).

2 $\Rightarrow$ 3. Let G_N be a NCOS and $p \in G_N$ if $p \notin NC G_N^* \Rightarrow \exists \mathcal{U}_N \in \mathcal{T}_{NC}$, $p \in \mathcal{U}_N$ and $\mathcal{U}_N \cap G_N \notin \mathbb{I}_{NC}$ but $p \in G_N \cap \mathcal{U}_N = NC \text{int}(G_N \cap \mathcal{U}_N)$, which contradiction part(2).

3 $\Rightarrow$ 4. Since $\mathcal{Y}_{NC}$ is NCOS, so by part (3) $\mathcal{Y}_{NC} = NC \mathcal{Y}_{NC}^*$.

4 $\Rightarrow$ 1. If possible $\exists \emptyset_N \neq G_N \in \mathcal{T}_{NC} \cap \mathbb{I}_{NC} \Rightarrow \exists p \in G_N$, but $G_N \cap \mathcal{Y}_{NC} = G_N \in \mathbb{I}_{NC}$. Then $p \notin NC \mathcal{Y}_{NC}^* = \mathcal{Y}_{NC}$, which contradiction.

- We defined the NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ is NC- codense iff $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \emptyset_N$.

2.7 Definition [9]: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTIS. For any NCS $\mathcal{A}_N$, the Neutrosophic crisp closure operator w.r.t. $\mathcal{T}_{NC}$ and $\mathbb{I}_{NC}$ of $\mathcal{A}_N$ by $NCCI^*(\mathcal{A}_N) = \mathcal{A}_N \cup NC \mathcal{A}_N^*$. And the NCT $\mathcal{T}_{NC}^*$ generated by $NCCI^*$, i.e. $\mathcal{T}_{NC}^* = \{\mathcal{A}_N \subseteq \mathcal{Y}_{NC}; NCCI(\mathcal{A}_N^c)^* = \mathcal{A}_N^c\}$ is finer than $\mathcal{T}_{NC}$ and for $\mathbb{I}_{NC} = \{\emptyset_N\}$, then $\mathcal{T}_{NC} = \mathcal{T}_{NC}^*$.

3.1 Definition : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTIS. A operator $\psi_{NC}: \text{NCSs} \rightarrow \mathcal{T}_{NC}$ for any NCS $\mathcal{A}_N$ is defined by $\psi_{NC}(\mathcal{A}_N) = ((\mathcal{A}_N^c)^*)^c$.

3.2 Example : Let $\mathcal{Y} = \{a, b, c\}$, $\mathcal{T}_{NC} = \{\emptyset_{NC}, \mathcal{Y}_{NC}, \langle \emptyset, \{a\}, \{b, c\} \rangle, \langle \{a\}, \{b, c\}, \emptyset \rangle\}$, and $\mathbb{I}_{NC} = \{\emptyset_{NC}, \langle \emptyset, \{a\}, \{b\} \rangle, \langle \emptyset, \{a\}, \{c, b\} \rangle, \langle \emptyset, \{c\}, \{b\} \rangle, \langle \emptyset, \{c\}, \{a, b\} \rangle, \langle \emptyset, \{a, c\}, \{b\} \rangle\}$. Then $((\{c\}, \emptyset, \{a, b\})^*)^c = \langle \mathcal{Y}, \{b, c\}, \emptyset \rangle$ and $\psi_{NC}(\langle \{c\}, \emptyset, \{a, b\} \rangle) = \langle \emptyset, \{a\}, \mathcal{Y} \rangle$.

3.3 Remark :

- From (Definition 3.1), for any $p \in \psi_{NC}(\mathcal{A}_N)$, $\exists G_N \in \mathcal{T}_{NC}$ s.t $G_N \cap \mathcal{A}_N^c \in \mathbb{I}_N$. Also we see that $(\mathcal{A}_N^c)^* = [\psi_{NC}(\mathcal{A}_N)]^c$ and $\psi_{NC}(\mathcal{A}_N^c) = (\mathcal{A}_N^*)^c$.
- It clear that $\psi_{NC}(\mathcal{A}_N)$ is NCOS because $(\mathcal{A}_N)^*$ is NCCS.
- $\psi_{NC}(\mathcal{A}_N \cup \mathcal{B}_N) \neq \psi_{NC}(\mathcal{A}_N) \cup \psi_{NC}(\mathcal{B}_N)$, from (Example 3.2) if $\mathcal{A}_N = \langle \{c\}, \{a, b\}, \emptyset \rangle$, $\mathcal{B}_N = \langle \{c\}, \emptyset, \{a, b\} \rangle$, $\psi_{NC}(\mathcal{A}_N \cup \mathcal{B}_N) = \emptyset_{NC}$ but $\psi_{NC}(\mathcal{A}_N) \cup \psi_{NC}(\mathcal{B}_N) = \langle \emptyset, \{a\}, \mathcal{Y} \rangle$.
- For any $\mathcal{A}_N^c \in \mathbb{I}_{NC}$. Then $\psi_{NC}(\mathcal{A}_N) = \mathcal{Y}_{NC} = \psi_{NC}(\mathcal{Y}_{NC})$.
- The ψ_{NC} - operator preserves the odor relative to $\subseteq$, in another meaning if $\mathcal{A}_N \subseteq \mathcal{B}_N$, then $\psi_{NC}(\mathcal{A}_N) \subseteq \psi_{NC}(\mathcal{B}_N)$.
- In the case of $\mathcal{A}_N \sum \mathbb{I}_{NC}$, calculating ψ_{NC} - operator to both $\mathcal{B}_N \cap (\mathcal{A}_N)^c$ and $(\mathcal{B}_N \cup \mathcal{A}_N)$, $\mathcal{A}_N$ has no effect on it, i.e. it equals to $\psi_{NC}(\mathcal{B}_N)$.

There are many properties of the ψ_{NC} - operator that have important and fundamental implications for many topological concepts, which are a common conclusion between (Definition 3.1) and the properties of the local function in defined on NCSs, as shown in the following theorem.

3.4 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTIS, then for any NCSs $\mathcal{A}_N, \mathcal{B}_N$, the following statement are hole:

- 1- $\psi_{NC}(\mathcal{A}_N \cap \mathcal{B}_N) = \psi_{NC}(\mathcal{A}_N) \cap \psi_{NC}(\mathcal{B}_N)$.
- 2- $\psi_{NC}(\mathcal{A}_N) \subseteq \psi_{NC}(\psi_{NC}(\mathcal{A}_N))$.
- 3- $\psi_{NC}(\mathcal{A}_N) = \psi_{NC}(\psi_{NC}(\mathcal{A}_N))$ iff $(\mathcal{A}_N^c)^* = [(\mathcal{A}_N^c)^*]^*$
- 4- If $[(\mathcal{A}_N \cap (\mathcal{B}_N)^c) \cup (\mathcal{B}_N \cap (\mathcal{A}_N)^c)] \in \mathbb{I}_{NC}$, then $\psi_{NC}(\mathcal{A}_N) = \psi_{NC}(\mathcal{B}_N)$.
- 5- $\mathcal{A}_N \cap \psi_{NC}(\mathcal{A}_N) = NCInt^*(\mathcal{A}_N)$.
- 6- If $G_N \in \mathcal{T}_{NC}$, then $G_N \subseteq \psi_{NC}(G_N)$.
- 7- $\psi_{NC}(\mathcal{A}_N) = \cup \{G_N \in \mathcal{T}_{NC}; G_N \cap \mathcal{A}_N^c \sum \mathbb{I}_{NC}\}$.

Proof (4). Put $\mathcal{A}_N \cap \mathcal{B}_N^c = \mathcal{C}_N$, $\mathcal{B}_N \cap \mathcal{A}_N^c = \mathcal{D}_N$, since $[\mathcal{C}_N \cup \mathcal{D}_N] \in \mathbb{I}_{NC} \Rightarrow \mathcal{C}_N, \mathcal{D}_N \in \mathbb{I}_{NC}$, but $\mathcal{B}_N = (\mathcal{A}_N \cap \mathcal{B}_N) \cup (\mathcal{A}_N \cap \mathcal{B}_N^c) = (\mathcal{A}_N \cap \mathcal{C}_N) \cup \mathcal{D}_N$ and by (Remark 3.3) $\psi_{NC}(\mathcal{B}_N) = \psi_{NC}[(\mathcal{A}_N \cap \mathcal{C}_N) \cup \mathcal{D}_N] = \psi_{NC}(\mathcal{A}_N)$.

Proof (5). Since $NCint^*(\mathcal{A}_N) = [NCCI^*(\mathcal{A}_N^c)]^c = [\mathcal{A}_N^c \cup \mathcal{A}_N^{c*}]^c = \mathcal{A}_N \cap \mathcal{A}_N^{c*c} = \mathcal{A}_N \cap \psi_{NC}(\mathcal{A}_N)$.

Proof (6). Let $G_N \in \mathcal{T}_{NC}$ and $\psi_{NC}(G_N) = [(G_N^c)^*]^c$ and $(G_N^c)^* \subseteq NCCI(G_N^c) = G_N^c$. Then $G_N \subseteq [(G_N^c)^*]^c = \psi_{NC}(G_N)$.

- It is noted that $\mathcal{T}_{NC}^* = \{G_N \subseteq \mathcal{Y}_{NC}; G_N \subseteq \psi_{NC}(G_N)\}$ is a NCTS finer than $\mathcal{T}_{NC}$.
- One of the properties that we deduce using (Theorem 3.4) is:
 - a) If $\mathcal{A}_N \cap \mathcal{B}_N = \emptyset_{NC}$, then $\psi_{NC}(\mathcal{A}_N) \cap \mathcal{B}_N^* = \emptyset_{NC}$ and $\mathcal{A}_N^* \cap \psi_{NC}(\mathcal{B}_N) = \emptyset_{NC}$.
 - b) If $\mathcal{A}_N \cap \mathcal{B}_N = \emptyset_{NC}$ and $\mathcal{A}_N, \mathcal{B}_N$ are NCOSs, then $\mathcal{A}_N^* \cap \mathcal{B}_N = \emptyset_N$ and $\mathcal{A}_N \cap \mathcal{B}_N^* = \emptyset_N$.

3.5 Proposition : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTS and $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} = \emptyset_{NC}$, then $\psi_{NC}(\mathcal{A}_N) \subseteq \mathcal{A}_N^*$.

Proof : Let NCP $p \in \psi_{NC}(\mathcal{A}_N) \Rightarrow \exists \mathcal{U}_N \in \mathcal{T}_{NC}$ s.t $\mathcal{U}_N \cap \mathcal{A}_N^c \in \mathbb{I}_{NC} \dots \dots 3-1$.

If possible $p \notin \mathcal{A}_N^c \Rightarrow \exists G_N \in \mathcal{T}_{NC}$ s.t $G_N \cap \mathcal{A}_N \in \mathbb{I}_{NC} \dots \dots 3-2$.

From 3.1 and 3.2, we get $G_N \cap \mathcal{U}_N = [(G_N \cap \mathcal{U}_N) \cap \mathcal{A}_N] \cup [(G_N \cap \mathcal{U}_N) \cap \mathcal{A}_N^c] \in \mathbb{I}_{NC}$. Then $\mathcal{T}_{NC} \cap \mathbb{I}_{NC} \neq \emptyset_{NC}$, which contradiction the assumption.

3.6 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{T_{NC}}$ be a NCTS. Then following are equivalent: اكتب المعادلة هنا.

- 1- NC- codense.
- 2- If $\mathcal{A}_N$ is NCCI, then $\psi_{NC}(\mathcal{A}_N) \subseteq \mathcal{A}_N$.
- 3- $NCint(NCCI(\mathcal{A}_N)) = \psi_{NC}(NCint(NCCI(\mathcal{A}_N)))$.
- 4- For $\mathcal{U}_N \in \mathcal{T}_{NC}$, then $\psi_{NC}(\mathcal{U}_N) \subseteq \mathcal{U}_N^*$.
- 5- If $H_N \in \mathbb{I}_{NC}$, then $\psi_{NC}(H_N) = \emptyset_{NC}$.

Proof 1 $\Rightarrow$ 2. By (Proposition 3.5) and (Theorem 2.4) $\psi_{NC}(\mathcal{A}_N) \subseteq NC\mathcal{A}_N^* \subseteq NCCI(\mathcal{A}_N) = \mathcal{A}_N$.

2 $\Rightarrow$ 3. Since $\text{NCint}(\text{NCCI}(\mathcal{A}_N)) \varepsilon \mathcal{T}_{NC} \Rightarrow$ by (Theorem 2.4) $\text{NCint}(\text{NCCI}(\mathcal{A}_N)) \subseteq \psi_{NC}(\text{NCint}(\text{NCCI}(\mathcal{A}_N)))$, but $\text{NCCI}(\mathcal{A}_N)$ is NCCS and by part (2) $\psi_{NC}(\text{NCint}(\text{NCCI}(\mathcal{A}_N))) \subseteq \psi_{NC}(\text{NCCI}(\mathcal{A}_N)) \subseteq \text{NCint}(\text{NCCI}(\mathcal{A}_N))$.

3 $\Rightarrow$ 4. By (Theorem 2.6). For $\mathcal{U}_N \varepsilon \mathcal{T}_{NC} \Rightarrow \mathcal{U}_N \subseteq \text{NC}\mathcal{U}_N^* \Rightarrow \text{NCint}(\text{NCCI}(\mathcal{U}_N)) \subseteq \text{NCCI}(\mathcal{U}_N) \subseteq \text{NCCI}(\text{NC}\mathcal{U}_N^*) = \text{NC}\mathcal{U}_N^*$ and $\psi_{NC}(\text{NCint}(\text{NCCI}(\mathcal{U}_N))) = \text{NCint}(\text{NCCI}(\mathcal{U}_N)) \subseteq \text{NC}\mathcal{U}_N^*$ but $\psi_{NC}(\mathcal{U}_N) \subseteq \psi_{NC}(\text{NCint}(\text{NCCI}(\mathcal{U}_N))) \subseteq \text{NC}\mathcal{U}_N^*$. Then $\psi_{NC}(\mathcal{U}_N) \subseteq \text{NC}\mathcal{U}_N^*$.

4 $\Rightarrow$ 5. By (Theorem 2.6). For $H_N \varepsilon \mathbb{I}_{NC}$, then $\psi_{NC}(H_N) = \mathcal{Y}_N \cap (\text{NC}\mathcal{Y}_N^*)^c = \mathcal{Y}_N \cap \mathcal{Y}_N^c = \emptyset_{NC}$.

5 $\Rightarrow$ 1. If possible $\exists \emptyset_N \neq G_N \varepsilon \mathcal{T}_{NC}$, $G_N \varepsilon \mathbb{I}_{NC}$. Then by part (5), $\psi_{NC}(G_N) = \emptyset_N$, and by (Theorem 1.4(4)) $G_N \subseteq \psi_{NC}(G_N) = \emptyset_N$, which contradiction.

3.7 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ be a NCTIS, if $\mathbb{I}_{NC}$ is NC-codense, then

- 1- $\psi_{NC}[\text{NC}[\psi_{NC}(\text{NC}\mathcal{A}_N^*)]^*] = \psi_{NC}[\text{NC}\mathcal{A}_N^*]$.
- 2- $\text{NC}[\psi_{NC}(\mathcal{A}_N)]^* = \text{NC}[\psi_{NC}[\text{NC}[\psi_{NC}(\mathcal{A}_N)]^*]]^*$.

Proof 1. Put $W_N = \psi_{NC}(\text{NC}\mathcal{A}_N^*)$ is NCOS, and by (Theorem 3.6) $W_N = \psi_{NC}(\text{NC}\mathcal{A}_N^*) \subseteq \text{NC}(\text{NC}\mathcal{A}_N^*)^* \subseteq \text{NC}\mathcal{A}_N^* \Rightarrow \psi_{NC}(\text{NC}W_N^*) \subseteq \psi_{NC}(\text{NC}\mathcal{A}_N^*) = W_N$, since $W_N \varepsilon \mathcal{T}_{NC}$, and by using (Theorem 2.4, 3.5), $W_N \subseteq \psi_{NC}(W_N) \subseteq \text{NC}W_N^* \Rightarrow W_N \subseteq \psi_{NC}(W_N) \subseteq \psi_{NC}(\psi_{NC}(W_N)) \subseteq \psi_{NC}(\text{NC}W_N^*)$. Then $W_N = \psi_{NC}(\text{NC}W_N^*)$, here the proof ends.

3.8 Theorem : Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ be a NCTIS, with $\mathbb{I}_{NC}$ is NC-codense. If $\mathcal{A}_N$ is a NCOS on $\mathcal{B}_N$ is a NCOS, then $\psi_{NC}(\text{NC}\mathcal{A}_N^*) \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*) = \psi_{NC}[\text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^*]$.

Proof. Since $\psi_{NC}[\text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^*] \subseteq \psi_{NC}(\text{NC}\mathcal{A}_N^*) \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*)$, from the fact $\mathcal{A}_N \cap \mathcal{B}_N \subseteq \mathcal{A}_N$ and $\mathcal{B}_N$ and (Theorem 2.4, 3.4) conversely, if $\mathcal{A}_N$ is a NCOS $\Rightarrow$ by (Theorem 3.4(4)) $\mathcal{A}_N \cap \text{NC}\mathcal{B}_N^* \subseteq \text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^* \Rightarrow \psi_{NC}(\mathcal{A}_N) \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*) \subseteq \psi_{NC}[\text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^*] \Rightarrow \psi_{NC}[\text{NC}[\mathcal{A}_N \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*)]^*] \subseteq \psi_{NC}[\text{NC}[\psi_{NC}[\text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^*]]^*] = \psi_{NC}[\text{NC}[\text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^*]]$ by (Theorem 3.7)(... 3-3). But $\text{NC}\mathcal{A}_N^* \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*) \subseteq \text{NC}[\mathcal{A}_N \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*)]^* \Rightarrow \psi_{NC}(\text{NC}\mathcal{A}_N^*) \cap \psi_{NC}[\psi_{NC}(\text{NC}\mathcal{B}_N^*)] \subseteq \psi_{NC}[\text{NC}[\mathcal{A}_N \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*)]^*]$ and $\psi_{NC}(\text{NC}\mathcal{B}_N^*) \subseteq \psi_{NC}[\psi_{NC}(\text{NC}\mathcal{B}_N^*)] \Rightarrow \psi_{NC}(\text{NC}\mathcal{A}_N^*) \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*) \subseteq \psi_{NC}[\text{NC}[\mathcal{A}_N \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*)]^*]$ (...3-4) from 3-3 and 3-4, we get that $\psi_{NC}(\text{NC}\mathcal{A}_N^*) \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*) \subseteq \psi_{NC}[\text{NC}(\mathcal{A}_N \cap \mathcal{B}_N)^*]$.

3.9 Corollary: Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ be a NCTIS, with $\mathbb{I}_{NC}$ is a NC-codense, the following are hold for any two NCSs $\mathcal{A}_N$ and $\mathcal{B}_N$.

- 1- $\psi_{NC}[\text{NC}[\psi_{NC}(\mathcal{A}_N) \cap \mathcal{B}_N]^*] = \psi_{NC}[\text{NC}[\psi_{NC}(\mathcal{A}_N)]^* \cap \psi_{NC}(\text{NC}\mathcal{B}_N^*)]$.
- 2- $\psi_{NC}[\text{NC}[\psi_{NC}(\mathcal{A}_N \cap \mathcal{B}_N)]^*] = \psi_{NC}[\text{NC}[\psi_{NC}(\mathcal{A}_N)]^*] \cap \psi_{NC}[\text{NC}[\psi_{NC}(\mathcal{B}_N)]^*]$.
- 3- $\text{NC}[\psi_{NC}(\text{NC}\mathcal{A}_N^*)]^* \cup \text{NC}[\psi_{NC}(\text{NC}\mathcal{B}_N^*)]^* = \text{NC}[\psi_{NC}[\text{NC}(\mathcal{A}_N \cup \mathcal{B}_N)^*]]^*$.

Proof (3). According to (Remark 3.2), (Theorem 3.4) and (Theorem 3.8) we will have the following:
First, $[\text{NC}[\psi_{NC}(\text{NC}\mathcal{A}_N^*)]^*]^c = \psi_{NC}[(\psi_{NC}(\text{NC}\mathcal{A}_N^*))^c] = \psi_{NC}[\text{NC}[(\text{NC}\mathcal{A}_N^*)^c]] = \psi_{NC}[\text{NC}(\psi_{NC}(\mathcal{A}_N^c))]^*$. This is true for $\mathcal{B}_N$ as well, and from then we conclude the following. $[\text{NC}(\psi_{NC}(\text{NC}\mathcal{A}_N^*))^*]^c \cap [\text{NC}(\psi_{NC}(\text{NC}\mathcal{B}_N^*))^*]^c = \psi_{NC}[\text{NC}(\psi_{NC}(\mathcal{A}_N^c))]^* \cap \psi_{NC}[\text{NC}(\psi_{NC}(\mathcal{B}_N^c))]^* = \psi_{NC}[\text{NC}(\psi_{NC}(\mathcal{A}_N^c \cap \mathcal{B}_N^c))]^* = \psi_{NC}[\text{NC}((\text{NC}(\mathcal{A}_N \cup \mathcal{B}_N)^c)^*)]^c = \psi_{NC}[(\psi_{NC}(\text{NC}(\mathcal{A}_N \cup \mathcal{B}_N)^*))^c]^c = (\text{NC}(\psi_{NC}(\text{NC}(\mathcal{A}_N \cup \mathcal{B}_N)^*))^*)^c$.

3.10 Definition : NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is called $\mathbb{I}_{NC}$ -resolvable iff $\exists$ two $\mathbb{I}_{NC}$ -dense NCSs $\mathcal{A}_N, \mathcal{B}_N$ s.t $\mathcal{A}_N \cap \mathcal{B}_N = \emptyset_{NC}$ and $\mathcal{A}_N \cup \mathcal{B}_N = \mathcal{Y}_{NC}$. Otherwise is called $\mathbb{I}_{NC}$ -irresolvable.

– NCS $\mathcal{A}_N$ is called $\mathbb{I}_{NC}$ -dense iff $\text{NC}\mathcal{A}_N^* = \mathcal{Y}_N$.

3.11 Example : Let $\mathcal{Y} = \{a, b, c\}$, $\mathcal{T}_{NC} = \{\emptyset_N, \mathcal{Y}_N, \langle \{a\}, \{b, c\} \rangle, \langle \{a\}, \{b, c\}, \emptyset \rangle\}$, and $\mathbb{I}_{NC} = \{\emptyset_N, \langle \{a\}, \emptyset, \emptyset \rangle\}$. Then $\langle \{c\}, \emptyset, \{a, b\} \rangle$ is $\mathbb{I}_{NC}$ -dense.

3.12 Remark :

1. If $\mathbb{I}_{NC} = \{\emptyset_N\}$, then $\mathcal{A}_N$ is $\mathbb{I}_{NC}$ -dense iff its Neutrosophic crisp dense ($NCCI(\mathcal{A}_N) = \mathcal{Y}_N$).
2. If $\mathcal{A}_N$ is $\mathbb{I}_{NC}$ -dense iff $\psi_{NC}(\mathcal{A}_N^c) = \emptyset_N$.
3. If $\mathcal{A}_N$ is $\mathbb{I}_{NC}$ -dense iff for each non- empty NCOS $\mathcal{U}_N \ni \mathcal{U}_N \cap \mathcal{A}_N \bar{\mathbb{I}}_{NC}$.
4. If $\mathcal{A}_N$ is $\mathbb{I}_{NC}$ -dense, then for any NCOS G_N , $G_N = G_N \cap NC(\mathcal{A}_N \cap G_N)^*$.
5. NCTIS $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ be a $\mathbb{I}_{NC}$ -irresolvable iff there is no $\mathbb{I}_{NC}$ -dense, which is complement is $\mathbb{I}_{NC}$ -dense.

3.13 Proposition : The following properties are correct:

1. If $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is $\mathbb{I}_{NC}$ -resolvable, then $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is NC-codense.
2. $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is $\mathbb{I}_{NC}$ -resolvable iff $\exists \mathbb{I}_{NC}$ -dense $\mathcal{A}_N \ni \psi_{NC}(\mathcal{A}_N) = \emptyset_N$.
3. $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is $\mathbb{I}_{NC}$ -resolvable iff $\exists$ NCS $\mathcal{A}_N \ni \psi_{NC}(\mathcal{A}_N^c) = \psi_{NC}(\mathcal{A}_N) = \emptyset_N$.

Proof(1). Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is $\mathbb{I}_{NC}$ -resolvable $\Rightarrow \exists$ two $\mathbb{I}_{NC}$ -dense, $\mathcal{A}_N$ and $\mathcal{B}_N \ni \mathcal{Y}_N = \mathcal{A}_N \cup \mathcal{B}_N, \mathcal{A}_N \cap \mathcal{B}_N = \emptyset_N \Rightarrow NC \mathcal{Y}_N^* = NC \mathcal{A}_N^* \cup NC \mathcal{B}_N^* = \mathcal{Y}_N \cup \mathcal{Y}_N = \mathcal{Y}_N \Rightarrow \mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is NC-codense.

Proof(3). Let $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is $\mathbb{I}_{NC}$ -resolvable $\Rightarrow \exists$ two $\mathbb{I}_{NC}$ -dense, $\mathcal{A}_N, \mathcal{B}_N \ni \mathcal{Y}_N = \mathcal{A}_N \cup \mathcal{B}_N, \mathcal{A}_N \cap \mathcal{B}_N = \emptyset_N \Rightarrow \mathcal{A}_N^c = \mathcal{B}_N, \mathcal{B}_N = \mathcal{A}_N^c$ and $NC \mathcal{A}_N^* = NC \mathcal{B}_N^* = \mathcal{Y}_N \Rightarrow (NC \mathcal{A}_N^*)^c = \emptyset_N \Rightarrow \psi_{NC}(\mathcal{A}_N^c) = [NC[(\mathcal{A}_N^c)^c]]^c = [NC \mathcal{A}_N]^c = \emptyset_N, \psi_{NC}(\mathcal{A}_N) = [NC(\mathcal{A}_N^c)^*]^c = [NC \mathcal{B}_N]^c = \emptyset_N$. Conversely. Let $\mathcal{A}_N$ be NCOS $\ni \psi_{NC}(\mathcal{A}_N) = \psi_{NC}(\mathcal{A}_N^c) = \emptyset_N \Rightarrow [NC(\mathcal{A}_N^c)^*]^c = \emptyset_N$ iff $NC(\mathcal{A}_N^c)^* = \mathcal{Y}_N$ and $[NC[(\mathcal{A}_N^c)^c]]^c = \emptyset_N$ iff $NC(\mathcal{A}_N)^* = \mathcal{Y}_N$ and $\mathcal{A}_N \cup \mathcal{A}_N^c = \mathcal{Y}_N$, then $\mathcal{Y}_{\mathbb{I}_{NC}}^{J_{NC}}$ is $\mathbb{I}_{NC}$ -resolvable.

3. Conclusions and future works

1. One of the future works that could be done is to generalize the local function and ψ - operator in punter sets and Multi Neutrosophic sets [10,11,17].
2. The concepts of NCLF and ψ_{NC} - operator are studied via stable Neutrosophic Crisp topological space instead of NCTS in order to demonstrate the difference between them as well as the characteristics they share [12,13]. On the other hand, we can also study these concepts on NCT- Sets, Neutrosophic Axial sets and n-valued Neutrosophic Crisp sets [14,15,16].

Reference

- [1] A.A. Salama, Neutrosophic Crisp Points & Neutrosophic Crisp Ideals, Neutrosophic Sets and Systems, 2013, Vol. L. PP. 1-5.
- [2] F-Smarandache, A Unifying Field in Logics, Neutrosophic Logic, Neutrosophy. Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.
- [3] Florentine Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA 2002.
- [4] K. Kuratowski, Topology, Warszawa, 1933.
- [5] K. Kuratowski, Topology, Vol. I, New York: Academic Press 1966.
- [6] Altal Kang, Y.K., Al-Swidi, L.A.A., On Some Types of Proximity ψ - set, Journal of Physics: Conference Series, 2021, 1963(1), 012076.
- [7] Abdulsada, D.A., Al-Swidi, L.A.A., Center Set Theory of Proximity Space, Journal of Physics: Conference Series, 2021, 1804 (1), 012130.
- [8] .Ali Abdu Isada, D., Al-Swidi. L.A.A., Compatibility of Center Topology. IOP Conference Series: Materials Science and Engineering, 2020.928(4), 042002.
- [9] A. A. Salama, Florentin Smarandache, Neutrosophic Crisp Set Theory, Educational Publisher Columbus 2015.
- [10] Al-Tamimi, T. Al-Swidi. L.A.A., . Al-Obaidi, A. H. M., New Concepts in Partner Multi Neutrosophic Topological space, International Journal of Neutrosophic Science, 2024 ,24(3), PP. 172-178.
- [11] Al-Tamimi. T.A. H., Al. Swidi, L.A.A., Al. Obaidi, A.H.M., Partner sets for Generalizations of Multi Neutrosophic Sets. International Journal of Neutrosophic Sciences 2024,24(1), PP. 8-13.
- [12] Tomma, D.N., Al- Swidi, L.A.A., Stable Neutrosophic Crisp Topological Space, International Journal of Neutrosophic Science, 2024, 23(4), PP. 154-159.

- [13] Tomma, D.N., Al-Swidi, L.A.A., Necessary and Sufficient Conditions for a Stability of the Concepts of Stable Interior and Stable Exterior via Neutrosophic Crisp Sets, *International Journal of Neutrosophic Science*, 2024, 24(1), PP. 87-93.
- [14] Hedi, M.H., Al-Swidi, L.A.A., The Neutrosophic Axial Set theory, *Neutrosophic Sets and Systems*, 2022, 51, PP. 295-302.
- [15] Al-Nafee, A.B., Broumi, S., Al-Swidi, L.A., n-Valued Refined Neutrosophic Crisp Sets, *International Journal of Neutrosophic Science*, 2021, 17(2), pp. 87-95.
- [16] Abdulsada. A., Al-Swidi, L.A.A., Hedi, M. H., On NCT-Set Theory, *Neutrosophic Sets and Systems*, 2024, 68.PP. 99-108.
- [17] F. Smarandache 2023; Introduction to the Multi Neutrosophic Sets, *Neutrosophic Sets and Systems*, 61(1) PP. 89-99.
- [18] A. H. M. Al-Obaidi and Q. H. Imran, on new types of weakly neutrosophic crisp open mappings. *Iraqi Journal of Science*, 62(8) (2021), 2660-2666.
- [19] A. H. M. Al-Obaidi, Q. H. Imran and M. M. Abdulkadhim, on new types of weakly neutrosophic crisp closed functions. *Neutrosophic Sets and Systems*, 50(2022), 239-247.
- [20] Q. H. Imran, A. H. M. Al-Obaidi, & Smarandache, F., On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets. *Neutrosophic Sets and Systems*, 32, (2020), 425-434.
- [21] Q. H. Imran, R. K. Al-Hamido and A. H. M. Al-Obaidi, on new types of weakly neutrosophic crisp continuity. *Neutrosophic Sets and Systems*, 38(2020), 179-187.
- [22] Q. H. Imran, K. S. Tanak and A. H. M. Al-Obaidi, on new concepts of neutrosophic crisp open sets. *Journal of Interdisciplinary Mathematics*, 25(2)(2022), 563-572.
- [23] Q. H. Imran, A. H. M. Al-Obaidi, F. Smarandache and Md. Hanif PAGE, on some new concepts of weakly neutrosophic crisp separation axioms. *Neutrosophic Sets and Systems*, 51(2022), 330-343.
- [24] Q. H. Imran, M. M. Abdulkadhim, A. H. M. Al-Obaidi and Said Broumi, Neutrosophic crisp generalized sg-closed sets and their continuity. *International Journal of Neutrosophic Science*, 20(4) (2023), 106-118.
- [25] M. M. Abdulkadhim, Q. H. Imran, A. H. M. Al-Obaidi and Said Broumi, On neutrosophic crisp generalized alpha generalized closed sets. *International Journal of Neutrosophic Science*, 19(1) (2022), 107-115.